Mathematical Formula of the Pauli Exclusion Principle

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Abstract: By studying the phase change of relativistic matter wave in the process of electron collision, the interaction formula of strongly correlated electron system is derived, a mathematic formula of the Pauli exclusion principle is proposed, its validity in superconductor is checked. The predicted superconducting gap is in good agreement with the experimental results of 21 typical superconductors. In addition, the interaction formula clearly shows that at very low temperature, the collision between some electrons and lattice will become neither energy gain nor energy loss, which provides a new insight for the study of superconductivity mechanism.

1. Introduction

In 1911, Dutch scientist H. K. Onnes and others found that mercury resistance disappeared at very low temperature, showing a superconducting state. Since then, the research on superconductivity has been widely concerned. On the one hand, a variety of superconducting materials with practical potential have been found, on the other hand, great progress has been made in the study of superconductivity mechanism [1,2]. Many efforts have been made to elucidate the mechanism of superconductors [3~6]. Scientists have put forward a variety of theories, among which BCS theory and GL theory are more important. However, BCS theory can not explain the reason for the existence of the second type superconductors, especially the limit temperature (the critical transition temperature of superconductors should not be higher than 40K), which has been broken through by the second type superconductors.

Relativistic matter wave provides a basic concept for the study of strongly correlated quantum systems in superconductors[7]. In this paper, by studying the phase change of relativistic matter wave in the process of electron collision, the interaction formula of strongly correlated electron system is derived, a mathematic formula of the Pauli exclusion principle is proposed, its validity in superconductor is checked. The predicted superconducting gap is in good agreement with the experimental results of 21 typical superconductors.

2. Relativistic matter waves in particle collision

Consider a particle in the electromagnetic vector potential A, the relativistic matter wave is given in path integral by

$$\phi = \exp\left(\frac{i}{\hbar}\int_{x_0(L)}^{x}(mu_\mu + qA_\mu)dx_\mu\right) \quad \Leftrightarrow \quad p_\mu \equiv -i\hbar\frac{\partial}{\partial_\mu} - qA_\mu \quad . \tag{1}$$

This is the definition of relativistic matter wave [7], where m is the mass of the particle q, u_μ is the 4-vector velocity field in the particle beam, A_μ is the electromagnetic 4-vector potential, μ=1,2,3,4, x_4=ict; the integral path takes on any mathematical path L from x_0 to x in the particle beam (the L is not particle track, it is a mathematical path in the velocity field).

Now consider two particles 1 and 2, as shown in Fig.1, their relativistic matter waves are given by

$$\begin{aligned}\phi^{(1)}&=\exp\left(\frac{i}{\hbar}\int_{0(L)}^{x}(p_\mu{}^{(1)}+q^{(1)}A_\mu{}^{(1)})dx_\mu\right)=\exp\left(\frac{i}{\hbar}\int_{0(L)}^{x}R_\mu{}^{(1)}dx_\mu\right)\\ \phi^{(2)}&=\exp\left(\frac{i}{\hbar}\int_{0(L)}^{x}(p_\mu{}^{(2)}+q^{(1)}A_\mu{}^{(2)})dx_\mu\right)=\exp\left(\frac{i}{\hbar}\int_{0(L)}^{x}R_\mu{}^{(2)}dx_\mu\right)\end{aligned}. \tag{2}$$

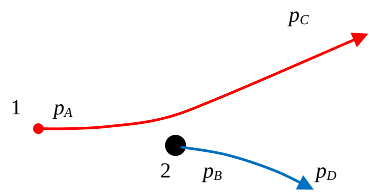

Fig.1 Particle 1 scatters off particle 2.

