RIEMANN HYPOTHESIS IS PROVEN ON ONE PAGE

DMITRI MARTILA
INDEPENDENT RESEARCHER
J. V. JANNSENI 6–7, PÄRNU 80032, ESTONIA

Abstract. A short research about the Riemann Hypothesis.
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There is a vivid interest in the Riemann Hypothesis, and there are no reasons to doubt Riemann Hypothesis. [1] Still, despite many attempts to prove the long-standing Millennium Prize problem, those have yet to be published in a reputable journal. Zeta function is $\zeta = \zeta(x + i\, y)$. The critical strip is $0 < x < 1$, the critical line is $x = 1/2$.

The number $N(T) = \Omega(T) + S(T)$ of zeroes of zeta function has jumps only when $S(T)$ has a jump $\Delta\, S(T) = S(T + \delta\, T) - S(T) = 1$ if $\delta\, T \to 0$, see Ref. [2, 3, 4], where $0 < x < 1$, $0 < y \leq T + \delta\, T$ area was studied. Therefore, $\Delta\, N(T) = N(T + \delta\, T) - N(T) = 1$. However, there are at least two counter-examples at a given y_0: $x_0 + i\, y_0$ and $1 - x_0 + i\, y_0$ due to Riemann's original paper. But $\Delta\, N(T) = 1 < 2$. From this contradiction, there cannot be counter-examples.

Why the $S(T)$ has $\Delta\, S(T) = 1$ jump? Because $S(T)$ is defined (see Refs. [2, 4]) on the critical line, and only one zero per $y = y_0$ can be on the critical line.

Why $\Omega(T)$ does not have a jump at $y = y_0$? Because it is expressed via [3, 4]

$$\Omega(T) = \frac{T}{2\pi} \log \frac{T}{2\pi} - \frac{T}{2\pi} + \frac{7}{8} + O(1/T)\,, \tag{1}$$

which can not have jumps $\Delta\, \Omega(T) > 0.1$ because $O(1/T) \ll 1$.

References

[1] David W. Farmer, "Currently there are no reasons to doubt the Riemann Hypothesis," arXiv:2211.11671 [math.NT], 2022AD.
[2] Timothy S. Trudgian, "An improved upper bound for the argument of the Riemann zeta function on the critical line II." J. Number Theory, 134: 280–292 (2014).
https://doi.org/10.1016/j.jnt.2013.07.017

eestidima@gmail.com.

[3] A. Selberg, Contributions to the theory of the Riemann zeta function, Arch. Moth. Naturuid. 48:5 (1946), 89–155.
[4] E. C. Titchmarsh, The theory of the Riemann zeta function, Clarendon Press, Oxford 1986; Aleksandar Ivic, The Riemann Zeta-Function: Theory and Applications (Dover Books on Mathematics), 2003. Pages of interest: 252–265.