

Gravity

Classical and Quantum Physics: *how-ever the twain meet.*

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Abstract

In this 3rd paper of a triptych in quantum mechanics, a description of quantum gravity is derived bottom up from potential theory of the scalar potential $\Phi(r)$ for validity at the Planck scale and validation top down using Einstein's general relativity results in gravity.

The description on quantum level results in an exact and identical description in the transformed curvature frequency (k, r) domain of the *Schwarzschild* and *Droste* metric and yields identical *curvatures* derived in the classical tensor description of general relativity of A.Einstein, i.e. where the twain meet.

The derivation (§4.4) does not provide support cq. mathematical evidence of space *deformation* by gravity field components (applicable in spacetime as well with a proper definition of time as *true* 4th dimension, however is not subject of this paper); deformation is in geodesic trajectories caused by properties of objects i.e. quantum particles and massive bodies, exposed to gravity fields caused by mass in space.

The treatment requires support of dimensions on quantum level in the Standard Model of fundamental particles e.g. in field descriptions; when in the mathematical description rigorous conditions for validity on quantum scale are accounted for, quantum-theory yields deterministic results in the transformed frequency domain with validity down to the Planck scale. Therefore, instead of statistical expectation values acquired in the natural (r, t) domain, the fields can be described with *exact* functions in the transformed mathematical domain.

The gravity field is a potential field interacting on quantum scale with mass as property of elementary massive particles, acquired by the Higgs mechanism. The field is emergent from two fundamental vector fields at spatial locations r with *scalar* field values, §4.

The quantum particle of gravity is argued to be a *mediating scalar gauge boson* of the Higgs-type instead of a force-carrying boson, by mediating the gravity force F_G locally on the mass property by the field values (scalar magnitudes) of the gravitation field. F_G then is a (very) weak fundamental force, generated on quantum scale with visible effects in the entire universe on extreme scale.

In the field descriptions (§4), 1.) *no support* is found for *singularities*; by introduction of dimensions and mass density on quantum Planck scale e.g. in the standard model (SM) as well as on all other scales, *singularities fully disappear*, §5. In the field description, 2.) evidence is found that singularities at r_s / r - appearing as well in the derivations by K. Schwarzschild and J. Droste of Einstein's tensor treatment - can be and are likely to be introduced by violations of *variable validity* and include description of far-field approximation (-terms) in equations for $r \rightarrow \infty$ in the field, e.g. in mathematical integrated *combinations* of natural- and transformed- domain mathematics, see §4.4.1 eq. (4.7a).

1. Introduction

Consistent descriptions connecting the results of Einstein's tensor treatment of gravity in general relativity with quantum physics have not surfaced despite major efforts in e.g. string theory, brane theory, manifold descriptions, holographic scenario's, entropic force proposals as well as attempts in mathematical non-Abelian groups and more.

The classical approach in strong gravity fields i.e. massive objects e.g. black holes, is to describe trajectories on surfaces by Killing -vectors (Wilhelm Killing, Germany) and vector fields as tensor metrics on a manifold. This approach e.g. often uses Lie algebras and matrix mechanics which both pose substantial restrictions in the use of these mathematics when this description is to remain valid in the quantum realm. Furthermore, the use of Killing vectors for descriptions of e.g. scalar fields is not found to required, even when the fields originate from vector fields.

The derivation by both K. Schwarzschild and J. Droste (1916) [21] of the field of gravitation and particle motion is a most ingenious as well as elaborate, however cumbersome trajectory (in the meaning of difficult to handle or to take into separate domain for near- and far-field description) to exact solutions of the Einstein tensor analysis, and seems directed to planetary motions and its corrections. It e.g. is including non-Abelian matrix mechanics and non-symmetrical (far-)field approximations with *many* substitutions and as well yields singularities where they are *unlikely* to exist e.g. possibly introduced by multiple substitutions of variables in the mathematics of the field descriptions (§4.2, §5). Terms in equations with $1/r$ may indicate curvature domain i.e. in a field description indicating zero curvature for $r \rightarrow \infty$ i.e. m (testmass) outside the field or in extreme weak field of M , which points at variable validity violation as well, or as J.Droste [21] phrased it: "this <variable> r is not the same as occurs in (4)".

