The Peculiar Case of *p*

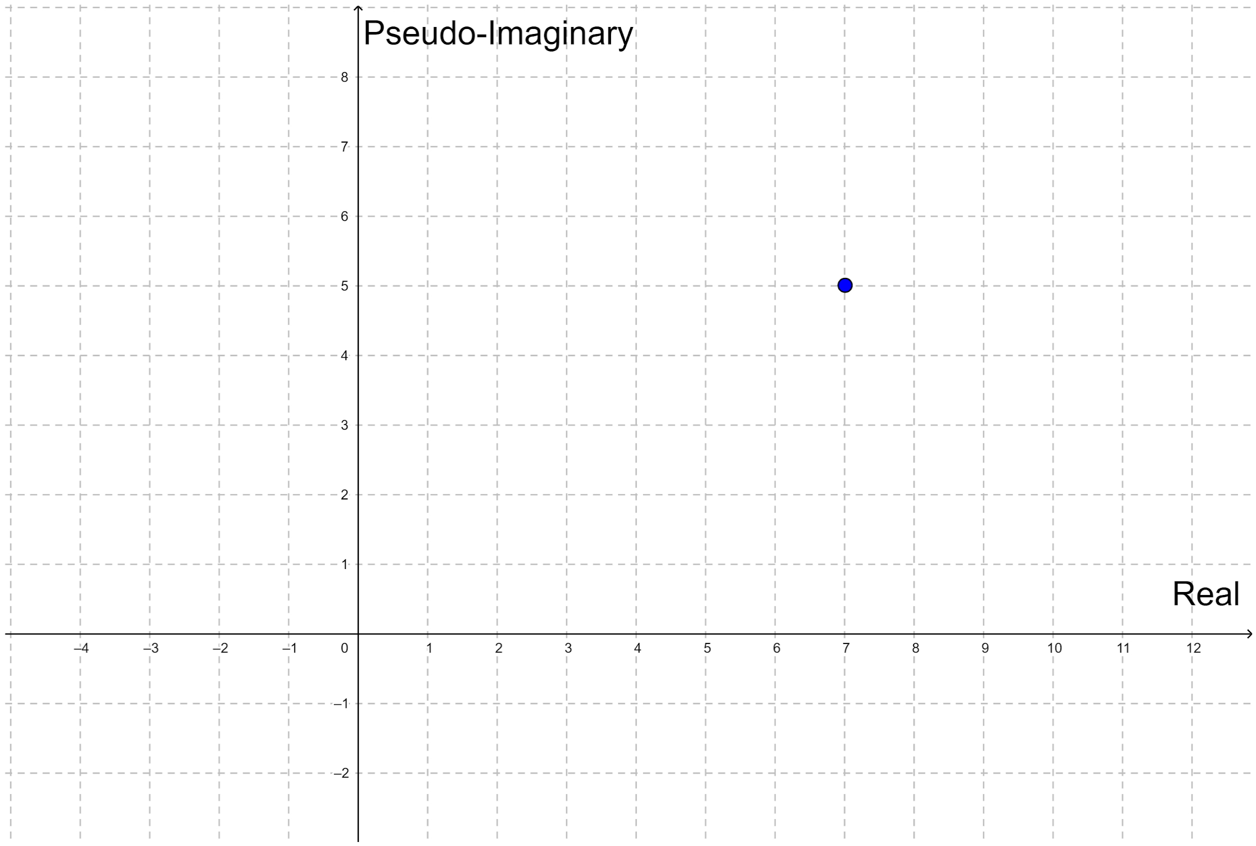
Simon A. Pritchett

Division by zero is a phenomenon described by confused mathematicians for hundreds of years. I would like to declare the value of one divided by zero to be equal to "*p*". I shall call *p* the pseudo-imaginary unit, as it shares several characteristics with *i*, the imaginary unit, as they both solve difficult equations that cannot be solved with basic algebraic and arithmetic mathematical operations, and they both extend real numbers into a new, larger set of numbers. In the case of *i*, the set is known as the complex numbers, whereas with *p*, it is known as the "pseudo-complex numbers".

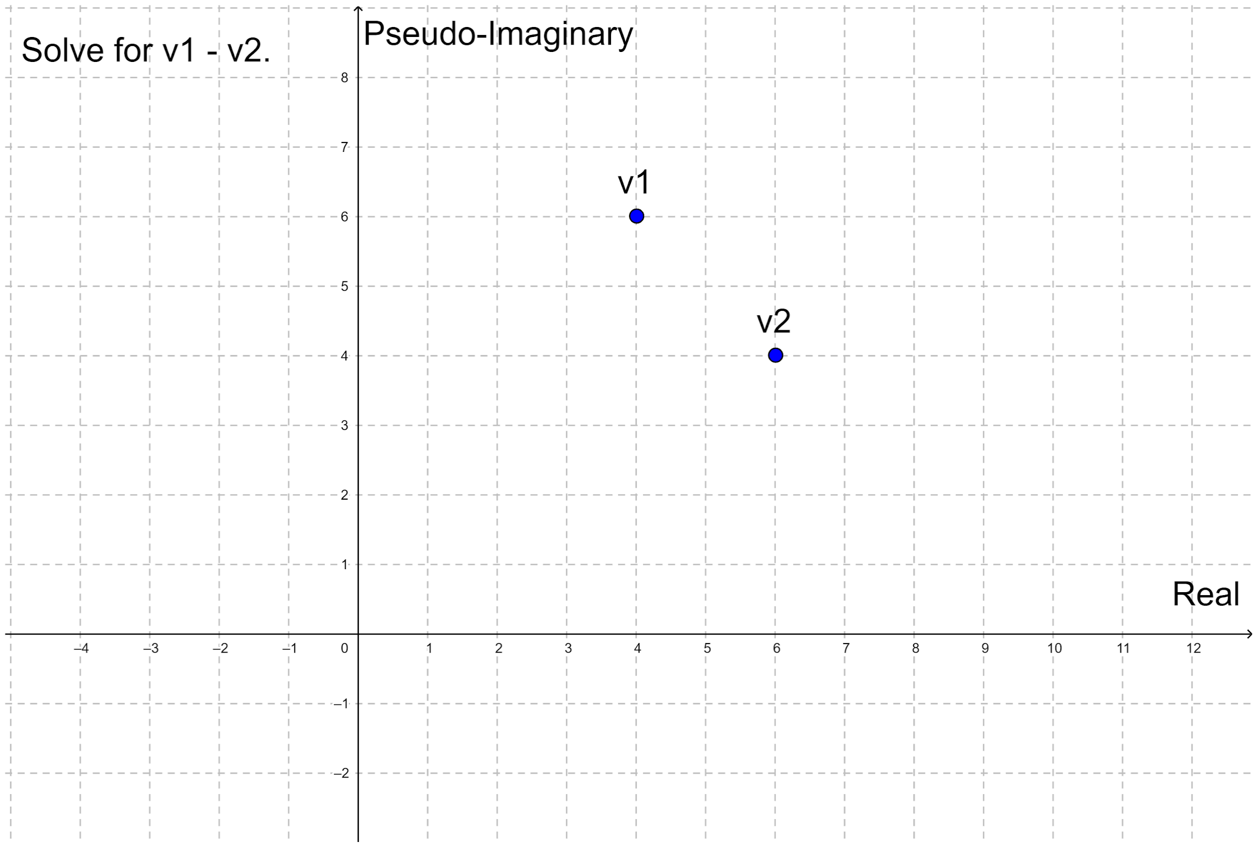
Simplifying division by zero expressions shares similarities with simplifying negative radicals to calculate pure imaginary numbers. For instance, 5/0 can be simplified to 5*p*.

What about pseudo-complex numbers? Here we go. Let's try 7 + 3*p*. This is a pseudo-complex number. The real part is seven and the pseudo-imaginary

part is 3. We can map this on the pseudo-complex plane like this:



Adding and subtracting pseudo-complex numbers is similar to adding complex numbers. Let's try this:

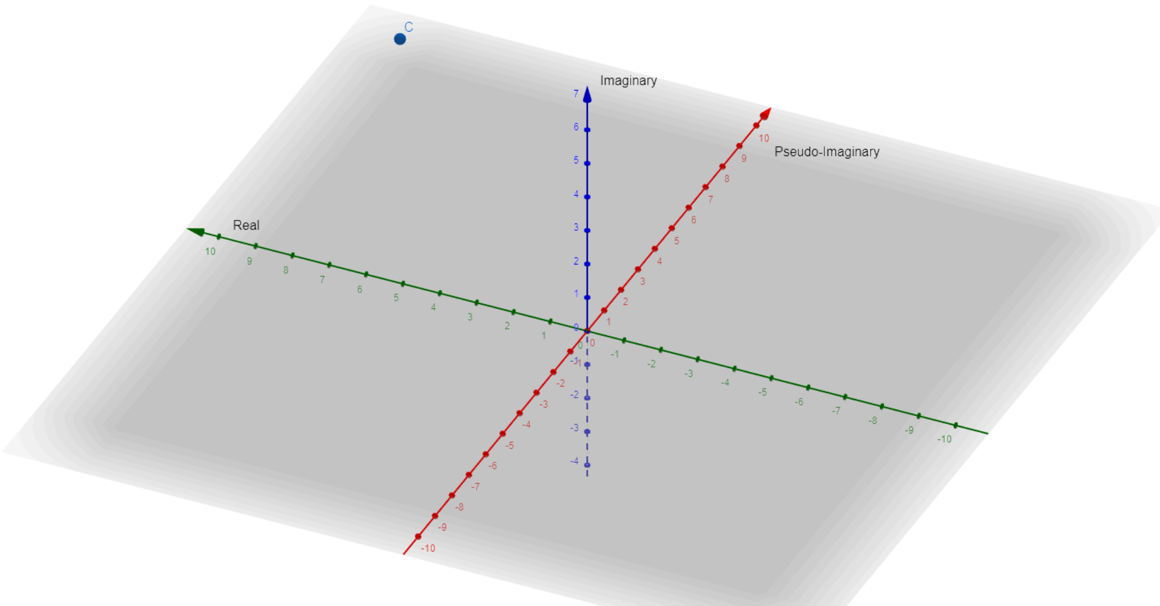


v1 = (4 + 6p)
v2 = (6 + 4p)

Subtract the two real parts and get -2. Now subtract the two pseudo-imaginary parts and get 2. We now have -2 + 2p, the answer to the problem.

We can also do what I call “interdimensional operations”. These operations combine real, pure imaginary, and pseudo-imaginary numbers. These create what I call “ultra-complex numbers”. We’re going to need to turn to the ultra-complex plane. In order for this to work, we’re gonna have to add a

third dimension. Let’s map (6 + 2p + 5i).



This is too complex. I don’t think my brain, nor my computer, nor the universe will be able to handle me adding a fourth dimension. Well, it’s good that I don’t have a fourth set of numbers.

On the bright side, we can now calculate all the negative factorials! Yay! I realized I wrote exclamation points, without realizing that that was ironic because the factorial sign is an exclamation point. Anyways, no more gamma function needed. Starting with

-1!

The formula for factorial goes like this:

n! = n-1! * n

Which can also be written as

n! = n+1! / n+1

In this case, n = -1. -1 + 1 = 0. Zero factorial is equal to 1. We can now plug in 1 in our equation:

-1! = 1/0

We can now replace the 1/0 with *p* and we get:

-1! = p

Next up we have

-2!

This is

-1! / -1

In other words:

(1/0)/-1

Which is equal to

p/-1

Or

-p.

What about 0/0? That is another pseudo-imaginary constant called "*s*", known as the pseudo-imaginary nullity. This constant also allows us to solve equations like this:

$$\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^2 - 9}$$

We can plug in three in place of x. We now have:

$$\lim_{x \to 3} \frac{3^2 - 6 - 3}{3^2 - 9}$$

We then solve the exponents and get:

$$\lim_{x \to 3} \frac{9 - 6 - 3}{9 - 9}$$

We then solve both sides and get:

$$\lim_{x \to 3} \frac{0}{0}$$

0/0 is *s*, so our answer is *s*.

Using *p* and *s*, we can solve all limit equations by just plugging in. All calculus professors will hate me after this.

What about the pseudo-complex plane? First of all, we must calculate the difference between *p* and *s* to determine where *s* should be on the pseudo-imaginary axis. If *p* is 1/0 and *s* is 0/0, then 1/0 - 0/0 = 1/0, meaning that *p* - *s* = p. We cannot calculate the position of *s* on the plane… unless we realize this. 2p is 2/0 and it is at 2 on the plane, meaning that 0p, or 0/0, should be at 0 on the plane, even though, as multiplication and division are inverses, 0p is 1. We now know that *s* is at zero on the plane. Let's map (5 + 2s). First of all, let's multiply 2 and *s*.

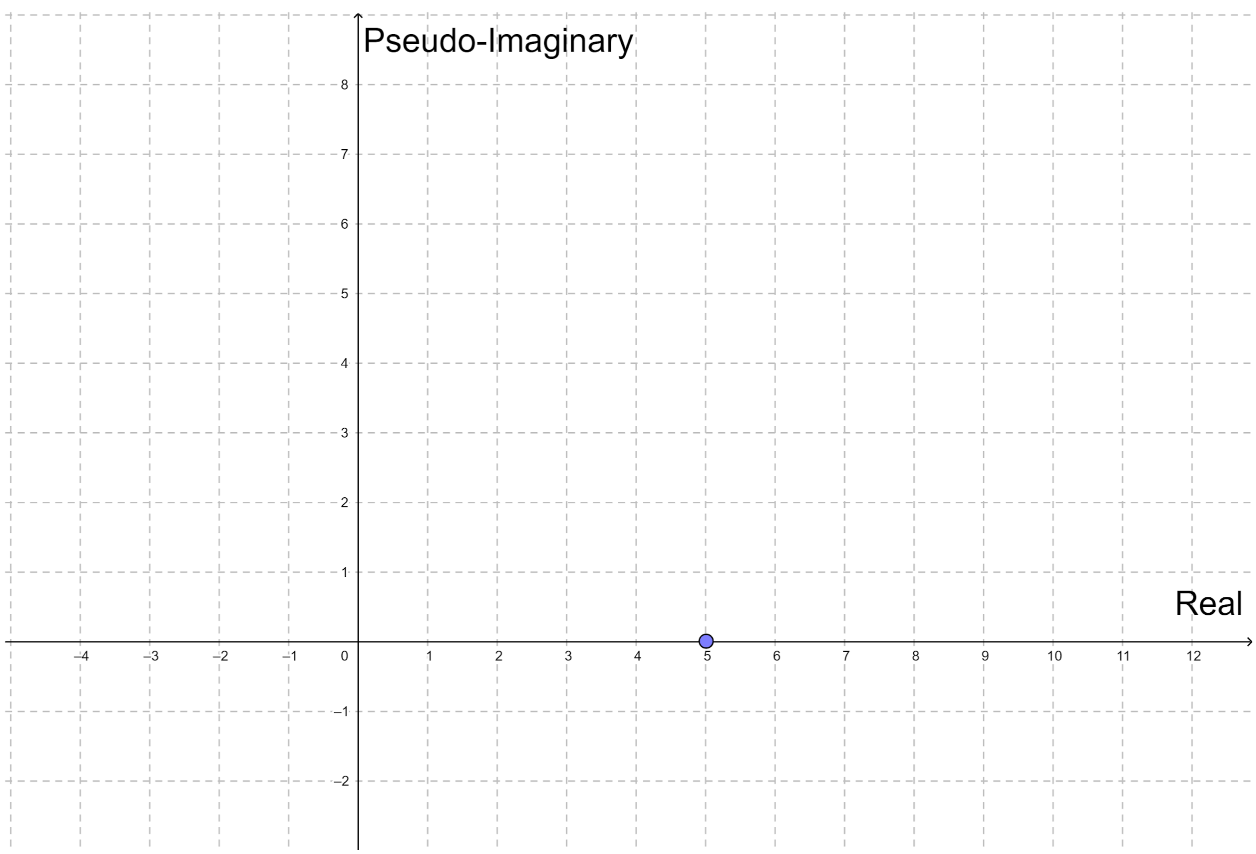
$$\frac{0}{0} \times \frac{2}{1}$$

0 * 2 = 0
0 * 1 = 0

This proves what I call the Nullity Law of Graphing:

$$s \times n = s$$

Let's now graph that number.



The real axis is at five, and the PI axis is at zero. This means that

$$s + n = n$$

s **is very similar to zero in that multiplying it by a natural number gives you it, adding it to a natural number gives you the natural number, and it is a number minus itself. We could even argue that *s* equals zero. Which would mean that 0/0 = 0. Is this mathematically possible? 0/0 is an indeterminate form, however, potentially, 0 is its hidden value.**

In conclusion, I aim to allow expressions involving division by zero to have a value of *p * n*. I also aim for negative factorials to be possible with basic arithmetic. I finally aim for 0/0 to be given a value of *s*, which may or may not equal 0. This will simplify limits for years to come. That has been my paper on mathematical analysis, part 1 to be specific.

Properties of *p*

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Here are some rules to remember when using *p* and *s*.

Product Rule

$$\frac{n}{0} \times \frac{x}{y} = \frac{nx}{0}$$

When you are multiplying *p* or anything multiplied by *p* by another fraction, you are essentially only multiplying the numerators, and keeping the denominators as 0. However, if your goal is not to simply multiply two fractions but instead to find the value of a fraction of a multiple of *p*, then multiply the entire fraction (without a denominator of zero) by the numerator (of the fraction with a denominator of zero). This "extension" of the Product Rule is called the Fraction Rule.

Quotient Rules

$$\frac{n}{0} \div \frac{x}{y} = \frac{ny}{0}$$

This rule is the inverse of the Product Rule.

$$\frac{x}{y} \div \frac{n}{0} = \frac{0}{ny}$$

This is the inverse of the other Quotient Rule. This rule also introduces us to a new constant: b_m. It is the multiplicative inverse of the pseudo-imaginary unit. It is, without a doubt, equal to 0. Quotients calculated with this rule are always multiples of b_m.

$$\frac{n}{0} \div \frac{m}{0} = \frac{0}{0}$$

This rule is more commonly used in addition to division.

