Dark Matter versus Dark Energy in Clusters using the Dark Matter by Gravitation theory

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1. ABSTRACT

This paper develops the Dark Matter by Gravitation theory, DMbG theory hereafter, in clusters of galaxies.
I have been developing this theory in galaxies since 2013 and I have published more than 20 papers most of them using rotation curves of galaxies, especially the ones belonging to M31 and Milky Way. So to understand this paper is compulsory to consult the paper *A DM theory by gravitation for galaxies and clusters-V2.* Vixra:2312.0002 where it is fully developed the DMbG theory.
An important results got by DMbG theory is that total mass associated to a galactic halo up to a specific radius depend on the square root of radius without limit. Apparently, this result would be absurd because of divergence of the total mass. However it is the Dark energy the responsible to counterbalance the DM, see the chapter 5.Also it is defined the zero gravity radius as the spherical space needed by the dark energy to counter balanced the total mass (baryonic and DM).
In the framework of DMbG it is possible to calculate such radius depending on virial mass and the amount of total mass enclosed into the sphere with zero gravity radius, being proportional to virial mass with a numerical factor approximated by 2.6657. As the virial mass is a value able to be measured into a cluster, these formulas are highly valuables.
In order to illustrate all the important formulas got in the paper, it has been chosen a sample of cluster of galaxies, whose virial radius and masses have been published recently. Namely calculus has been made for Virgo and Coma clusters.
But the most important result found in this work, see the last chapter, is the ratio zero gravity radius versus virial radius which is equal to the fraction of 100 divided by the universal fraction of DE, both of them to the power 0.4. This ratio is 7.277. This ratio is universal as it does not depend of virial mass or any other local parameter. For example if Virgo virial radius is 1.7 Mpc then its zero gravity radius will be 12.4 Mpc and if Coma virial radius is 2.8 then its zero gravity radius will be 20.4Mpc. This ratio is quite easy to check experimentally as it is related to distances between neighbour clusters and the virial radius associated to the cluster.
Consequently this result is a crucial test in order to check the rightness of the DMbG theory.

2. INTRODUCTION

The basis of this paper are developed in [1] Abarca, M. 2023, so it is highly recommended to read it to understand the meaning of this paper. The dark matter by gravitation theory, DMbG theory hereafter, is an original theory developed since 2013 through more than 20 papers, although in [1] Abarca, M. 2023 is published the best version as physical as mathematically. Therefore is not possible to understand this paper if reader have not at least a general knowledge about the DMbG theory.

In fact in [1] Abarca, M. 2023, the chapters 16 and 17 are dedicated to cluster where it is introduced the formula for R_{ZG} and some others to extend the theory from galaxies to clusters. The newness of the important results got in this paper are due to the possibility to approximate the virial radius to R_{200} and the virial mass to M_{200} , the chapter 3 is dedicated to explore this approximation using recent data published for some important clusters such as Virgo, Coma or some others.

The most important result of this work is got in the last chapter. This is the formula $\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5}$ =7.277

The importance of this formula is its universality as it is general for any cluster but it does not depend on virial mass or any other local parameter.

Reader knows the virial radius and virial mass concepts at cluster scale, but perhaps does not know the zero gravity radius concept, which is introduced and developed in chapter 5. The R_{ZG} is related to distances between clusters, because each cluster has a sphere with radius R_{ZG} where the gravitational field associated to the cluster dominates. For example if virial radius of Virgo is 1,7 Mpc then Virgo R_{ZG} is 12.4 Mpc, so it is not possible to have another cluster of galaxies in dynamic equilibrium inside this sphere, by the contrary inside this sphere there are galaxies which are falling towards the Virgo cluster.

So it is possible to state that thanks to the approximation of virial radius and virial mass for R_{200} and M_{200} it has been possible to get some impressive general results for clusters in the framework of Dark Matter by Gravitation theory.