Where, R=p+qA is known as the canonical momentum in the analytical mechanics[8]. The superscripts denote particle 1 or 2. The two particles with the separation r, provides the Coulomb 4-vector potentials by each other, that are given by

$$\begin{aligned}A_\mu{}^{(1)}&=\frac{1}{4\pi\varepsilon_0}\frac{q^{(2)}u_\mu{}^{(2)}}{c^2r}\propto p_\mu{}^{(2)}\\ A_\mu{}^{(2)}&=\frac{1}{4\pi\varepsilon_0}\frac{q^{(1)}u_\mu{}^{(1)}}{c^2r}\propto p_\mu{}^{(1)}\end{aligned}. \tag{3}$$

According to Eq.(2), the R can be generalized into matrix form

$$\begin{bmatrix}R_\mu{}^{(1)}\\ R_\mu{}^{(2)}\end{bmatrix}=(1+\begin{bmatrix}S^{(11)} & S^{(12)}\\ S^{(21)} & S^{(22)}\end{bmatrix})\begin{bmatrix}p_\mu{}^{(1)}\\ p_\mu{}^{(2)}\end{bmatrix}. \tag{4}$$
$$or\quad R=(1+S)p$$

For a pure Coulomb interaction, the matrix S would be a simple matrix whose $S^{(11)}$=$S^{(22)}$=0 in accordance with Eq.(3); for a complicated interaction, it is easy to prove that the interaction matrix S must be a Hermitian matrix: S^+=S due to $|\phi^{(1)}|$=$|\phi^{(2)}|$=1, see the proof in appendix A.

According to the theory of group, the Hermitian S is a linear combination of Pauli matrix set in terms of SU(2) symmetry:

$$\begin{bmatrix}R^{(1)}\\ R^{(2)}\end{bmatrix}=(1+S)\begin{bmatrix}p^{(1)}\\ p^{(2)}\end{bmatrix}=(1+c_1\sigma_1+c_2\sigma_2+c_3\sigma_3)\begin{bmatrix}p^{(1)}\\ p^{(2)}\end{bmatrix}. \tag{5}$$

where the Pauli matrices (SU(2) group) are given by

$$\sigma_1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\quad \sigma_2=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\quad \sigma_3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}. \tag{6}$$

and c_1, c_2, c_3 are three independent, first order small, real parameters. Now, we discuss every term in Eq.(5) and clarify their physical meanings as follows.

(1) c_1 works, c_2=c_3=0, this case is

$$\begin{bmatrix} R_\mu^{(1)} \\ R_\mu^{(2)} \end{bmatrix} = (1 + c_1 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}) \begin{bmatrix} p_\mu^{(1)} \\ p_\mu^{(2)} \end{bmatrix} . \tag{7}$$

Using Eq.(3) to determine the coefficient c_1, it turns out that this equation represents the pure Coulomb interaction between the two particles.

$$\begin{bmatrix} R_\mu^{(1)} \\ R_\mu^{(2)} \end{bmatrix} = \begin{bmatrix} p_\mu^{(1)} \\ p_\mu^{(2)} \end{bmatrix} + \begin{bmatrix} q^{(1)} A_\mu^{(1)} \\ q^{(2)} A_\mu^{(2)} \end{bmatrix} . \tag{8}$$

In this case, R is simply the canonical momentum of the particles.

(2) c_1 and c_3 work, c_2=0, this case is

$$\begin{aligned} \begin{bmatrix} R_\mu^{(1)} \\ R_\mu^{(2)} \end{bmatrix} &= (1 + c_1 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}) \begin{bmatrix} p_\mu^{(1)} \\ p_\mu^{(2)} \end{bmatrix} \\ &= (1 + c_3 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}) \begin{bmatrix} p_\mu^{(1)} \\ p_\mu^{(2)} \end{bmatrix} + \begin{bmatrix} q^{(1)} A_\mu^{(1)} \\ q^{(2)} A_\mu^{(2)} \end{bmatrix} \end{aligned} . \tag{9}$$

Let us observe their total canonical momentum R, they are

$$\begin{aligned} R_4 &= R_4^{(1)} + R_4^{(2)} \\ \mathbf{R} &= \mathbf{R}^{(1)} + \mathbf{R}^{(2)} \end{aligned} . \tag{10}$$