***p*!**

To solve p! we must introduce a fourth constant, b_a. This is not the multiplicative inverse of *p*, but rather the additive inverse. In other words, it is *-p*. We also must return to the formula for the factorial function.

$$p! = p \times b_a!$$

Wait a second. Is *p* - 1 equal to *-p* or *0p*? Because *0p* is equal to 1. But then, you realize that you must use the gamma function as *p* is a fraction. Hold up, we can use the pi function instead of the gamma function, as that will solve our problem of not knowing what p - 1 is. The formula for x! Is

$$\int_0^\infty x^n e^{-x} dx$$

. We also see a power of e, so we must remember Euler's formula, but we can plug in p instead of i. The modified formula is $e^{p\theta} = cos(\theta) + psin(\theta)$**. Plug in p for n in the integral and get**

$$\int_0^\infty x^p e^{-x} dx$$

. Use the formula and you get

$$\int_0^\infty e^{-x} cos(lnx) dx + p \int_0^\infty e^{-x} sin(lnx) dx$$

. I doubt these functions are analytic*, so I just plugged these into integral-calculator.com. Even that site doubts the functions are analytic, so it used numerical analysis to compute this definite integral. It got 0.498016, approximately. For the second integral, we got -0.15495p. Our final answer is 0.498016 - 0.15495p. This is actually just i!, but with p instead of i.

CLARIFICATION: The functions are, indeed, analytic. They just don't have elementary antiderivatives.

Let's return to some rules.

Like Sum Rule

$$\frac{n}{0} + \frac{y}{0} = \frac{n+y}{0}$$

Unlike Sum Rule

$$\frac{n}{0} + \frac{y}{z} = undefined, \equiv z \neq 0$$

This means that if you add a multiple of *p* to a normal fraction, it will be undefined, if and only if the denominator does not equal zero. This is because you can't find a common denominator as zero is not a factor of any number

besides zero itself. However, this is only on the surface. If you think about it, the reason zero is not a factor of any number is that you can't divide by zero. Now that *p* is a thing, we can divide both sides by zero. We can change this equation to this:

$$\frac{n}{0} + \frac{y \div 0}{z \div 0}$$

Which we can rewrite as

$$\frac{n}{0} + [(\frac{y}{0}) \div (\frac{z}{0})]$$

Now apply the third quotient rule to the section in brackets and then perform standard addition to solve the equation.

This also means that the answer to an unlike addition equation is simply the augend.

Like Difference Rule

$$\frac{n}{0} - \frac{y}{0} = \frac{n-y}{0}$$

This is the inverse of the Like Sum Rule.

Unlike Difference Rule

$$\frac{n}{0} - \frac{y}{z} = undefined \equiv z \neq 0$$

The inverse of the Unlike Sum Rule is what this is.
Use the same techniques in that section to solve these equations.

Pseudo-Imaginary Equations

Simon A. Pritchett

Basic Addition

1. **4 + 3p**

$$\frac{4}{1} + \frac{3}{0}$$

Addition commutes, so let's swap the fractions to make this easier to work with.

$$\frac{3}{0} + \frac{4}{1}$$

Apply the Unlike Sum Rule.

$$\frac{3}{0} + \frac{4 \div 0}{1 \div 0}$$

$$\frac{3}{0} + (\frac{4}{0}) \div (\frac{1}{0})$$

Apply the third quotient rule

$$\frac{4}{0} \div \frac{1}{0} = \frac{0}{0}$$

We now just add the two and get:

$$\frac{3}{0} + \frac{0}{0} = \frac{3}{0}$$

Which is our final answer.

Basic Multiplication

$$\frac{4}{0} \times \frac{5}{3}$$

Apply the product rule and get your final answer:

$$\frac{4}{0} \times \frac{5}{3} = \frac{20}{0}$$

Basic Subtraction

$$\frac{4}{0} - \frac{2}{3}$$

Apply the 0/0 divisor rule after expanding and you get this:

$$\frac{4}{0} - \frac{0}{0}$$

Your final answer is:

$$\frac{4}{0} - \frac{0}{0} = \frac{4}{0}$$

The answer is always equal to the minuend.

Basic Division

$$\frac{5}{0} \div \frac{3}{4}$$

Apply the first quotient rule and get this as your final answer:

$$\frac{5}{0} \div \frac{3}{4} = \frac{20}{0}$$

Basic Exponentiation, Roots & Logarithms

This section will have <u>THREE</u> problems, not one.

Exponentiation:

$$3p^5$$

Remove the *p* and do 3^5, which equals 243. Now add the *p* and your answer is 243p. Or you could simply multiply 3p or 3/0 five times and get 243p.

Roots:

$$\sqrt[3]{8p}$$

Remove the *p* and take the cube root of eight, which is 2. Add *p* after 2 and we get the answer to the problem, which is 2*p*.

Logarithms:

$$\log_{4p} 1024p$$

Again, remove the *p* and take the base 4 logarithm of 1024. It is 5. Add the *p* back and you get 5*p* as your final answer.

Basic Interdimensional Operations

5 + 4p + 3i

Represent 5 and 4p as a fraction and get

$$\frac{5}{1} + \frac{4}{0} + 3i$$

***i* cannot be represented as a fraction. We can now swap 4p and 5 to make it easier to work with.**

$$(\frac{4}{0} + \frac{5}{1}) + 3i$$

We can now use the Unlike Sum Rule to calculate 5's… wait. It is equal to 0/0 every time ever. You should have memorized that by now. We add 0/0 to 4/0 and get

$$\frac{4}{0} + 3i$$

Which can also be written as

$$4p + 3i$$

Which is our final answer.

Insight on Plotting Pseudo-Complex Numbers

Simon A. Pritchett

Pseudo-imaginary numbers include *p*, 3*p*, 5*p* and *s*. Combining them with real numbers via addition or subtraction creates a pseudo-complex number. In order to distinguish between a pseudo-complex number (PCN) and an equation, PCNs will appear in parentheses. Also, PCNs will more than likely ask you to graph the expression, while equations will ask you to evaluate the expression. If we, for instance, tell you:

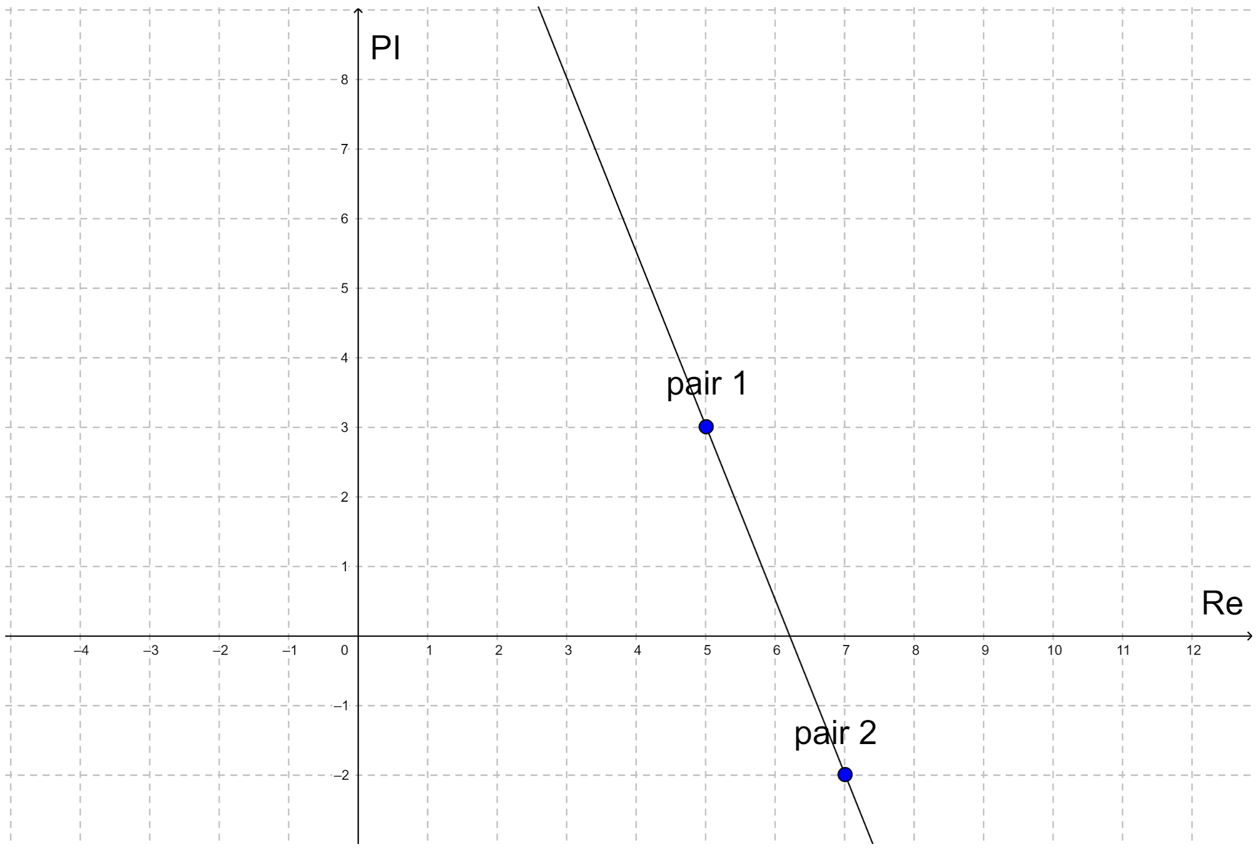
Plot $5 + 3p$ **and** $7 + 2b_a$ **and then connect the points on the coordinate plane.**

The way the plot these is we first look at the first pair's real number, which is 5. That means, for that pair, x = 5. We can now look at the imaginary part, which is 3p. That means that y = 3. Now to the second pair. As 7 is the x number in the pair, x = 7. $2b_a$ is the hard part. I told you that b_a is the additive inverse of p, so it is -p. -p is -1 on the y-axis, so y = -1. But then you realize it was $\underline{2}b_a$. Multiply -1 by 2 and get -2, which is the true value of y. Therefore, y = -2. Let's create a function table to represent this relationship.

Pair 1: Re(1) = 5	**x = 5**
Pair 1: PI(1) = 3	**y = 3**
Pair 2: Re(2) = 7	**x = 7**
Pair 2: PI(2) = -2	**y = -2**

First of all, let's call this function f(p).

We can now plot these points.



The graph is a straight line which means… YOU GUESSED IT! f(p) is a linear function! Which is cool I guess. Let's try 3 points now.

Graph (3 + 5p) + (4 + s) + (14 + 6p) and connect the points.

For the first pair, we have x = 3 and 5 = y. For the second pair we have x = 4, and y = s. As you know s = 0p. Therefore, y = 0. For the third pair, we have x = 14 and y = 6. Function table time!

Pair 1	**x = 3**
Pair 1	**y = 5**
Pair 2	**x = 4**
Pair 2	**y = 0**
Pair 3	**x = 14**

Pair 3	**y = 6**

Let's call this g(p):

That's a triangle! This is a piecewise linear function. Alright, next.

Let's try graphing interdimensional operations on the ultra-complex plane. I still don't think my brain, nor my computer nor the universe will be able to handle a fourth dimension, but this is only one more. I think the only thing that won't be able to handle the third dimension is my brain.

This is the equation:

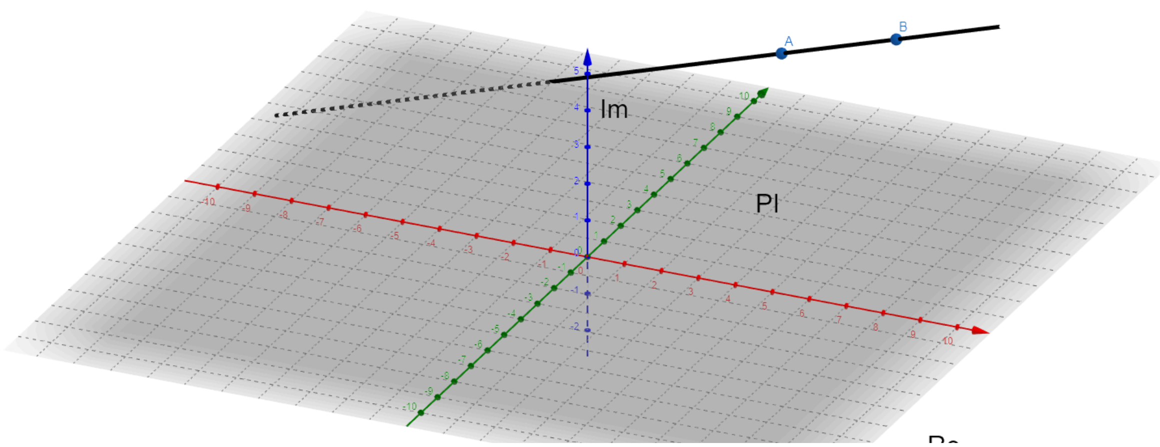
Graph (3 + 5p + 4i) + (7 + 3p + 6i) and connect the points.

We'll go straight to the function table this time. By the way the imaginary part is the z-axis.

"Pair" 1	**x = 3**

“Pair” 1	**y = 5**
“Pair” 1	**z = 4**
“Pair” 2	**x = 7**
“Pair” 2	**y = 3**
“Pair” 2	**z = 6**

It’s about time to graph.



Another linear function, but this time, in 3D. This is even cooler, but it’s just okay to me. No function is extremely cool to me. They are all a little warm.

Plotting Pseudo-Complex Numbers

Simon A. Pritchett

Single Point

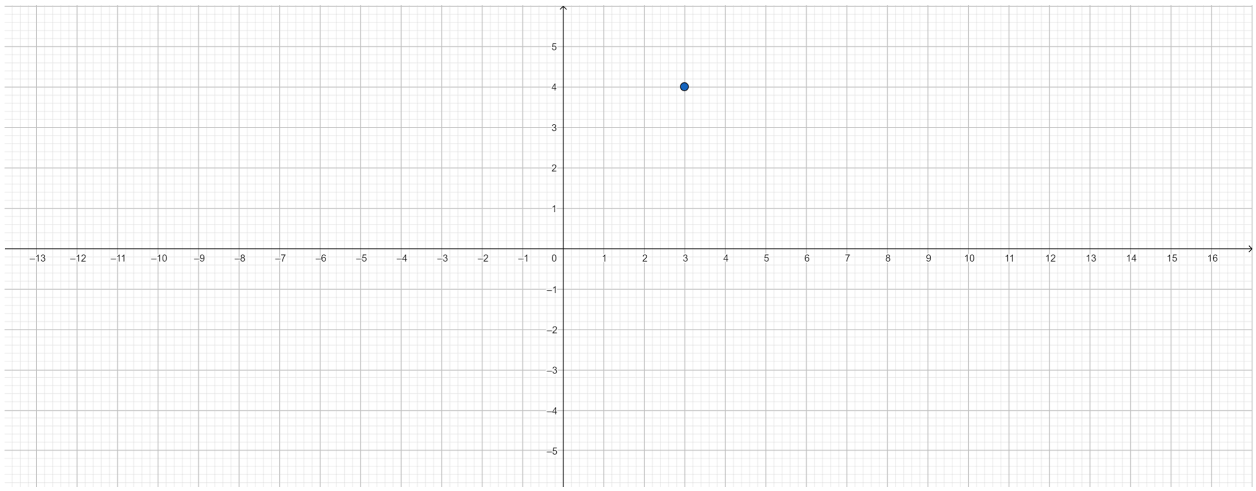
This section has three equations.

Graph (3, 4p).

Function table:

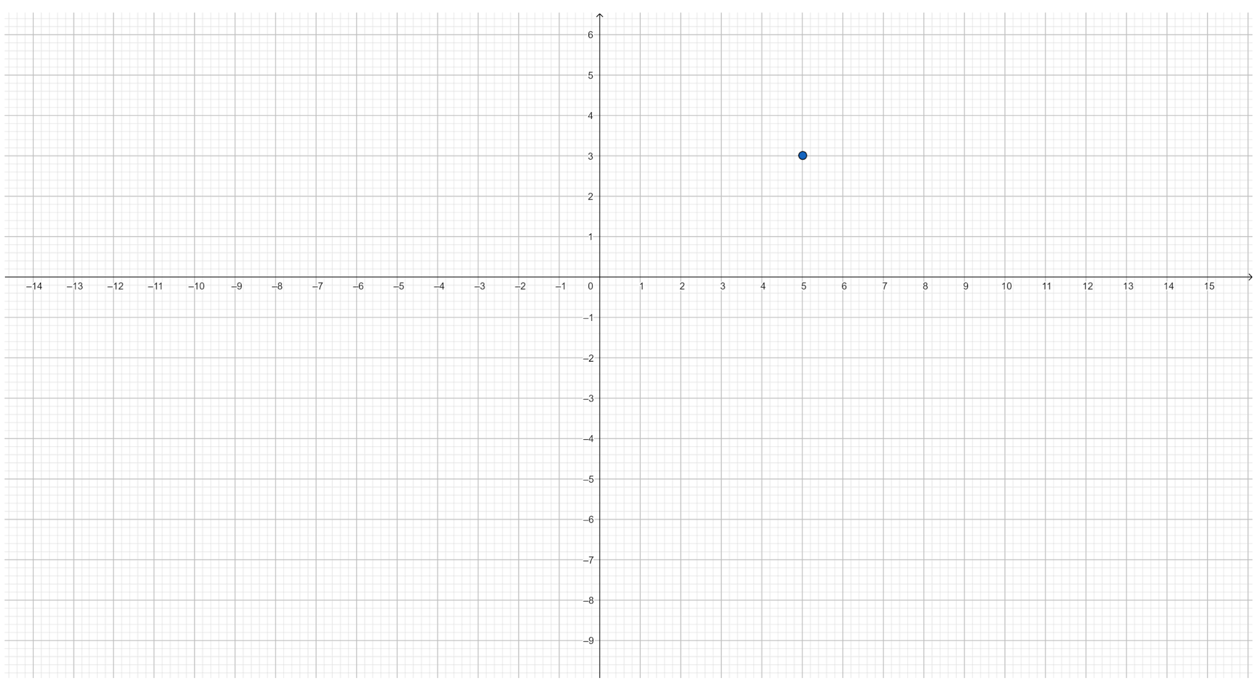
Re = 3	**x = 3**

PI = 4	**y = 4**



Graph (5, 3p).

