Periodic Function Approach to Prime Number Analysis with Graphical Illustrations

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Abstract

This paper introduces a novel approach employing periodic functions for the comprehensive analysis of prime numbers. The method encompasses primality testing, factor counting and listing, prime distribution calculation, and the determination of the Nth prime. The exposition of the technique is presented in a clear and sequential manner, guiding the reader through each step with explicit equations. Graphs are strategically incorporated between crucial stages to facilitate a rapid and intuitive visualization of the rationale and outcomes of each maneuver. The paper concludes with concise reflections and ongoing inquiries into the potential applications and refinements of the proposed method.

keypoints:periodic functions ,prime number ,Graph

1 Introduction

In the realm of number theory, the exploration and comprehension of prime numbers have been longstanding pursuits, captivating the curiosity of mathematicians for centuries. This paper introduces a pioneering approach that harnesses the power of periodic functions to conduct a thorough and comprehensive analysis of prime numbers. Covering a spectrum of mathematical tasks, ranging from primal testing to factor counting and listing, prime distribution calculation, and even the determination of the Nth prime, the proposed method unfolds as a systematic and innovative solution to long-standing challenges in the field.

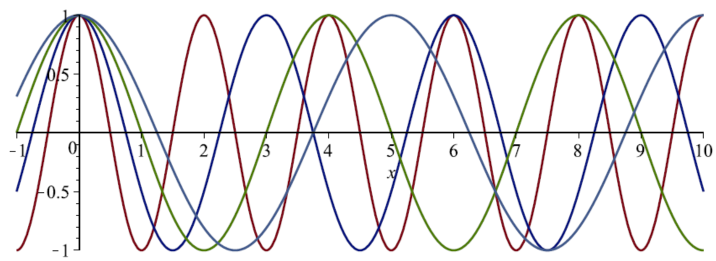
The exposition of this novel technique unfolds in a clear and sequential manner, guiding the reader through each analytical step with explicit equations. Throughout the presentation, strategically placed graphs serve as visual aids, facilitating a rapid and intuitive comprehension of the rationale behind each maneuver and offering insightful glimpses into the outcomes of the analysis.

As the paper delves into its final sections, it culminates with concise reflections, providing valuable insights into the potential applications and avenues for refinement of the introduced method. This exploration not only contributes to the theoretical understanding of prime numbers but also hints at practical implications and avenues for further research. In essence, this paper represents a significant stride in the ongoing quest to unravel the mysteries of prime numbers, offering a fresh perspective through the integration of periodic functions in a comprehensive analytical framework. [1] [2] [3] [5] [4]

2 A Novel Approach Unveiling Periodic Functions for Initial Functions and Strategy in Comprehensive Analysis

Begin with the set of basic cos functions of this shape $cos(\frac{2\pi x}{q})$. here wave of number q, so that q is an ,$q \geq 1$ waves 2 through 5 are shown

$$plot(cos(\frac{2\pi x}{q})(q = 2...5), x = -1, 1...10)$$



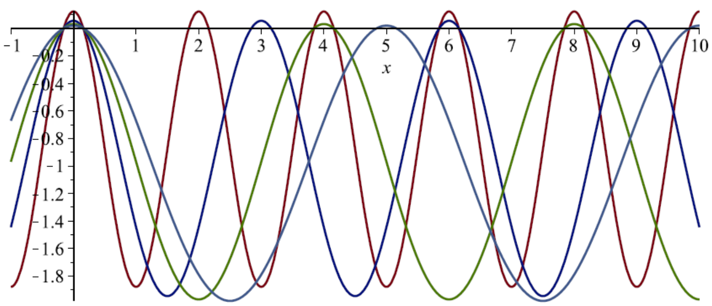
In the preliminary stage of our innovative approach, the first step serves a dual purpose, initiating a trans formative process that holds significance for the subsequent analytical maneuvers. Firstly, this initial step orchestrates a shift in the waves' positions, ensuring that their values persist as +ve above the multiples of each distinct wave numbers. Simultaneously, the same shift renders these values negative below integers that do not align with each wave's corresponding number. This manipulation establishes a nuanced dynamic, creating a dichotomy that plays a crucial role in the forthcoming stages of our analysis.

