# A NEW NUMERICAL INTERPRETATION OF THE CONCEPT OF EXPONENTORY (Θ NOTATION)

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**0- Abstract**

In this paper I show a possible change in the theory of series beyond product. Instead of a resolution Bottom-to-Top we will see a necessary application of the method for exponents that is a process Top-to-Bottom. That implies a change in the numerical results in a same proposition of a series.

**1- Introduction**

The idea developed by me [1] about serial operator of exponents has shown recently some problems in the numerical interpretation if we approach the series of functions to hyper-operations algebras. It is not in fact possible an interpretation of resolution of the exponential serial operation in a Bottom-to-Top resolution of the resultant exponential tower. It was firstly my idea to get a simplification of the computations and obtain with simple series not too big numbers. But it is a contrary interpretation of the classic literature in mathematics since Euler [3] (fifth Fermat number $2^{(2^{(5)})}+1=4{,}294{,}967{,}297$ is not prime) obviously using Top-to-Bottom resolution, and maybe before him.

**2- The interpretation change**

In my first approach of the operator exponentory I defined it as

$$\underset{n=a}{\overset{b}{\Theta}} f(n)=\underbrace{f(a)\uparrow f(a+1)\uparrow\ldots\uparrow f(b-1)\uparrow f(b)}_{\rightarrow} \quad (1)$$

The arrow indicates the order of resolution. But adapting it to the theories like tetration [2] (itered exponentiation) that is why the order of resolution have to change to:

$$\underset{n=a}{\overset{b}{\Theta}} f(n) = \underbrace{f(a) \uparrow f(a+1) \uparrow \ldots \uparrow f(b-1) \uparrow f(b)}_{\leftarrow} \quad (2)$$

**3- Numerical examples**

First we are going to view my own order of resolution (which was wrong in a classic point of view).

$$\underset{n=3}{\overset{5}{\Theta}} n = 3 \uparrow 4 \uparrow 5 = 81 \uparrow 5 = 3486784401 \quad (3)$$

Now, in the other hand, the correct operation order following tradition of exponential towers:

$$\underset{n=3}{\overset{5}{\Theta}} n = 3 \uparrow 4 \uparrow 5 = 3 \uparrow 1024 = 3{,}73 \cdot 10^{488} \quad (4)$$

**4- First property of exponentory operator**

I want to express here something related to the topic as an extra. Exponentory has neutral element in single lineal variable:

$$\underset{n=1}{\overset{b}{\Theta}} n = 1 \quad (5)$$

The proof is very simple, any power with 1 in the basis has a result of 1. ( $1^n = 1 \forall n \in \mathbb{C}$ )
Which implies that every finite or infinite significant series should start in a number $n > 2$ on single lineal variable.

**5- Conclusions**

In my way to obtain more reasonable results in numeric applications of exponentory I misunderstood tradition in process of resolution of exponential towers, but if the mathematician can assume that very large numbers will be obtained in the use of the $\Theta$ notation, te resolution Top-to-Bottom is more accurate.

**6- References**

**[1] Millas Vera, Juan Elias. - New notation in series of functions. (vixra.org/abs/2101.0070) [2021]**

**[2] Trappman, Henryk. Robbins, Andrew. - Tetration reference. [2008]**

**[3] Euler, Leonhard. - Observations on a theory of Fermat and others on looking at prime numbers. Commentarii academiae scientiarum Petropolitanae, Volume 6, pp. 103-107. [published 1738]**