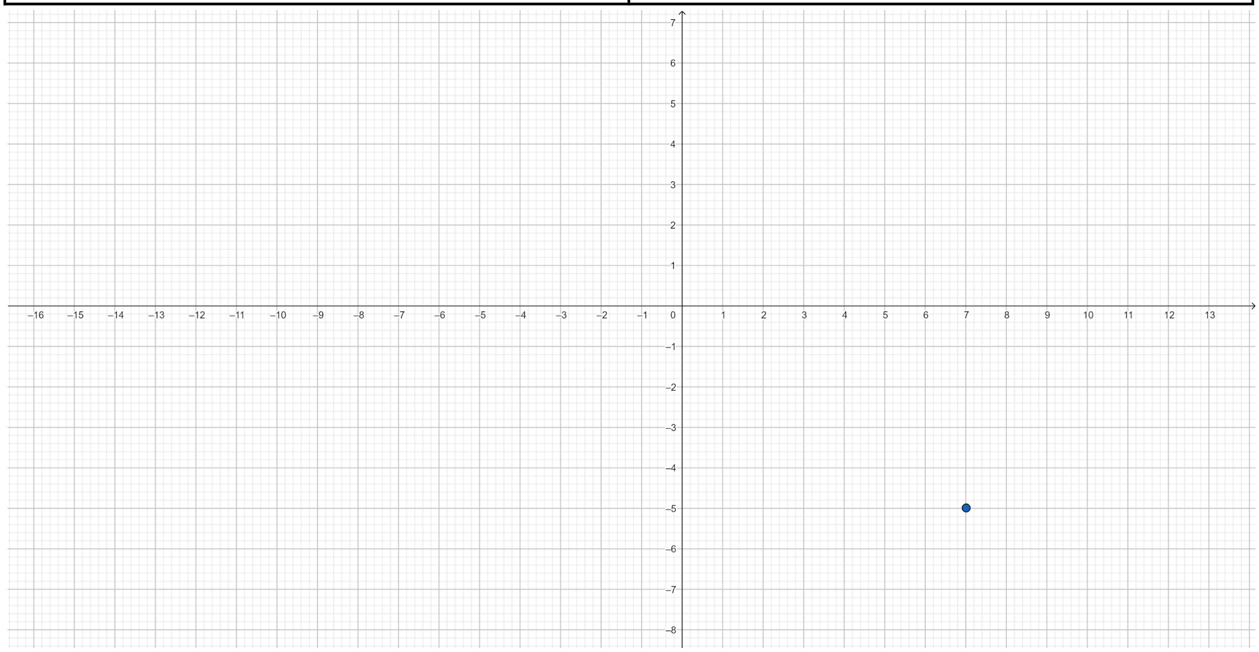
Re = 5	**x = 5**
PI = 3	**y = 3**



Graph (7, $5b_a$)

$5b_a$ = -5p

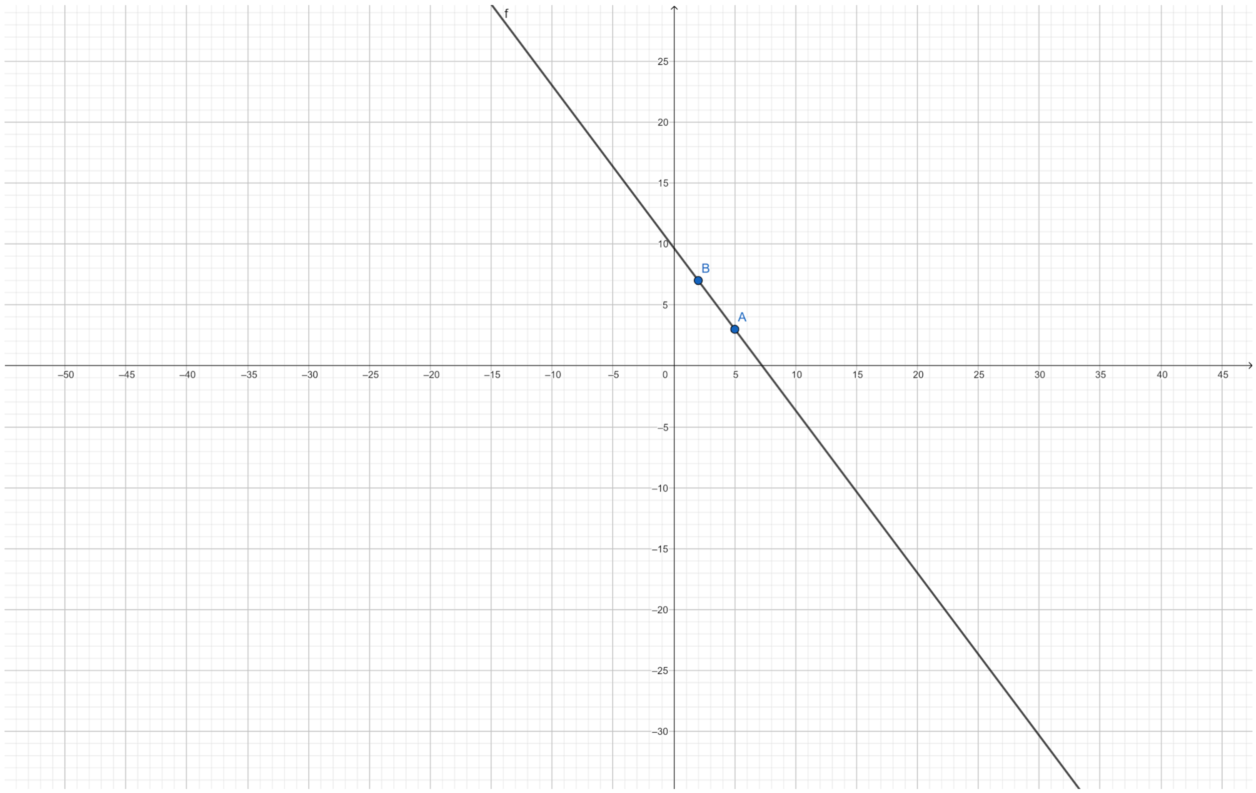
Re = 7	**x = 7**
PI = -5	**y = -5**



Double Point
This section has three equations.

Graph (5 + 3p) + (2 + 7p) and connect the points.

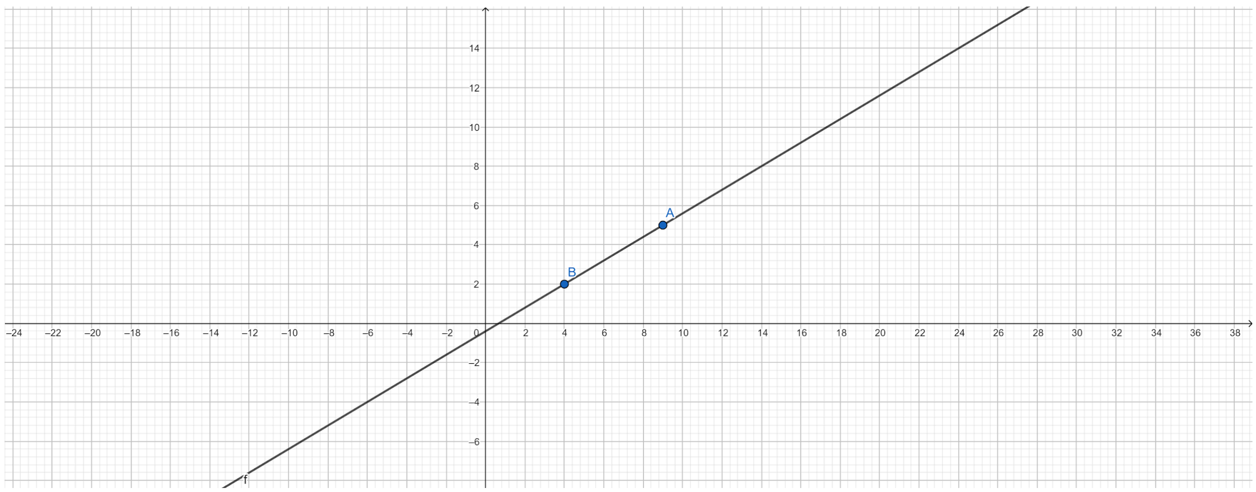
Pair 1: Re(1) = 5	**x = 5**
Pair 1: PI(1) = 3	**y = 3**
Pair 2: Re(2) = 2	**x = 2**
Pair 2: PI(2) = 7	**y = 7**



This is another linear function.

Graph (9 + 5p) + (4 + 2p)

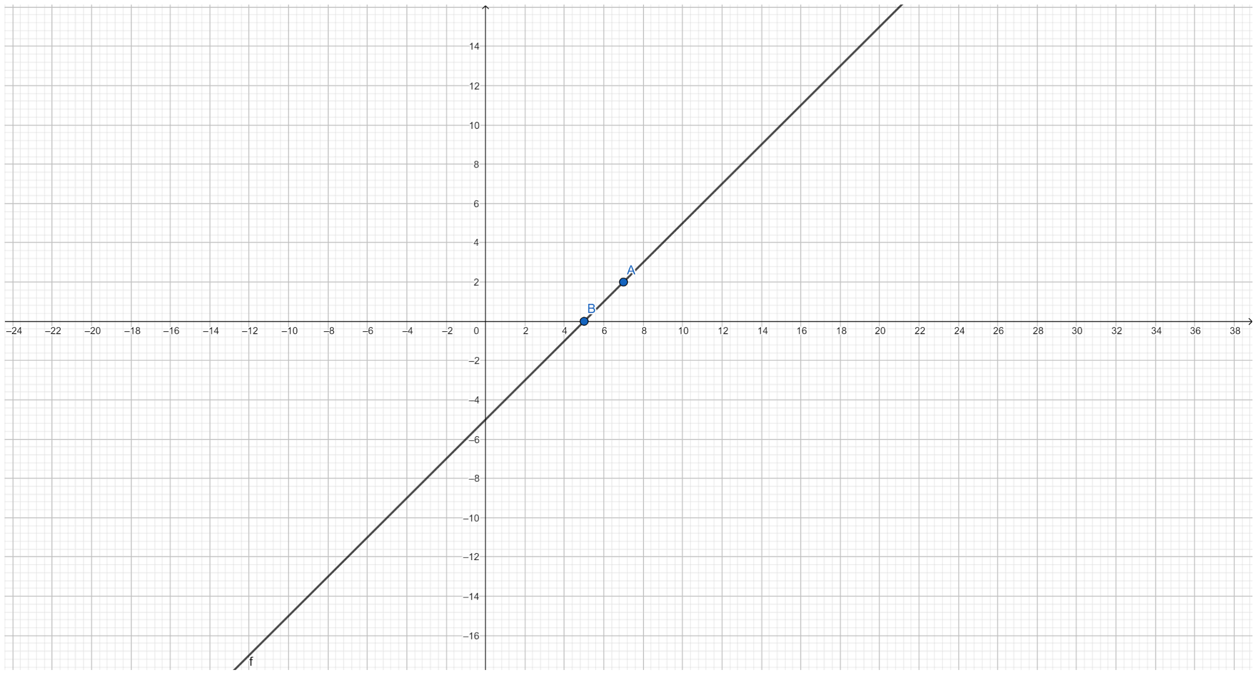
Pair 1: Re(1) = 9	**x = 9**
Pair 1: PI(1) = 5	**y = 5**
Pair 2: Re(2) = 4	**x = 4**
Pair 2: PI(2) = 2	**y = 2**



Every one of these is a linear function.

Graph (7 + 2p) + (5 + s)

Pair 1: Re(1) = 7	**x = 7**
Pair 1: PI(1) = 2	**y = 2**
Pair 2: Re(2) = 5	**x = 5**
Pair 2: PI(2) = 0	**y = 0**

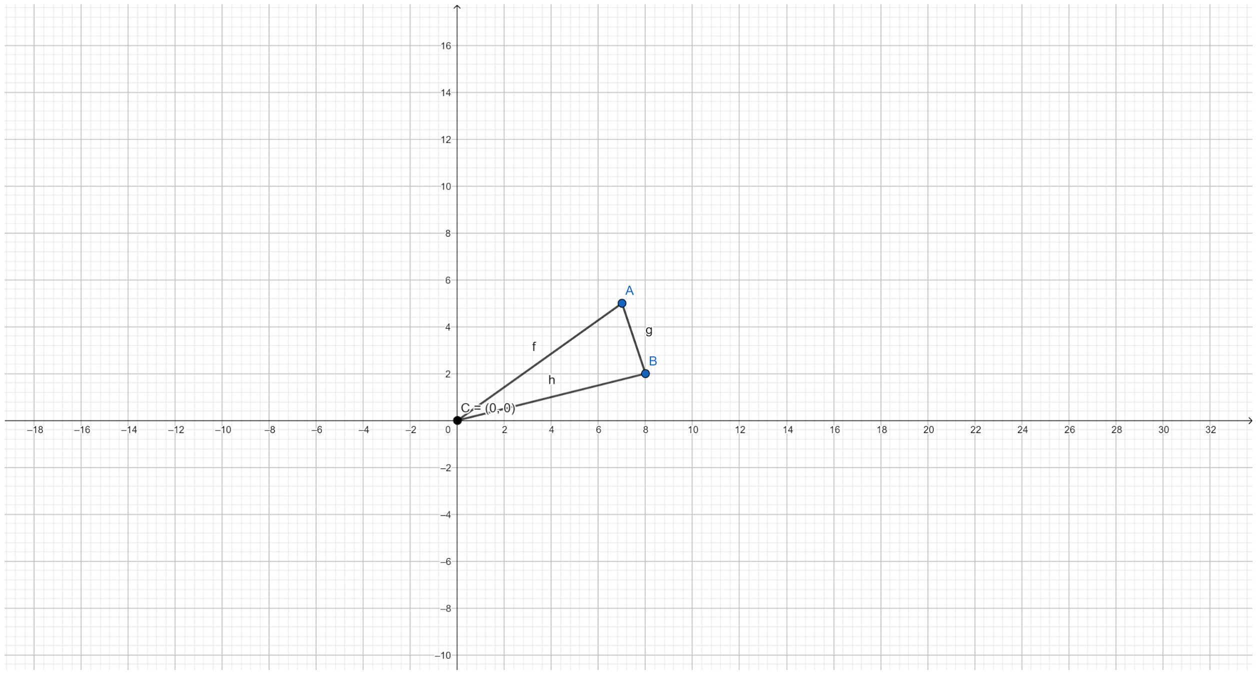


OMG WILL WE EVER NOT HAVE A LINEAR FUNCTION HUH!?!?!?!?!?!?

Triple Point
This section has one equation.

Graph (7 + 5p) + (8 + 2p) + (0 + s)

Pair 1: Re(1) = 7	**x = 7**
Pair 1: PI(1) = 5	**y = 5**
Pair 2: Re(2) = 8	**x = 8**
Pair 2: PI(2) = 2	**y = 2**
Pair 3: Re(3) = 0	**x = 0**
Pair 3: PI(3) = 0	**y = 0**

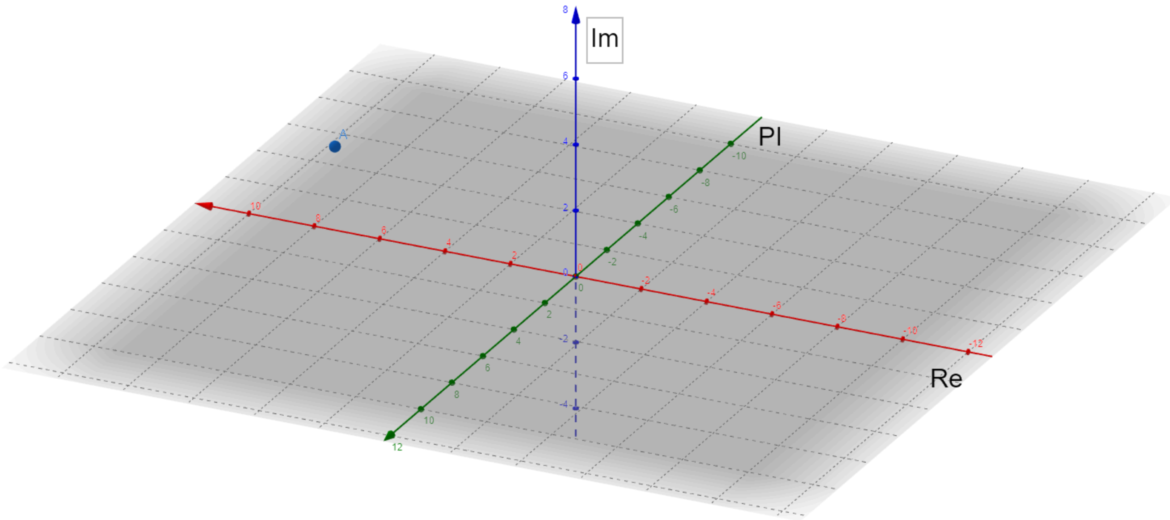


Finally! A non-li… wait. This is a PIECEWISE LINEAR FUNCTION! NOOOOOOOO!

Ultra-Complex Plane - Single Point
This section has two equations.

Graph (5 + 5p + 5i)

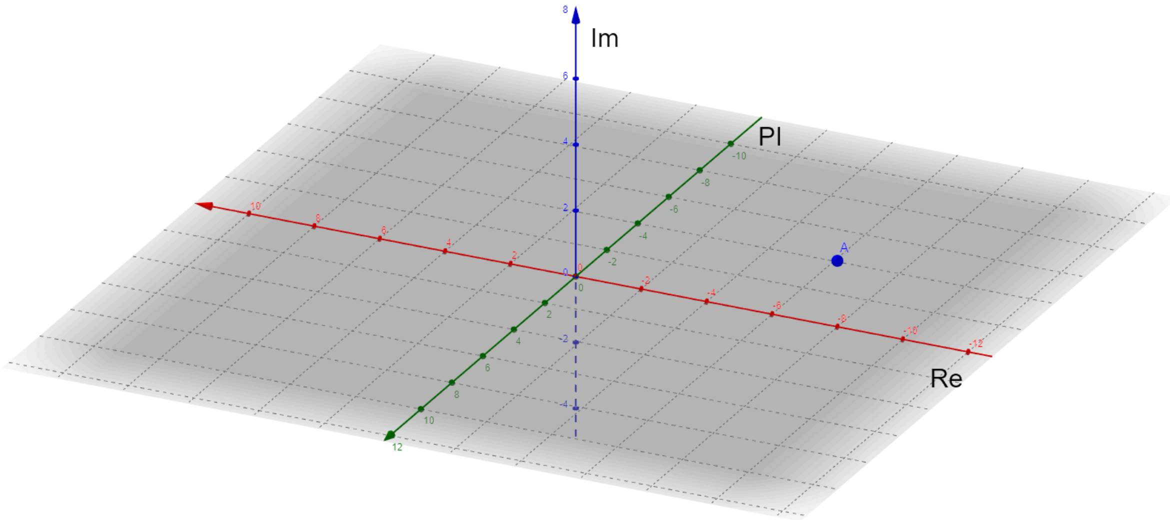
Re(1) = 5	**x = 5**
PI(1) = 5	**y = 5**
Im(1) = 5	**x = 5**



One lone dot. I feel very sorry for you.

