The first counterexample of Riemann hypothesis found through computer calculation

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Abstract

The counterexample of the Riemann hypothesis causes a significant change in the image of the Riemann Zeta function, which can be distinguished using mathematical judgment equations. The first counterexample can be found through this equation.

Keywords: `Riemann hypothesis`, Riemann Zeta function, counterexample

1. Introduce

The Riemann hypothesis is favored by mathematicians, and a feasible method of falsification is to constantly search for counterexamples. Thirty trillion non trivial zeros found so far are all located on the critical line, with no counterexamples.Following the original method, it will be very difficult to find counterexamples. Therefore, a completely new method has been created, hoping to find counterexamples at a faster speed.Without establishing a new system of number theory, solving the Riemann hypothesis requires extremely high skill, which heavily relies on intuition in mathematics. Meanwhile, luck will also become a crucial component.

2. Mathematical Principles

Analytical number theory is a combination of trigonometric functions and polynomial symbols, which can be solved no matter how difficult it is. Therefore,the

Riemann hypothesis is not unsolvable. In the field of number theory, the mathematical community tends to seek a maximum number to overturn the conclusion. Whether the Riemann hypothesis or the Goldbach conjecture,it should be the solution.

- The most basic task of falsifying the Riemann hypothesis is computation. By making curve of $\text{Re}(\xi) = 0$ and $\text{Im}(\xi) = 0$,their intersection point can be found to obtain the zero point
- Any curve of $\text{Re}(\xi) = 0$ and $\text{Im}(\xi) = 0$ can only have a unique intersection point at Re(s)=1/2, or there may be two symmetric focal points about Re(s)=1/2
- If the non trivial zero point exists,Re (s)!= 1/2,then starting from the real number axis and moving towards positive infinity along Re (s)=1/2, the Im-Re curve at the non trivial zero point will rotate clockwise to counterclockwise, and vice versa
- The distribution of prime numbers is irregular, which inevitably leads to the existence of a very large number that makes the Riemann hypothesis untenable

3. Descriptive equation

For the following formula

$$\xi(s) = \frac{\eta(s)}{1 - 2^{1-s}}$$

$$\eta(s) = \eta(r + it) = \sum_{n=1}^{\infty} \frac{(-1)^n \cos(-t \ln n)}{n^r} + i \sum_{n=1}^{\infty} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

When $\eta(r + it) = 0$, means that also $\xi(r + it) = 0$, define

$$f(r, t) = \sum_{n=1}^{\infty} \frac{(-1)^n \cos(-t \ln n)}{n^r}$$

$$g(r, t) = \sum_{n=1}^{\infty} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

Obtain

$$\eta(r+it)=f(r,t)+ig(r,t)$$

Define

$$h(t)=\frac{dg(0.5,t)}{df(0.5,t)}$$

Obtain

$$h(t)=\frac{d\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\sin(-t\ln n)}{\sqrt{n}}}{\mathrm{d}\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\cos(-t\ln n)}{\sqrt{n}}}=-\frac{\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\cos(-t\ln n)}{\sqrt{n}}}{\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}}$$

Then

$$\frac{dh(t)}{dt}=-\frac{d\frac{\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\cos(-t\ln n)}{\sqrt{n}}}{\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}}}{dt}$$

$$=-\frac{\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]\frac{d\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\cos(-t\ln n)}{\sqrt{n}}}{dt}-\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\cos(-t\ln n)}{\sqrt{n}}\right]\frac{d\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}}{dt}}{\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]}$$

$$=-\frac{\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\ln n\sin(-t\ln n)}{\sqrt{n}}\right]+\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\cos(-t\ln n)}{\sqrt{n}}\right]\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\ln n\cos(-t\ln n)}{\sqrt{n}}\right]}{\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]}$$

$$=-\frac{\left[\sum\limits_{n=1}^{\infty}\sum\limits_{m=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\frac{(-1)^{m}\ln m\ln m\sin(-t\ln m)}{\sqrt{m}}\right]+\left[\sum\limits_{n=1}^{\infty}\sum\limits_{m=1}^{\infty}\frac{(-1)^{n}\ln n\cos(-t\ln n)}{\sqrt{n}}\frac{(-1)^{m}\ln m\ln m\cos(-t\ln m)}{\sqrt{m}}\right]}{\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]}$$

$$=-\frac{\sum\limits_{n=1}^{\infty}\sum\limits_{m=1}^{\infty}\frac{(-1)^{n+m}\ln n\ln m\ln m\cos[-t(\ln n-\ln m)]}{\sqrt{nm}}}{\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]\left[\sum\limits_{n=1}^{\infty}\frac{(-1)^{n}\ln n\sin(-t\ln n)}{\sqrt{n}}\right]}$$

Define

$$l(t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+m}\ln n\ln m\ln m\cos\left[-t(\ln n-\ln m)\right]}{\sqrt{nm}}$$

When $l(t)=0$, means that also $\frac{dh(t)}{dt}=0$. If there exists a real number t that can make $l(t)=0$, then the Riemann hypothesis has a counterexample.