Substituting Eq.(9) into Eq.(10), we get

$$\begin{aligned} R_4 &= p_4^{(1)} + c_3 p_4^{(1)} + q^{(1)} A_4^{(1)} + p_4^{(2)} - c_3 p_4^{(2)} + q^{(2)} A_4^{(2)} \\ \mathbf{R} &= \mathbf{p}^{(1)} + c_3 \mathbf{p}^{(1)} + q^{(1)} \mathbf{A}^{(1)} + \mathbf{p}^{(2)} - c_3 \mathbf{p}^{(2)} + q^{(2)} \mathbf{A}^{(2)} \end{aligned} . \tag{11}$$

so

$$c_3 = \frac{p_4^{(1)} + q^{(1)} A_4^{(1)} + p_4^{(2)} + q^{(2)} A_4^{(2)} - R_4}{p_4^{(2)} - p_4^{(1)}} , \tag{12}$$

$$c_3 = \frac{|\mathbf{p}^{(1)} + q^{(1)} \mathbf{A}^{(1)} + p^{(2)} + q^{(2)} \mathbf{A}^{(2)} - \mathbf{R}|}{|\mathbf{p}^{(2)} - \mathbf{p}^{(1)}|} . \tag{13}$$

Recall that c_3 is a real coefficient of SU(2) group symmetry, c_3 can be evaluated using the energy exchange in Eq.(12); or c_3 can be evaluated using the momentum exchange in Eq.(13). Both Eq.(12) and Eq.(13) have mathematical singularity: if the numerator is non zero, the denominator of Eq.(12) does not allow the two particles to have the same relativistic energy, otherwise the SU(2) group parameter c_3 will blow up; the repulsion between the two particles will blow up (assume that attraction and resonance make no sense). In the same way, the two particles will not be allowed to have the same momentum vector in Eq.(13). This is just what the Pauli exclusion principle implies. In terms of the above formalism, the Pauli exclusion principle means that the two particles (Fermions, identical) cannot share the same relativistic energy E_r and momentum, i.e.

$$p_4 = \frac{mic}{\sqrt{1-v^2/c^2}} = \frac{iE_r}{c}; \quad p_4{}^{(2)} \neq p_4{}^{(1)}; \quad \mathbf{p}^{(2)} \neq \mathbf{p}^{(1)} \quad . \tag{14}$$

If the two particles are in a overall stationary state, their overall wave function is in the form

$$\phi^{(1)}\phi^{(2)} = \phi(\mathbf{x}^{(1)}, \mathbf{x}^{(2)}) \exp\left(-\frac{E}{\hbar}t\right) \ , \tag{15}$$

while

$$\begin{aligned} \phi^{(1)}\phi^{(2)} &= \exp\left(\frac{i}{\hbar}\int_{0(L)}^{x} R_\mu{}^{(1)} dx_\mu + \frac{i}{\hbar}\int_{0(L)}^{x} R_\mu{}^{(2)} dx_\mu\right) \\ &= \exp\left(\frac{i}{\hbar}\int_{0(L)}^{x} \mathbf{R}^{(1)}\cdot d\mathbf{x}^{(1)} + \frac{i}{\hbar}\int_{0(L)}^{x} \mathbf{R}^{(2)}\cdot d\mathbf{x}^{(2)} + \frac{i}{\hbar}\int_{0(L)}^{x} (R_4{}^{(1)} + R_4{}^{(2)}) dx_4\right) \end{aligned} \ , \tag{16}$$

Comparing Eq.(15) with (16), we find the energy conservation law as follows

$$R_4 = R_4{}^{(1)} + R_4{}^{(2)} = \frac{iE}{c} = const. \ . \tag{17}$$

This is just the physical meaning of R_4, we can infer that $\mathbf{R}$ is responsible for the total canonical momentum conservation. To note that the R in Eq.(12) contains not only the electromagnetic potential A but also other new ingredients, the numerator of the group parameter c_3 should be non zero in general.