Graph (-8 + s + 2i)

Re(1) = -8	**x = -8**
PI(1) = 0	**y = 0**
Im(1) = 2	**x = 2**

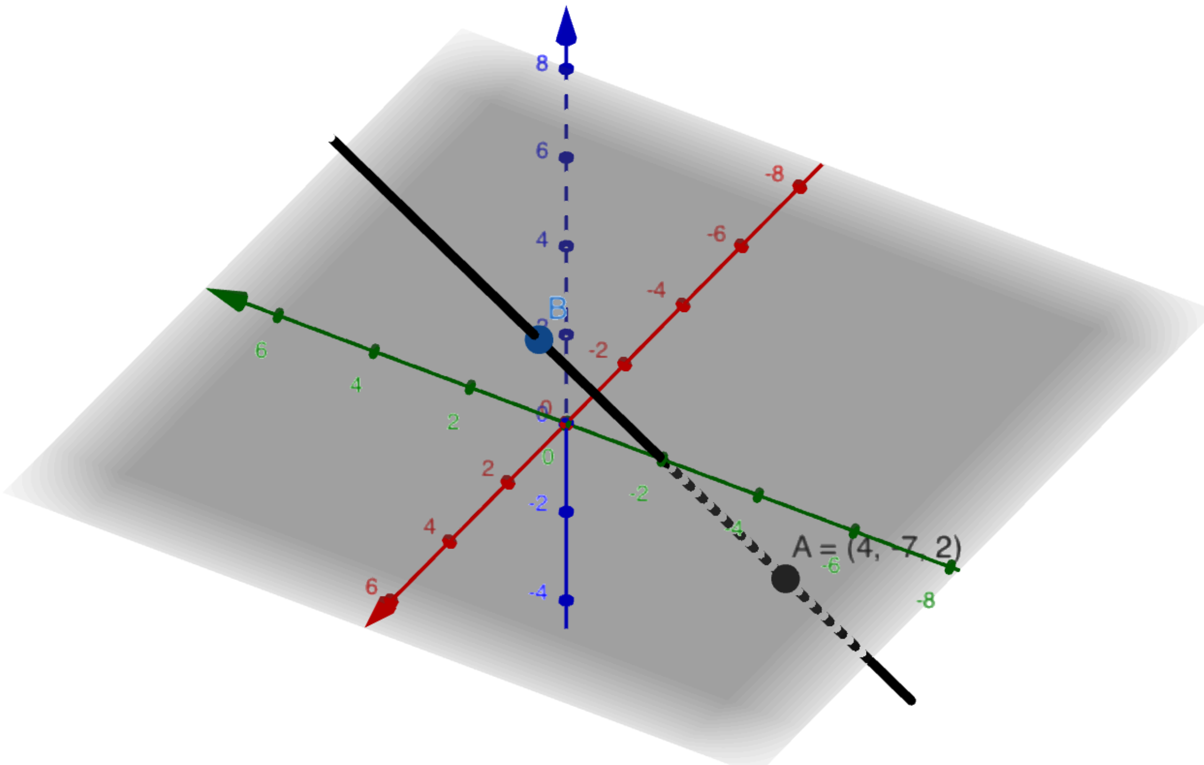


Ultra-Complex Plane - Double Point
This section has one equation.

Graph $(4 + 7b_a + 2i) + (-4, 3p + -2i)$

$7 * -p = -7p$

Pair 1: Re(1) = 4	**x = 4**
Pair 1: PI(1) = -7	**y = -7**
Pair 2: Im(1) = 2	**x = 2**
Pair 2: Re(2) = -4	**y = -4**
Pair 3: PI(2) = 3	**x = 3**
Pair 3: Im(2) = -2	**y = -2**



Oh no. A 3D LINEAR FUNCTION!?!?!?!?!?! This function will kill us all! Everyone, TAKE COVER IMMEDIATELY!

Insight on Arithmetic with Pseudo-Complex Numbers

Simon A. Pritchett

By "with", I mean between. I mean arithmetic <u>between</u> PCNs. I don't mean equations like

$\sqrt{4p}$

Since those only include a PCN. The answer is 2p, by the way. I mean a problem where you add or subtract two PCNs. Or multiply/divide them. PCNs are made up of a pseudo-imaginary part and a real part. Equations like the one below are what I mean.

Evaluate the sum of $(7+4p)+(9+2p)$.

Arithmetic with PCNs is very similar to arithmetic with complex numbers, however, arithmetic with PCNs has another major concept: the simple answer.

The simple answer is an answer determined by the following steps:

1. **Add the real parts to the pseudo-imaginary parts. Subtract it if the PCNs are joined via subtraction (like** $(4-2p)+(5-6p)$**). We now have** $(11p)+(11p)$.
2. **Convert the new expression into two division by zero expressions. We now have** $\frac{11}{0}+\frac{11}{0}$.
3. **Add/subtract with the Like Sum/Difference Rule and your final answer is** $22p$.

If you need to find the "complex" answer, solve the equation like you would solve the same equation with complex numbers. Here are the steps:

1. **Substitute the *p* with *i*.**
2. **Solve the equation like an ordinary complex number equation.**
3. **Change the *i* back to a *p*.**
4. **If asked, evaluate the answer to your new expression.**

Let's now evaluate the complex sum of

$(7+4p)+(9+2p)$.

1. **Let's do *i* substitution first of all. We now have** $(7+4i)+(9+2i)$.
2. **We now add the two like a normal complex number. We first add the real parts and then add the imaginary parts. We now have** $(16+6i)$.
3. **Change the *i* to a *p*.** $(16+6p)$.
4. **If we evaluate the expression we get our final answer:** $(22p)$.

We got the same answer as last time. The first method is known as "direct calculation" and the second one is called *i*-substitution. But what about division and multiplication? Well, here you go. This is the answer to that.

You can use *i*-substitution with multiplication and division too, but you add and multiply instead. Let's try direct calculation for multiplication. Our equation is $(4+3p)\times(9+2p)$**.**

1. **Add the real parts to the pseudo-imaginary parts. We now have** $(7p)\times(11p)$**.**
2. **Convert the new expression into two division by zero expressions. We now have** $(\frac{7}{0}\times\frac{11}{0})$**.**
3. **We can now use the Product Rule and get our final answer:** $77p$**.**

Let's now compare the answer with the *i*-substitution answer.

1. **Substitute the *p* with *i*.**
2. **Solve this like a normal equation and we get $6i^2 + 35i + 36$.**
3. **Change it back to *p* and solve the equation. As $p^2 = p$, we can replace it with p. We now have 77p.**

Same answer! Anyways, let's try both methods for a subtraction equation. The equation is $(4 + 9p) - (2 + 6p)$.

1. **Add the real and imaginary parts together. We now have:**
2. **Convert it to two division by zero expressions. We now have** $\frac{13}{0} - \frac{8}{0}$.
3. **Apply the Like Difference Rule and get your final answer of 5p.**

We can now apply *i*-substitution.

1. **Change the *p* to *i*.**
2. **Subtract like you would a complex number. We now have** $2 + 3i$.
3. **Change the *i* back to *p*. We now have** $2 + 3p$.
4. **Add the two and we now have our final answer which also happens to be** $5p$.

The same answer again! That is a rule of addition and subtraction with pseudo-complex numbers. We get the same simple and complex answer. If you are still confused, there are more practice problems in the chapter below.

Arithmetic with Pseudo-Complex Numbers

Simon A. Pritchett

Direct Solving

Addition: Calculate the simple value of $(5 + 3p) + (2 + 7p)$ **using the direct calculation technique.**

1. **Add the real and pseudo-imaginary parts together: the answer is** $8p + 9p$.
2. **Convert to division by zero: the answer is** $\frac{8}{0} + \frac{9}{0}$.
3. **Evaluate the expression: the answer is** $17p$**.**

Subtraction: Calculate the simple value of $(2 + 3p) - (7 - 4p)$ **using the direct calculation technique.**

1. **Subtract the real and pseudo-imaginary parts. We now have** $5p - 3p$
2. **This time, let's not convert it to division by zero. We can just subtract the two coefficients. We now have our final answer which is 2*p*.**

Multiplication: Calculate the simple value of $(5 + 2p) \times (8 + 5p)$ **using the direct calculation technique.**

1. **Add the real and pseudo-imaginary parts. We now have** $7p \times 13p$**.**
2. **Multiply the two coefficients. We now have our final answer of** $91p$**.**

***i*-Substitution**

Addition: Calculate the complex value of $(8 + 3p) + (14 + 4p)$ **using the *i*-Substitution technique.**

1. **Replace all instances of *p* with *i*. We now have** $(8 + 3i) + (14 + 4i)$
2. **Add this like a complex number. We now have** $(22 + 7i)$**.**
3. **Change *i* back to *p* and evaluate the expression. Our final answer is** $(29p)$**.**

Subtraction: Calculate the complex value of $(4 + 6p) - (7 + 2p)$ **using the *i*-Substitution technique.**

1. **Replace all instances of *p* with *i*. We now have** $(4 + 6i) - (7 + 2i)$**.**
2. **Subtract this like a complex number. We now have** $-3 + 4i$**.**
3. **Change *i* back to *p* and evaluate the expression. Our final answer is** p**.**

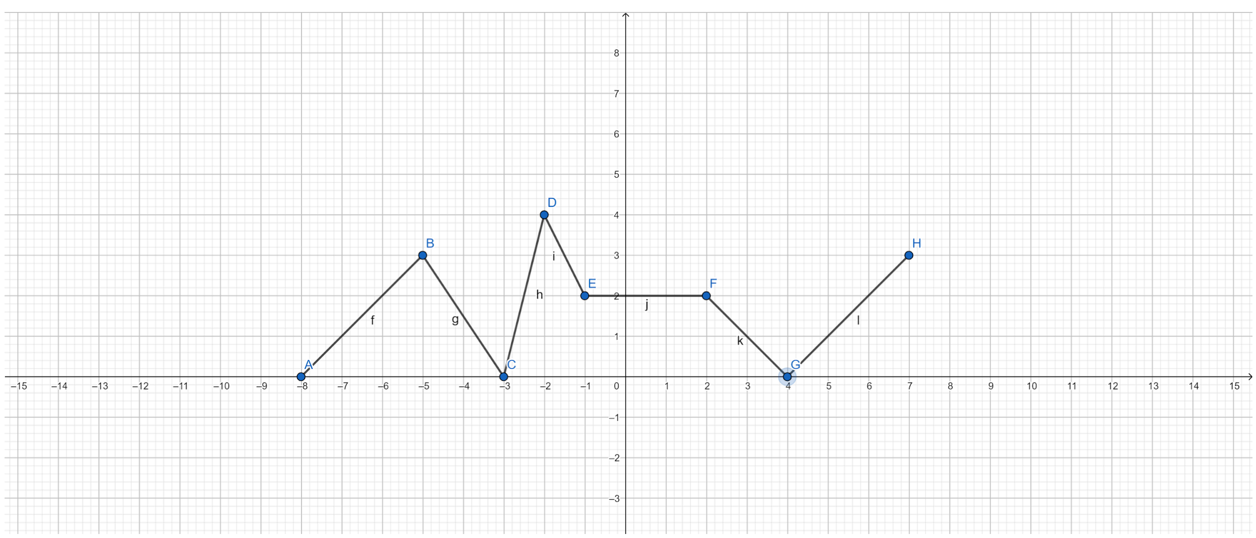
I just wanted to tell you that you can check your answers for addition and subtraction ***i*-Substitution** problems by using the direct calculation method and comparing both answers you get. If they are equal, your answer is correct. If they are not, then your answer is incorrect.

p is for Piecewise

Simon A. Pritchett

First of all, we need to talk… about how p is algebraic. It took me a whole month to realize this, but it turns out p is the solution to the linear equation of $0x - 1 = 0$. Similarly, *s* is algebraic, as it is the solution to the polynomial $0x^2 + 0x + 1$.

Anyways, it's about time to talk about *p* and piecewise functions. First of all, what type of function is this?



If you guessed "piecewise linear function" you were correct. We will talk about those.

Here is a law for graphing:

Any graph of multiple points on the pseudo-complex plane can be represented as a piecewise linear function.

This is kind of obvious when you think about it, so that's it for us talking about piecewise linear functions. This section is all about applying our constants to algebraic equations.

Let's say we have this equation:

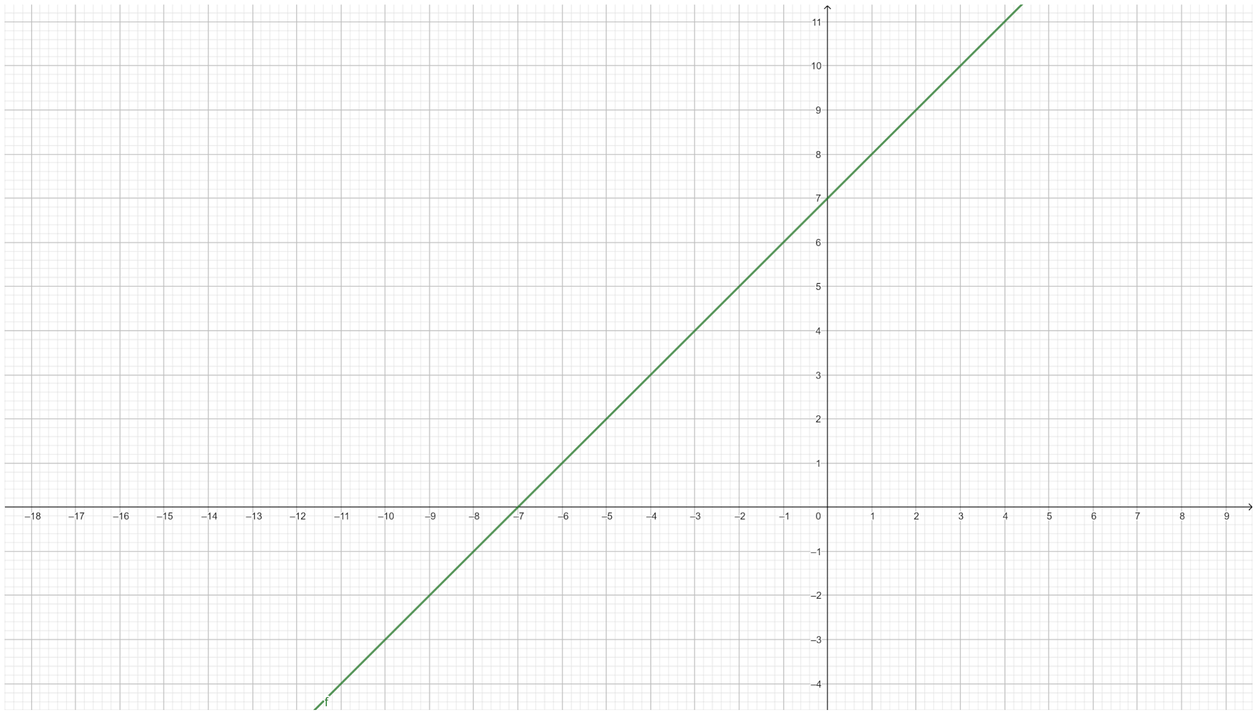
Solve for x.

$$px + 7p^2 = 54p$$

So, how we would handle this equation is as follows:

1. **Determine what $(7p)^2$ is. It is 49p.**
2. **Replace the *p*s with 1s. We now have $1x + 49(1) = 54(1)$. In other words: 1 times what + 49 = 54.**
3. **Evaluate the above expression. The answer is 5. You can now change the 1's back to *p*. x = 5.**

Let's graph that equation. First, let's define f(x) = px + 7p^2. Alright, time to graph.

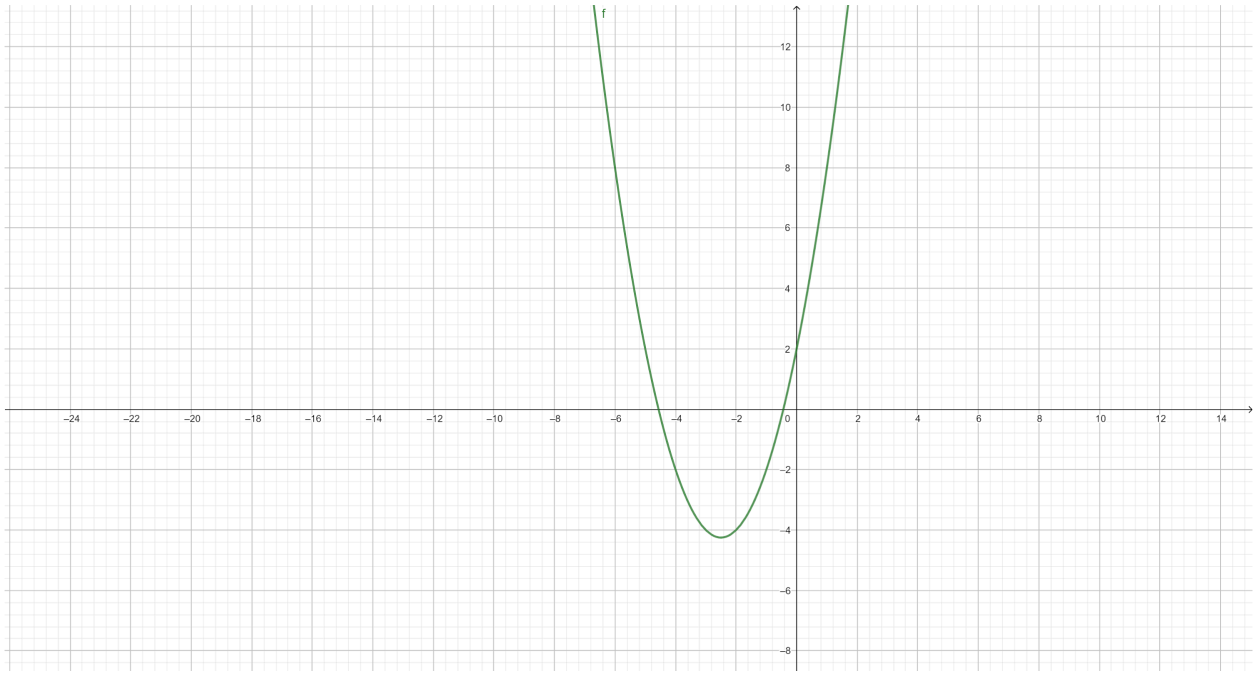


Alright, seems go… OH NO! WE HAVE ANOTHER LINEAR FUNCTION! SAVE ME PARABOLA! SAVE ME!

