Langenscheidt's Pocket Japanese Dictionary and the Onsager's solution

Anindya Kumar Biswas*

Department of Physics;

North-Eastern Hill University,

Mawkynroh-Umshing, Shillong-793022.

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Abstract

We study the entries of Langenscheidt's Pocket Japanese Dictionary written in a Romanized pronunciation system. We draw the natural logarithm of the number of entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We find that the entries underlie a magnetisation curve. The magnetisation curve i.e. the graph of the reduced magnetisation vs the reduced temperature is the exact Onsager solution of the two dimensional Ising model in the absence of external magnetic field.

* anindya@nehu.ac.in

I. INTRODUCTION

"Mera juta hai Japani,

shor pe lal topi,

phir bhi dil Hindusthani,.."...Raj Kapoor...

So we take a Japanese dictionary. This is Langenscheidt's Pocket Japanese Dictionary, [1]. This is written both in Japanese characters and in a Romanized pronunciation system-a modified version of the standard Hepburn. We follow the Romanized pronunciation system to go through the Japanese words. To introduce we note few Japanese words with their English meanings, from the Dictionary, [1]. asameshi means breakfast, ayu means sweetfish, bachi means drumstick, bīru means beer, boirā means boiler, bōkaru means vocalist, bui means buoy, buki means weapon, bubun means component, buttō means pagoda, chawan means bowl, chi means blood, chiku means district, dorama means drama, dōshi means comrade, dōkan suru means agree, fuchi means frame, fukai means deep, garō means art gallery, hamaguri means calm, jōshi means superior, kake means bet, karada means body, kao means face, kawaii means cute, kazan means volcano, kazu means count, kigō means symbol, kita means north, kona means powder, kono means this, kondo means this time, kusari means chain, matsu means pine, me means eye, mimi means ear, mondai means problem, moru means leak, muki means direction, naka means inside, nana means seven, nani means what, nishi means west, otto means husband, ranchi means lunch, renga means brick, renzu means lens, rōmaji means Roman script, sagan means sandstone, satō means sugar, shindai means berth(bunk), sora means sky, sōchi means device, sugata means image, suji means action, suki means plow, tanka means unit cost, tanpa means high-frequency, tera means temple, tōron means debate(discussion), tsuchi means earth, wan means bay, yagi means goat, yama means hill, yoko means side, so on and so forth.

We count all the entries one by one, beginning with each letter. The result is the table, tableI. To visualise we plot the number of entries against the respective letters in the dictionary sequence, [1] , in the adjoining figure, fig.1.

Looking for the Graphical Law in this dictionary, we proceed narrating the precedence. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found existence of a magnetisation

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
509	614	454	583	179	464	562	1386	562	512	3031	0	1109	725	595	223	0	505	2828	1498	355	0	160	0	691	247

TABLE I. Entries of Langenscheidt's Pocket Japanese Dictionary

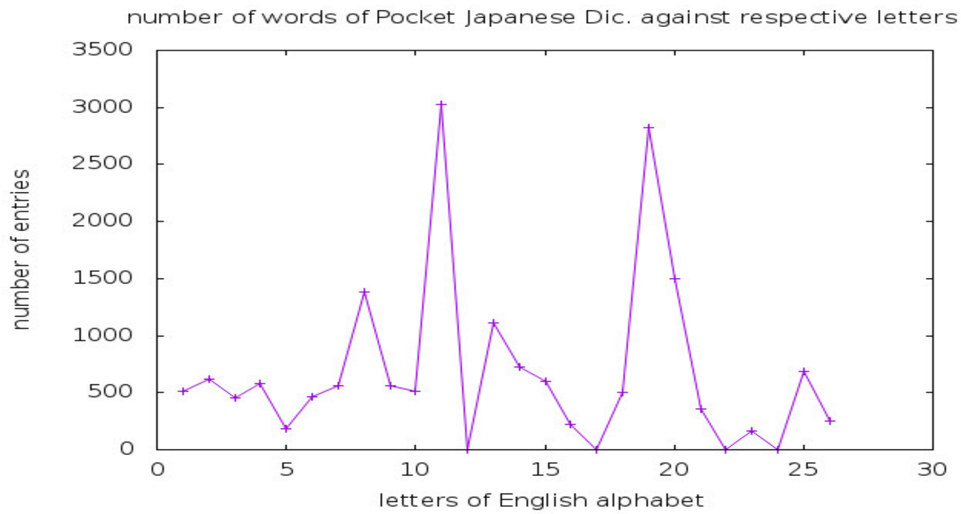

FIG. 1. The vertical axis is number of entries in the Langenscheidt's Pocket Japanese Dictionary,[1]. The horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

curve under each language. We have termed this phenomenon as the Graphical Law.