The numerator of the group parameter c_3 in Eq.(12) involves the sum of the electric interactions of the two particles between each other; substituting the Coulomb potential of Eq.(3) into it, we get

$$q^{(1)}A_4{}^{(1)} + q^{(2)}A_4{}^{(2)} = \frac{1}{4\pi\varepsilon_0}\frac{q^{(1)}q^{(2)}u_4{}^{(2)}}{c^2 r} + \frac{1}{4\pi\varepsilon_0}\frac{q^{(1)}q^{(2)}u_4{}^{(1)}}{c^2 r} \propto \frac{1}{r} \ , \tag{18}$$

where r is the distance between the two particles. In Fermi electron gas, the numerator of c3 should be associated with temperature T, with combination of Eq.(12) and (13), so that the group parameter c3 is written in the form as

$$c_3 = \frac{H(T,1/r)}{p_4{}^{(2)} - p_4{}^{(1)}} + \frac{M(T,1/r)}{|\mathbf{p}^{(2)} - \mathbf{p}^{(1)}|} \ , \tag{19}$$

For the Fermi electron gas, let experiments determine the numerator factors $H(T,1/r)$ and $M(T,1/r)$. Only observing the particle 1, its matter wave R is given by

$$R_\mu{}^{(1)} = p_\mu{}^{(1)} + \left(\frac{H(T,1/r)}{p_4{}^{(2)} - p_4{}^{(1)}} + \frac{M(T,1/r)}{|\mathbf{p}^{(2)} - \mathbf{p}^{(1)}|}\right) p_\mu{}^{(1)} + q^{(1)}A_\mu{}^{(1)} \ . \tag{20}$$

It is equivalent to the situation in which the particle 1 is posed in a new electromagnetic filed as

$$
\begin{aligned}
&R_\mu^{(1)} = p_\mu^{(1)} + q^{(1)} A_\mu^{(1,new)} \\
&A_\mu^{(1,new)} = \left(\frac{H(T,1/r)}{p_4^{(2)} - p_4^{(1)}} + \frac{M(T,1/r)}{|\mathbf{p}^{(2)} - \mathbf{p}^{(1)}|} \right) \frac{p_\mu^{(1)}}{q^{(1)}} + A_\mu^{(1)}
\end{aligned} \quad . \tag{21}
$$

The first term in $A_\mu^{(1,new)}$ is just what we proposed: the interaction formula of the Pauli exclusion principle. The second term represents the pure Coulomb interaction. The Eq.(21) is called as the mathematic formula of the Pauli exclusion principle.

(3) c_1 and c_2 work, c_3=0, this case is

$$
\begin{aligned}
\begin{bmatrix} R_\mu^{(1)} \\ R_\mu^{(2)} \end{bmatrix} &= (1 + c_1 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}) \begin{bmatrix} p_\mu^{(1)} \\ p_\mu^{(2)} \end{bmatrix} \\
&= (1 + c_2 \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}) \begin{bmatrix} p_\mu^{(1)} \\ p_\mu^{(2)} \end{bmatrix} + \begin{bmatrix} q^{(1)} A_\mu^{(1)} \\ q^{(2)} A_\mu^{(2)} \end{bmatrix}
\end{aligned} \quad . \tag{22}
$$

Since the entries of the interaction matrix S have imaginary numbers, we have to consider the momentum having real-part momentum and imaginary-part momentum as follows

$$
\mathbf{p}^{(1)} = \mathbf{p}^{(1)\mathrm{Re}} + i\mathbf{p}^{(1)\mathrm{Im}} \quad . \tag{23}
$$