Speaking of parabolas, let's try this equation:

Graph $px^2 + 5x + 2 = 0$.

Hmm. I recognize that form… FINALLY! DR. PARABOLA HAS COME TO SAVE ME! IT'S A QUADRATIC! TIME TO GRAPH IT!



Phew. That sure is a good-looking parabola. Time to find the roots of the quadratic.

The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Let's try using it.

Substituting *p* for 1 is the easiest way to plug it into algebraic equations. For arithmetic purposes, you use the rules for unlike addition and subtraction, but for algebraic purposes you substitute the real number for *p* * the real number.*

Using these methods, we now have $x = \frac{-5 \pm \sqrt{17p}}{2p}$.

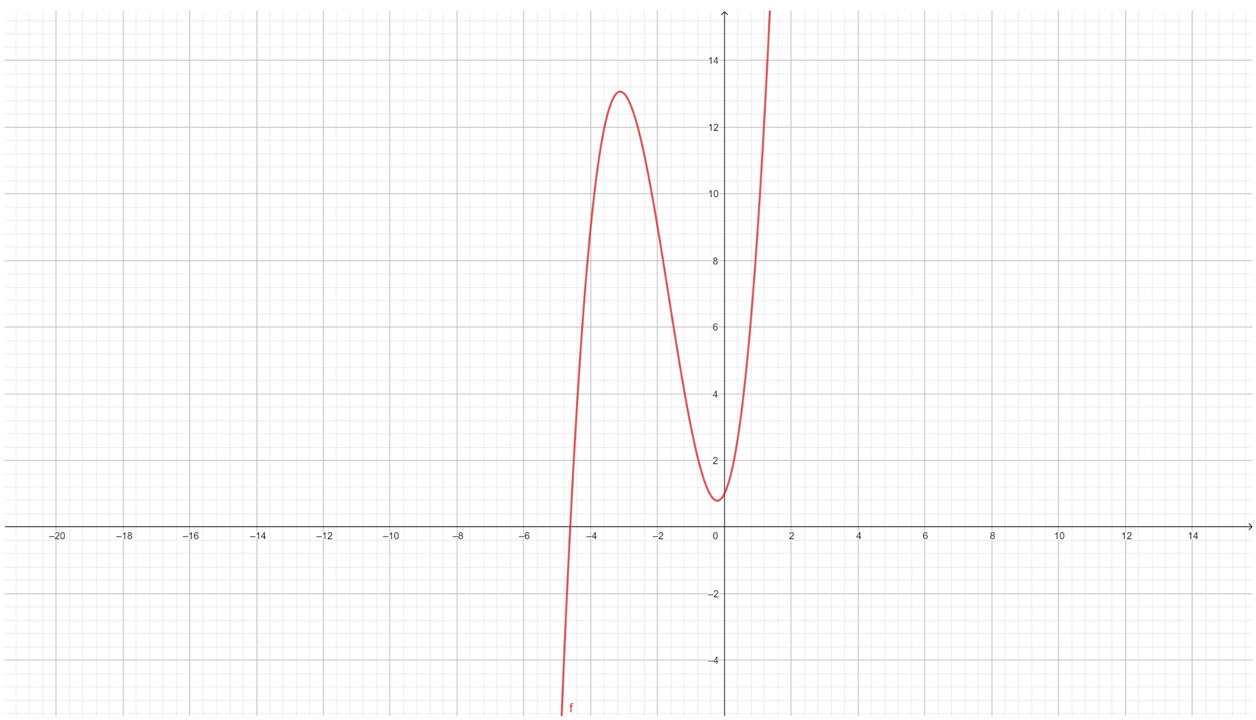
Our roots are $x = \frac{-5}{2p} + \frac{1}{2p}\sqrt{17}$ **and** $x = \frac{-5}{2p} + \frac{-1}{2p}\sqrt{17}$ **using the quadratic formula.**

***Clarification: We're supposed to multiply both sides by *p*, so we would have 8p^2, but order of operations says to do exponents first, and p^2 = p, so we would still have 8p. This may seem wrong, but if we substitute p for 1, it will make more sense. What we now have when we substitute is 8(1^2). We do the exponents first and get 1. We now have 8(1). Now change 1 back to p and we get 8p.**

Let's try a different equation, but no roots, just the graph.

Graph px^3 + 5x^2 + 2x + 1.

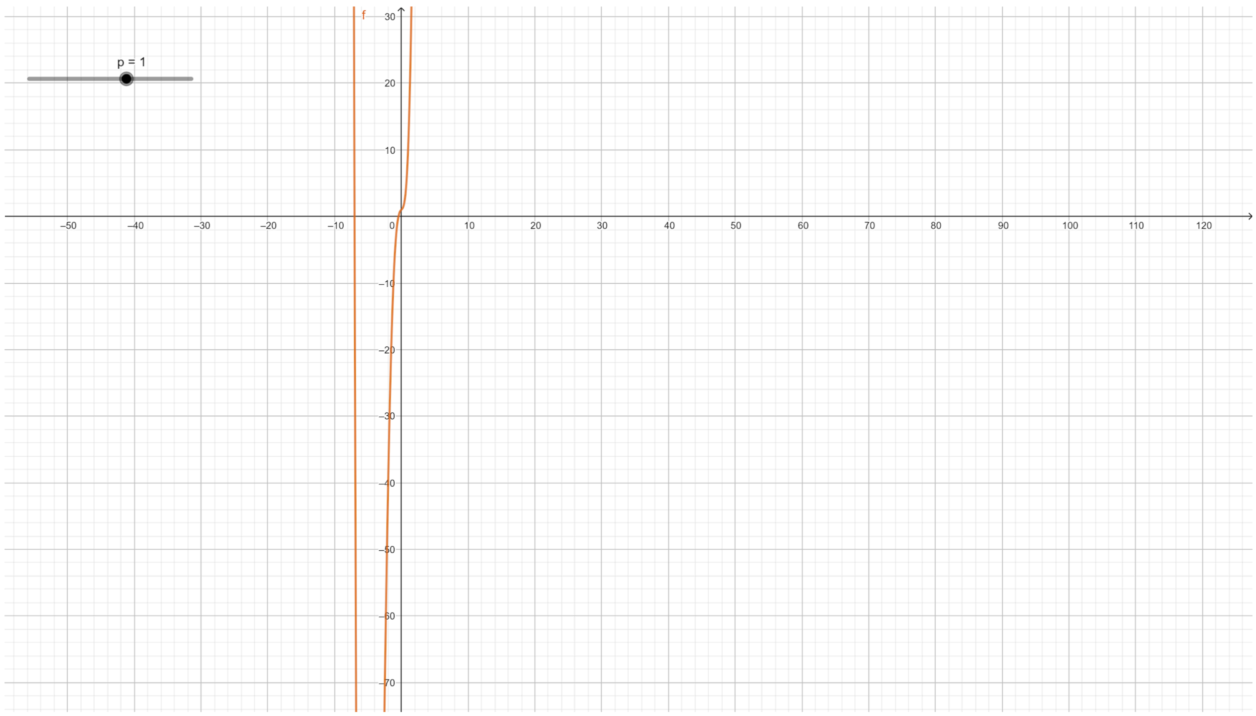
This is a cubic function, not a quadratic. But it's cool anyways. Time to graph!



In my opinion, the graph of the cubic function is like the sine wave graph but cooler.

Let's try graphing a quartic function for no reason. We have

$px^4 + 5x^3 + 2x^2 + x + 1$



This was pointless. This graph is lame.

Let's talk about matrices. Let's add these matrices:

$$\begin{bmatrix} p & 2 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 2 & 7 \end{bmatrix}$$

Let's do this. Add all these terms and get this:

$$\begin{bmatrix} 6p & 2 \\ 5 & 12 \end{bmatrix}$$

Keep in mind that *p* = 1*p*, so *p* + 5 = 6*p*. Substituting *p* for *1* may also help.

That is basic matrix addition with *p*. Let's try multiplication.

Let's multiply together the same two matrices we added last time.

The equation is:

$$\begin{bmatrix} p & 2 \\ 3 & 5 \end{bmatrix} \times \begin{bmatrix} 5 & 0 \\ 2 & 7 \end{bmatrix}.$$

The solution is

$$\begin{bmatrix} 9p & 14 \\ 25 & 35 \end{bmatrix}$$

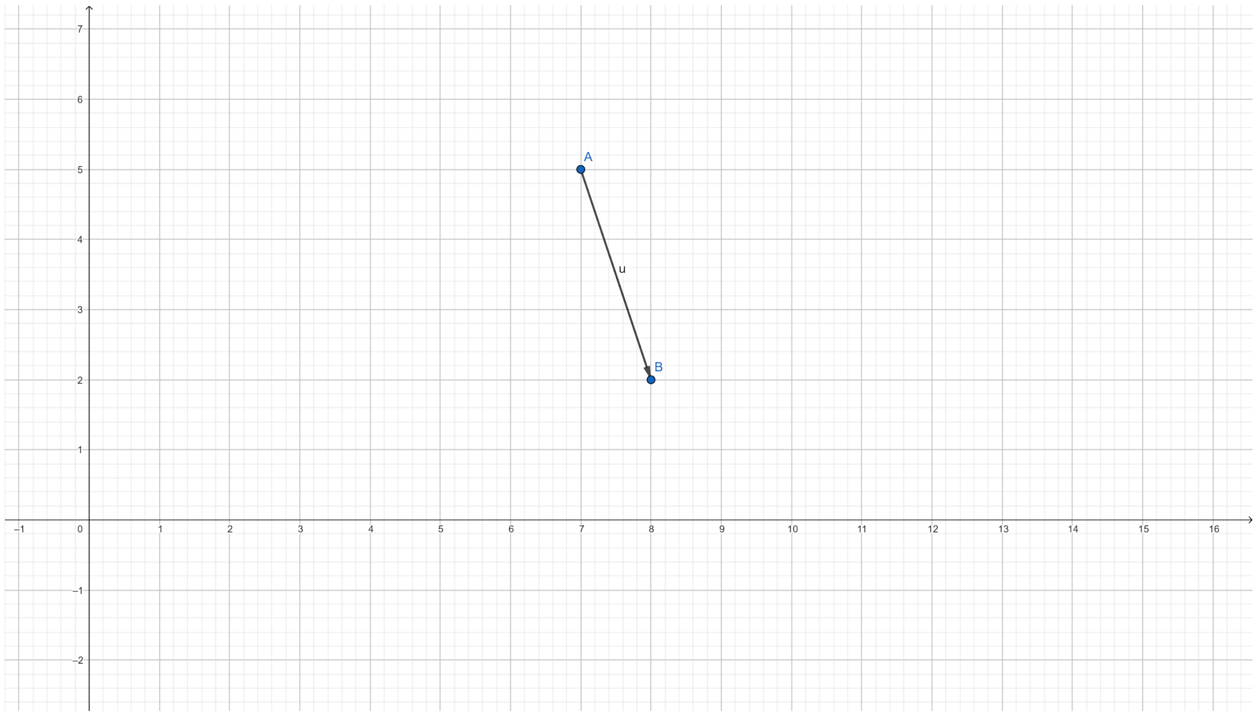
You just treat *p* as 1. This works for all algebraic and matrix-related operations and equations.

Now, it's time to talk about vectors.

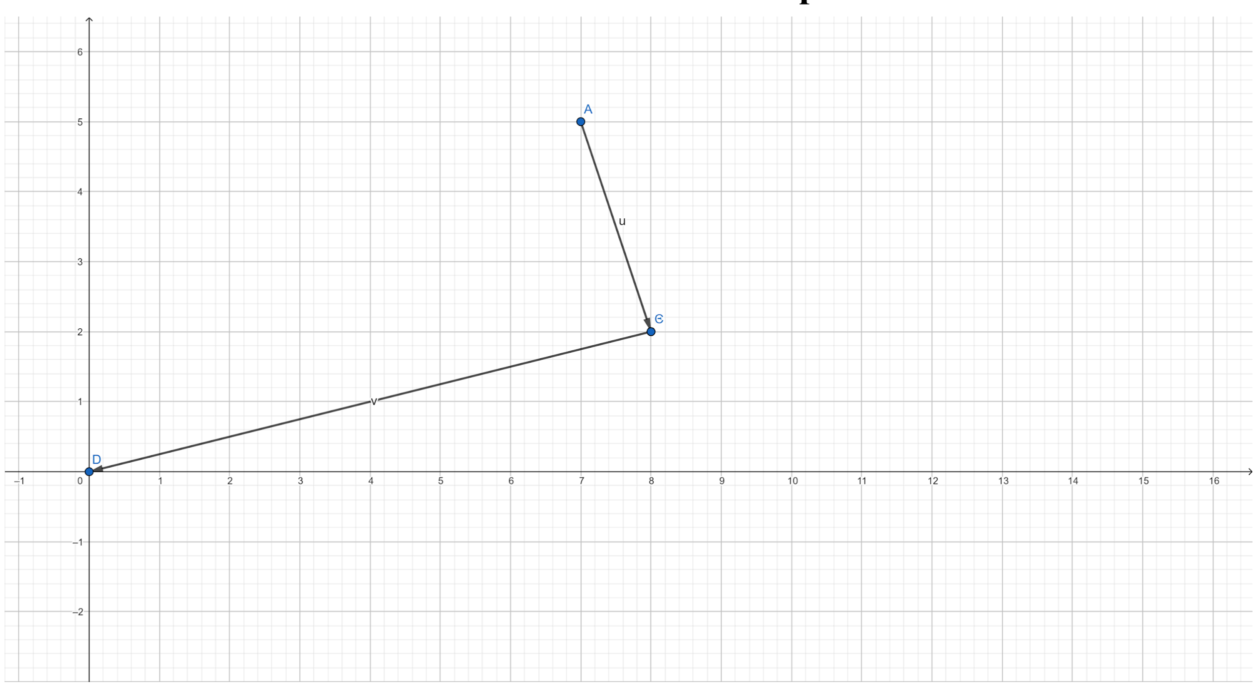
Let's return to the very last 2D plotting equation we did.

(7 + 5p) + (8 + 2p) + (0 + s)

Let's remove the third point. Let's connect the first point to the second point using a vector.

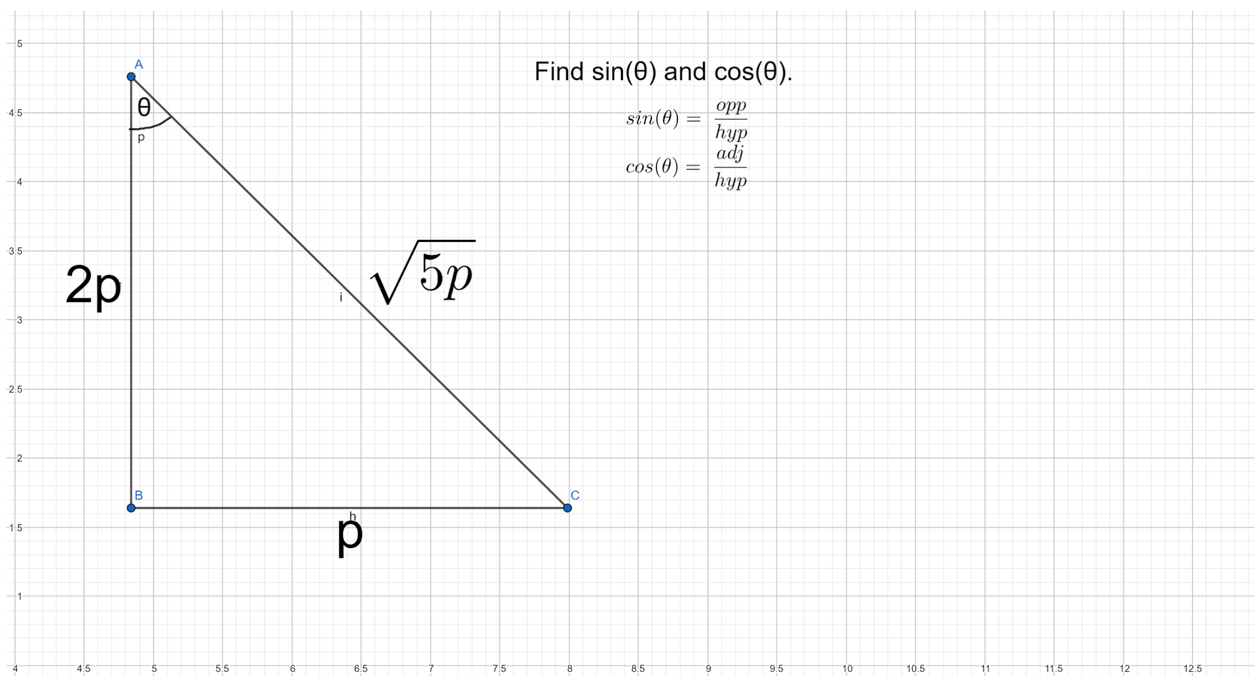


Let's look at this vector's x and y-components. Its x-component is 1, and it's y-component is -3. Let's add this to another vector using the last two pairs instead. These are both of the vectors on the same plot.



This new vector's x-component is -8 and its y-component is -2. Let's call the first vector "$\overrightarrow{AB}$" and the second vector "$\overrightarrow{CD}$". We add the two x and y-components. We now have $(1-8),(-8-2)$. **These are the vectors written in traditional vector notation. Let's now add the components. Our final answer is** $-7, 10.$

Earlier, I talked about sine graphs, let's use them for something. It's trig time!



First of all, this is (almost) a 45-45-90 triangle. Anyways, let's find the sine and cosine of θ. As we know from SOHCAHTOA,

$$sin = \frac{opp}{hyp}.$$

The side that is opposite of θ has a side length of *p*. The hypotenuse has a side length of $\sqrt{5p}$**, therefore, our sine is** $\frac{p}{\sqrt{5p}}$**. Let's rationalize the denominator.**

To do this, we multiply both sides by $\sqrt{5p}$. We now have $\frac{\sqrt{5p^2}}{5p}$. Let's now simplify a little. Remember my clarification earlier?