In *classical physics* the result is considered exact, however from quantum mechanical (QM) perspective, the Planck scale behaviour (near zero, curvature, singularities) and the non-Abelian mathematics (even only partially), don't acquire full validity on this scale; and when with non-commutative operators in descriptions, a different approach is required.

This prompted the (Poisson related) direct scalar potential approach taken into the complex vector space and Green's function based system-model as a *starting point*.

A quantum mechanical description of gravity then is proposed as a *causal relation* in the natural and transformed *domains* by the curvature as frequency-intensity parameter i.e. the transformed (k, r) domain. In this domain mathematically exact result functions in quantum system causality descriptions can be derived instead of statistical probability functions, e.g. as outlined and illustrated in [17, 20]. Therefore, all natural domain mathematical (r, t) descriptions resulting in expectation values in a statistical, or non-Abelian group treatment are not suitable for exact results in qm.

Mass, causing curvature by gravitation is modeled by the frequency $(1/r, \text{ at location})$ of *curvature occurrence* at location r i.e. in the (k, r) domain. The circle frequency $\kappa = 2\pi.k(r)$ with $k(r) = 1/r$. In contrast with treatment in the natural (r, t) domain prone to mathematical and in (experimental) reality physical collapses with expectation values, the descriptions in the transformed frequency domain do not violate uncertainty relations in quantum properties and retain full validity in the space domain.

In the treatment on quantum scale, *stringent conditions* apply in the mathematical structure at the current local energy level: the theory is formally restricted to Abelian groups (Nils Henrik Abel, 1802-1829, Norway; Mathematician) e.g. algebras, operators, vector + scalar -spaces, -fields and requires *operator commutativity*, a fundamental condition. Where a classical approach top-down is used for a mathematical description in the quantum realm, the mathematics therefore must be *validated* for compliancy.

Mass is considered a property of a quantum or classical object. On quantum scale, creation of mass is on a much higher energy level and is caused by (spontaneous) symmetry breaking [22, 23] resulting in status quo states at lower energy level. Generation of mass is manifest by a Higgs' (type) boson coupling to other elementary particles e.g. quarks, resulting in mass of elementary quanta i.e. the source of a gravitation field distributing in space. This field as function of location settles instantly in space at max. c m/s.

Matter then is the result of concentrations of elementary particles of energy as result of coupling and forming elementary quanta with mass, accounting for mass -density and -dimension at quantum scale. Although the creation has taken place at much higher average energy levels than in the universe currently, this level had to be achieved in experiments (LHC) with proton experiments being carried out to find the Higgs particle. The found 125GeV is established as the value to acquire mass of the proton, consisting of three quarks; less massive fundamental particles may require less energy, i.e. more Higgs-type particles might exist.

The creation of mass on quantum level in the model then is taken as a system source that interacts (couples) via a Higgs mechanism-function as cause, resulting in a massive elementary particle. The mass generating effect for elementary particles resulted instantly in gravity potentials around them and in different local conditions, chemical elements and matter could be created at suitable energy levels.

The basis of the quantum gravity field then is identified in two *vector-fields*, and from these fields, in pairs of vectors for locations in space, a gravity source description with pre-determined fieldstrength by inner vector (i.e, scalar) product is derived.

Gravity thus is considered to evolve in space in a *causal system-theoretical description* i.e. then constitutes *the description of gravity effects in space* starting from quantum sources.

Top-down, this paper takes the classical results of the Einstein based solutions of the Schwarzschild radius as starting point i.e. shown in the in §4.4 derived identical curvature results; then treats singularities upscaled, in general avoiding algebraic associations with variables addressing individual quanta i.e. quanta in curved space or manifolds directly or by integral domain transformation.

In [20] is argued that individual quantum behaviour remains hidden from observation or measurement and this includes for example degrees of freedom or internal energy of quanta. Side-effects and other indirect information of experiments can be gathered to reveal quantum behaviour, e.g. propagation of quanta in materials [20], and LHC high energy experiments. This treatment of gravity does not provide support for the hypothesis of the occurrence of (*naked*) *singularities* in large enough massive objects, nor in the abstract point-source of mass (§4.3) with a

field of gravitation around it; the generic results are as well independent of (properly defined) coordinate systems.

2. The Higgs (-type) boson

The Higgs boson has been predicted since 1961 by Higgs [22], Brout and Englert [23]; as a free detectable particle has an extreme short time to live at current energy level. Experiments with the LHC in 2012 revealed the existence of particle (energy app. 125 GeV) and the achievement is being regarded a formidable breakthrough.