3. VIRIAL MASS AND VIRIAL RADIUS IN CLUSTER OF GALAXIES

As reader knows it is a good estimation about virial radius and virial mass for cluster of galaxies to consider Rvir = R_{200} and Mvir= M_{200}. Where R_{200} is the radius of a sphere whose mean density is 200 times bigger than the critic density of Universe $\rho_C = \frac{3H^2}{8\pi G}$ and M_{200} will be the total mass enclosed by the radius R_{200}.

In this epigraph will be shown some data published by prestigious researchers that confirm this approximation.

It is right to get the following relation between both concepts $R_{VIR}^3 \approx R_{200}^3 = \frac{G \cdot M_{200}}{100 \cdot H^2}$ or $M_{VIR} \approx M_{200} = \frac{100 H^2 R_{200}^3}{G}$

The checking process will begin with the bigger cluster in the Local Universe. The graph below comes from [7] Seong –A Oh.2023.At the foot notes, they inform that virial radius is 2.8 Mpc, so using the above formula it is got Mvirial = 2.5E15 Msun, that match with mass published.

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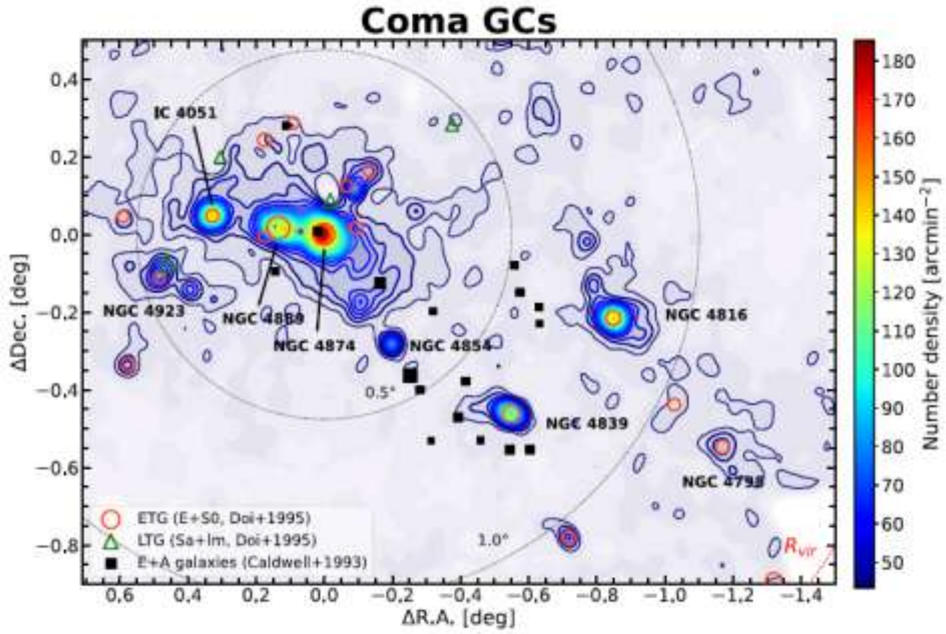

Figure 3. Spatial number density contour map of GCs in the Coma field including NGC 4839 and NGC 4816 (see S. Oh et al. 2023, in preparation, for details). Dotted line circles represent $R = 0°.5$, $1°.0$, and $R_{vir}(=2.8$ Mpc) from NGC 4874 at the Coma center. Red circles and green triangles mark early-type galaxy members, and late-type galaxy members (Doi et al. 1995). Black boxes mark E+A galaxies (Caldwell et al. 1993). The contour levels denote $2\sigma_{bg}$ and higher with an interval of $1\sigma_{bg}$ where σ_{bg} denotes the background fluctuation. The contour maps were smoothed using a Gaussian filter with $\sigma_G = 1'$. The color bar represents the GC number density.