Secondly, the first step imposes a constant width for each wave crest, strategically aligning them with integer points. By doing so, we ensure that wave of crests sharing an integer points also know x-intercepts. This unique definition and imposition of constant crest width create a structured foundation, setting the stage for a more precise and coordinated exploration of the periodic functions in our analytical strategy. This dual-functionality of the first step not only primes the subsequent stages but also establishes a mathematical framework that enhances the interpret ability and coherence of our approach.

3 Harmony in Limitation: Navigating Wave Peak Restrictions

The process involves selecting the half-width of the crest, denoted as 'r,' and assessing each wave at this specific width. Subsequently, the wave is systematically shifted downward by the determined amount. It's important to note that the chosen value for 'r' must adhere to the condition $0 < r \leq 1/2$. This iterative procedure is repeated to achieve the desired outcome. So we using $cos(\frac{2\pi r}{q})$ now we selected r by simplification are $\frac{1}{2\pi}$,$\frac{1}{\pi}$ and $\frac{1}{2}$ For this we used r=$\frac{1}{2\pi}$. So waves 2 through 5 are shown as a reference.

$$plot(cos(\frac{2\pi x}{q}) - cos(\frac{1}{q}))(q = 2...5), x = -1, 1...10)$$

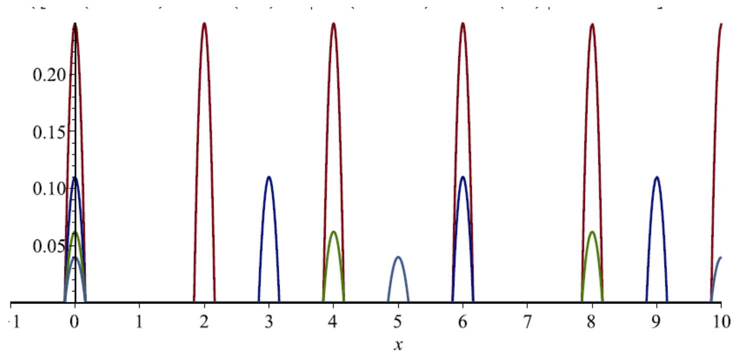


The restriction on the parameter "r" stems from a series of considerations. Selecting r=0 results in a lack of information above the axis, leaving no meaningful data to analyze. On the other hand, opting for $r > 1/2$ introduces extraneous information above the axis, where it shouldn't exist, introducing unwanted noise into the analysis. Therefore, the limitation on the range of r is essential for maintaining the integrity and relevance of the information being processed.

4 Noise Reduction

In order to enhance the clarity of subsequent summations, any disruptive information located below the axis is proactively mitigated. This is achieved by incorporating the absolute values of the functions into their own magnitudes, thereby preemptively eliminating unwanted noise during the summation process.

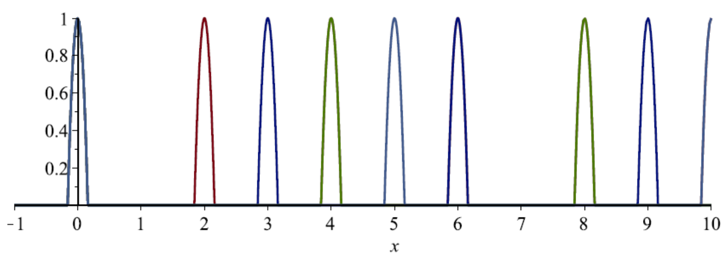
$$plot(cos(\frac{2\pi x}{q}) - cos(\frac{1}{q}) + |cos(\frac{2\pi x}{q}) - cos(\frac{1}{q}|))(q = 2...5), x = -1, 1...10)$$



5 Scaling Peaks to 1.

Following the current procedure, where the peaks no longer retain a value of 1, a normalization step is introduced to bring all values back to 1. This involves dividing each data point by the quantity $1 - cos(\frac{1}{q})$ to counteract the effects of the scaling process. Additionally, a further division is implemented to address the constraints imposed on the wave peaks. This second division is performed by dividing each data point here. The outcome of these normalization steps .

