Analysis of k-calculus from Introducing Einstein's Relativity by Ray d'Inverno

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2022-01-07

Abstract
Einstein's theory of special relativity, SR, is a generally accepted theory that analyses, for instance, relationships between two inertial reference systems moving at a constant speed against each other. This relationship between the coordinates of an event in the two inertial reference systems is made using so-called Lorentz Transformations, LT. These transformations constitute the most central concept within SR. It is from these transformations that other concepts within SR are derived, concepts such as time dilation, length contraction.

Keywords
Special Relativity, Reference System, Event, Light Signal, Lorentz Transformations, Mathematical model, Paradox, Reality, k-calculus, k-factor

In this work, we will analyze some aspects of the concepts above. We will show that no matter what method and model you use, you will always come across a contradiction. The contradiction is obtained if one carefully verifies the model with the reality, the physics, the mathematics, and the logic. We follow the book [1], chapter 2.

2.1 Model building

"The activity consists of constructing a mathematical model which we hope in some way capture the essentials of the phenomena we are investigating."

Yes, this is the most important moment in the explanation of a physical phenomenon. If the model is done correctly, if it correctly reflects the physical phenomenon, then the following calculations, conclusions, results should not contradict either the model or the existing mathematical or physical laws.

2.2 Historical background

Here the author of [1] goes through some of the steps made by different physicists, researchers, which ultimately resulted in the creation of the special theory of relativity. We mention some of them:

- 1865, James Clark Maxwell: theory of electromagnetism; light-bearing ether
- 1887, Michelson - Morley experiment; negative result
- 1904, Hendrik A. Lorentz: Lorentz transformations; Lorentz factor
- 1905, Albert Einstein: The theory of special relativity

Author Ray d'Inverno says:
”In fact, the essence of the special theory of relativity is conteined in the Lorentz transformations.”

This is true but it is from these transformations that the contradictions emerge!

2.3 Newtonian framework

Here they talk about events, about space-time diagrams, world-line, observers. We show our own picture of this.

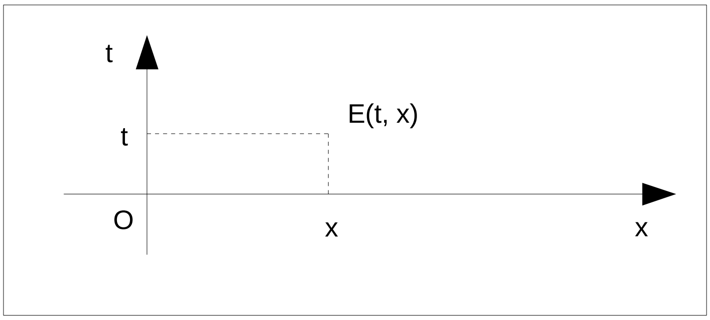

Fig. 1

We depict an event E that takes place at the time *t* on the t-axis and at the point *x* on the x-axis. Say that this figure represents an inertial reference system S. The point O, at which the t-axis and the x-axis intersect, represents the origin of the reference system.

2.4 Galilean transformations

”N1: Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces acting on it.”
This is logical, I see no problem with this statement.

The author of the book [1] says:
”Thus, there exists a privileged set of bodies, namely those not acted on by forces.”
I do not think this conclusion can be drawn from the N1.

2.5 The principle of special relativity

Here they address, among other things:
”Many fundamental principles of physics are statements of impossibility, and the above statement of the relativity principle is equivalent to the statement of the impossibility of deciding, by performing dynamical experiments, whether a body is **absolutely in rest** or in uniform motion.”

I argue that one can determine if an object is at absolute rest or if it is moving at a constant speed. This is shown in the book [2].

Furthermore, reference is made to the first postulate
Principle of special relativity: All inertial observers are eqvivalent.

I'm just asking the following question: if they are equivalent then why are their clocks ticking differently? This is a legitimate question!

2.6 The constancy of the velocity of light

Second postulate of SR, constancy of velocity of light:
The velocity of light is the same in all inertial systems.

I accept this, how else? Maxwell has come to this conclusion in his work.
If $c = 1/(\mu_0\varepsilon_0)^{1/2}$ then c is not dependent on anything other than the properties of the medium:

the permittivity of free space, ε_0
the permeability of free space, μ_0.

The speed of light is one thing and **the relative speed** between two objects or between one object and the wavefront of the light signal is another.