Coma Cluster parameters according [7] Seong –A Oh.et al. 2023

Parameter	Main Cluster
Heliocentric galaxy velocity, v_h	7167 km s^{-1}
Heliocentric group velocity, v_h	6853 km s^{-1}
Velocity dispersion, σ_v	1082 km s^{-1}
Virial mass (dynamics),[b] M_{vir}	$2.7 \times 10^{15} M_\odot$
Weak-lensing mass,[c] M_{WL}	$1.2 \times 10^{15} M_\odot$

Data from Dynamic method
M_{VIR} = 2.7E15 Msun and R_{VIR} = 2.8 Mpc

Using in the approximation formula 2.8 Mpc as R_{200} is got M_{200} =2.5E15 Msun whose relative difference versus M_{VIR} is 7.4%

The author [7] Seong, shows that the formula to calculate Virial mass is M_{VIR}= 1.5E6.$h^{-1}\cdot\sigma_V$ Msun where σ_V =1082 km/s So the formula gives the mass in Msun units on condition that velocity dispersion σ_V units are km/s and h = 0.7

3.1 CHECKING THE VIRIAL MASS APROXIMATION ON A SAMPLE OF CLUSTERS AND GROUP OF G.

Data [4] R.Ragusa et al.2022				
Group of galaxies G. Or Clusters C.	Virial Radius	Virial Mass	Mass calculated	Relative diff for M
Name	Mpc	10^{13} Msun	1E13Msun	%
Antlia C.	1,28	26,3	2,39E+01	-9,21E+00
NGC596/584 G.	0,5	1,55	1,42E+00	-8,18E+00
NGC 3268 G.	0,9	8,99	8,30E+00	-7,67E+00
NGC 4365Virgo SubG.	0,32	0,4	3,73E-01	-6,73E+00
NGC 4636 Virgo SubG.	0,63	3,02	2,85E+00	-5,73E+00
NGC 4697Virgo Sub G.	1,29	26,9	2,44E+01	-9,14E+00
NGC 5846 G.	1,1	16,6	1,52E+01	-8,71E+00
NGC 6868 G.	0,6	2,69	2,46E+00	-8,57E+00

Data beside in green have been taken from [4] R.Ragusa et al. 2022 and using the formula $M_{200} = \frac{100H^2R_{200}^3}{G}$ it is calculated its mass associated for each radius. The yellow column shows the relative difference for masses, always under 10 %. The mass calculated are lower than mass published. With these examples it is shown that the consideration of R_{200} and M_{200} as virial radius and virial mass is an acceptable approximation for a wide range of celestial bodies, group of galaxies or clusters.

As the Virgo cluster is the nearest between the big clusters it is crucial to check the approximation for virial mass and radius with its data.

According [12] Karachentsev I.D. et al. 2014.In page 5 it is shown that Rvir = 1.8 Mpc and Mvir = 7E14 Msun. And using the approximate formula Mvir= M_{200} it is got Mvir = 6.64E14 Msun which is quite close to published value.

According [15] Olga Kashibadze, I. Karachentsev 2020, see pag 9, Rg=Rvir= 1.7 Mpc and Mvir= (6.3±0.9)E14 Msun. Using formula for M_{200} for 1.7 Mpc it is got 5.59E14 Msun which match with mass published.

In table below are summarized the results for the two most prominent cluster of galaxies.

Cluster of galaxies	Virial Radius	Virial mass	Calculated M_{200}	Mass Relative diff.
	Mpc	x 10^{14} Msun	x 10^{14} Msun	%
Virgo [13]Kashibadze 2020	1.7	6.3±0.9	5.59	11
Coma [7] Seong-A. 2023	2.8	27	25	7.4

In conclusion R_{200} and M_{200} are a very good estimation for Virial radius and Virial mass for galaxies, group of galaxies and cluster of galaxies, when they are in dynamical equilibrium, as it is well known by the astrophysicist researchers.

