On quantum-spacetime linelement with scalar curvature from first principles

Holger Döring
DPG-departement matter and cosmos
section: GRT and gravity
IQ-Wissen Berlin
Germany
ORCID: 0000-0003-1369-1720

e-mail: holger.doering@alumni.tu-berlin.de

Abstract:

In an uncomplicated manner a quantum-lineelement of the local tangent-spacetime of Minkowski-type can be derived from a fundamental form of physical analysis. This term includes a variable that can be interpreted as a combination of Ricci -scalar and cosmological constant.

In this method, the Planck - constant is included in the lineelement-form in a natural manner. If this ansatz is seen as real from first principles, then classical Minkowski-lineelement is wrong.
So because this is the limiting case for GRT without gravity, quantum gravity-equations must also be false and modified. Local invariant Lorentz-Planck- transformations can be derived for certain special conditions of the velocity values. This exploration of quantum in tangential-spacetime may lead to a deeper understanding of gravity quantization.

Key-words: Planck-Lorentz-transformation; Planck-Minkowski-linelement; quantized flat spacetime; Ricci-scalar; scalar-curvature; cosmological constant; minimal Planck-length; first principles; quantizing uncurved spacetime; advanced Hamilton-function; quantized formulated Lagrangian for SRT.

1. Introduction:

There are two methods for constructing a quantum-spacetime theory from classical limits. Going from $0 \Rightarrow \hbar$ for quantum states against continuity and going from $G \Rightarrow R_k^i$ for gravity and curvature from Newton-model to Einstein Tensor-description and then going from zero to $\hbar$. (G is the gravitational constant according to Newton's law). The first means going from flat Minkowski spacetime to a sort of quantized tangential spacetime and the second to go from spacetime without curvature to GRT with gravity-force and then to quantum $\hbar$. The second attempt was mostly used. [1.],[2.].
This study only constructs a quantum flat Planck- Minkowski spacetime from fundamental planck-length squares. If this attempt is physically, logically, mathematically consistent and successful, it should be considered. From this level a consistent quantum gravity could be constructed, which limiting case this construction here would be, but would not be real flat because Ricci-scalar is also included as a scalar curvature term as the cosmological constant is in local area.

Citation Wheeler [3.] : *"Space-time geometry is no longer high above the battle between matter and energy. It is part of this struggle. Geometry indicates matter how it should move, but mass in turn dictates the curvature to geometry."*
This is also a fact for quantum tangential spacetime, where it is shown, that a gravity-free local spacetime TM of a manifold M is not really flat as normally assumed and empty [4.] but depends on physical variables. The method used to calculate this problem in this study is shown in the following diagram marked in red (left arrow down), where the conventional method is marked in black (middle arrows to the right and then down):

$$\begin{array}{lcl} \text{SRT/TM} & \rightarrow G/R^i_k \rightarrow & \text{GRT} \\ \downarrow \hbar & & \downarrow \hbar \\ \text{QSRT/ } T_Q(M) & \rightarrow G/R^i_k \rightarrow & \text{QGRT} \end{array} \tag{0.}$$

<u>*Diagram 1: relations of SRT and GRT to their quantized systems.*</u>

2. Methods/Calculation:

From first principles is defined:

If $$\frac{1}{r^2{}_{PL}} := R_{Fund} \pm \Lambda := \frac{m^2{}_{PL} \cdot c^2}{\hbar^2} \quad , \tag{1.}$$

where R is the Ricci-scalar curvature and Λ is cosmological constant, then this definition can be developed to its general case of:

$$R \pm \Lambda = \frac{m^2 \cdot c^2}{\hbar^2} \tag{2.}$$

and of course R is defined over the two curvature-mainaxes (in two dimensions) as scalar invariance of curvature:

$$R := \left| \frac{2}{\rho_1 \cdot \rho_4} \right| \quad \text{resp.} \quad R := R_{i,k} \cdot g^{i,k} \quad . \tag{3.}$$

This leads to a modification of Einsteins local energy-equation for flat spacetime of:

$$E = m \cdot c^2 - \frac{\hbar^2 \cdot (R \pm \Lambda)}{m} \tag{4a.}$$

with $B := \frac{\hbar^2 \cdot (R \pm \Lambda)}{m}$ defined. (4b.)