2.7 The *k*-factor

In the figures and models here, one takes $c = 1$ *(c = 1 light-second/1 second)*.
In normal cases, *c = 299,792.458 km/s*.
What does it mean?
This means that, in the 2-dimensional space-time diagram, you have the same "unit of length" both on the x-axis and on the t-axis. e.g. if you use on the t-axis as unit *1 (one) second* then the unit on the x-axis becomes *1 (one) light-second*.
Light-second, light-year are units of length.

But if you set $c = 1$, the corresponding conversion must also be made for v. Say we approximate c to *300,000 km/s* and the Earth's speed around the Sun to *30 km/s*.
Then these two speeds in the model from [1] will be as follows:

c = 1 light-second/second
v = 1/10,000 light-second/second

A legitimate question: why complicate it, what were the purposes of creating this k-calculus model?

In such model, the world-line of a point from the wavefront of light will be at an angle to the x-axis and the t-axis of $\pi/4 = 45°$. See the next figure.

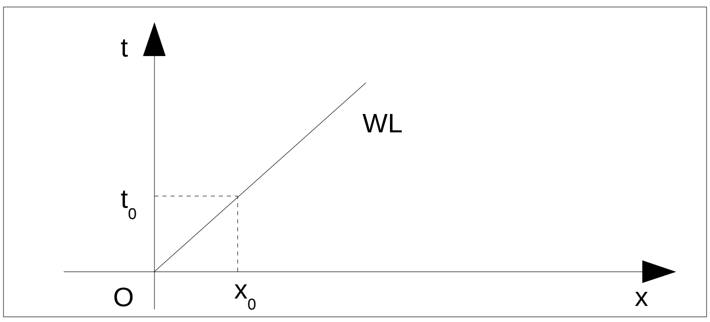

Fig. 2

The line WL represents world-line for a light signal starting from the origin of the reference system. t_0 is time and is measured in e.g. seconds, $t_0 = 1$ *second*. x_0 is the distance on the x-axis between the origin of the reference system and the point where the light front reaches in the meantime t_0. This distance is $x_0 = 1$ *light-second*.

Remember that all events occur on the x-axis, that even the light signal we observe moves on the x-axis!

An inertial reference system considered within SR moves at a speed less than that of light, $v < 1$, this means that the world-line of such a system will be at an angle less than 45° to the t-axis. See the next figure.

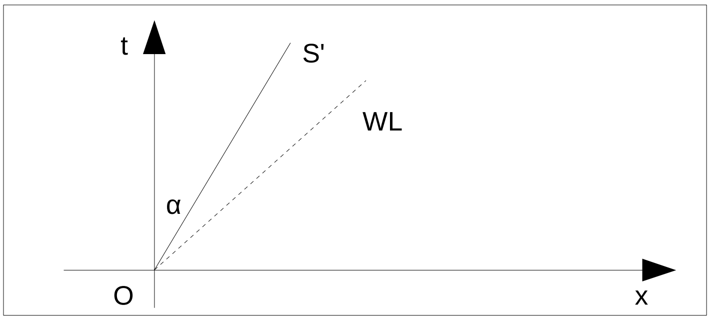

Fig. 3

We also show what it would look like if S' is at rest relative to S, $v = 0$. Then the angle of world-line of S' to the t-axis will also be zero, they become parallel. See the next figure.

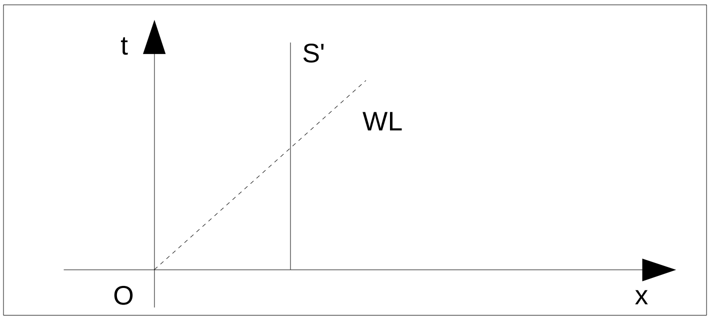

Fig. 4

The world-line of the reference system with the origin at the point O becomes a line that coincides with the t-axis.

Quote from [1]:
”Let us assume we have two observers, *A* at rest and *B* moving away from *A* with uniform (constatnt) speed.”

This approach is used in almost all situations when treating SR, doing derivation of LT.
It is said that *A* is at rest. But I ask a question here that is very important:

Relatively what is A at rest? Can proponents of SR provide an example of such a reference system?

You can have this set of reference systems to investigate a physical phenomenon but not when using light signals!