4. VIRIAL THEOREM AS A METHOD TO CALCULATE THE DIRECT MASS IN CLUSTERS

This chapter is based on the 16 chapter in [1]Abarca,M.2023., so there is a long way to get the formula for masses by the parameter a^2 by the formula called Direct mass.

In chapter 9, epigraph 9.8 of paper [1]Abarca,M.2023was demonstrated that direct formula $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ is the most suitable formula to calculate the total mass depending on radius in the galactic halo region.

The units of parameter a^2 are $m^{5/2}/s^2$

4.1 PARAMETER a^2 FORMULA DEPENDING ON VIRIAL RADIUS AND VIRIAL MASS

Due to the fact that the Direct mass formula has one parameter only, is enough to know the mass associated to a specific radius to be able to calculate parameter a^2. That is the situation when it is known the virial mass and the virial radius for a cluster of galaxies.

This formula is only a way to estimate parameter a^2 because outside the virial radius always there will be a fraction of the galaxies belonging to cluster. Anyway, this method may estimate a lower bound of parameter a^2 associated to clusters.

The Virial theorem states that $M_{VIRIAL}(<r) = M_{DYNAMICAL}(<r) = \frac{V^2 \cdot r}{G}$ is a formula right for a cluster of galaxy on condition that velocity and radius are calculated for galaxies in dynamical equilibrium.
If it is considered that the virial radius is the border of halo cluster where galaxies are in dynamical equilibrium and at the same time is negligible the amount of Baryonic matter outside the sphere with such radius, then it is possible to apply the formula of $M_{DIRECT}(< R_{VIRIAL})$. Then by equation of both formulas will be possible to clear up a^2.

$M_{VIRIAL}(< R_{VIRIAL}) = \frac{a^2 \cdot \sqrt{R_{VIRIAL}}}{G}$ so $a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$, this formula will be called parameter a^2(M_{VIR}, R_{VIR}) because depend on both measures.

4.2 PARAMETER a^2 FORMULA DEPENDING ON VIRIAL MASS ONLY

Using the formula got in previous paragraph $R_{VIR}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ it is right to get parameter a^2 depending on M_{VIR} only $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$. This formula will be called parameter a^2 (M_{VIR}) as depend on M_{VIR} only.

With the virial data for some important clusters such as Virgo or Coma cluster will be calculated its parameter $\mathbf{a^2}$ with the formula $a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$ and with the formula $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$.

The last formula is an approximation of the previous formula as it is supposed that $R_{VIR} = R_{200}$. Below are calculated both formulas and fortunately its relative difference is negligible.

Cluster	Virial Radius	Virial mass	Parameter a^2(M_{VIR}, R_{VIR})	Parameter a^2 (M_{VIR})	Relativ diff.
	Mpc	$\cdot 10^{14}$ Msun	I.S. units $m^{5/2}/s^2$	I.S. units $m^{5/2}/s^2$	%
Virgo	1.7	6.3±0.9	3.6527E23	3.581E23	2
Coma	2.8	27	1.2198E24	1.2042E24	1.3

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh,2023

Notice how close are both results for parameter a^2 especially when relative differences for masses are 11% and 7.4%

5. DARK MATTER IS COUNTER BALANCED BY DARK ENERGY AT ZERO GRAVITY RADIUS

This chapter is based on the 17 chapter in [1]Abarca,M.2023

The basic concepts about DE on the current cosmology can be studied in [9] Chernin,A.D.

According [11] Biswajit Deb. Plank satellite data (2018) give a new updated, Hubble constant, H = 67.4$\pm$ 0.5 km/s/Mpc and a new $\Omega_{DE} = 0.6889 \pm 0.0056$. However currently there is a tension regarding Hubble constant as there are published by prestigious researchers others measures for H bigger than 70 Km/s/Mpc. In this paper will be used H= 70 Km/s/Mpc and $\Omega_{DE} = 0.7$ as the fraction of Universal density of DE.