The Higgs field is considered to have a permanent ubiquitous invisible presence in nature, however this starting point does not identify a cause (source) for the field. A source should be right at the basis of a spatial field distribution and 'no-source' is postulated not being in line with the fundamental causal evolvement of (events in) nature. The obvious source is the 'big bang' in which the distribution of energy and consequently of mass quantum particles likely was far from uniform. Mass can be regarded as an energy state of e.g. concentrations Higgs coupled elementary particles leading to e.g. protons at the energy level to form nucleï and to other Higgs type particles in less massive quantum elementary particles, the basis for different types of matter. Where concentrations and energy levels were favourable enough, currently visible matter has been created, estimated around 5% of total mass in the universe.

Assuming that 100% of quantum mass has been created during the 'big bang', then 95% would have be around still in quanta, at creation less concentrated and without favourable conditions to start forming elements, and *not ever aggregating anymore* at current energy level and scale in 'empty' space.

The assumption is that on quantum level, once a boson couples to form standard model elementary particles with mass, with *all* the generated entities/particles instantly gravity fields were created, and matter and gas clusters could be formed with elementary particles at locations where concentration and energy levels were favourable.

The gravity field generated from the Higgs field(s) filled space at max. c m/s, i.e. each coupled Higgs boson is a source of mass generation by interaction, and the therewith created gravity field distributes instantly in space. The total energy of the massive particle is contained in the field and mass of the particle, the field decaying with distance, and the total energy is limited to the Higgs energy or less when other less massive elementary particles are created.

Its maximum field value is assumed to be located in the centre in free space directly at the surface of the Higgs coupled (dimensional) particle, and decays losing density by expansion on a spatial spherical surface.

As the Higgs mechanism interacts with elementary particles acquiring mass, which particles make up elementary SM particles, *quantum gravity sources* therefore consist of discrete quantum masses i.e. mass as property in matter is quantized. This applies to the total particle energy value of mass and field. The fieldstrength represents the potential energy as function of the dimension r in space i.e. by dimension is not quantized.

The potential field on quantum level reacts identical as argued (see §4, massive black hole M), by storage of the energy of an external particle m_1 entering and moving in the field with a velocity component in the direction of the source m_0 , meaning that energy is given to the particle and vice

versa. There are 3 options, 1.) the particle escapes by taking a trajectory escaping the source, returning the taken field energy of the conservative field, or 2.) interacts, meaning that all of its own energy + the energy taken from the field are coupled to form a massive particle, illustrating *energy exchange* i.e. the interaction of the potential field and mass on quantum level.

The 3rd option of orbiting by gravity is rejected due to the weak quantum gravity fields and required extreme curvature value required at the Planck scale, i.e. a stronger force is required to allow orbits at this scale, obviously the electromagnetic force between protons and electrons.

The foregoing fundamentally means that the gravity mechanism is valid at *extremely different* scales: the potential field (temporarily) loses energy and gains the same amount later-on by particles or massive objects, when *not* aggregating at quantum level or merging with e.g. a black hole. An established interpretation in quantum physics is that the particles are a manifestation of excitations of a (Higgs) field in the structure of/with other particle fields, and a more classical view is that the Higgs particle interacts with other standard model particles to provide mass in building blocks of matter, while in this paper the cause = boson coupling *status quo* as the source, and the effect c.q. result = mass + gravity field i.e. a fundamental causality relation in a system-theoretical, description is proposed.

This system theoretical approach fully *contrasts* with the unitarity of the Schrödinger wavefunction of Hamiltonian evolution of quantum states, as it *transfers* by transformation ****the evolution to *cause* and *effect* mathematical (r, t) relations in the integral *transformed domains* leading to *exact functions* instead of probability relations: which is the basic reason to explore and use *different mathematics* in quantum physics e.g. in gravity as presented here. This approach increases the distance to - and minimizes anomalies created by - *interpretations*. In the 2nd paper [20] of the triptych, information is considered 'ordered' energy (that may become fully dis-ordered e.g. in a black hole), arguing that 'information' only can be contained/preserved in some form of memory i.e. atoms, molecules, dna, (human) brain, books, computer chips etc. as argued in [20], taking away the sting of the information paradox in black holes.

