Set theory can’t be directly representative of algebraic constructions in Goldbach conjecture and other NT problems.

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0- Abstract

In this paper I want to express my thoughts on the non possible link between set theory arguments in Number Theory. It is maybe good a first approximation to a problem to think in relation to sets, but my actual thinking is that you can’t solve an algebraic construction with a direct implication between the set and the algebraic variable. As example I will analyze my past trying to prove strong Goldbach conjecture. Finally I explain other version of this topic also with Goldbach conjectures as examples.

1- Introduction

In 2021 I pre-published (then published in a monograph in LAP editorial that year) a paper named Proof of the Goldbach Conjecture. In that paper I mention the possibility of being the variables of the expression $2n=p+q$ analyzed as sets we can do an approach to a parity result in right side of the equation which in any case implies that the even part of left equation will be a consequence of equality between parts. Maybe this was original in a common sense, but my thought of be a necessary condition to proof the conjecture has change. As we will see in the next part, there is no necessary implication between the description of the set in which is the variable and the behavior of the variable (or variables) in a determinate formulae.

2- Set theory and algebra. A difficult connection.

A lot of Caculus book start with a brilliant exposition of the aspects of the sets, for example Analysis I Chapter 3 [Terence Tao]. It is obviously important to know what is the meaning of characterization a number in a set and in which sets can a variable moves.

But, my experience trying to solve problems just in the range of doing equations with sets tells me that you can't have a direct connection of the results of manipulation of sets to the correct conclusions you can obtain doing manipulation with variables.
In that case will be enough correct an expression like: $2t+2t=4t$. If we characterize for example $t \in \mathbb{Q}$. But it will be not correct an algebraic and characterization and the same time. For example: $\mathbb{Z}+\mathbb{Z}$ to obtain something logic. It will be in any case necessary doing a "set of sets" theory to apply logically all determine algebra to manipulation of sets. By now, the only possibility is use expression of basic logic (like union, disunion, negative, etc). For example:
$\mathbb{N}^{+} \cup \mathbb{N}^{-} \cup 0=\mathbb{Z}$. So the first conclusion and the main topic of this paper is that you can not use algebraic operations as tool between sets. For example will be a non-sense the expression $\mathbb{N}-\mathbb{Z}$ or $\mathbb{Q} \times \mathbb{N}$. This part of mathematics still belong to imagination and not proof tools, of course you can write it, but as I had discover it has not enough implication in logic to be a good tool. So we can assume that your proof trying will not be good.

3- The strong and week Goldbach conjecture. An amazing perspective if the Algebra could be applied to Set Theory.

Taking less seriously the concept of non linked between set description and algebraic variables, and doing an abstraction of the future applications of Set theory we can for a moment imagine a panoramic in which the set theory can be useful to describe Number Theory problems.
There could be a link between the two aspects following a medium path. First describing sets (this part will be not explained here) and then applying logic to form an algebraic relations of the description of that sets. For example if we take a look to strong and week Goldbach conjecture, we can describe first the composition of the variables and the link them to a belonging to Naturals and Primes (P).

If $\mathbb{N}=n=(2n) \cup (2n+1)$,

and $P \in \mathbb{N}$,

Then describing strong Goldbach conjecture as: $2n=p+q$; $n \in \mathbb{N}-\{1\}; p \in P; q \in P$

then $P+P=2P \approx \mathbb{N}/2=2n-\{2\}$.

But week Goldbach conjecture as $2n-1=p+q+r$; $n \in \mathbb{N}-\{1,2,3\}; p \in P; q \in P; r \in P$.

then $P+P+P=3P \approx \mathbb{N}/2=(2n-1)-\{1\}\{3\}\{5\}$.

So $2P=3P$ when $\mathbb{N}$ tend to infinity, which is impossible.

The conclusion of this is that using sets and operations we find paradoxes, so maybe is not a help this approximation and we should still using sets just to define variables and not to do operations with them. But by now, this is not useful mathematics process, so we can not trust in the possible conclusion of results obtained with algebraic use of Set Theory.