In the 4 dimensional space-time, it is

$$
p^{(1)} = p^{(1)\mathrm{Re}} + ip^{(1)\mathrm{Im}} \quad . \tag{24}
$$

In this case, using the perturbation theory, if one only observes the particle 1, from Eq.(22) we have

$$
\begin{aligned}
\mathbf{R}^{(1)\mathrm{Re}} &= \mathbf{p}^{(1)\mathrm{Re}} + c_1 \mathbf{p}^{(2)\mathrm{Re}} + c_2 \mathbf{p}^{(2)\mathrm{Im}} \\
\mathbf{R}^{(1)\mathrm{Im}} &= \mathbf{p}^{(1)\mathrm{Im}} + c_1 \mathbf{p}^{(2)\mathrm{Im}} - c_2 \mathbf{p}^{(2)\mathrm{Re}}
\end{aligned} \quad . \tag{25}
$$

Obviously, the c_2 terms in Eq.(25) stand for a perturbation. The particle 1 whose complex momentum is $p^{(1)}$ can be regarded as a pair of particles whose momenta are $p^{(1)Re}$, and $p^{(1)Im}$ respectively, there is a long story about the pairing mechanism for the dual particles; to cut a long story short, in the author's earlier papers[7], $p^{(1)Re}$ and $p^{(1)Im}$ are connected to spin up and spin down when c_2 terms in Eq.(25) stand for a perturbation.

3. Application to Superconductor

In this section, we will check the validity of $A_\mu^{(1,new)}$ for the Fermi electron gas in superconductor. Consider two electrons, their matter waves with SU(2) group symmetry are given by

$$
\begin{aligned}
\phi^{(1)} &= \exp\left(\frac{i}{\hbar} \int_{0(L)}^{x} R_\mu^{(1)} dx_\mu \right) \\
\phi^{(2)} &= \exp\left(\frac{i}{\hbar} \int_{0(L)}^{x} R_\mu^{(2)} dx_\mu \right)
\end{aligned} \quad , \tag{26}
$$

$$\begin{bmatrix} R^{(1)} \\ R^{(2)} \end{bmatrix} = (1 + c_1\sigma_1 + c_2\sigma_2 + c_3\sigma_3)\begin{bmatrix} p^{(1)} \\ p^{(2)} \end{bmatrix} . \tag{27}$$

The phase angles of the matters waves consist of three interacting parts: (1) the Coulomb interaction between the two particles by the group parameter c_1. (2) the spin interaction between the two particles by the group parameter c_2. (3) the Pauli exclusion principle by the group parameter c_3.

In this paper we only focus on the Pauli exclusion principle formula, leaving the spin coupling effect (Cooper pair) as an opening problem, i.e., in this paper we always take c_2 =0.

Consider two neighboring electrons 1 and 2 on the Fermi energy surface of a superconductor, with energy E_1 and E_2 respectively, if the electron 1 collides with the crystal lattice duo to its thermal motion and hope jump to a higher energy E_3, suppose that $E_1 < E_2 < E_3$, then the electron 1 will get a trouble: its energy will equal to E_2 at some moment when increasing its energy from E_1 to E_3, the Pauli exclusion formula will blow up at that moment due to the denominator being E_1-E_2=0; the new electromagnetic field in Eq.(21) will blockade the jump, because its denominator becomes zero and the Pauli exclusion term blows up to ban the electron 1 having energy across the energy E_2 of its neighboring electron 2, as shown in Fig.2(a).

$$A_\mu^{(1,new)} = \left(\frac{H(T,1/r)}{p_4^{(2)} - p_4^{(1)}} + \frac{M(T,1/r)}{|\mathbf{p}^{(2)} - \mathbf{p}^{(1)}|} \right) \frac{p_\mu^{(1)}}{q^{(1)}} + A_\mu^{(1)} . \tag{28}$$

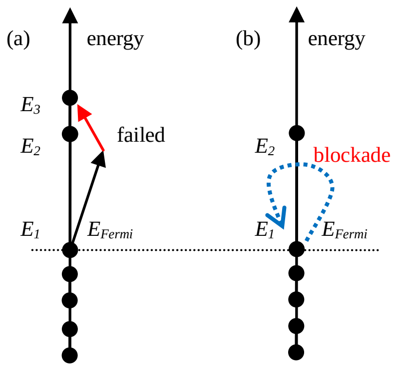

Fig.2 Electron energy levels in the Fermi electron gas.