Then, we moved on to investigate into, [3], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of the graphical law behind the bengali language,[4] and the basque language[5]. This was pursued by finding of the graphical law behind the Romanian language, [6], five more disciplines of knowledge, [7], Onsager core of Abor-Miri, Mising languages,[8], Onsager Core of Romanised Bengali language,[9], the graphical law behind the Little Oxford English Dictionary, [10], the Oxford Dictionary of Social Work and Social Care, [11], the Visayan-English Dictionary, [12], Garo to English School Dictionary, [13], Mursi-English-Amharic Dictionary, [14] and Names of Minor Planets, [15], A Dictionary of Tibetan and English, [16], Khasi English Dictionary, [17], Turkmen-English Dictionary, [18], Websters Universal Spanish-English Dictionary, [19], A Dictionary of Modern Italian, [20], Langenscheidt's German-English Dictionary, [21], Essential Dutch dictionary by G. Quist and D. Strik, [22], Swahili-English dictionary by C. W. Rechenbach, [23], Larousse Dictionnaire De Poche for the French, [24], the Onsager's solution behind the Arabic, [25], the graphical law behind Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung, [26], the graphical law behind the NTC's Hebrew and English Dictionary by Arie Comey and Naomi Tsur, [27], the graphical law behind the Oxford Dictionary Of Media and Communication, [28], the graphical law behind the Oxford Dictionary Of Mathematics, Penguin Dictionary Of Mathematics, [29], the Onsager's solution behind the Arabic Second part, [30], the graphical law behind the Penguin Dictionary Of Sociology, [31], behind the Concise Oxford Dictionary Of Politics, [32], a Dictionary Of Critical Theory by Ian Buchanan, [33], the Penguin Dictionary Of Economics, [34], the Concise Gojri-English Dictionary by Dr. Rafeeq Anjum, [35], A Dictionary of the Kachin Language by Rev.O.Hanson, [36], A Dictionary Of World History by Edmund Wright, [37], Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool, [38], A Dictionary of Plant Sciences by Michael Allaby, [39], respectively. The graphical law was pursued more in Along the side of the Onsager's solution, the Ekagi language ,[40], Along the side of the Onsager's solution, the Ekagi language-Part Three, [41], Oxford Dictionary of Biology by Robert S. Hine and the Graphical law, [42], A Dictionary of the Mikir Language by G. D. Walker and the Graphical law, [43], A Dictionary of Zoology by Michael Allaby and the Graphical Law, [44], Dictionary of all Scriptures and Myths by G. A. Gaskell and the Graphical Law, [45], Dictionary of Culinary Terms by Philippe Pilibossian and the Graphical law, [46], A Greek and English Lexicon by H.G.Liddle et al simplified by Didier Fontaine and the Graphical law, [47], Learner's Mongol-English Dictionary and the Graphical law, [48], Complete Bulgarian-English Dictionary and the Graphical law, [49], A Dictionary of Sindhi Literature by Dr. Motilal Jotwani and the Graphical Law, [50], Penguin Dictionary of Physics, the Fourth Edition, by John Cullerne, and the Graphical law, [51], Oxford Dictionary of Chemistry, the seventh edition and the Graphical Law, [52], A Burmese-English Dictionary, Part I-Part V, by J. A. Stewart and C. W. Dunn et al, head words and the Graphical Law, [53], The Graphical Law behind the head words of Dictionary Kannada and English written by W. Reeve, revised, corrected and enlarged by Daniel Sanderson, [54], Sanchayita and the Graphical Law, [55], Samsad Bangla Abhidan and The Graphical Law, [56], Bangiya Sabdakosh and The Graphical Law, [57], Samsad Bengali-English Dictionary and The Graphical Law, [58], Rudyard Kipling's Verse and the Graphical Law, [59], W. B. Yeats, The Poems and the Graphical Law, [60], The Penguin Encyclopedia of Places by W. G. Moore and the Graphical law, [61], The Poems of Tennyson and the Graphical Law, [62], Khasi-Jaintia Jaids(Surnames) and the Graphical law, [63], Age, Amplitude of accommodation and the Graphical law, [64], Dictionary of Ayurveda by Dr. Ravindra Sharma and the Graphical law, [65], The Practical Sanskrit-English Dictionary by Vaman Shivram Apte and The Graphical Law, [66], The Langenscheidt's Pocket Russian Dictionary and The Graphical Law, [67], The Scholar Dictionary Portuguese and The Graphical Law, [68], respectively.