> **"Clarification: We're supposed to multiply both sides by *p*, so we would have $8p^2$, but order of operations says to do exponents first, and $p^2 = p$, so we would still have 8p. This may seem wrong, but if we substitute p for 1, it will make more sense. What we now have when we substitute is $8(1^2)$. We do the exponents first and get 1. We now have 8(1). Now change 1 back to p and we get 8p."**

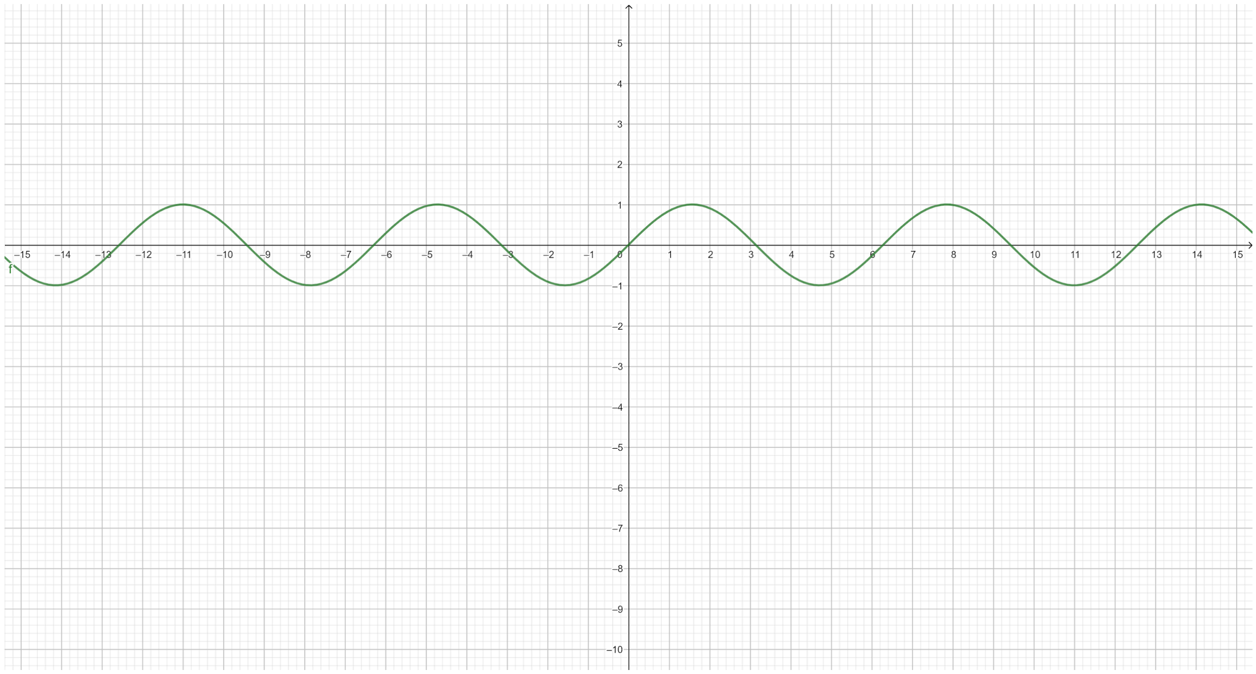
We can apply a similar thing here. We can simplify $5p^2$ to $5(1^2)$. $1^2 = 1$, so we have 5(1). Change the one back to *p* and we have 5p. We now have our final answer: $\frac{\sqrt{5p}}{5p}$. What about cos(θ)? According to SOHCAHTOA:

$$cos = \frac{adj}{hyp}$$

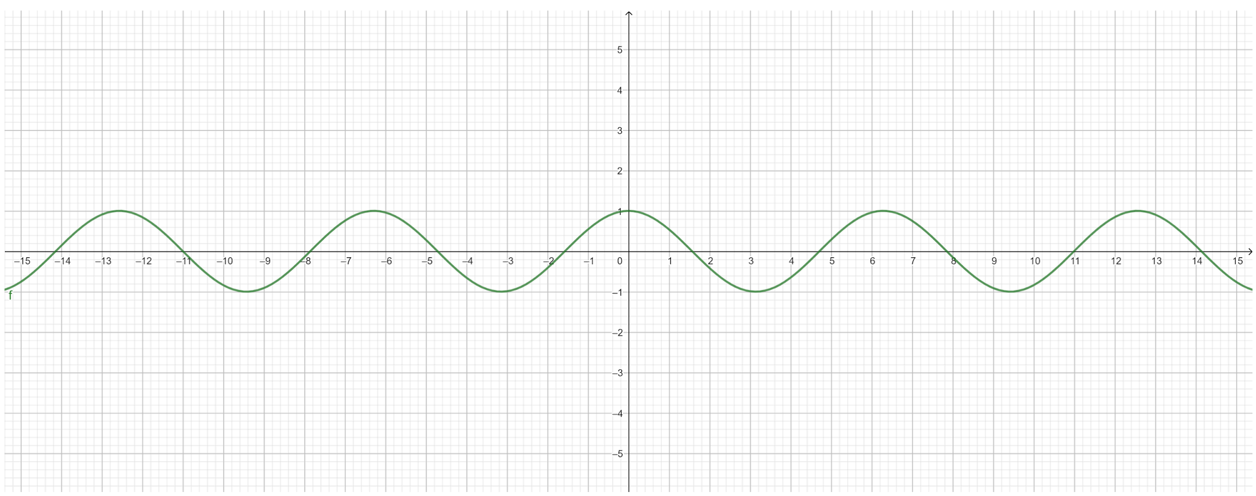
The side adjacent to θ has a side length of 2*p*. The opposite side is $\sqrt{5p}$, so we now have $\frac{2p}{\sqrt{5p}}$. Time to rationalize! After rationalizing we are left with $\frac{\sqrt{2p^2}}{5p}$. We can apply the rule previously mentioned, and we get a final answer of $\frac{\sqrt{2p}}{5p}$.

Let's graph both equations. We'll graph each one twice. The first time, we'll graph sin(x) and cos(x), and then we'll graph sin(45 degrees) and cos(45

degrees) as those are our angle measures. First we got sin(x):

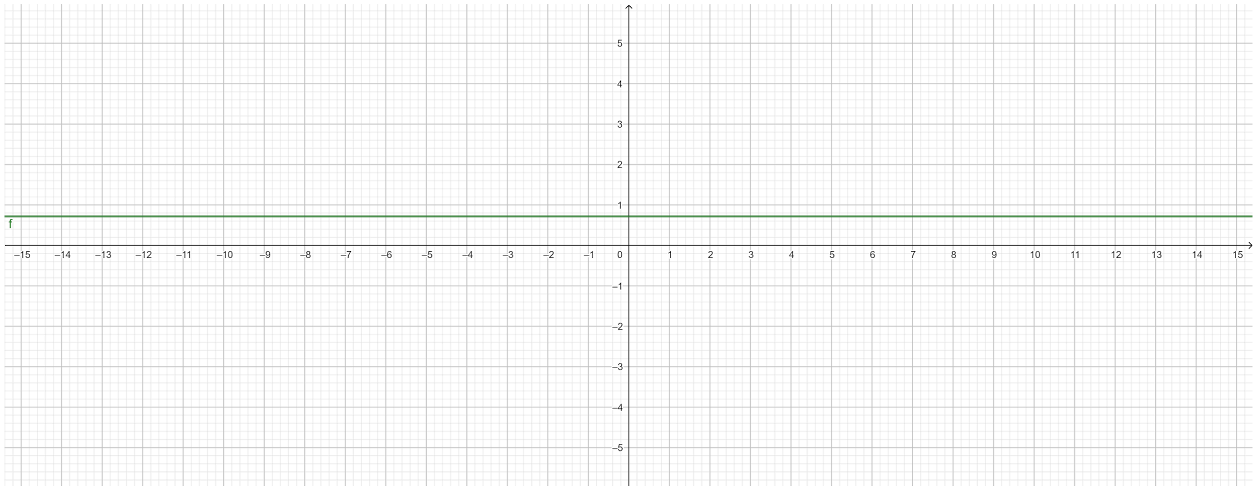


The classic sine wave is in my sight. Let's swap the "sin" for "cos" in GeoGebra.

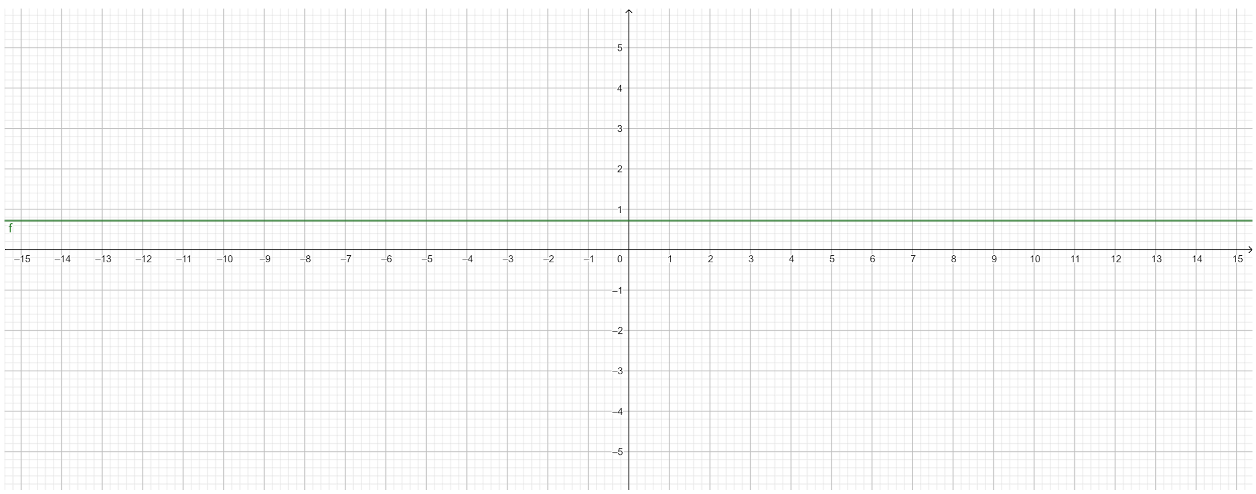


Another cool I guess.

Cool, I guess. What about sin(45)? Let’s try that.



Looks fine... OH NO! A STRAIGHT LINE! GOTTA HIDE! SWITCH TO COSINE IMMEDIATELY!



OH NO! IT’S THE EXACT SAME! SWITCH TOPICS IMMEDIATELY!

But before we switch topics, you should know that that is the pseudo-complex sine of 45 degrees. That is the official name. Anyways, let’s switch topics.

Let’s talk about logarithmic equations. When we graph those, we don’t get any straight lines. At least make sure that we don’t have any base *p* logarithms.

Let’s start with a relatively simple equation:

Solve for x.

$$log_{xp}4p = 2p$$

This is asking "What squared = 4p?" and the answer to that is 2p. The answer is 2p. x = 2.

Here is a more complex equation:

Solve for x.

$$log_{xp}16p + log_{4p}xp = 3p$$

Start off by looking at the roots of 16. The square root of 16 is a whole number: 4. Let's try plugging in 4. We now have

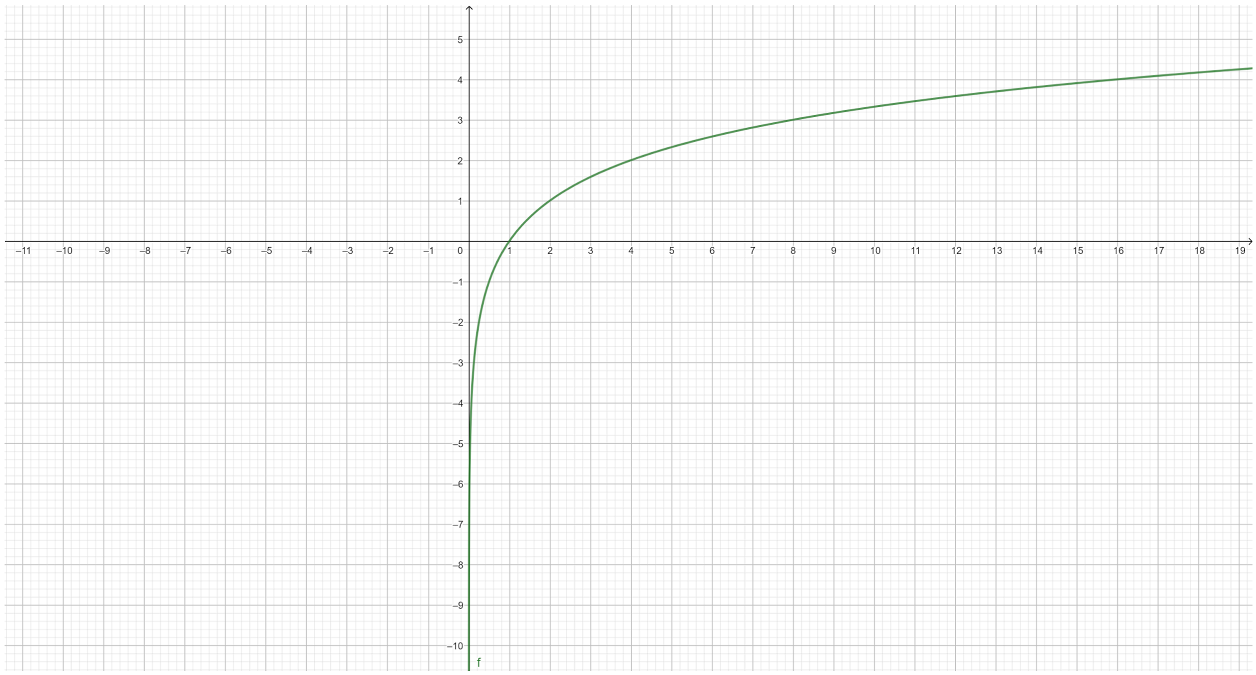
$$log_{4p}16p + log_{4p}4p = 3p$$ **.**

We can now calculate both sides of the equation. We now have

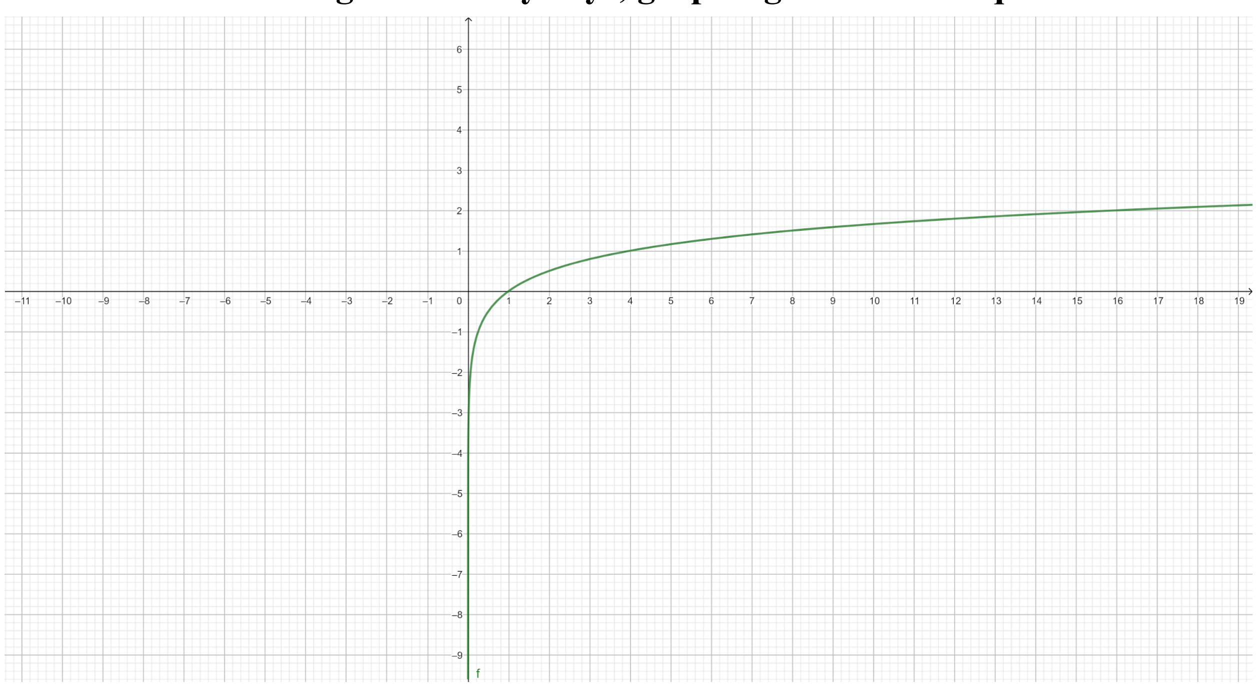
$$2p + p = 3p$$

Which is a true expression! x = 4.

Let's graph those equations now. Starting with the first one. The first one is in the form of $\log_{2p}(x)$. By the way, don't forget to substitute *p* for 1.



This was the graph of the first equation, and it's pretty good, even though I can sort of see a straight line. Anyways, graphing the second equation time.



It's like the other one, but less steep. Let's calculate the derivative of these to find the exact slope. The derivative of log(x) = $\frac{1}{xln(base)}$**. The base for the first one is 2. We now have** $\frac{1}{xln(2)}$**. The base for the second one is 4, so we have** $\frac{1}{xln(4)}$**. If we solve both of these, we get the following (don't forget to add back the *p*):**

$$\frac{d}{dx}log_{2p}(x) = \frac{1}{0.69314718056p}$$

and

$$\frac{d}{dx}log_{4p}(x) = \frac{1}{1.38629436112p}.$$

Let's do the math to see which one is larger. Starting off with the first equation, we divide the two and get 1.44269504089. With the second one, we have 0.72134752044. We can compare the two values and we get

1.44269504089 > 0.72134752044.

That means the slope of the first equation was higher.

What about natural logarithms? The natural log, like an ordinary log, requires you to multiply the base by *p*. Let's try solving this:

Solve for x.