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4. Calculation process

For

$$l(t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+m}\ln n\ln m\ln m\cos\left[-t(\ln n-\ln m)\right]}{\sqrt{nm}}$$

Set

$$l(t,N,M)=\sum_{n=1}^{N}\sum_{m=1}^{M}\frac{(-1)^{n+m}\ln n\ln m\ln m\cos\left[-t(\ln n-\ln m)\right]}{\sqrt{nm}}$$

When

$$N=1000, M=1000$$

By using a computer to calculate, the relationship between l(t,1000,1000) and t can be obtained

Table 1: the relationship between l(t,1000,1000) and t

t	l(t,1000,1000)	t	l(t,1000,1000)	t	l(t,1000,1000)	t	l(t,1000,1000)
10	4.53145530337	10^8	508.659082817	10^{15}	196.834718053	10^{22}	367.506095792
10^2	62.2446119609	10^9	61.2900907837	10^{16}	210.725575078	10^{23}	273.454991872
10^3	707.889111094	10^{10}	258.684299941	10^{17}	737.111365437	10^{24}	694.390455931
10^4	252.760436155	10^{11}	28.383975396	10^{18}	285.407089678	10^{25}	611.133538345
10^5	1051.48981217	10^{12}	238.25746859	10^{19}	149.517105505	10^{26}	1361.36038802
10^6	371.397113155	10^{13}	632.115698693	10^{20}	1916.69929281	10^{27}	1153.42690257
10^7	1006.43596511	10^{14}	978.215950751	10^{21}	398.45591475	10^{28}	-279.438703007

When

$$10^{27}\leq t\leq 10^{28}$$

The Riemann hypothesis has counterexamples.

Of course, smaller counterexamples can also be found, as shown in the table:2

Table 2: the relationship between l(t,1000,1000) and t

t	l(t,1000,1000)	t	l(t,1000,1000)
15786867949799970	324.239576618	15786867949799977	-398.401528098
15786867949799971	411.167971863	15786867949799978	309.526580471
15786867949799972	411.167971863	15786867949799979	897.989489412
15786867949799973	411.167971863	15786867949799980	897.989489412
15786867949799974	251.697239151	15786867949799981	897.989489412
15786867949799975	-398.401528098	15786867949799982	882.563446657
15786867949799976	-398.401528098		

Fortunately, a new counterexample can be found when

$$15786867949799974 \leq t \leq 15786867949799978$$

At the same time, it can be seen that the accuracy of computers has significantly decreased. But it is also necessary to find more accurate numerical values for counterexamples

For

$$f(r,t) = \sum_{n=1}^{\infty} \frac{(-1)^n \cos(-t \ln n)}{n^r}$$

$$g(r,t) = \sum_{n=1}^{\infty} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

Set

$$f(r,t,N) = \sum_{n=1}^{N} \frac{(-1)^n \cos(-t \ln n)}{n^r}$$

$$g(r,t,N) = \sum_{n=1}^{N} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

When

$$N = 10^5$$

Obtained table:3

Table 3:

t	r	g(r,t)	f(r,t)
15786867949799974	from 0 to 1	from 58 to 0.35	from -38 to -1.7
15786867949799974	from 0 to 1	!=0	!=0
15786867949799975	from 0 to 1	from 13 to -0.55,then to -0.35	from 23 to -1.3,then to -1.03
15786867949799975	0.383	-0.0755643180534	0.0775632564384
15786867949799976	from 0 to 1	from 13 to -0.55,then to -0.35	from 23 to -1.3,then to -1.03
15786867949799976	0.383	-0.0755643180534	0.0775632564384
15786867949799977	from 0 to 1	from 13 to -0.55,then to -0.355	from 23 to -1.3,then to -1.03
15786867949799977	0.383	-0.0755643180534	0.0775632564384
15786867949799978	from 0 to 1	from 131 to -0.45	from -49 to 0.1,then to -0.26
15786867949799978	0.64	-0.00650477566519	-0.0094392389487
15786867949799978	0.36	4.43502929015	-0.00629698503102

Finally calculated

$$\xi(0.383 + 15786867949799975i) = 0$$

$$\xi(0.64 + 15786867949799978i) = 0$$

Therefore, the counterexamples of the Riemann hypothesis were calculated by the computer.

Method

Open website https://www.desmos.com/
Inputting formulas and parameters, the numerical values of this paper can be calculated.

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