Quote from [1]:
"In fact, there is a hidden assumption here, since how do we know that B's world-line will be a straight line as indicated in the diagram?"

There is no assumption, if B moves with constant velocity then results from the model, from the geometry, that B's world-line **is** a straight line.

2.8 Relative speed of two inertial observers

We consider two inertial reference systems, S and S'. S' moves (to the right in the figure) at a constant speed $v > 0$ relatively S.

In this section, the following **thought experiments** are performed. At time t_0 a light signal is sent from S to S'. When the light signal reaches S', the signal is sent back to S. The world-line of the reflected light signal becomes a line that is symetrical with WL relative to the t-axis. The angle between these two world-lines will then be 90°.

We mark a number of points on the t-axis and on the x-axis.
Two and two of these points form distances and we will calculate their length. They also form some right-angled triangles and we will calculate the length of their sides. Here are the calculations for some of the distances found in Fig. 5:

Distance between two points P and Q, we denote by d(PQ).
- the distance between T_0 and T_0' is the distance that S' moved during the time t_0.

$$d(T_0T_0') = x_0 = vt_0$$

- the distance between P and P' is the distance between S and the point where S' is when the light signal arrives to it.

$$d(PP') = x_1 = vt_1$$

We calculate the time t_1 based on the figure Fig. 5.
The time interval $t_1 - t_0$ is the time the light signal from S needs to reach S'.

$x_1 = c(t_1 - t_0)$; We have $c = 1 \rightarrow \boldsymbol{x_1 = t_1 - t_0} \rightarrow$

$vt_1 = t_1 - t_0 \rightarrow t_0 = t_1 - vt_1 = t_1(1 - v) \rightarrow t_0 = t_1(1 - v) \rightarrow$

$\boldsymbol{t_1 = t_0/(1 - v)}$

This reasoning, to compare distances that during the same time are passed both by the light signal and by the reference system in motion, I have not seen in any literature, only in [2-6].

This is the key to finding contradictions in the models that deal with the derivation of LT.

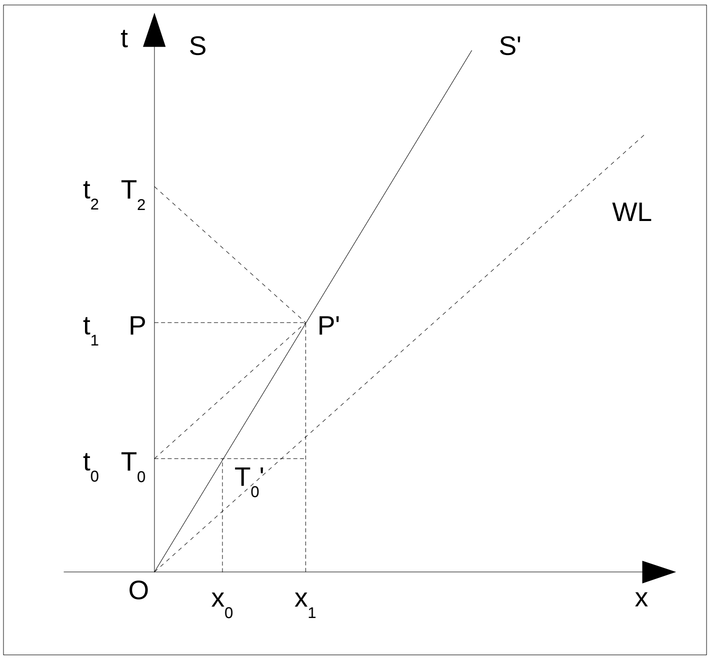

Fig. 5

In the triangle OPP' we have the following relations. We denote the angle between OP and OP' by α. Then we have:

$tan\,\alpha = d(PP')/d(OP) = x_1/t_1 = v$ (tan = tangent)

This angle represents the slope of world-line for S' relative to the t-axis.

$d(OP')^2 = d(PP')^2 + d(OP)^2$ (Pythagoras' theorem) $\rightarrow$

$d(OP')^2 = (vt_1)^2 + t_1^2 \rightarrow \mathbf{d(OP')^2 = t_1^2(1 + v^2)}$

In the triangle $PP'T_0$ is $d(PP') = d(PT_0)$ because the opposite angles are 45°. The same goes for the triangle $PP'T_2$, $d(PP') = d(PT_2)$. All these distances are equal to $\mathbf{t_1 - t_0}$. See calculations above.

Now we can calculate t_2, the time when the light signal returns to S.