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of entries of the Langenscheidt's Pocket Japanese Dictionary, [1]. The section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

II. THE GRAPHICAL LAW ANALYSIS

For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by f and the respective rank, [69], denoted by k. k is a positive integer starting from one. Moreover, the minimum non-zero number of entries is one hundred sixty. Hence, we attach a limiting entry number one. The limiting rank is maximum rank plus one, here it is twenty two. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table,II, and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.3. We then ignore the letter with the highest number of entries, tabulate in the adjoining table,II,and redo the plot, normalising the lnfs with lnf_{n-max}, and starting from $k=2$ in the figure fig.4. Normalising the lnfs with lnf_{2n-max}, we tabulate in the adjoining table,II, and starting from $k=3$ we draw in the figure fig.5. Normalising the lnfs with lnf_{3n-max} we record in the adjoining table,II, and plot starting from $k=4$ in the figure fig.6. In this way we obtain up to the figure fig.7.

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{nmax}	lnf/lnf_{2nmax}	lnf/lnf_{3nmax}	lnf/lnf_{4nmax}	lnf/lnf_{5nmax}
1	0	0	3031	8.017	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.223	2828	7.947	0.991	1	Blank	Blank	Blank	Blank
3	1.10	0.356	1498	7.312	0.912	0.920	1	Blank	Blank	Blank
4	1.39	0.450	1386	7.234	0.902	0.910	0.989	1	Blank	Blank
5	1.61	0.521	1109	7.011	0.875	0.882	0.959	0.969	1	Blank
6	1.79	0.579	725	6.586	0.822	0.829	0.901	0.910	0.939	1
7	1.95	0.631	691	6.538	0.816	0.823	0.894	0.904	0.933	0.993
8	2.08	0.673	614	6.420	0.801	0.808	0.878	0.887	0.916	0.975
9	2.20	0.712	595	6.389	0.797	0.804	0.874	0.883	0.911	0.970
10	2.30	0.744	583	6.368	0.794	0.801	0.871	0.880	0.908	0.967
11	2.40	0.777	562	6.332	0.790	0.797	0.866	0.875	0.903	0.961
12	2.48	0.803	512	6.238	0.778	0.785	0.853	0.862	0.890	0.947
13	2.56	0.828	509	6.232	0.777	0.784	0.852	0.861	0.889	0.946
14	2.64	0.854	505	6.225	0.776	0.783	0.851	0.861	0.888	0.945
15	2.71	0.877	464	6.140	0.766	0.773	0.840	0.849	0.876	0.932
16	2.77	0.896	454	6.118	0.763	0.770	0.837	0.846	0.873	0.929
17	2.83	0.916	355	5.872	0.732	0.739	0.803	0.812	0.838	0.892
18	2.89	0.935	247	5.509	0.687	0.693	0.753	0.762	0.786	0.836
19	2.94	0.951	223	5.407	0.674	0.680	0.739	0.747	0.771	0.821
20	3.00	0.971	179	5.187	0.647	0.653	0.709	0.717	0.740	0.788
21	3.04	0.984	160	5.075	0.633	0.639	0.694	0.702	0.724	0.771
22	3.09	1	1	0	0	0	0	0	0	0

TABLE II. Entries of the Langenscheidt's Pocket Japanese dictionary: ranking, natural logarithms, normalisations