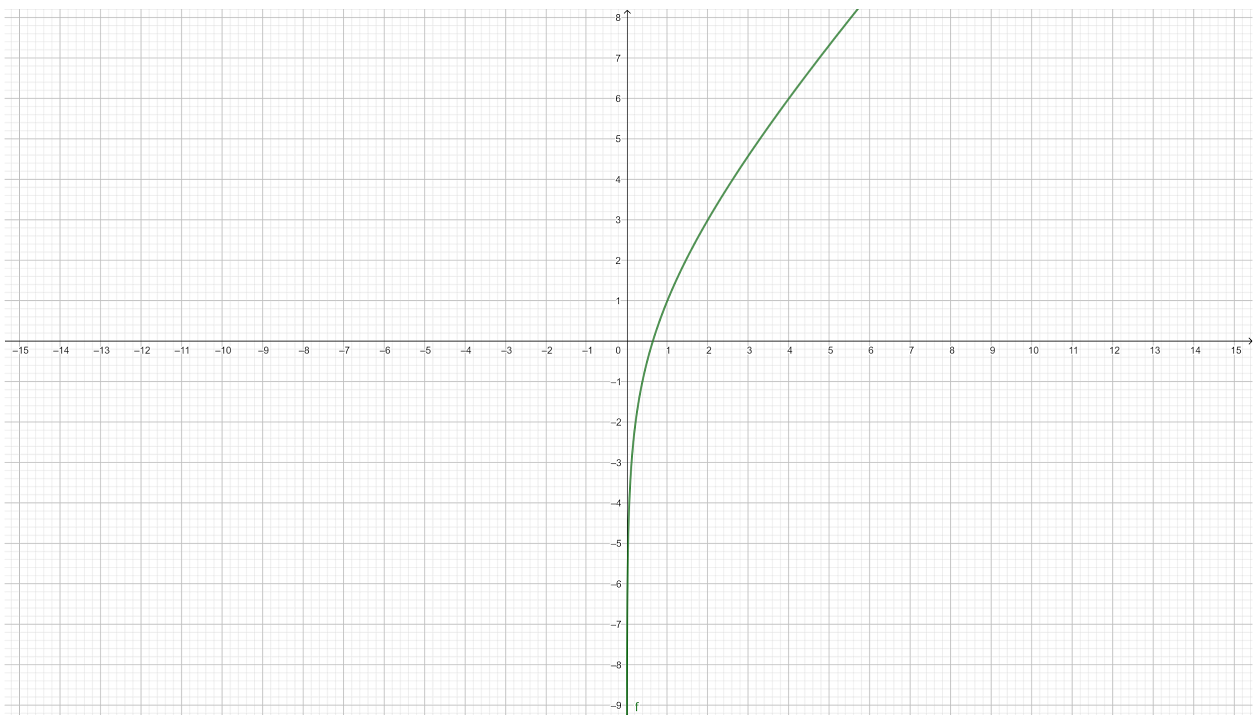
$$ln_p ep^x + log_{xp}16p = 6p$$

Let's see the natural logs of various powers of *e*.

The first power is e^1. That doesn't make sense, as the base 1 log of 16 is undefined. We would have a value of *p*, as the natural log of $e^1 = 1 + 0 = 1$. What about two? Let's try $\ln(ep^2) + \log_{2p}16p = 6p$. Starting with the first term. The natural log of $ep^2 = 2$. The base 2 log of $16 = 4$. $2p + 4p = 6p$. $x = 2$. Let's graph this. We have the natural log of the exponential function plus the base 2 logarithm of another function. This is an equation of the form

$f(x) = ln(e^x) + log_2(x)$.

Let's graph this as that.



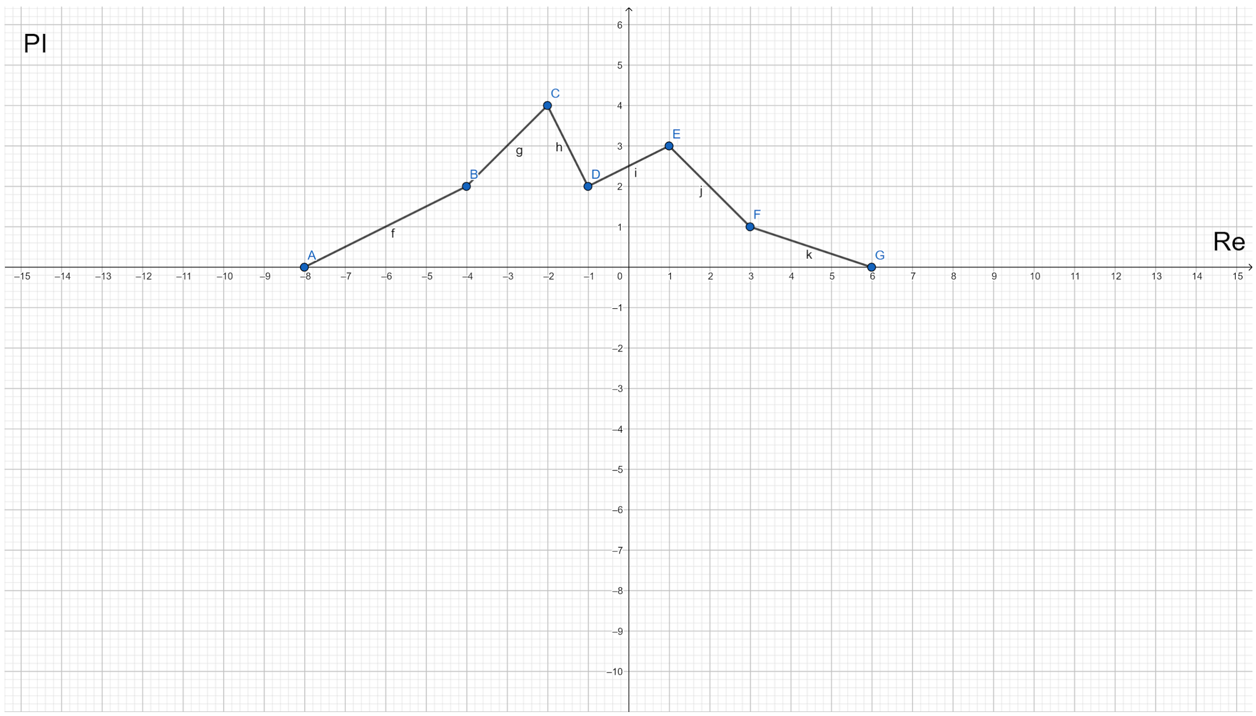
This looks strange. I don't know how to describe this shape.

Algebraic Mystery

Simon A. Pritchett

Identifying Piecewise Linear Functions
1 problem

Is this a piecewise linear function?



This is a piecewise linear function as it is made of line segments.

Solving for x
3 problems

1. $px + 3p = 7p$

You should know at this point that 4 * p = 4p. 4p + 3p = 7p. Therefore, x = 4.

2. $3px + 2p = 23p$

This is asking "3 times what multiple of *p* + 2*p* = 23p?" The closest multiple of 3 to 23 is 21. And 3 * 7 = 21. Let's try 7. We now have $3(7)p + 2p = 23p$. 3 * 7 = 21, and 21p + 2p = 23p. x = 7

3. $(57p + 37p) + 7p(x^2) = 941p$

This equation is much more complicated. Let's start by adding the first two terms. We now have $(94p) + 7p(x^2) = 941p$. We know that x must be something greater than 10, as $7(10^2) = 700$, and 700 + 94 = 794. Let's try 11.

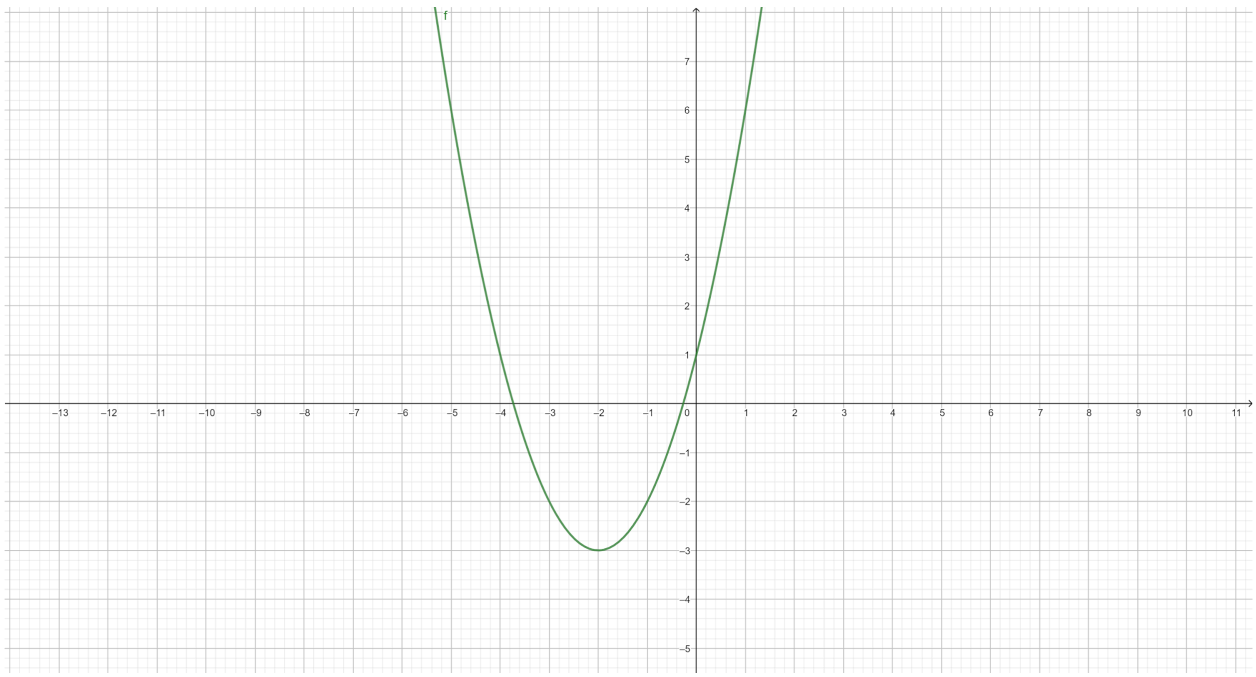
$7(11^2) = 7(121) = 847$. Add the *p* back, and we now have
$(94p) + 847p = 941p$**. This expression is true. Therefore, x = 11.**

Graphing Algebraic Equations

3 problems

1. $px^2 + 4x + 1$

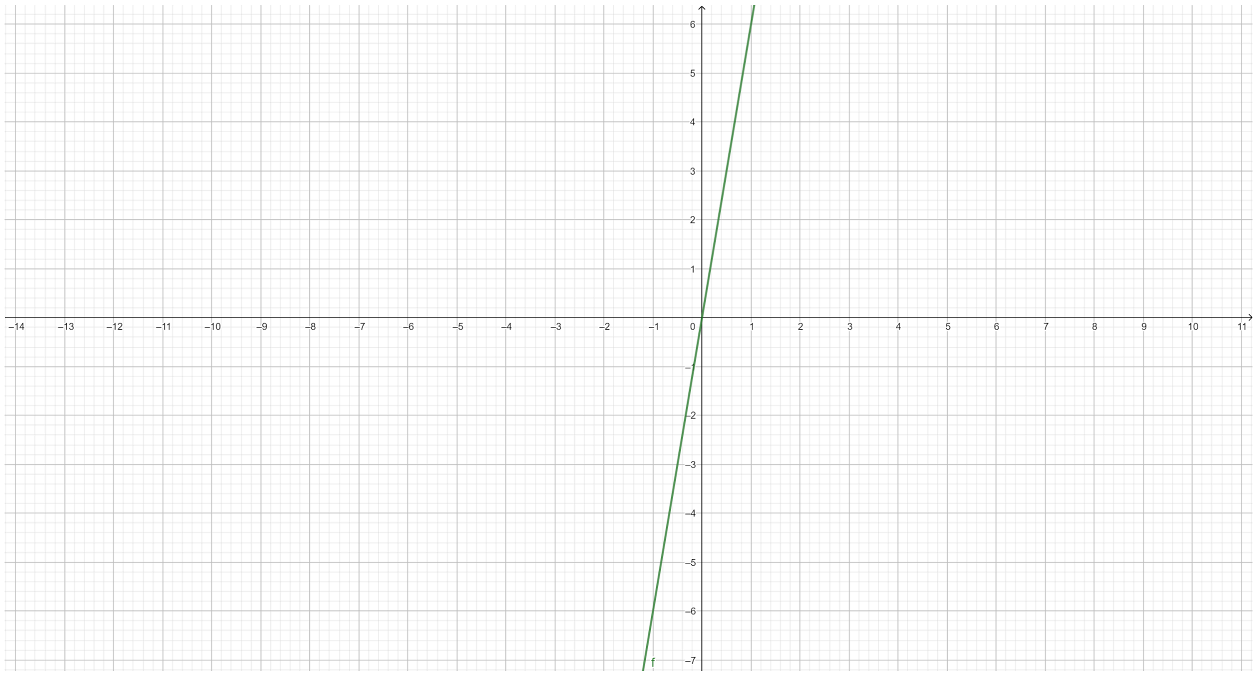
According to this form, this is a quadratic equation. This means the graph is a parabola. Let's see.



I was right.

2. $4x + px$

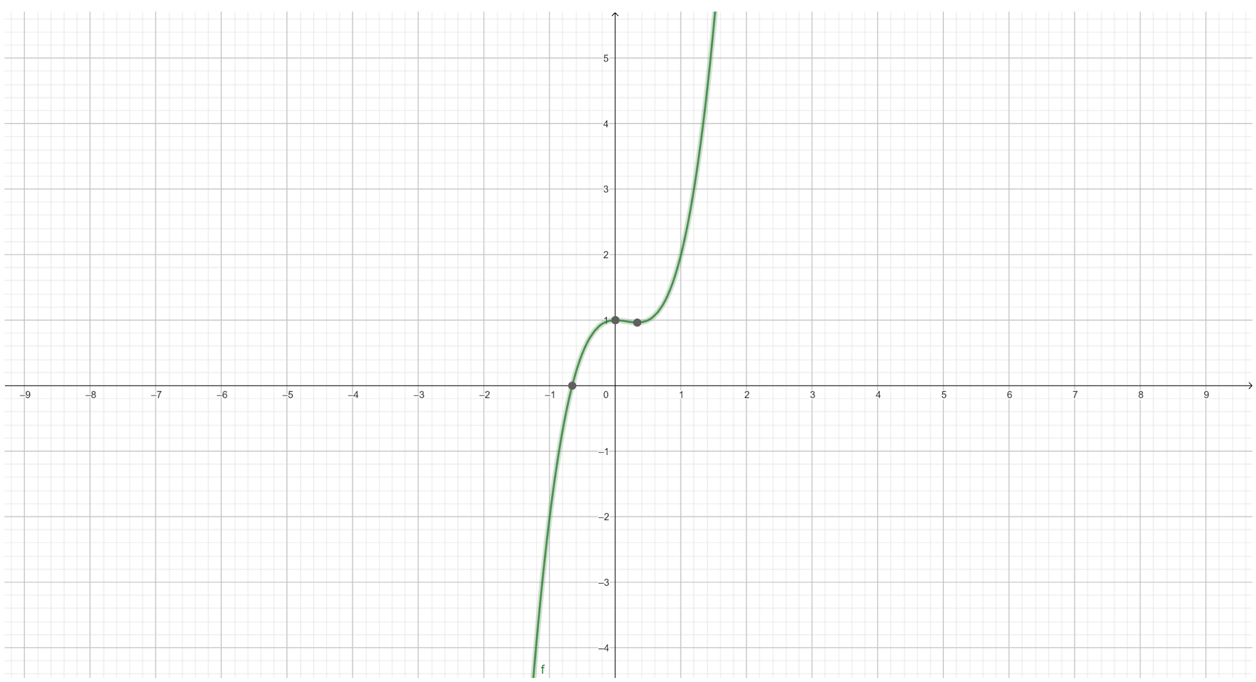
Oh no. This is the standard form of a… LINEAR EQUATION! I DON'T WANT TO SEE THE GR...



COVER MY EYES IMMEDIATELY PLEASE!

3. $2x^3 - px^2 + 1$

Phew. This is a cubic function. Time to graph:



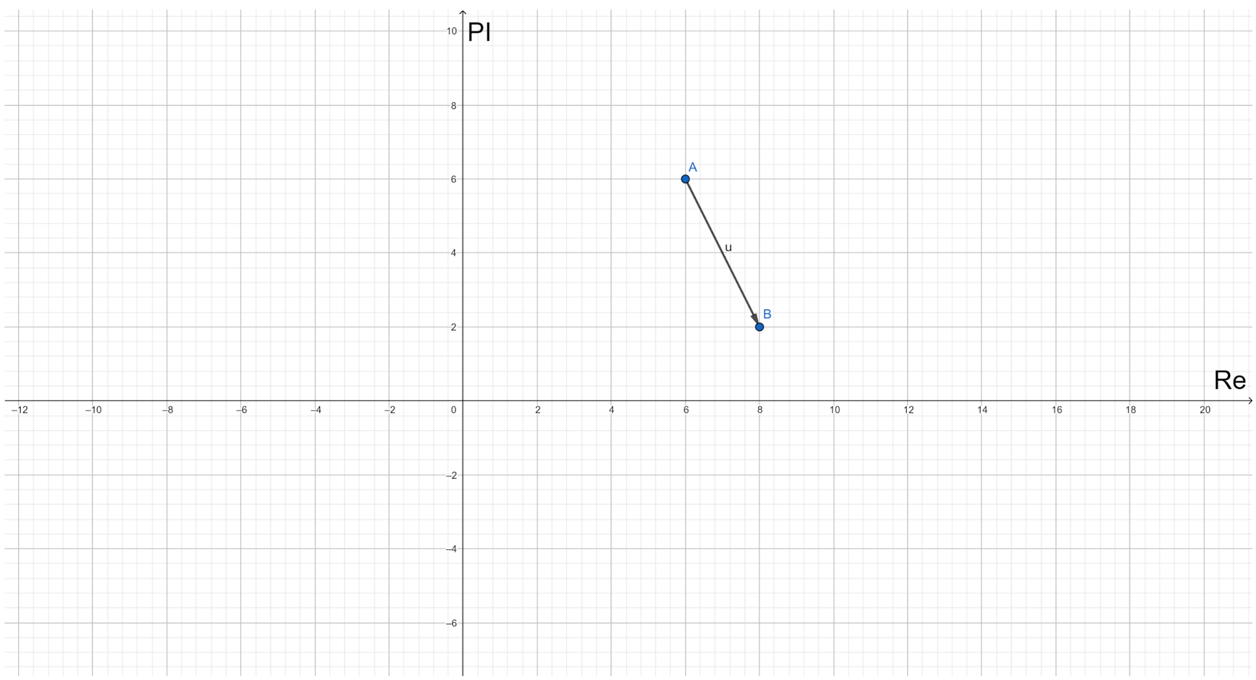
The cooler sine wave is now the lamer sine wave.