$$plot(\frac{(cos(\frac{2\pi x}{q}) - cos(\frac{1}{q}) + |cos(\frac{2\pi x}{q} - cos(\frac{1}{q})|)}{2(1 - cos(\frac{1}{q})})(q = 2...5), x = -1, 1...10)$$

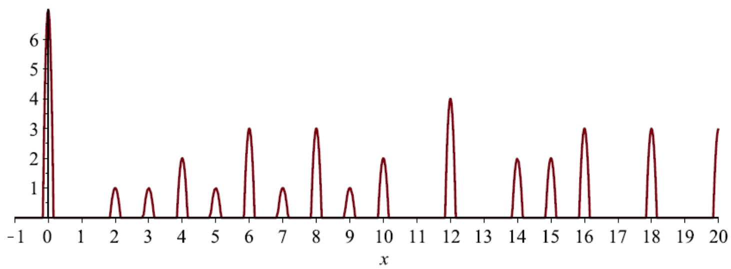


6 Summation

It appears you're describing a function that assigns a value of 1 to integers that are either factors of x or multiples of their wave numbers. To clarify the definition, let's denote the set of factors of x as F(x)

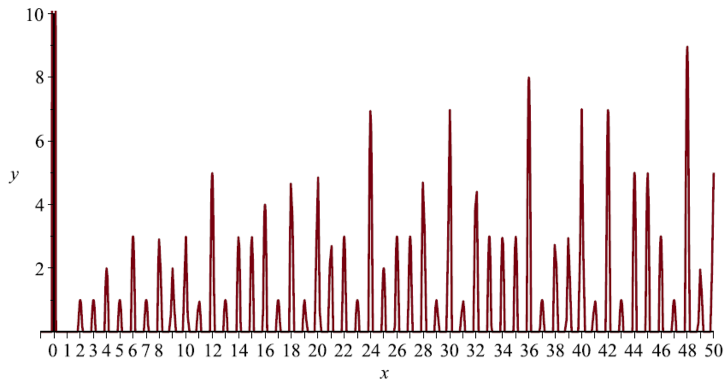
$$F(x) = \sum_{q=2}^{i}(\frac{(cos(\frac{2\pi x}{q}) - cos(\frac{1}{q}) + |cos(\frac{2\pi x}{q}) - cos(\frac{1}{q})|)}{2(1 - cos(\frac{1}{q}))})(q = 2...5), x = -1, 1...10)$$

The result of adding the 2 through 8 waves for a reference is:



In order to guarantee the function correctness up to an Integer x for all x, the summation must include all waves up to x. Here is q from 2 to 50.

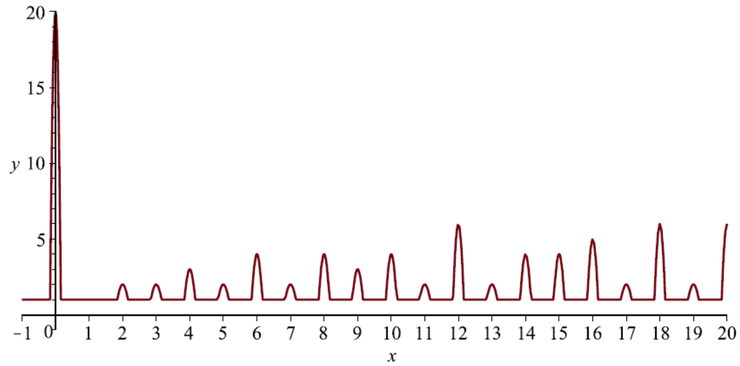
$plot(F(x), x = -1, 1...50, y = 0...10)$