This is called as the singularity blockade effect. This leads to that the electron 2 blockades any neighboring electrons jumping to higher energies (higher than energy E_2), as shown in Fig.2(b). This singularity blockade effect also means that the electrons on the Fermi energy surface have the energy gap ΔE=E_2 -E_1 in Fig.2(b) which allows electron 1 to do thermal activity, while the electrons inside the Fermi energy surface have no thermal activity because their singularity blockade effect for each other allow only very weak thermal activity (jail effect). The Fig.2(b) tells us that the energy gap ΔE=E_2 -E_1 should be at least the order of thermal energy $3kT/2$ in magnitude to support the thermal activity of electron 1. Considering the kinetic energy distribution of thermal electrons as the Maxwell's distribution, so the energy gap in a superconductor is estimated by the singularity blockade effect as in the range

$$\Delta E \approx \frac{3}{2}kT \sim \frac{6}{2}kT \quad . \tag{29}$$

where k is the Boltzmann constant, the energy gap mounts on the Fermi energy surface of the electron gas, only a few electrons with the higher energy E_2 become local leaders bullying over other neighboring electrons in its vicinity, as shown in Fig.3

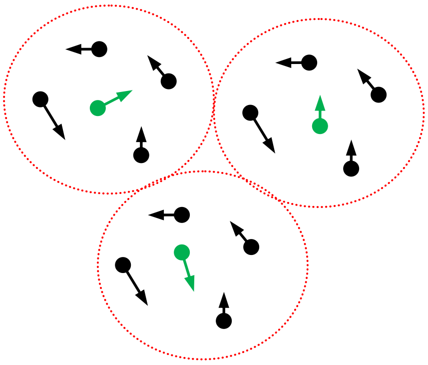

Fig.3 Only a few electrons become local leaders bullying over other neighboring electrons.

How to reduce the Pauli exclusion effect? According to the formula in Eq.(21) and reality of physics, the numerator of c_3 must go to zero if the two electrons separate far apart. Therefore, the numerator can eliminate the singularity of the formula. The Pauli principle only works for the neighboring electron pairs. Another way reducing the singularity is to let temperature go down enough, the strength of interaction between the electrons 1 and 2 becomes so weak that the numerator may go close to zero, as a result, the energy gap on the Fermi energy surface also becomes narrow.

Consider the electron 1 colliding with the crystal lattice with the gain in energy through a phonon

$$E_D = \hbar\omega_D \quad . \tag{30}$$

where ω_D is the Debye frequency of the crystal lattice, by definition, the phonon has the maximal energy in the crystal lattice, we doubt: whether the electron 2 as local leader allows the electron 1 to absorb up the phonon? If the energy gap $\Delta E = E_2 - E_1$ is small than the phonon energy E_D, then, arming with the singularity blockade effect, the electron 2 will ban the electron 1 to absorb the phonon. In order to avoid the singularity of $E_1 = E_2$, the electron 1 will give up the phonons with energy larger than ΔE, which happens at the condition

$$\Delta E < \hbar\omega_D \quad . \tag{31}$$

As the result, the electron 1 has no energy gain or loss during collision with the crystal lattice through touching a E_D phonon; this is equivalent to a kind of elastic collision with the crystal lattice, it means the zero resistance for the electron 1 scattering of the crystal lattice. Therefore, as the temperature falls down to a critical T_c, the energy gap equals to or becomes less than the maximal energy E_D of phonons, some electrons have no energy gain or loss during collision with the crystal lattice under the control of the singularity blockade effect, thus, the superconductivity occurs. The critical temperature is determined by

$$\Delta E = \frac{g}{2} kT_c \approx \hbar\omega_D; \quad g = 3 \sim 6 \ . \tag{32}$$

According to the singularity blockade effect, our calculation of energy gap formula Eq.(32) agrees well with experimental data for 21 typical superconductors[1] as in the Table 1.