Whichever interpretation is the right one, in this *system model* of quantum *causal* relations is not an issue [20], as in the invisible reality of quanta, the energy and transitions of individual quantum particles cannot be observed (quanta remain 'invisibles' with regard to their original state) i.e. the description of the phenomena is hypothetical and based on thought experiments and assumptions, abstracted in the mathematics, and is to be *verified* by experiment.

Quantum behaviour thus is described in systems-theory [11, 13, 14] by a source and a result function in a causality description which identifies this relation only by cause- and effect-functions (system input-output relations).

This paper thus fundamentally considers Higgs-type bosons as the causal quantum of mass, and assumes spatial dimensions at the Planck scale i.e. in the standard model SM. The Higgs' type boson coupling creates mass and field, manifest in a natural domain. A transform of the mathematical description into the transformed domain [11, 12], yields quanta based mass distributions in the frequency (k , r) domain, valid in space as in classical physics. Then results finally link with the results of Einstein's tensor-analysis of gravity.

First, attention is being paid to the standard model.

3. The Standard Model & Dimensions

Mass of fundamental particles of the standard model (SM) e.g. an electron or a neutron, is accepted as a property, without a definition in space apart from location.

Standard model particles are usually considered as entities without dimension, which - as in this paper is assumed - is a drawback of the model occasionally.

Therefore as a starting point of the description, elementary particles are not regarded dimensionless and are considered in reality to '*take up some amount of space*', i.e. in a 3 dimensional 3D description. A second reason for 3D is that gravity does not give rise to very dynamic events at quantum level requiring a mandatory 'spacetime' fundamental description of the nature of gravity. At current energy level quantum gravity dynamics are low, however in (very large scale) galaxy centres of extremely large black holes and mergers of large massive objects, this requirement changes. However, the vectors in definition of the source of the gravity field can be complex, therefore e.g. *phase-differences* of the field relative to a source when required can be accounted for, e.g. with local time as *parameter* (when not yet defined and implemented as a true dimension). Three dimensions in order to expand, or rather shrink, i.e. *define* dimensions in the quantum realm are in this paper fundamental to be described mathematically in emergent gravitational fields from QM-perspective.

As a consequence, the property 'mass' is to be described in terms of mass-density i.e. specific mass property with a *volume, surface or line-curve* in a dimensional description.

For quantum particles in the standard model in 3D-space, the description of mass m is substituted with a product of *specific-mass* m_s (kg/m^3) mass density and its dimensional volume V (m^3), i.e. $m = m_s \cdot V$ (kg) as treated in §4.

This at first sight does not seem to have substantial impact in the SM or in classical physics where a massive object is characterized with mass M , however has the effect of fundamentally changing the perspective in physics as *dimensions* are *embedded* on *all* scales of the description.

4. Field Description

An electromagnetic description requires 2 fields in a vector environment of E and H i.e. both with attracting and repelling components *and* both have direction and magnitude and as well include potential E-field theory (showing the brilliance of J.C. Maxwell). This is mandatory in EM fields as energy transport is required to be described (i.e. RH system with Poynting vector in classical EM theory and by a vector *gauge boson* in the SM). A description of gravity in a unified field theory therefore fails in case the field of gravity is a scalar field: it does not (need to) transport energy to be converted at large distance, only conserves and returns energy and with 1 component of mass is attractive only. Gravitons as carrier of a gravity force in analogy with photons of the electromagnetic force, thus never have been found.

A deviating hypothesis for the influence of gravity in space is postulated by identification two basic *fields* in (complex) vectorspace with a resulting field manifest with *scalar* complex fieldstrength values of the potential conservative field, illustrated by a non-rotating, non-charged black hole.

By introduction of mass density in the dimensional description in §3 - to explore very small or even

quantum black holes - mass density would have to be extreme i.e. beyond limits of required energy of creation to be supported and is therefore *rejected*: from eq. (4.4) and eq. (5.1) follows $m_{DS} = 3 \cdot c^4 / (16\pi \cdot G \cdot a_s \cdot r_s^3)$, with a_s acceleration at Schwarzschild radius r_s (order of magnitude $r_s \sim 10^{-35}$ m), m_{DS} specific mass of quantum, c (m/s) vacuum speed of light: i.e. black holes only exist at *star scales*.