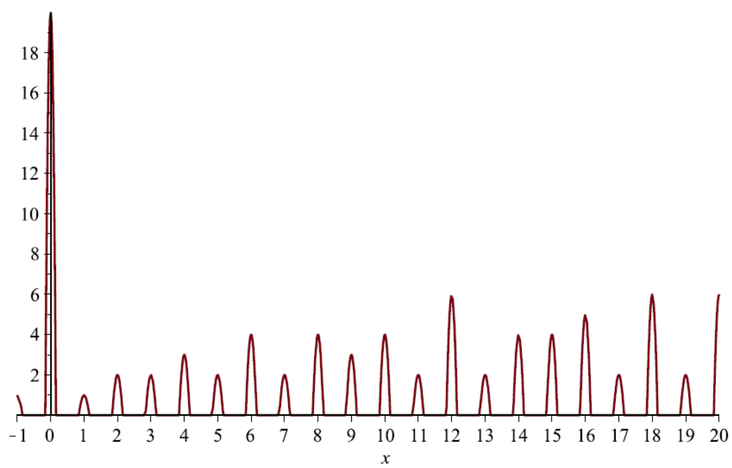
7 Unveiling Patterns with a Waveform Prime Sieve and Exploring Factor Counting Techniques

At this juncture, the function's value at an integer coincides with the count of factors pertaining to that integer, encompassing the integer itself while excluding 1 . Prime Sieve and Exploring Factor Counting Techniques F(x)=1 such that $x \leq i$ and that $F(x) > 1$ for all composites. There are two ways to make the values of the function equal to "the number of factors of a number including one and the number". The first, is to simply add a baseline of 1 to the function. The second is to include the q = 1 wave. Both are shown below.

$plot[F(x) + 1, j = 20...20, x = 1...20, y = -1..20]$



$$\sum_{q=1}^{i} \frac{(cos(\frac{2\pi x}{q}) - cos(\frac{1}{q}) + |cos(\frac{2\pi x}{q}) - cos(\frac{1}{q})|)}{2(1 - cos(\frac{1}{q}))})(i = 20...20), x = -1, 1...20)$$



8 Decoding Factors: An In-Depth Exploration

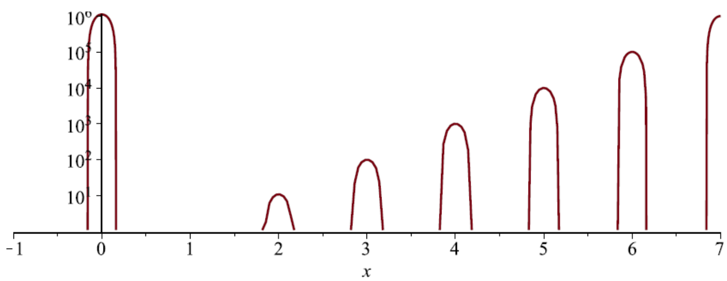
To ascertain the specific factors of a given number, it's important to recognize that each factor contributes a distinct value to the function F(x) at the corresponding integer. Moreover, each number possesses a unique set of factors with no duplication's within that set. Instead of every factor contributing a value of 1, if each factor imparts a unique value, and the sums of these values within the sets are also distinct, then the resulting output at any integer becomes a unique representation of that specific set.

To designate the unique value for each factor, we introduce the concept of a "Tag" for that factor. For instance, for a wave number q, consider a tag of $10^{(q-1)}$for 2's, 100 for 3's, 1000 for 4's, and so forth. Incorporating the sum of tag values for each factor, as opposed to all factors contributing 1, yields an exclusive output. It's important to note that while there are certainly other tags that meet the mentioned criteria, the chosen approach is one of simplicity, as demonstrated in the following equation.Define a Factor Tagging Function, T(x), such that

$$T(x) = \sum_{q=2}^{i}(10^{q-1})(\frac{(cos(\frac{2\pi x}{q}) - cos(\frac{1}{q}) + |cos(\frac{2\pi x}{q}) - cos(\frac{1}{q})|)}{2(1 - cos(\frac{1}{q}))})(q = 2...5), x = -1, 1...10)$$

A log plot of T(x) up to 7 for reference is as follows. *Note, the x = 1 value, is existent, but not visible at this graph's resolution.