From this equation (4a.) the advanced Hamilton-function for energy-momentum can be derived in dependence from B :

$$m^2{}_0 \cdot c^4 + p^2 \cdot c^2 - 2 \cdot B \cdot \gamma \cdot m_0 \cdot c^2 \cdot (1 - \frac{v}{c}) = E^2 - 2 \cdot B \cdot (E - p \cdot c) \tag{4c.}$$

whith the usual notation for Gamma:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad . \tag{4d.}$$

This equation (4c.) gives the natural completeness of local SRT-Hamiltonian from Pythagoras-set to advanced, developed cosine-set.
Equation (4c.) can be solved for Hamilton function H resp. for energy E :

$$H = E = B \pm \sqrt{m^2{}_0 \cdot c^4 + p^2 \cdot c^2 + B^2 - 2 \cdot B \cdot \left(pc + \gamma \cdot m_0 \cdot c^2 \cdot \left(1 - \frac{v}{c}\right)\right)} \tag{4e.}$$

This result leads directly to corrected line-element of quantized Planck-Minkowski-spacetime in its new form of:

$$ds^2 = x^2 \cdot \frac{\hbar^2 \cdot (R \pm \Lambda)}{c^2 \cdot m_0{}^2} - c^2 \cdot t^2 \cdot \left(\frac{\hbar^2 \cdot (R \pm \Lambda)}{c^2 \cdot m_0{}^2} - 1\right) \tag{5a.}$$

which also can be written as:

$$ds^2 = c^2 \cdot t^2 + \frac{\hbar^2 \cdot (R \pm \Lambda)}{c^2 \cdot m_0{}^2} \cdot (x^2 - c^2 \cdot t^2) \tag{5b.}$$

As is seen, the timelike coordinate is quantized *<u>and</u>* exists in a classical form, which gives this system of description a similiar partial form of a bimetric whereby the spacelike dimension-coordinate appears only in a quantum description. If the ansatz of spacetime- signature in coordinates is changed, then the conditions of classical and quantum description are reversed.

This equation (5a./5b.) is from now on called „Planck-Minkowski-linelement“.

As is seen, this new lineelement is conform to classical Minkowski- description of local spacetime and differs from it only for the conformal term of:

$$K := \frac{\hbar^2 \cdot (R \pm \Lambda)}{c^2 \cdot m_0{}^2} \quad . \tag{6.}$$

In this form the new local linelement can be written as:

$$ds^2 = c^2 \cdot t^2 + K \cdot (x^2 - c^2 \cdot t^2) \tag{7a.}$$

or

$$ds^2 = K \cdot x^2 - c^2 \cdot t^2 \cdot (K - 1) \quad , \tag{7b.}$$

which comes from [4.]:

$$ds^2 = d\,x^i \cdot dx^k \cdot \eta_{i,k}$$

with to the classic Minkoski tensor varied compared metric fundamental tensor for uncurved tangential spacetime :

$$\eta_{i,k} = \begin{pmatrix} K & 0 \\ 0 & 1-K \end{pmatrix} \quad (7c.)$$

This term leads directly to the Planck-Lorentz-transformations, described with its transformation-matrix A (formulated for two dimensions) extended from [5.]:

$$A = \begin{pmatrix} (\sqrt{K}\cdot\gamma\cdot\Gamma) & \Gamma \\ \Gamma & (\sqrt{K}\cdot\gamma\cdot\Gamma) \end{pmatrix} \quad (8.)$$

with $\begin{pmatrix} x' \\ c\cdot t' \end{pmatrix} = \begin{pmatrix} x \\ c\cdot t \end{pmatrix} \cdot A$ (9.)