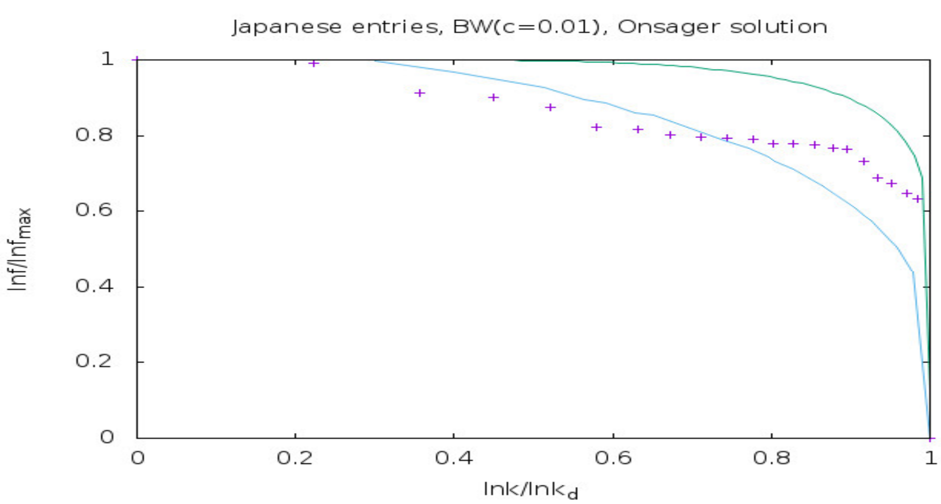

FIG. 2. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Langenscheidt's Pocket Japanese Dictionary, with the fit curve being the Bragg-Williams curve, BW(c=0.01), in the presence of external magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$. The uppermost curve is the Onsager solution.

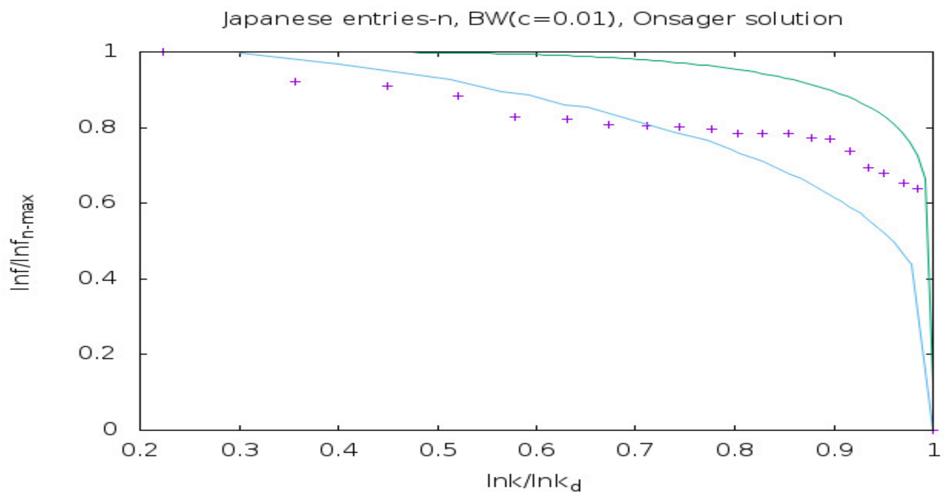

FIG. 3. The vertical axis is $\frac{lnf}{lnf_{n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Langenscheidt's Pocket Japanese Dictionary, with the fit curve being the Bragg-Williams curve, BW(c=0.01), in the presence of external magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$. The uppermost curve is the Onsager solution.