$$plot[T(x), (i = 7...7), (x = 1...7)]$$

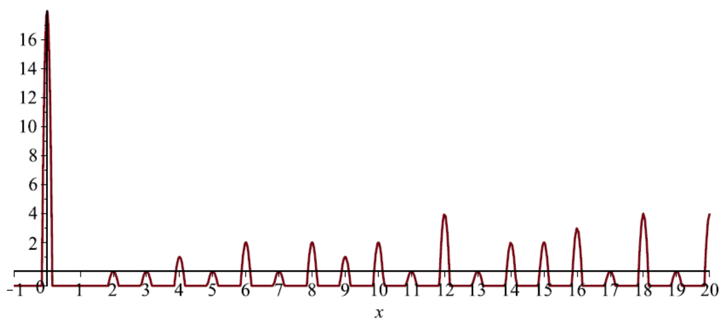


This function generates an output in binary such that the 1s correspond to the factors from right to left. For example T(6) = 100111 shows the factors of 6 to be 1, 2, 3, and 6. While not discussed here, further associations can now be made between the decimal value of each binary string and its associated set. That is, [1] = 1 = [1], [11] = 3 = [1,2], [101] = 5 = [1,3], [1011] = 11 = [1,2,4], [10001] = 17 = [1,5], and so on. It is interesting to note, that it seems all the decimal values are primes, and that they span a subset of the primes. Questions on this are included in the afterthoughts section.

9 Discerning Numbers:Unraveling the Composite from the Natural through Fluctuations

The next step involves the manipulation of the function F(x) to distinguish between composite and natural numbers. This process includes further wave peak restriction, de-noising, and re normalization, similar to the previous steps. To achieve this, shift F(x) downward by 1 unit, effectively positioning information above the axis exclusively above the composite numbers. This strategic adjustment aids in the separation of composites and naturals, facilitating a clearer distinction in the analyzed data.

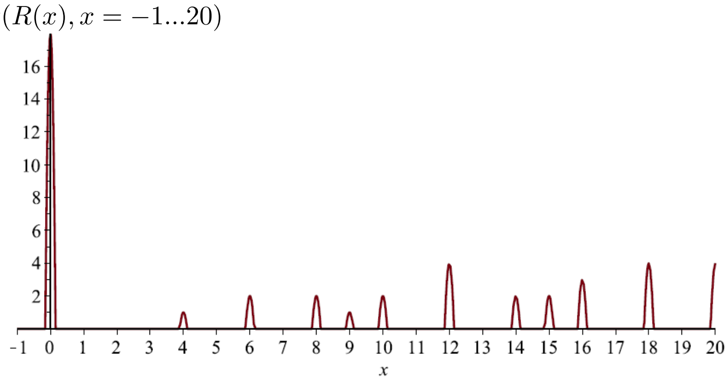
$$plot[F(x) - 1, (i = 20...20), (x = -1...20)]$$



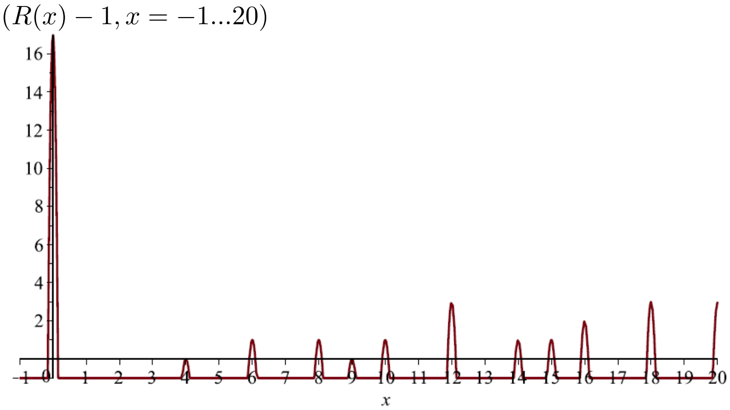
Then, remove the data below the axis via absolute value, and divide by 2 to counter that manipulation. Define that new function to be R(x).

$$R(x) = (\frac{(F(x) - 1 + |F(x) - 1|)}{2})$$

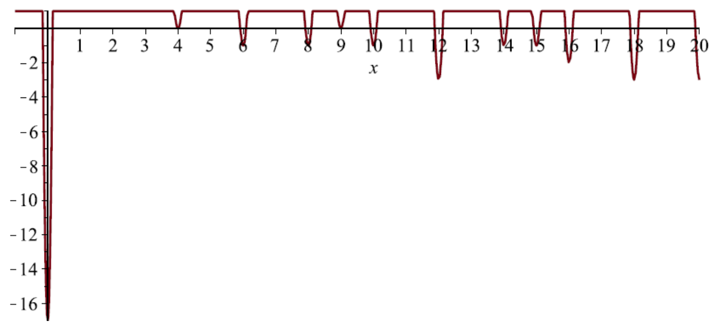
$plot(R(x), x = -1...20)$



Now, the goal is to get all the composite peaks to have the same value, namely 1. This is accomplished by first shifting the function down by 1. $plot(R(x) - 1, x = -1...20)$