Arithmetic with Matrices involving *p*
1 problem

$$\begin{bmatrix} 2 & 3 \\ p & 6 \end{bmatrix} + \begin{bmatrix} 9 & 6 \\ 4 & p \end{bmatrix}.$$

Add each entry. The answer is $\begin{bmatrix} 11 & 9 \\ 5p & 7p \end{bmatrix}$**.**

Arithmetic with Vectors on the Pseudo-Complex Plane
1 problem

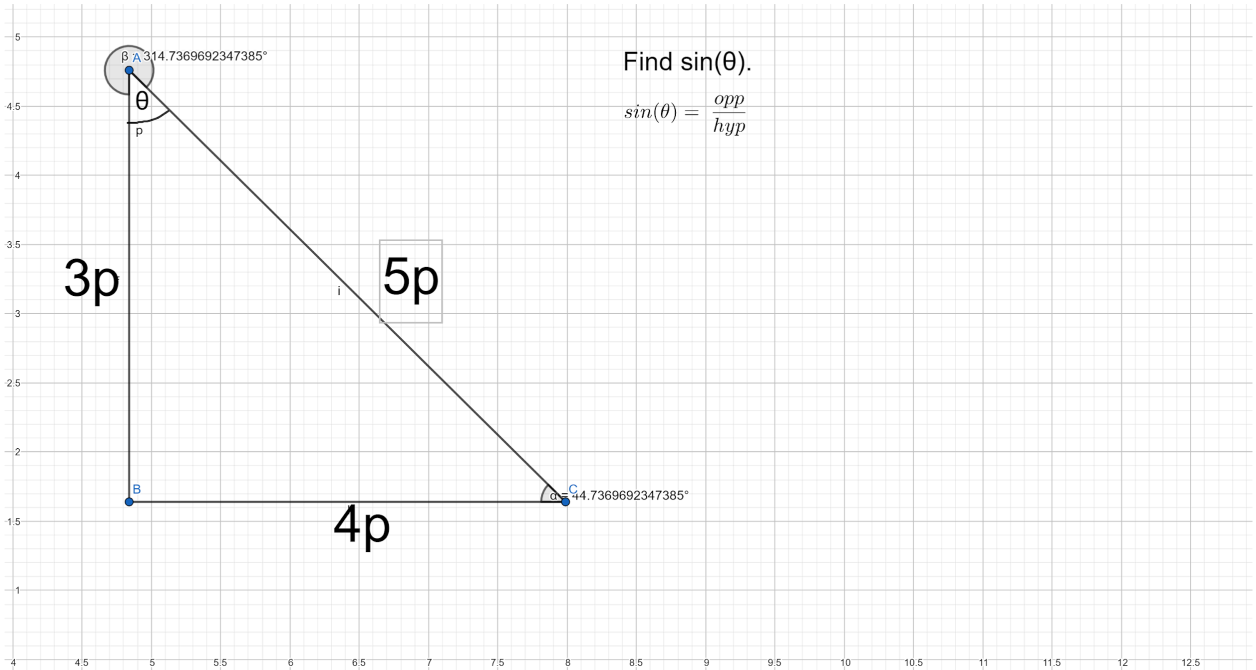


What are the x and y-components of this vector?

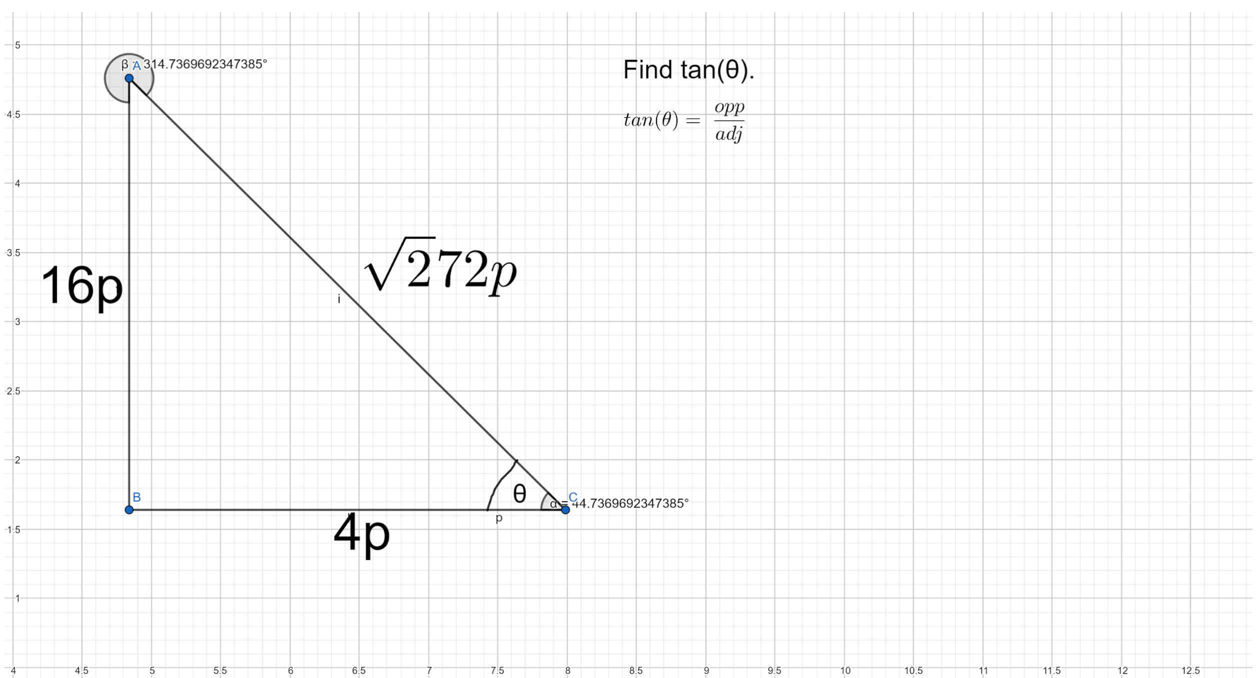
$\overrightarrow{AB}$ **= (2, -4)**

Working with the Trigonometric Functions

2 problems



Sine of θ using SOHCAHTOA is $\frac{4p}{5p}$**. We cannot simplify this.**



The opposite side to θ is 16p. The adjacent to θ is 4p. We have our answer of $\frac{16p}{4p}$ **, which can be simplified to** $4p$**.**

Solving Logarithmic Equations
2 problems

Solve for x.

1. $log_{xp}1024p = 5$

Remember this equation from Chapter 3? If you do, then this is x = 4.

Solve for x.

2. $log_{4p}xp + log_{xp}4096p = 5p$

Let's look at the factors of 4096. They are 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, and 4096. One of these is the cube root of 4096, which is 16. Let's try plugging that in. We now have $log_{4p}16p + log_{16p}4096p = 5p$**.** $log_{4p}16p$ **= 2p, and** $log_{16p}4096p$ **= 3p, and 2p + 3p = 5p. This means that x = 16.**

Natural Logarithms
1 problem

Solve for x.

$ln_pe^{xp} + log_{xp}25p = 7p$

Starting with 1. The natural log of e^1 = 1. But you see, the base 1 log of anything is undefined, so it doesn't work. Let's try e^2. But for this, you see that the answer is a whole number, not a rational or irrational decimal number. The base 2 log of 25 is an irrational number, We need to only look at the factors of 25. The next factor of 25 is 5. Let's try 5. The natural log of e^5 = 5.

Add that to the base 5 log of 25, which is 2. 5 + 2 = 7, and add the *p* back and you get 7p as your final answer. That means that x = 5.

The Verdict of the Case of *p*

Simon A. Pritchett

This case is coming to a close. The verdict is "GUILTY" of violating all the laws of mathematics. We covered basic arithmetic and algebra regarding pseudo-complex numbers. We also learned how to plot these numbers on the pseudo-complex plane. We also learned various rules applying to these mysterious numbers. We, most importantly, defined 1/0 to be equal to *p*, the pseudo-imaginary unit. Anyways, thanks for reading. And as always, stay peculiar. That is the en… wait, I think I forgot something.

That Fifth Constant

Simon A. Pritchett

There was a fifth constant I forgot to mention. This constant is called *A*. We haven't talked about 0^0 yet. First of all, let's get something out of the way. 0^{-1} = p, and $0^1 = 0$. This means that 0^0 must be a value between p and 0. We can write this as two inequalities where *A* is the supposed value of this equation.

$A \geq p$

And

$A \leq 0$

We must also remember that

$x^n = x^{n-1} \times x$

To calculate 0^0 we must apply this rule. x = 0 and n = 0. $0^{0-1} = 0^{-1}$ which is p. We multiply p by 0 and get s. This means that

A = s.

I guess this chapter was pointless.

OBJECTION! There is a contradiction in this statement. We said that A is greater than p and less than 0, however, s is less than p. We have actually proved that A cannot be s, more than proving that A = s. Let's try to calculate a more precise value. We only know that A is between p and 0. Writing these in the standard form of a pseudo-complex number gives us:

A is between 0 + p and 0 + s. Let's try 0.000001p as our value for A. If we take the square root of this, we get 0.001p. As the value of A approaches s, the square root of it approaches s also. Let's write this as a limit.

$\lim_{A \to s} \sqrt{A} = s$.

Let's return to standard form. 0.000001p = 0 + 0.000001p, which is extremely small. This indicates that A is an infinitesimally small quantity multiplied by p, but greater than s. However, A is greater than p. It's also less than zero. But p is greater than zero. There is still a chance that A is a negative real quantity plus a positive pseudo-imaginary quantity. That is extremely likely.

Even though what I said in the previous paragraph may be true, we often don't and shouldn't try to assign A a value, we should just accept A to be a constant with a value, no matter if it is known or not. This is like how we aren't trying to define i as a real quantity.

Just for fun, let's calculate the square roots of all the pseudo-complex constants we've covered. They are

p	**0^{-1}**
s	**0^{-1} - 0^{-1}**
b_a	**-|0^{-1}|**
b_m	**1/0^{-1}**
A	**0^{-1+1}**

First, the square root of p. We know that p = 1/0. We can use a radical rule for square roots of rational numbers. It goes like this:

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

We take the square root of both the numerator and the denominator. We get 1/0. The square root of p is p.

A similar thing applies to s. The square root of 0/0, gives us back.

What about b_m? Same thing. The square root of zero is zero, and b_m = 0, so the square root of b_m is itself.

When we look at b_a, things get more complicated. This constant is equal to -1/0. If we take the square root of both sides, we get i/0. That means the square root, for the first time, is not itself. It is i times p, or ip. I like to call this constant *Q*.

The square root of A is the one that takes the longest to compute. A is not a rational number. A is not even truly a pseudo-complex number, as it can't be expressed in the form of a + bp. It is a type of number called a hypo-complex number. The square root of A must be solved with a quadratic equation. The equation is

$x^2 = A$.

We can set this equation to be equal to zero by subtracting A from both sides.

$x^2 - A = 0$

We can use the quadratic formula and eventually it simplifies to

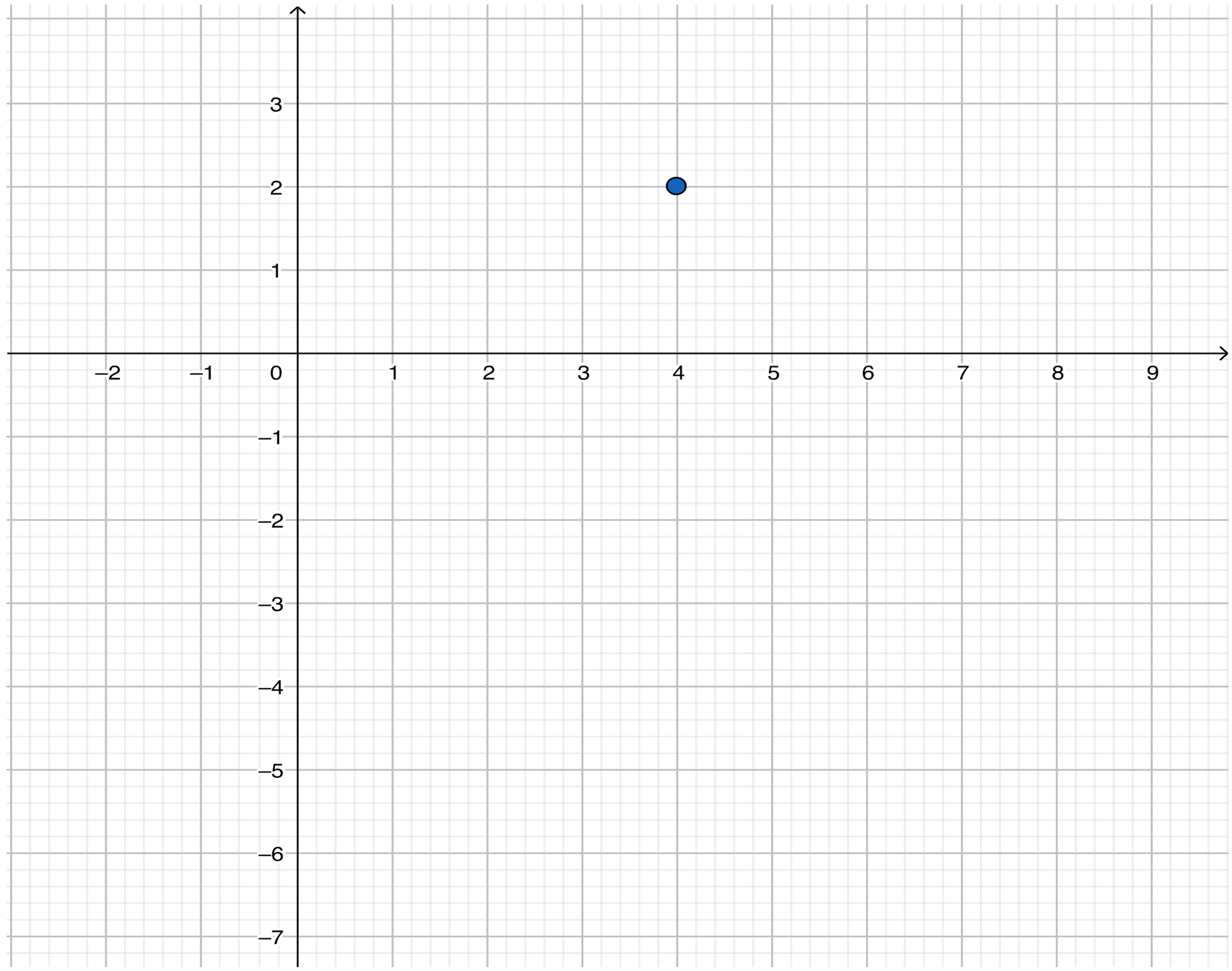
$\pm A$

Which is our square root.

What about the pseudo-complex plane? We cannot graph multiples of A on the pseudo-complex plane without adding a third axis. Either that, or removing one of the axes and replacing it with a hypo-complex axis. I would like a plane with four axes, one for real, one for imaginary, one for pseudo-imaginary, and one for hypo-imaginary, however, as I have said several times, neither my brain nor my computer would be able to handle that.

Let's try a two-axis one with real and hypo-imaginary components.

We will plot 4 + 2A.



Cool.

The same rules as p apply to arithmetic and algebra with these numbers.

There is one thing I must try, interdimensional operations.

Solve 3 + 4p + 7i + 13A using the direct calculation technique.

I honestly forgot how to do this myself, so I must check my previous notes.

I have finished, so let's try this.

Addition is associative, so let's group this.

(3 + 4p) + (7i + 13A)

Add the real part to the PI part. We now have 7p + (7i + 13A). Add the imaginary part to the HI part to get our final answer of 7p + 20A. Just for fun, let's use i-Substitution as well. We get 10i + 17A. As long as the coefficients of both results add to the same value, this does not violate what I call the fundamental theorem of pseudo-complex arithmetic, the rules that I mentioned in the arithmetic chapters regarding addition and subtraction. The second part of the theorem is the rules for multiplication and division that I covered.

Now, this is the true end. Thanks for reading. We analyzed pseudo-complex numbers, starting with p, followed with s, the b constants, and then A. See you later. Cheers.

More Properties of *p*

Simon A. Pritchett

Oh, you're still here. There's just one last thing, or a little more than one last thing, that I want to get off my chest. These numbers violate so many laws of mathematics, it sort of is crazy. First law to violate:

0x = 0.

The definition of a multiplicative inverse (I) of a number (x) is

Ix = 1.

As p is the multiplicative inverse of 0,

0p = 1.

Second law:

$1^x = 1$.

If we plug in p for x we get 1^p. We also know that the definition of exponentiation using e is

$a^b = e^{b \ln a}$.

Plug in p for b and 1 for a and get $e^{p \ln 1}$. The natural logarithm of 1 is 0, so plug in 0 for that and get e^{0p}. This is the problem. As p is the multiplicative inverse of 0, 0p = 1, so we get

$1^p = e$.

Next law:

$x^0 = 1 \Leftrightarrow x \neq 0$

Use the same property as last time. We get

$e^{0 \ln p}$

Now, what is ln(p)? To do this, we must thieve a trick from the complex numbers. It's called "polar form". The definition of a complex number in polar form is

$r(\cos \theta + i \sin \theta)$.

This comes from Euler's formula. The "exponential" definition of polar form is

$re^{i\theta}$.

Plug in p instead of i and get.

$re^{p\theta}$.

r is the absolute value of the number and θ is the angle. The angle of p is the same as the angle of i in the pseudo-complex plane and complex plane respectively. The angle of p and i are both π/2. The radius is similarly the same for p and i. It is 1. Simplify.

$e^{(\pi/2)p}$.

Polar form is another way to express complex and pseudo-complex numbers. Therefore

$e^{(\pi/2)p} = p$.

Take the natural logarithm of both sides of the equation.

$(\pi/2)p = \ln(p)$.

We did it!

Let's plug in this for ln(p).

$e^{0\ (\pi/2)p}$.

We can apply this identity. The proof of this identity is trivial and is left as an exercise to the reader.

$0np = n$

Apply this identity to the exponent and get.

$p^0 = e^{\pi/2}$.

Hold up. This violates this other law of exponents:

$a^b a^c = a^{b+c}$.

Plug in p for a and -1 and 1 for b and c respectively.

$p^{-1}p^1 = p^0$.

You should know that any number raised to the power of negative one is the multiplicative inverse. If you raise a number to the power of 1, you get the number itself. We get

$0p = p^0$

Plug in 1 for 0p and get.

$1 = p^0$.

Wait… can we prove that this is not true? Why, yes we can!

1. **Assume $p^0 = 1$.**
2. **Rewrite this expression in radical form.** $\sqrt[0]{1} = p$**.**
3. **Rewrite this with fractional exponents. $1^{1/0} = p$.**
4. **$1/0 = p$. $1^p = p$.**
5. **$1^p = e$. $e = p$.**
6. **This is a contradiction. Therefore, the original statement is wrong. Q.E.D.**

I can't think of anything else to write here, so for now, this is the end. Cheers.