5.1 ZERO GRAVITY RADIUS DEPENDING ON PARAMETER a^2 FORMULA

According [9] Chernin,A.D. in the current cosmologic model ΛCDM , dark energy has an effect equivalent to antigravity i.e. the mass associated to dark energy is negative and the dark energy have a constant density for all the Universe equal to $\varphi_{DE} = \varphi_C \bullet \Omega_{DE} = -6.444 \cdot 10^{-27} kg/m^3$ being $\Omega_{DE} = 0.7$ and $\rho_C = \frac{3H^2}{8\pi G}$ = 9.2E-27 kg/m^3 the critic density of the Universe.

As DE density is constant, the total DE mass is proportional to Radius with power 3, whereas DM mass grows with radius power 0.5 so it is right to get a radius where DM is counter balanced by DE.

According [9] Chernin,A.D. $M_{DE}(< R) = -\frac{\rho_{DE} 8\pi R^3}{3}$, and using the values for H = 70 Km/s/Mpc and $\Omega_{DE} = 0.7$ it is got the value $M_{DE}(< R) = -\varphi_{DE} \frac{8\pi R^3}{3} = -5.3984 \cdot 10^{-26} \cdot R^3\, kg$. It is important to highlight that this formula is proposed by [9] Chernin,A.D. Notice that this author multiply by two the volume of a sphere i.e. he considers that the effective density of dark energy is two times the $\varphi_{DE} = \varphi_C \bullet \Omega_{DE}$.

[9] Chernin defines gravitating mass $M_G = M_{DE} + M_{TOTAL}$, where M_{TOTAL} is baryonic plus dark matter mass, and defines R_{ZG} , Radius at zero Gravity as the radius where $M_{DE} + M_{TOTAL} = 0$. i.e. when the gravitating mass is zero.

This leads to equation $M_{TOTAL} = \varphi_{DE} \frac{8\pi R_{ZG}^3}{3}$. As Direct mass $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{R_{ZG}}}{G} = M_{TOTAL}(<r)$, in the framework of ***DM by gravitation theory*** it is possible to clear up rightly $R_{ZG} = \left[\frac{3a^2}{8\pi G \rho_{DE}}\right]^{2/5}$ and as $\varphi_{DE} = \frac{3 \cdot H^2}{8\pi G}\Omega_{DE}$ then by substitution $R_{ZG} = \left[\frac{a^2}{H^2 \cdot \Omega_{DE}}\right]^{2/5}$ This formula will be called R_{ZG} (parameter a^2).

For example, see [1] Abarca, M. 2023. In epigraph 14.1 was estimated parameter $a^2 = 4.428 \cdot 10^{21}$ (I.S. units) associated to Local Group, adding the four ones associated to MW, M31,LMC and M33, that as it is known they are the most massive galaxies in the Local Group. So using such value, the R_{ZG} for Local Group is 2.19073 Mpc

5.2 ZERO GRAVITY RADIUS FORMULA DEPENDING ON VIRIAL MASS

In previous chapter was got the value for $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$ depending on M_{VIR} as local parameter only, so by substitution in R_{ZG} formula it is right to get $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ where the only local parameter is M_{VIR} so this formula will be called $R_{ZG}(M_{VIR})$ and $R_{ZG} = K \cdot (M_{VIR})^{1/3}$ where $K = 3.683309948 \cdot 10^8$ (I.S. units).

In table below will be calculated R_{ZG} using both formulas.

Cluster	Virial mass	Parameter a^2(M_{VIR} , R_{VIR})	R_{ZG} (parameter a^2)	R_{ZG} (M_{VIR})	Relatif diff.
	$\cdot 10^{14}$ Msun	I.S. units $m^{5/2}/s^2$			%
Virgo	6.3±0.9	3.6527E23	12.97	12.87	0.7
Coma	27	1.2198E24	21.015	20.9	0.55

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh,2023

Below are calculated the same concepts: R_{ZG} (parameter a^2) and R_{ZG} (M_{VIR}) using Parameter a^2(M_{VIR}). It is remarkable how both calculus are mathematically equivalents as it was expected, i.e. when it is used the Parameter a^2(M_{VIR}) to calculate R_{ZG} (parameter a^2) then match mathematically with R_{ZG} (M_{VIR}). See in table below how the grey values match perfectly.