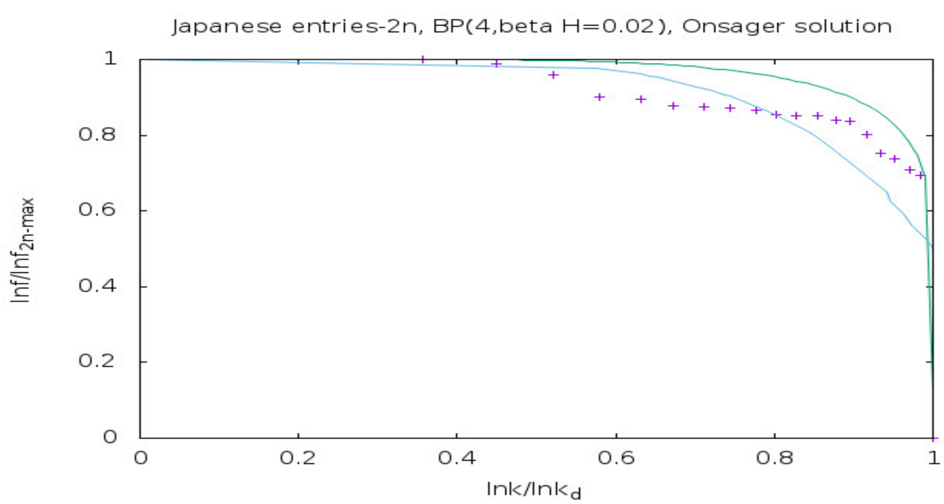

FIG. 4. The vertical axis is $\frac{lnf}{lnf_{2n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Langenscheidt's Pocket Japanese Dictionary, with the fit curve, BP(4,$\beta H = 0.02$), being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.01 or, $\beta H = 0.02$. The uppermost curve is the Onsager solution.

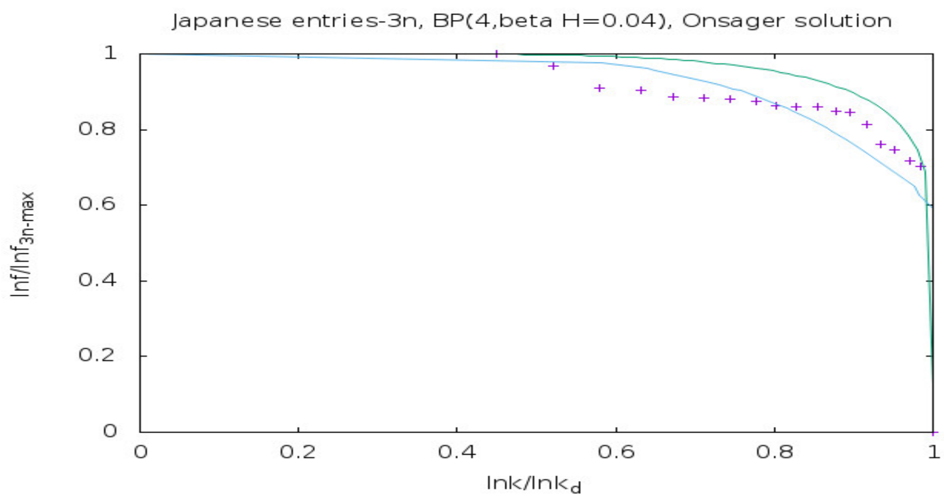

FIG. 5. The vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Langenscheidt's Pocket Japanese Dictionary, with the fit curve, BP(4,$\beta H = 0.04$), being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.02 or, $\beta H = 0.04$. The uppermost curve is the Onsager solution.

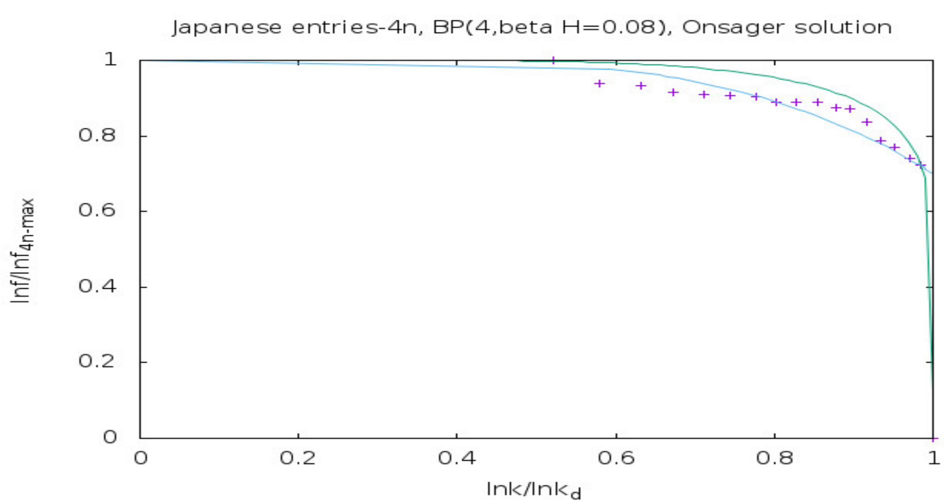

FIG. 6. The vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Langenscheidt's Pocket Japanese Dictionary, with the fit curve, BP(4,$\beta H = 0.08$), being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.04 or, $\beta H = 0.08$. The uppermost curve is the Onsager solution.