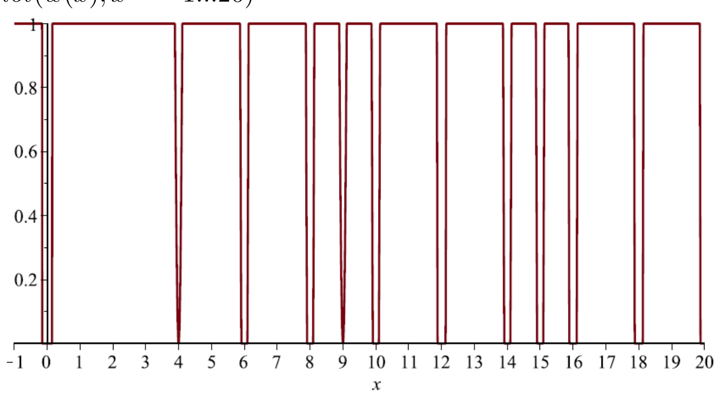
Second, flip the function over the x axis. $plot(-1R(x) - 1, x = -1...20)$



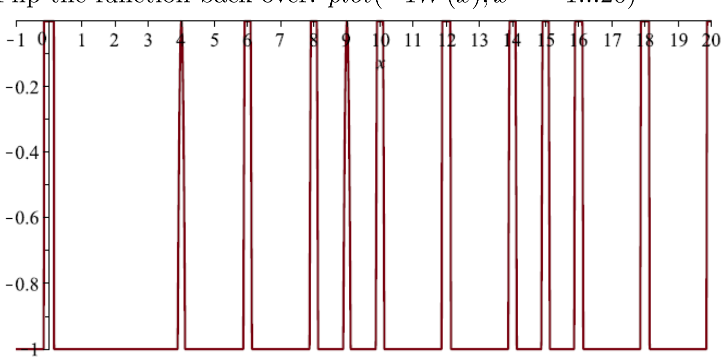
To mitigate the impact of magnitude changes, incorporate the absolute value and subsequently divide by 2. This operation effectively eliminates all peaks that now reside below the axis. In the paper, this function was denoted as w(x), serving as a provisional name for organizational purposes.

$$W(x) = (\frac{(1 - R(x) + |1 - R(x)|)}{2})$$

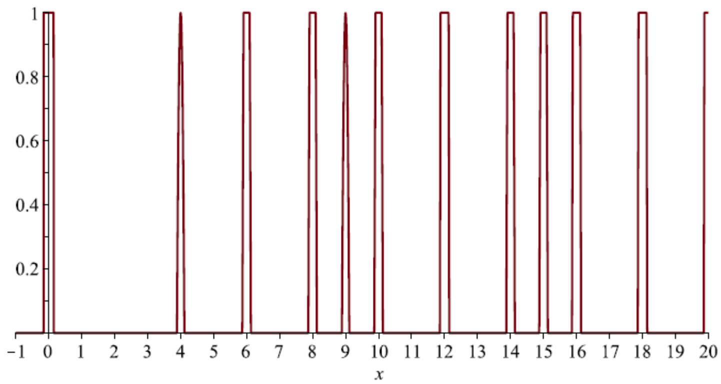
$plot(w(x), x = -1...20)$



Flip the function back over. $plot(-1W(x), x = -1...20)$



Finally, shift it back up by 1. $plot(-1W(x) + 1, x = -1...20)$



This function now has a peak value of 1 for all composites and only the composites, and is labeled Z(x).

Z(x)=-W(x)+1 Sum taken over this function can be used if the composites $\leq a$ such that it used to determine by prime distribution.