Cluster	Virial mass	Parameter a^2(M_{VIR})	R_{ZG} (parameter a^2)	R_{ZG} (M_{VIR})	Relatif diff.
	$\cdot 10^{14}$ Msun	I.S. units $m^{5/2}/s^2$			%
Virgo	6.3±0.9	3.581E23	12.871	12.871	0
Coma	27	1.2042E24	20.9069	20.9069	0

With this important cluster of galaxies, it has been illustrated how the total mass, calculated by $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$, is counter balanced by dark energy at mega parsecs scale, and precisely this Radius at zero gravity determines the region size where the cluster has gravitational influence.

Below has been calculated the zero gravity radius for the sample of group of galaxies or cluster published by [4] R.Ragusa et al.2022. See the blue columns.

The column in green shows the R_{ZG} using the formula by the virial mass $R_{ZG} = K \cdot (M_{VIR})^{1/3}$ where $K{=}3.683309948 \cdot 10^8$ (I.S. units).

Data [4] R.Ragusa et al.2022			
Celestial	VirialRadius	Virial Mass	Zero Grav R
Body	Mpc	1E13Msun	Mpc
Antlia cluster	1,28	26,3	9,62E+00
NGC596/584	0,5	1,55	3,74E+00
NGC 3268	0,9	8,99	6,73E+00
NGC 4365	0,32	0,4	2,38E+00
NGC 4636	0,63	3,02	4,68E+00
NGC 4697	1,29	26,9	9,69E+00
NGC 5846	1,1	16,6	8,25E+00
NGC 6868	0,6	2,69	4,50E+00

5.3 TOTAL MASS ASSOCIATED TO THE SPHERE WITH RADIUS ZERO GRAVITY

Thanks the previous epigraphs it is right to get the total mass associated to a specific cluster, by two ways: using the formula of DE density or using the direct formula, using for both formulas the zero gravity radius.

The first one is by $M_{TOTAL} = \varphi_{DE} \frac{8\pi R_{ZG}^3}{3}$ so $M_{TOTAL}(< R_{ZG}) = M \cdot (R_{ZG})^3$ being $M = 5.3984 \cdot 10^{-26}$ (I.S.units).

The second one, which is mathematically equivalent, is by the direct mass formula $M_{TOTAL}(< R_{ZG}) = \frac{a^2 \cdot \sqrt{R_{ZG}}}{G}$.

In table below are shown calculus of total mass for Virgo and Coma clusters and the Local Group of galaxies.

Cluster	Radius ZG	Param a^2 (M_{VIR})	$M_{TOTAL}(<R_{ZG})=M \cdot R^3_{ZG}$	Direct $M_{TOTAL}(<R_{ZG})$	Relati diff
	Mpc		Msun	Msun	%
Virgo	12.871	3.581E23	1.699945E15	1.69946E15	0
Coma	20.9069	1.2042E24	7.28353E15	7.2836E15	0
Local group	2.19073	4.28E21	8.379931E12	8.37994E12	0

5.4 TOTAL MASS ASSOCIATED TO THE SPHERE RADIUS ZERO GRAVITY USING THE VIRIAL MASS

From this formula $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ it is right to calculate its cubic power and by substitution in

$M_{TOTAL}(< R_{ZG}) = \varphi_{DE} \frac{8\pi R_{ZG}^3}{3}$ being $\varphi_{DE} = \frac{3 \cdot H^2}{8\pi G} \bullet \Omega_{DE}$ it is right to get that

$M_{TOTAL}(< R_{ZG}) = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/6}} \cdot M_{VIR} = U \cdot M_{VIR}$. Being U= 2.665735 this is a magnificent result¡ Below are compared the masses for Virgo and Coma clusters using this formula and the previous one by the R_{ZG} to the cubic power.