Table 1 Comparison of energy gaps for 21 typical superconductors

Material	T_c,K	$2\Delta E$, mV	g=$2\Delta E/kT$	g, this prediction	g, BCS prediction
Hf	0.13	0.044	3.9	3~6	3.53
Cd	0.52	0.14	3.2	3~6	3.53
Zn	0.85	0.23	3.2	3~6	3.53
Al	1.2	0.35	3.4	3~6	3.53
In	3.4	1.05	3.6	3~6	3.53
Hg	4.2	1.7	4.6	3~6	3.53
Pb	7.2	2.7	4.3	3~6	3.53
Nb	9.3	3.0	3.8	3~6	3.53
V_3Ge	11.2	3.1	3.2	3~6	3.53
V_3Si	17.1	5.4	3.7	3~6	3.53
Nb_3Sn	18.1	4.7	3.0	3~6	3.53
K_3C_{60}	19	5.9	3.6	3~6	3.53
Rb_3C_{60}	29	7.5	3.0	3~6	3.53
$Ba_{0.6}K_{0.4}BiO_3$	18.5	5.9	3.7	3~6	3.53
$(Nd_{0.925}Ce_{0.075})_2CuO_4$	21	7.4	4.4	3~6	3.53
$(La_{0.925}Sr_{0.075})_2CuO_4$	36	13	4.3	3~6	3.53
$YBa_2Cu_3O_{7-\delta}$	87	30	4.0	3~6	3.53
$Bi_2Sr_2Ca_2Cu_3O_{10}$	108	53	5.7	3~6	3.53
$Tl_2Ba_2Ca_2Cu_2O_8$	112	44	4.5	3~6	3.53
$Tl_2Ba_2Ca_2Cu_3O_{10}$	105	28	3.1	3~6	3.53
$Hg_2Ba_2Ca_2Cu_3O_8$	131	48	4.3	3~6	3.53

The proposed interaction formula for strong correlated system has a striking advantage: it provides us a new mechanism for explaining why the electrons can collide with the crystal lattice without gain or loss in energy, although this paper has not discussed the spin pair. In the same way, the singularity blockade effect can also be used to explain the super-fluidity at an extreme low temperature: collision without gain or loss in energy, although some are in fermion system (^{3}Hellium liquid, superconductor) while others are in boson system (^{4}Hellium liquid, excitons) [2].

In fact, the singularity blockade effect belongs to three-body problem (e.g. particle 1 and 2, phonon), for which the relativistic matter waves seem to be suitable, rather than traditional quantum wave differential equations. In fact, there are many new properties of relativistic matter wave that remains to be studied[7,9].

4. Conclusions

By studying the phase change of relativistic matter wave in the process of electron collision, the interaction formula of strongly correlated electron system is derived, a mathematic formula of the Pauli exclusion principle is proposed, its validity in superconductor is checked. The predicted superconducting gap is in good agreement with the experimental results of 21 typical superconductors. In addition, the interaction formula clearly shows that at very low temperature, the collision between some electrons and lattice will become neither energy gain nor energy loss, which provides a new insight for the study of superconductivity mechanism.

Appendix A

Theorem 1: The interaction matrix S is a Hermitian matrix: $S^{+}=S$.

Proof: The wave function of the j-th particle is given by

$$\phi^{(j)} = \exp\left(\frac{i}{\hbar}\int_{x_0(L)}^{x} R_{\mu}{}^{(j)} dx_{\mu}\right) = \exp\left(\frac{i}{\hbar}\int_{x_0(L)}^{x} [p_{\mu}{}^{(j)}(\mathbf{x},t) + S^{(jk)} p_{\mu}{}^{(k)}] dx_{\mu}\right), \tag{33}$$

where the duplicated indices imply summation over (Einstein summation convention). We define

$$\begin{aligned} \psi^{(j)} &= \exp\left(\frac{i}{\hbar}\int_{x_0(L)}^{x} p_{\mu}{}^{(j)} dx_{\mu}\right), \\ |\psi^{(j)}|^2 &= \psi^{(j)}\psi^{(j)*} = 1 \quad \text{(no sum over j)} \end{aligned} \tag{34}$$