$t_2 = d(T_2P) + d(PT_0) + d(T_0O) = (t_1 - t_0) + (t_1 - t_0) + t_0 \rightarrow$

$t_2 = (t_1 - t_0) + (t_1 - t_0) + t_0 \rightarrow$

$t_2 = 2t_1 - t_0 = 2t_0/(1 - v) - t_0 = (2t_0 - t_0(1-v))/(1 - v) = (2t_0 - t_0 + vt_0))/(1 - v) \rightarrow$

$\mathbf{t_2 = t_0(1 + v)/(1 - v)}$

We see that we can calculate all the distances between the different points formed in the model.

These distances depend **only** on

c – the speed of light, $c = 1$

v – the speed of S' relative to S, $0 < v < 1$

t_0 – the time when the light signal is transmitted from S to S'

We don't need to make any other assumptions!

We summarize:

$$\boldsymbol{t_1 = t_0/(1 - v)}$$
$$\boldsymbol{t_2 = t_0(1 + v)/(1 - v)}$$

The factors $(1 - v)$ and $(1 + v)$ occur abundantly in the book [2], although where the author uses c, the speed of light. There, they become equal to $(c - v)$ and $(c + v)$.

In the book [1] that we analyze, one denotes

$$k = ((1 + v)/(1 - v))^{1/2} \rightarrow k^2 = (1 + v)/(1 - v) \rightarrow$$
$$t_2 = k^2 t_0$$

It is the same relation as stated in [1]. Also the calculation that $(k^2 - 1)/(k^2 + 1) = v$ is correct.

But the **assumption** that the time in S' is proportional to the time in S is not correct! In the book [1] it is stated that the distance between the points O and P' is kt_0.

$$d(OP') = kt_0$$

We have seen before that $d(OP')^2 = t_1^2(1 + v^2)$.
We replace t_1 with $t_0/(1 - v) \rightarrow$

$$t_0^2(1 + v^2)/(1 - v)^2 = k^2 t_0^2 \rightarrow (1 + v^2)/(1 - v)^2 = (1 + v)/(1 - v) \rightarrow$$
$$(1 + v^2) = (1 + v)(1 - v) \rightarrow 1 + v^2 = 1 - v^2 \rightarrow 2v^2 = 0 \rightarrow$$
$$\boldsymbol{v = 0}$$

This represents a contradiction to the original condition that S' moves at a speed $v > 0$ relative to S.
We got this contradiction from **the only assumption** in our mathematical model, the assumption that $d(OP') = kt_0$, that the time in S' is proportional to the time in S with the factor k!

Why do you do that? You create a mathematical model, you use it to a certain point, but you do not pursue thinking. Why make a **assumption** here?
All distances can be calculated from the model!

I can not believe that the researchers who created k-calculus did not see that the OP' can be calculated from the triangle OPP' and that its length is $d(OP') = t_1(1 + v^2)^{1/2}$.
We skip 2.9 and 2.10.

2.11 The clock paradox

We make our own figure here too, to be able to explain better.

The reference system S' moves to the right in the figure at speed v relatively S. When S' is in O, a third reference system S'' starts at the point $2x$ from O. S'' moves to the left at speed v relatively S. After time t, S' and S'' hit togather at the point P (at the point x on the x-axis). Then they send a light signal to S. This signal has a world-line that is perpendicular to the WL.

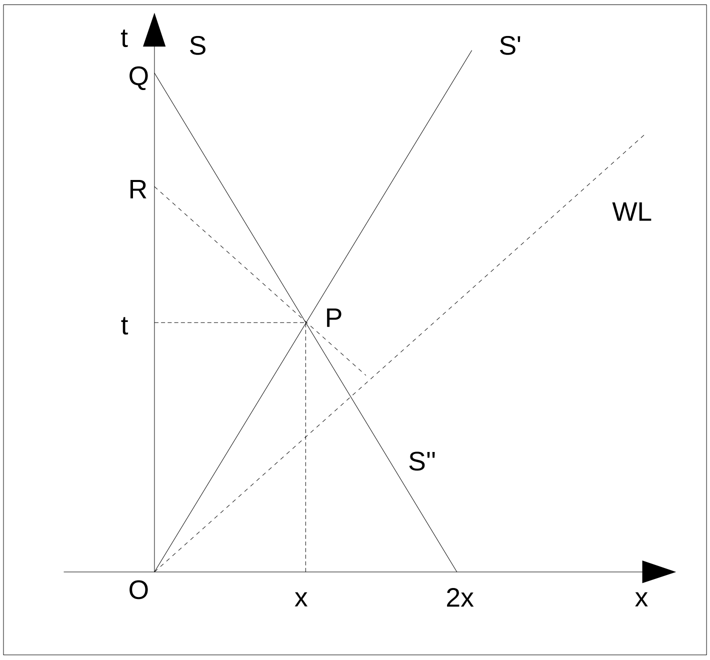

Fig. 6

Now we have all the parameters in place. Based on the figure, we can now calculate all distances.