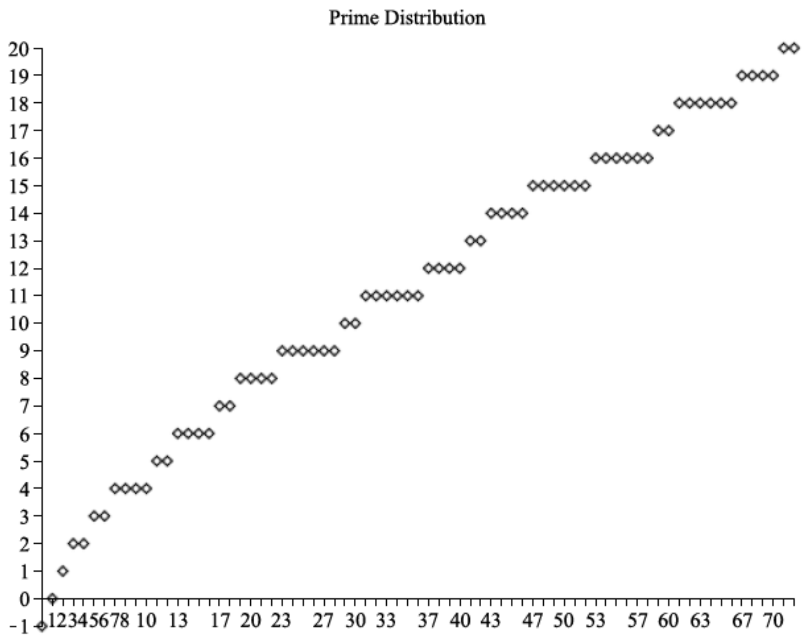
10 The Prime number Distribution

The number of primes $\leq a$ number is equal to that number , negative number of composites $\leq$ *number* negative number one ,the formula for the Prime Distribution, P(x), is:

$$P(x) = (x-1) - \sum_{n=1}^{x} Z(x)$$

As an example, P(72) outputs 20, which coincides with 71 being the 20th prime. Point plotting P(x) shows the familiar Prime Distribution. P(x) gives the exact distribution for all x as long as the initial restriction on j in F(x) is abided throughout the calculation.

point plot(seq[x,p(x)],x=0,1...72)



11 Unveiling the Recursive Sequence to Compute the Nth Prime Number

Using the formula for the exact distribution of the primes, a recursive sequence, $Q_s(n)$ can be fashioned to determine the nth prime. Given that $Q_0 = (0)$, then

$$Q_s(n) = n + Q_{(s-1)} - p(Q_{(s-1)})$$

The variable x was used in the math program that generated this paper due to the convenience of how the formula was formerly input, hence the change in variable from input to output. This sequence always equals the nth prime for some term s where $s < n$. Every subsequent term will also be that prime. The sequence prior to it repeating will always be the numbers from the formerly prime the term Q_2 through $Q_{(16)}$ for $Q_{(16)}(20)$ current prime. An illustration, the terms through for are shown as below. The reason the sequence is calculated shown as, and the reason why the term is not in the list, so it is covered in the next section.

$$q|| := 0$$

$$q||1 := x$$

for s from 2 to 16 do

$$q||s := x + q||(s-1) - p(q||(s-1)) \ end\ do;$$

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12 A Expression as to illustration P(0)

TheQ_1term of the sequence is equal to $x + Q_0 - p(Q_0)$. Given thatQ_0 is defined as 0 leads to P(0). Logically, the function of prime number $\leq a$ we say p(o) to 0. so Q_1 is equal to 0.So that p(x) for all $x \geq 1$ with p(x)=-1 This can be addressed in at least 2 ways. One, is the method used in the previous section, where the Q_1 and run throughQ_s with Q_1 give as equal to x sequence is started at and ran through with given as equal to x. The other method is to actually adjust p(x) for all $x \geq 1$ but p(0)=0 stage of the process. Multiplying the original R(x) by x, and then continuing from there through the entire process, gives the desired results. Thus, as a starting point, the new function $R_b(x)$would be.

$$R(x) = (\frac{x.(F(x) - 1 + |F(x) - 1|)}{2})$$

13 Reflections and Unanswered Queries

Let's address some of the questions and considerations mentioned:

13.1 Method Improvement and Streamlining

Choice of Periodic Functions The effectiveness of starting with different periodic functions depends on their mathematical properties and how well they align with the desired outcomes. Exploring different functions and analyzing their impact on the results could lead to improvements.