regarding S as smaller quantities for the interaction, then we have

$$\begin{aligned} \phi^{(j)} &= \exp\left(\frac{i}{\hbar}\int_{x_0(L)}^{x} [p_{\mu}{}^{(j)}(\mathbf{x},t) + S^{(jk)} p_{\mu}{}^{(k)}] dx_{\mu}\right) \\ &= \psi^{(j)} \exp\left(\frac{i}{\hbar}\int_{x_0(L)}^{x} S^{(jk)} p_{\mu}{}^{(k)} dx_{\mu}\right) \\ &= \psi^{(j)}\left(1 + \frac{i}{\hbar}\int_{x_0(L)}^{x} S^{(jk)} p_{\mu}{}^{(k)} dx_{\mu}\right) + O(S^2) \end{aligned} \tag{35}$$

Typically, we demand the matter wave function to meet the normalization:

$$|\phi^{(j)}|^2 = \phi^{(j)}\phi^{(j)*} = 1 \quad \text{(no sum over j)}\ . \tag{36}$$

where

$$\begin{aligned} \phi^{(j)} &= \psi^{(j)}\left(1 + \frac{i}{\hbar}\int_{0(L)}^{x} S^{(jk)} p_{\mu}{}^{(k)} dx_{\mu}\right), \\ \phi^{(j)*} &= \phi^{(j)+} = \left(1 - \frac{i}{\hbar}\int_{0(L)}^{x} [p_{\mu}{}^{(k)}]^* [S^{(kj)}]^* d[x_{\mu}]^*\right)\psi^{(j)*} \end{aligned} \tag{37}$$

The transpose operation used in the above expression is a preparation for the consistency of matrix calculation in it, we have

$$
\begin{aligned}
&|\phi^{(j)}|^2 = \phi^{(j)*}\phi^{(j)}|_{\text{(no sum over j)}} \\
&= \left(1 - \frac{i}{\hbar}\int_{x_0(L)}^{x} [p_\mu{}^{(k)}]^* [S^{(kj)}]^* d[x_\mu]^*\right) \psi^{(j)*}\psi^{(j)} \left(1 + \frac{i}{\hbar}\int_{x_0(L)}^{x} S^{(jk)} p_\mu{}^{(k)} dx_\mu\right) \\
&= \left(1 - \frac{i}{\hbar}\int_{x_0(L)}^{x} [p_\mu{}^{(k)}]^* [S^{(kj)}]^* d[x_\mu]^*\right) \left(1 + \frac{i}{\hbar}\int_{x_0(L)}^{x} S^{(jk)} p_\mu{}^{(k)} dx_\mu\right) \\
&= 1 - \frac{i}{\hbar}\int_{x_0(L)}^{x} [p_\mu{}^{(k)}]^* [S^{(kj)}]^* d[x_\mu]^* + \frac{i}{\hbar}\int_{x_0(L)}^{x} S^{(jk)} p_\mu{}^{(k)} dx_\mu + O(S^2)
\end{aligned} \quad . \qquad (38)
$$

We know

$$
[p_\mu{}^{(k)}]^* d[x_\mu]^* = p_\mu{}^{(k)} dx_\mu \quad . \qquad (39)
$$

thus

$$
\phi^{(j)+}\phi^{(j)} = 1 + \frac{i}{\hbar}\int_{0(L)}^{x} [-S^{(kj)*} + S^{(jk)}] p_\mu{}^{(k)} dx_\mu \quad . \qquad (40)
$$

The integral path is an arbitrary mathematical path; therefore, the normalization of ϕ leads to the conclusion

$$
\begin{aligned}
&\phi^{(j)+}\phi^{(j)} = 1 \quad \to \quad -S^{(kj)*} + S^{(jk)} = 0 \\
&S^+ = S
\end{aligned} \quad . \qquad (41)
$$

Proof finished.

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