Cluster	Virial mass	$M_{TOTAL}(< R_{ZG})$=U·M_{VIR}	$M_{TOTAL}(<R_{ZG})$=M·R^3_{ZG}	Relatif diff.
	· 10^{14} Msun	Msun	Msun	%
Virgo	6.3±0.9	1.6794E15	1.699945E15	1.2
Coma	27	7.1975E15	7.28353E15	1.2

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh,2023

As the reader may see the matching is very good.

5.5 TOTAL DARK ENERGY ASSOCIATED TO THE SPHERE WITH RADIUS ZERO GRAVITY

According [9] Chernin,A.D. $M_{DE}(< R_{ZG}) = -\frac{\rho_{DE} 8\pi R_{ZG}^3}{3}$ that it is just the opposite value to

$M_{TOTAL}(< R_{ZG}) = \varphi_{DE} \frac{8\pi R_{ZG}^3}{3}$ so $M_{DE}(< R_{ZG}) = -\frac{\sqrt[5]{100}}{\Omega_{DE}^{1/6}} \cdot M_{VIR} = -U \cdot M_{VIR}$ As it was expected. Therefore the total gravitating mass $M_G(<R_{ZG})=M_{TOTAL}(<R_{ZG})+ M_{DE}(<R_{ZG})$ enclosed into the sphere of zero gravity radius is zero, as it was postulated as definition . $M_{TOTAL}(<R_{ZG})+ M_{DE}(<R_{ZG}) = 0$

6. ZERO GRAVITY RADIUS VERSUS VIRIAL RADIUS

In chapter 3 was got $R_{VIR}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ or $R_{VIR} = \left(\frac{G \cdot M_{VIR}}{100 \cdot H^2}\right)^{1/3}$ as a good approximation of R_{VIR} as R_{200}. By other side in previous chapter has been got

$R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ so it is right to get the ratio $\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5}$ =7.277 This is an awesome result ¡

Because this ratio is Universal, it is not depend of virial mass. Below is shown a sample of clusters.

Celestial	VirialRadius	Virial Mass	Zero Grav R	Ratio
Body	Mpc	1E13Msun	Mpc	R_{ZG} / R_{VIR}
Antlia cluster	1,28	26,3	9,62E+00	7,52E+00
NGC596/584	0,5	1,55	3,74E+00	7,49E+00
NGC 3268	0,9	8,99	6,73E+00	7,47E+00
NGC 4365	0,32	0,4	2,38E+00	7,45E+00
NGC 4636	0,63	3,02	4,68E+00	7,42E+00
NGC 4697	1,29	26,9	9,69E+00	7,51E+00

NGC 5846	1,1	16,6	8,25E+00	7,50E+00
NGC 6868	0,6	2,69	4,50E+00	7,50E+00

Columns in blue come from [4] R.Ragusa et al.2022

The second columns shows the virial radius for each celestial body.

Column in green is the R_{ZG} calculated and column in pink is the ratio.

It is clear that the ratio R_{ZG} / R_{VIR} got in this sample of celestial bodies match very well with the value got by the theory.

Cluster	VirialRadius	Zero Grav R	Ratio
	Mpc	Mpc	R_{ZG} / R_{VIR}
Virgo C.	1.7	12.871	7.57
Coma C.	2.8	20.9069	7.467

The results for the most prominent clusters are impressive as well.

7. CONCLUSION

Thanks to the approximation of virial radius and virial mass for R_{200} and M_{200} it has been possible to get some impressive general results for clusters in the framework of Dark Matter by Gravitation theory. The most important result of this work is

$$\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5} = 7.277$$

This formula states a quite simple method to check the rightness of Dark Matter by Gravitation theory.

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