Unfringing Interference of Cross-Polarized Slits

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I project a diagonally-polarized (D) monochromatic complex plane-wave[1] electric field onto cross-polarized slits. (1) What is the instantaneous behavior of the electric field versus diffraction angle in the far field[2]? (2) The time-parametric vector sum of crossed-polarized slits' electric fields in a fixed plane about normal to the slits' rays traces Lissajous curves[3] particular to the diffraction[4] angle. (3) These uniform-brightness, diffraction-angle-dependent lines, circles, and ellipses constructively constitute unfringing interference of transverse field undulations[5] summed in parallel and falsify the first Fresnel-Arago Law[6,7] since the vector sum[1], somewhat ironically, predicts the outcome. This result essentially retro-extends Fresnel optics (transverse vibrations) upon the Young's Double-Slit via the (parallel and direct) lineages of Maxwell, Heaviside, and Poynting, supporting Young's original assertion[8.9].

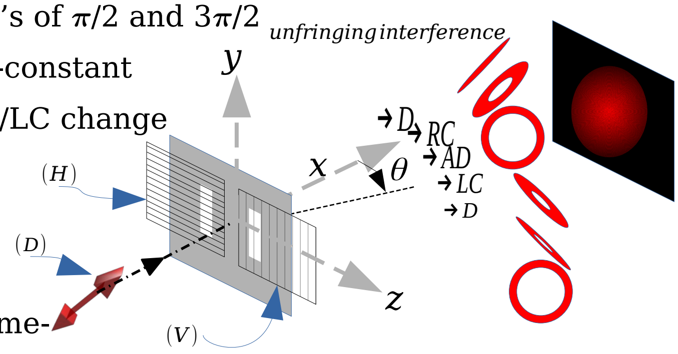
In the diffraction plane beyond linearly crossed-polarized optical slits, assuming a plane-wave E-field or wavefunction[11], the parametric vector sum, neglecting the diffraction envelope, is[10]:

$$\psi=\begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix}\begin{bmatrix} E_{0y} \\ E_{0z} \end{bmatrix} e^{[-i\omega t]} \quad \text{so that} \quad \Re(\psi)=\begin{bmatrix} E_{0x}\cos(-\omega t) \\ E_{0y}\cos(-\omega t+\Delta\phi) \end{bmatrix} \qquad (1)$$

(where ϕ_1, $\phi_2 \equiv \phi_1(\theta,+d), \phi_2(\theta,-d)$, $\Delta\phi = \phi_2 - \phi_1$, $\omega = 2\pi\lambda/c$, θ is the diffraction angle, and d is the slit spacing/2). This vector sum traces Lissajous patterns[3] which are cyclic versus θ. The time-average of

$$\vec{S}=(\Re\vec{B})\times(\Re\vec{E})=(\hat{i}/\mu_0)\left(E_{y0}^2\cos^2(\omega t)+E_{z0}^2\cos^2(\omega t-\Delta\phi)\right) \qquad (2)$$

(the instantaneous real irradiance[12, 13]), is independent of $\Delta\phi(\theta)$. (1) cycles in polarization and (2) in pulsed versus constant irradiance as functions of $\Delta\phi(\theta)$ (e.g., $\Delta\phi$'s of 0 and π contain D/A[nti]D with 2x-peak power pulses at 2ν , and $\Delta\phi$'s of $\pi/2$ and $3\pi/2$ contain R[ight]C[ircular-polarization]/L[eft]C with 1x-constant power.) For $E_{0y} = E_{0z}$, (fixed θ) sums constructing RC/LC change in direction only and sums constructing D/AD change in magnitude only with t. Comparing the present to parallel-polarized[8.9] (fringing) slits gives the two-slit geometry information encoded in polarization (and time-domain irradiance) in the former and in brightness (fringes) in the latter.



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