Parameter "r The choice of the parameter 'r' likely depends on the specific function and its role in the method. Experimenting with different values of 'r' might reveal patterns or optimizations.

Number of Flip Flops Minimizing the number of flip flops can enhance efficiency. Analyzing the role of each flip flop and whether it's essential for the method can guide optimization efforts.

Absolute Value Portion Whether considering the absolute value portion as the positive roots of squared quantities is beneficial depends on the specific mathematical properties and objectives. It's worth exploring alternative approaches to see if they lead to simplifications or improvements.

13.2 Efficiency and Time Complexity

Efficiency The efficiency of the method depends on various factors, including the chosen functions, parameter values, and implementation details. Profiling the algorithm and identifying bottlenecks can help optimize its efficiency.

Time Complexity Determining the time complexity would require a detailed analysis of the algorithm. The efficiency may vary depending on the specific operations involved, such as summation, factor counting, and prime determination.

Convergence Formula The formula for the number of terms 's' needed for convergence likely depends on the characteristics of the chosen functions. Analyzing convergence properties and deriving a formula could provide insights into optimization.

13.3 Decimal Values and Tag Multipliers

Decimal Values Verifying whether the decimal values obtained from the output of the tag function are prime involves checking their primality. This can be done using established primality testing algorithms.

Tag Multipliers Exploring different tag multipliers and their impact on the method could lead to alternative approaches or optimizations. Considering reciprocal multipliers is a valid avenue for investigation. The Twin Prime Conjecture suggests that there are infinitely many twin primes (pairs of primes that have a difference of 2, such as (3, 5), (11, 13), etc.). One approach to prove the conjecture is to find an infinite number of integer solutions to the system $F(x) = F(x+2) = 1$, as you mentioned. This system seems related to a periodic function, and you hinted at using trigonometric functions for this purpose.

To explore this further, you might consider using periodic functions, such as sine or cosine, to construct a function F(x) that satisfies the given conditions.

For example, you could define $F(x) = sin^2(x) + cos^2(x)$, which is identically equal to 1 for all x. However, constructing a periodic function that guarantees the existence of twin primes might be a non-trivial task.

Similarly, the Mersenne Prime Conjecture involves numbers of the form $2^m - 1$, where m is a positive integer. To prove the conjecture, you suggested finding an infinite number of solutions to $F(2^m - 1) = 1$. This implies looking for periodic behavior in the values of $F(2^m - 1)$..

Again, trigonometric functions or other periodic functions could be explored to create F(x) such that $F(2^m - 1) = 1$.for infinitely many m. However, constructing such a function would require careful consideration and mathematical analysis.

14 Conclusion

In conclusion, the presented approach seeks to address the Twin Prime Conjecture and the Mersenne Prime Conjecture by utilizing periodic functions and manipulating sequences. The method involves defining a function $F(x)$ with specific properties and exploring the behavior of its values to establish the existence of twin primes and Mersenne primes.

Several considerations and questions were raised during the exploration of this method. The choice of periodic functions, the parameter 'r' the number of flip flops, and the handling of absolute value portions were highlighted as areas for improvement and further investigation. The efficiency and time complexity of the algorithm were acknowledged as critical factors that could benefit from optimization and detailed analysis.

Additionally, the relevance of decimal values obtained from the tag function and the exploration of different tag multipliers were discussed, indicating the need for thorough examination and experimentation.

While the method presents a unique perspective on approaching these conjectures, further mathematical analysis and experimentation are necessary to validate its effectiveness. The intricate nature of number theory, especially in the context of prime numbers, requires a meticulous exploration of mathematical properties and potential optimizations.

The presented method provides a foundation for future research and refinement. It opens avenues for exploring various periodic functions, adjusting parameters, and analyzing the convergence properties of the proposed sequences. Continued collaboration and scrutiny within the mathematical community are essential to refine this approach and contribute to the broader understanding of twin primes and Mersenne primes.

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