From Fig. 6 we have

$$x = vt;\ d(tP) = vt;\ d(Rt) = d(tP) \rightarrow$$
$$R = vt + t \rightarrow \boldsymbol{R = t(1 + v)}$$
$$d(OP) = T = (t^2 + (vt)^2)^{1/2} = t(1 + v^2)^{1/2} \rightarrow \boldsymbol{T = t(1 + v^2)^{1/2}}$$

If we make the assumption that $R = kT$ as one do in [1], we get a contradiction!

$$R = kT \rightarrow t(1 + v) = kt(1 + v^2)^{1/2} \rightarrow (1 + v) = k(1 + v^2)^{1/2} \rightarrow$$
$$(1 + v)^2 = k^2(1 + v^2) \rightarrow (1 + v)^2 = (1 + v)(1 + v^2)/(1 - v) \rightarrow$$
$$(1 + v)(1 - v) = (1 + v^2) \rightarrow 1 - v^2 = 1 + v^2 \rightarrow \boldsymbol{v = 0}$$

This represents a contradiction to the original condition that S' moves at a speed $v > 0$ relative to S (the same calculations and result as before**).**

We see clearly from the figure that (all lengths in the figure are only dependent on the time t when S and S' meet)

$$\boldsymbol{Q = 2t}$$

The author of [1] says that $Q = (k + k^{-1})T$, where $k = ((1 + v)/(1 - v))^{1/2}$.

Calculation: $2t = (k + k^{-1})t(1 + v^2)^{1/2} \rightarrow$

$2/(1 + v^2)^{1/2} = (1 + v)^{1/2}/(1 - v)^{1/2} + (1 - v)^{1/2}/(1 + v)^{1/2} \rightarrow$

$2/(1 + v^2)^{1/2} = ((1 + v) + (1 - v))/(1 + v)^{1/2}(1 - v)^{1/2} \rightarrow$

$2/(1 + v^2)^{1/2} = 2/(1 - v^2)^{1/2} \rightarrow 1 + v^2 = 1 - v^2 \rightarrow \boldsymbol{v = 0}$

Quote from [1]:
”For k ≠ 1, this is greater than the combined time intervals 2T recorded between events OP and PQ by B and C. But should not the time lapse between two events agree? This is one form of the so-called clock paradox.” (In Fig 6. B = S', C = S'').

How can you think like that? T, the time laps in S' between the events O and P in the model, is not the real time that the clock in S' shows, but it is **the mathematical time from the model**.
If you make a model in which you convert physical quantities, then you must take these transformations into account all the way!
In the k-calculus model we have $c = 1$, $v = 1/10{,}000$ (e.g. Earth's velocity around the Sun) and e.g. ***t = 1 second* in S becomes $t' = (1 + v^2)^{1/2}$ *seconds* in S'**.

To compare quantities from reality, one must convert these quantities from the mathematical model to the real model, to reality!

In the book [1], we continue on to section 2.12 The Lorentz transformations.
This section is also nonsense because you base your calculations on incorrect grounds.

The error originates in the assumption that the time in the reference system in motion, t', is of the expression

$t' = kt$, or $Q = (k + k^{-1})t'$ there
$k = ((1 + v)/(1 - v))^{1/2}$ and t is the time in stationary reference system!

According to Fig. 5 and Fig. 6, the relationship between the time intervals in S and S' is as follows:

$$\boldsymbol{t' = t(1 + v^2)^{1/2}}$$

This is a relation between time laps in S and S' that results from the k-calculus model!

2.12 The Lorentz transformations

”We have derived a number of important results in special relativity”.
What results?
From Fig. 2.17, according to the author of [1], the following relations result:

$$t' - x' = k(t - x),\ t + x = k\,(t' + x') \qquad (2.7)$$

We have seen before that the relation $t' - x' = k(y - x)$ is wrong. According to calculations in 2.18 above, this relationship would look like this:

$$t' - x' = (t - x)(1 + v^2)^{1/2}$$

and then $t' - x' = k(y - x)$ can take place only if $v = 0$.