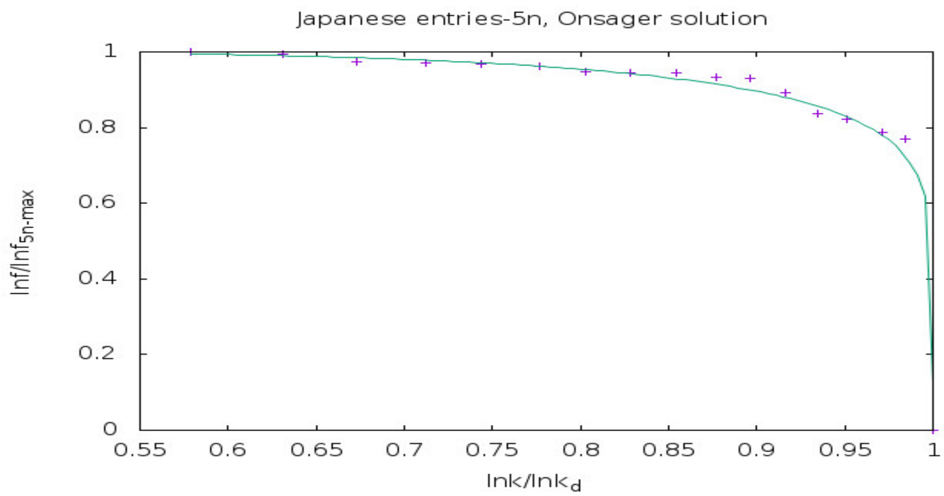

FIG. 7. The vertical axis is $\frac{lnf}{lnf_{5n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Langenscheidt's Pocket Japanese Dictionary, with the fit curve being the Onsager solution.

A. conclusion

From the figures (fig.2-fig.7), we observe that the entries of the Langenscheidt's Pocket Japanese Dictionary,[1], underlies the Onsager solution.
Moreover, the associated correspondence is,

$$\frac{lnf}{lnf_{5n-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [79].

III. APENDIX: MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, $\mu N L$, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[70], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [71], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [72]. In the Bragg-Williams approximation,[73], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L+c}{\frac{T}{T_c}} \tag{1}$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [74]. $\frac{T}{T_c}$ is referred to as reduced temperature.
Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [71]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [70],[71],[72],[73],[74], due to Bethe-Peierls, [75], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{2}$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe

BW	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

TABLE III. Reduced magnetisation vs reduced temperature data s for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours.

data s generated from the equation(1) and the equation(2) in the table, III, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.8. Empty spaces in the table, III, mean corresponding point pairs were not used for plotting a line.

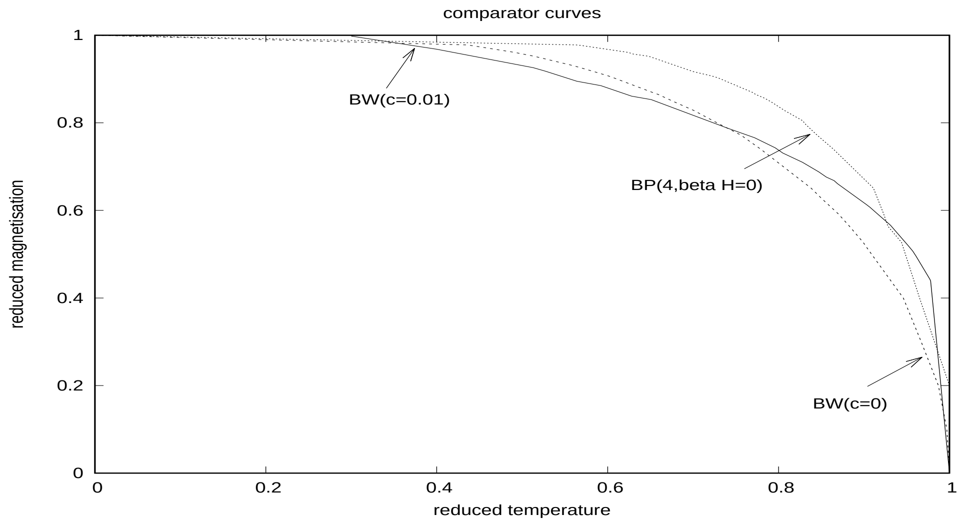

FIG. 8. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

C. Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field

In the Bethe-Peierls approximation scheme , [75], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{3}$$

Derivation of this formula ala [75] is given in the appendix of [7].