This shows directly the Planck-Lorentz-transformations for two dimensions of uncurved but quantized local spacetime in its conformal form:

$$x' = \Gamma\cdot(\gamma\cdot\sqrt{K}\cdot x + c\cdot t)$$
$$c\cdot t' = \Gamma\cdot(x + \gamma\cdot\sqrt{K}\cdot c\cdot t)$$

,with $\beta = \frac{v}{c}$ (10a./10b)

where

$$\sqrt{K} := \frac{\hbar}{c\cdot m_o}\cdot\sqrt{R\pm\Lambda} \quad (11a.)$$

and $\Gamma = \left(\sqrt{K\cdot(1-\beta^2)-1}\right)^{-1}$ (11b.)

where in this notation γ is defined as :

$$\gamma := \sqrt{\left(1-\frac{v^2}{c^2}\right)} \quad (11c.)$$

This leads in first order to a new formulated, quantized Lagrange-function of SRT resp. quasi-flat tangential spacetime $T_Q(M)$:

$$L = -\frac{m_0\cdot v^2}{\Theta}\cdot\sqrt{1-\Theta} \quad (12a.)$$

where Θ is defined as:

$$\Theta = K\cdot(1-\beta^2) \quad \text{and} \quad \beta = \frac{v}{c} \quad . \quad (12b.)$$

The Lagrange-function also can be written (in more common form) as:

$$L = \frac{-m_0\cdot c^2}{\Theta}\cdot\sqrt{\beta^4\cdot(1-\Theta)} \quad . \quad (12c.)$$

Since there is the Planck- condition $\Gamma^{-1} \geq 0$ as a substitute of new advanced Lorentz-factor to the old factor, which means that this factor itself is quantized and therefore also velocity-term, this then leads finally to the corrected maximal limiting condition in physical velocity for material fermion and boson bodies of:

$$v \leqslant c \cdot \sqrt{1 - \frac{c^2 \cdot m_0{}^2}{\hbar^2 \cdot (R \pm \Lambda)}} \tag{13.a}$$

which also can be written as:

$$v \leqslant c \cdot \sqrt{1 - \frac{m_0 \cdot c^2}{B}} \tag{13b.}$$

instead of:

$$v \leqslant c \tag{13c.}$$

like in classical, non-quantized SRT [6.].

From this all follows,that new, corrected light-cone conditions can be written as:

$$x = \pm c \cdot t \cdot \sqrt{\left(1 - \frac{c^2 \cdot m^2{}_0}{\hbar^2 \cdot (R \pm \Lambda)}\right)} \tag{14a.}$$

which likewise can be written as:

$$x = \pm c \cdot t \cdot \sqrt{1 - \frac{E_0}{B}} \quad \text{with} \quad E_0 = m_0 \cdot c^2 \quad . \tag{14b.}$$

instead of:

$$x = \pm c \cdot t \tag{14c.}$$

like in classical, non-quantized SRT [6.] .

Comment:
Since there is the intervall-condition of $K^{-1} \in [0;1]$, factor K^{-1} can be used for gauging of probability ψ like Lorentz-factor $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$, which also is defined for $\frac{v}{c} \in [0;1]$. This leads to a natural way of gauging probability. Probability therefore is already inherent build in in formulation of classical SRT resp. tangential spacetime TM and in $T_Q(M)$ also.

3. Conclusion:

It is possible to construct a quasi-flat quantum-spacetime in Minkowski-like coordinates with its local quantum invariance conditions for coordinate transformations. If this description would be true, so every real quantum-gravity must lead in its limit without gravity- force to this Planck-Minkowski-metric, like classical gravity of GRT must lead without force in its limit to classical flat spacetime of Minkowski-linelement. In all formulas above either Ricci-scalar term R or cosmological constant Λ can be set to zero. If both are set to zero, classical, local Minkoswki spacetime remains at tangential spacetime to manifold M .In this way the transition to classical theories is consistent formulated.