Furthermore, they come to

$$t' = (t - vx)/(1 - v^2)^{1/2},\ x' = (x - vt)/(1 - v^2)^{1/2} \quad (2.8)$$

and these relations are called **special Lorentz transformation** (in this model). In chapter **3 The key attributes of special relativity** they make further calculations and then they come to the usual LT:

$$t' = (t - vx/c^2)\gamma \text{ , } x' = (x - vt)\gamma \qquad (3.12)$$

where $\gamma = 1/(1 - v^2/c^2)^{1/2}$ is called the Lorentz factor.

But I have to ask another question here:
If the formula (2.8) represents Lorentz transformations from k-calculus, the model we have analyzed, and (3.12) represents Lorentz transformations from reality then we should be able to derive (3.12) from (2.8) by applying a reverse procedure than the one we did than we created the k-calculus model! Is not it like that?
Why else has the k-calculus model been built? To what use?

We compare one more time the physical quantities from the two models

- $t' = (t - vx)/(1 - v^2)^{1/2}$ from (2.8)
- $t' = (t - vx/c^2)/(1 - v^2/c^2)^{1/2}$ from (3.12)

- $x' = (x - vt)/(1 - v^2)^{1/2}$ from (2.8)
- $x' = (x - vt)/(1 - v^2/c^2)^{1/2}$ from (3.12)

Say we have a conversion method from (2.8) to (3.12) and one from (3.12) to (2.8).

- $c = 1$ from (3.12) to (2.8), this works
- $1 = c$ from (2.8) to (3.12), this does not work

You have created k-calculus, you have derived LT in it but you can not translate the result back to the real model? Why?

My answer is the following.
No model of reality that uses Lorentz transformations can be without contradiction!

Clarification

We explain once again how the model from k-calculus works. See the next figure.
If we compare this model, M_k, with reality, we get the following:
(all quantities from the model are marked with an index $_k$).

$c_k = c/c = 1$; speed of light in M_k
$v_k = v/c$; the velocity in M_k at which S' moves relative to S
$t_k = t$; the only physical quantity that is the same as in reality
x_k is expressed in light-units, e.g. light-seconds if t is measured in seconds

This forms the basis of model M_k. Light signals transmitted from S to S' move in a line that is 45° to the t-axis. Based on these assumptions that we build into the M_k model, we can calculate all other elements such as distances, time intervals, etc.

Once we have decided which mathematical model to use, only mathematics

applies!

We calculate the distance between S and S' at time t_k.

$$x_k = v_k t_k$$

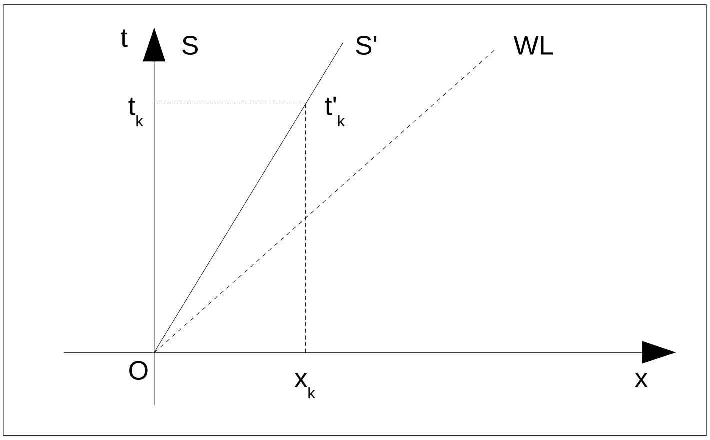

Fig. 7

From the model we see clearly that the time interval in S', t'_k, is greater than the same time interval t_k in S, (only in M_k). It appears from the triangle $Ot_kt'_k$.

$$t'_k = (t_k^2+v_k^2t_k^2)^{1/2} = t_k(1+v_k^2)^{1/2}$$

We see that there is a conversion factor between time intervals in S and S'. But that is not the factor mentioned in the book [1]. Conversion factor is

$q = (1+v_k^2)^{1/2}$ in the mathematical model M_k

$k = ((1 + v)/(1 – v))^{1/2}$ in the book [1]

It is because of this factor k that the contradiction arises in the model!

Conclusion

If we build our mathematical model correctly, if we apply mathematics, physics and logic correctly, then we shall not come to any contradiction!

Similar analysis of different concepts within SR is also done in [3-6].

References

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