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{4}$$

In the following, we describe datas in the table, IV, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that

$\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.9. Empty spaces in the table, IV, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE IV. Bethe-Peierls approx. in presence of little external magnetic fields

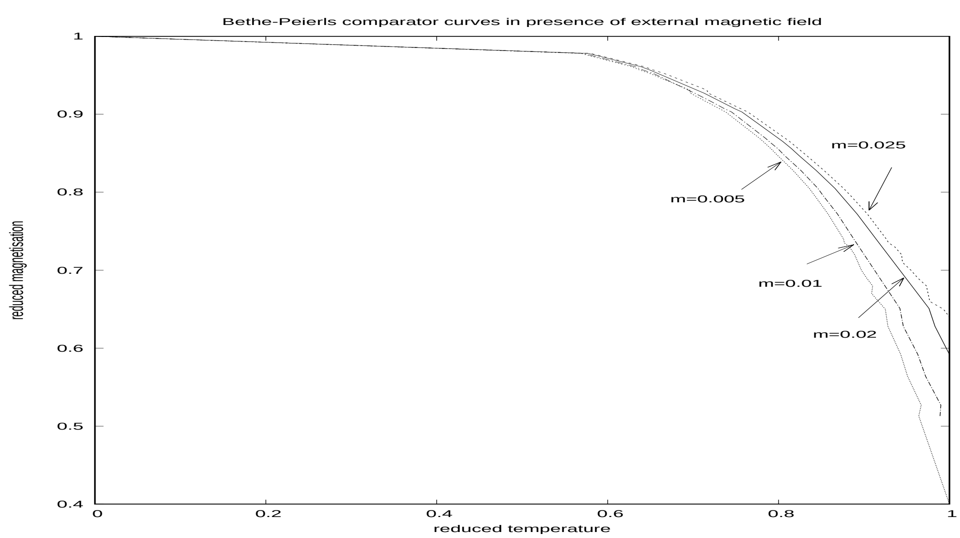

FIG. 9. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

D. Onsager solution

At a temperature T, below a certain temperature called phase transition temperature, T_c, for the two dimensional Ising model in the absence of external magnetic field i.e. for H equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [76], [77], [78], [75],

$$\frac{M}{M_{max}} = [1 - (sinh\frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

Graphically, the Onsager solution appears as in fig.2.

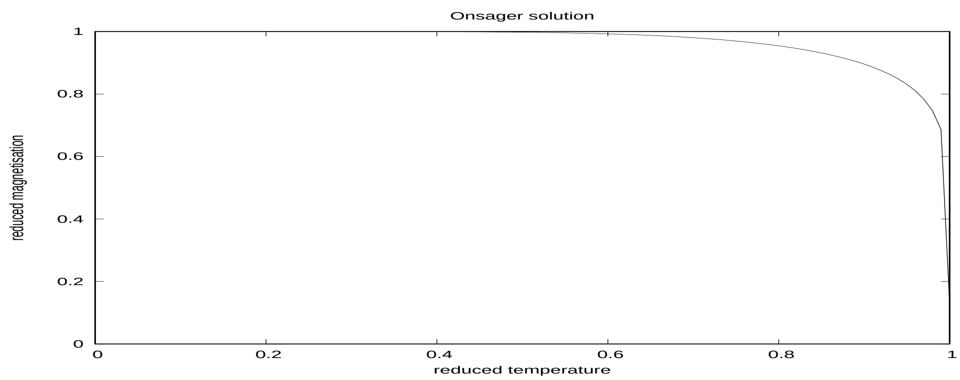

FIG. 10. Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in the absence of external magnetic field

IV. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper.

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