Also the local geometric invariance form is reviewed: the light cone will change its form a little bit, which size-changing depends from now on, controlled by R, Λ and m_0 . Not only in gravity-determined spaces [7.],[8.] but even in tangential spacetime, it is no longer a static form [9.],[10.]. At the same fourevent place light-cone differs now only by different m_0 but by constant moving v of a particle with constant m_0 and constant assumed Λ the light-cone form depends at its worldline from changing of local Ricci-scalar R even in tangential spacetime $T_Q(M)$. Every material object gets its own „light-cone", whereby classical light-cone only remains zerolike for photons resp. massless particles as a maximum description limit for inherent timelike movements.

4. Summary:

From first principles of a fundamental length-square with identifying square of inverse Planck-length with Ricci-scalar and cosmological constant there can be constructed a local quasi-uncurved quantum-spacetime without gravity as a corrected form of tangential spacetime TM_Q for manifold M . But this construction will include Ricci curvature scalar R and may lead to a consistent form of a description of quantum-gravity which in limit for zero gravity is this description of quantum tangential spacetime mentioned above.

5. Discussion:

A quantum form of tangential spacetime TM_Q can be constructed. Now it is the aim to show, if this description is a consistent physical form and if it is possible to take this description serious for a limit of gravity quantization in M without real field forces. From this limiting case then there can be developed and elaborated a consistent form of quantum gravity. The conclusion, in any case, is that spacetime without gravity field is no longer empty (see Wheelers statement avove) but already contains quantum states, which would have to be taken into account in the process of the further course of solving the problem. Also the role of Ricci-scalar and its connection with cosmological constant in descriptions of local regions of spacetime has to be further discussed.

6. Acknowledgements:
Many thanks to Prof. Philip E.A. Mortimer (PhD) ,former member of universities of Glasgow and Edinborough (Edinburgh) for help- and colourful ideas, discussions and suggestions.

7. References:

. [1.] Döring, H.A.W., some brief qualitative letter-comments on quantizing gravity: thirteen hypotheses.**2023**.hal-04278409.https://hal.science/hal-04278409

. [2.] Giulini,D., Kiefer,C.,Lämmerzahl,C.,(eds.):Lecture Notes in Physics, quantum-gravity – from theory to experimental research. Springer-Verlag, Berlin-Heidelberg-New York, **2003**

. [3.] Wheeler,J. A., Einsteins Vision, Springer Verlag Berlin-Heidelberg-New York, **1968.**

. [4.] Minkowski, H., Raum und Zeit,Vortrag; Leipzig und Berlin, Druck und Verlag von B.G . Teubner, **1909**

. [5.] Einstein, A.: Zur Elektrodynamik bewegter Körper, Annalen der Physik,**322**,10,**1905**. **doi:https://doi.org/10.1002/andp.19053221004**

. [6.] Misner, C.W., . Thorne, K.S., and Wheeler, J.A.: *Gravitation*. Freeman, San Francisco **1973**, ISBN 0-7167-0334-3.

. [7.] Penrose, R.: Techniques of Differential Topology in Relativity. Society for Industrial and Applied Mathematics (SIAM): CBMS-NSF Regional Conference Series in Applied Mathematics, **1972**; ISBN 0-89871-005-7, doi:10.1137/1.9781611970609.

. [8.] Minguzzi,E.: Lorentzian causality theory. In: Living Reviews in Relativity, Band 22, Nr.3, 3. Juni **2019**; doi:10.1007/s41114-019-0019-x.

. [9.] Papadopoulos, K., Acharjee,S., Papadopoulos,B.,K.: *The order on the light cone and its induced topology*. In: *International Journal of Geometric Methods in Modern Physics*. 15. Jahrgang, Nr.5, 1.Mai **2018**, S.1850069–18](arxiv.org [PDF]).

. [10.] Giulini, D.: Globale versus lokale Strukturen von Raum-Zeiten. Tutorium der AGjDPG, DPG-Frühjahrstagung **2017**, Bremen, 13. März 2017.

8. Verification:

This paper is written without help from a chatbot like Chat-GPT4 or other AIs. It‘s fully human work.

Juli 2024

Technische Universität Berlin