The black hole then is a massive object not ‘radiating’ other energy than its established field of gravitation (except Hawking radiation; which is not considered to influence a semi-static model and is neglected), and has restricted energy transport capabilities. In a description of transporting force-fields, a carrier boson is required as the transported energy may be used in transformation of energy far away from the source e.g. for recovery of information i.e. ordered energy or in strong fields, e.g. lightning strikes. The black hole has a gravity field attracting mass in space and therefore is not truly black with respect to mass and gravity, meaning that an energy exchange i.e. temporary energy transport exists between the massive hole and mass particle. Obviously this cannot be of em-nature radiated from the hole, and with the gravity field attractive for all types of energy, the energy is contained as well in the gravity field outside the massive object core as a direct function of its mass M .

This field therefore is capable of restricted exchange of energy with particles having mass or energy (equivalence relation), leaving a particle in the field with only 3 options: it can escape the source M , is attracted with motion towards the hole in the entire trajectory and collides with M , or the particle can orbit a massive black object.

In case it escapes, as soon as distance to the hole increases, it *returns* the field energy it gained in reducing distance and when captured, all of its energy including the gain from the field is delivered to the hole i.e. the field energy is returned and by increase of captured mass in the core, also the total energy (and horizon) of the hole increases.

For photons in the field of a massive object, with constant $v = c$ (m/s), this means a change of wavelength in mass equivalent energy $h \cdot v$ i.e. uv-shift in approaching M and red-shift when distance to M increases; for an observer of an image of the ‘lensing’ effect, the shifts are not observed as energy is exactly returned.

In case of a black hole the temporary lost energy cannot be compensated by em-radiation.

The conclusion therefore is that the energy temporary must be delivered by motional changes, the momentum or by rotation; the hole experiences a tiny force by acceleration of the field of m in the direction of m , which is more obvious to an observer when m is in the order of magnitude of M : the field is able to re-actively exchange energy i.e. a conservative potential field.

4.1 Acceleration and Force

The field of acceleration usually is considered to be the cause of ‘action at a distance’ by gravity; geo-metrically the fieldstrength values at a location \underline{r} derived on quantum scale in the qm-perspective are *pre-determined* in an acceleration field, i.e. locally the fieldstrength is determined in space and the field values are the manifestations of the established field where local mass is present, thereby mediating the gravity force F_G experienced by mass at the particular location.

The acceleration vectorfield however is argued in §4.2 *not* to be the source field of gravity.

Still, the force of gravity is not ‘*carried*’ but is truly ‘*mediated*’ on a local mass property in space i.e. the boson-coupling in *local mass* m with the local fieldstrength fundamentally results in the *local* gravity force F_G on mass, and is manifest as force directly after the fields of M and m are established in space, which is assumed for both m and M to distribute directly after mass creation at quantum level.

The ‘graviton’ as quantum of gravity thus constitutes a *mediating boson type* .

Particles have different masses and may require different coupling energy, therefore it cannot be excluded that several types of Higgs bosons exist, and if so, all *with* mass.

Higgs boson: $E = h \cdot \nu = 125 \text{ GeV}$, equivalent mass $125 \text{ GeV}/c^2$; the energy value is relatively high as it provides the energy to create mass of elementary particles e.g. of quark-composite protons in the LHC experiments.

As the gravity field cannot act as fundamental source of *energy*, the *energy* only in restricted way can be transported by the field (although astronomy reports have been made interpreting energy transport by gravity fields by assumed loss of energy in interactions of heavy objects); comment author: (free, independent) transport may be a wrong description; conserved potential field energy in the strong gravity fields converting into kinetic energy of motion may be a consistent explanation; gained energy from the field is temporary and is returned. The field is conservative i.e. gravity ‘uses’ mass *and* field to exchange energy and the trajectory closed integral in the end equals zero: $\int_C F \cdot dS = 0$.

The property to exchange re-actively and locally internal energy of the potential field is of major importance in (quantum) physics, for example in local temporal energy exchange with photons enabling propagation in materials [20].

The gravity force F_G thus is a fundamental force manifest by a potential fieldstrength of acceleration magnitudes of a quantum gravity source, and is the one (and only) force with the most significant visible impact in the universe, and unlike other fundamental force-vectorfields, the quantum boson is a *mediating scalar boson*, i.e. F_G is not being ‘carried’ by a vector boson.

The gravity potential field thus *acts* as an acceleration *scalar*field, and this is because matter consists of two inseparable components of energy: mass and field.

The cause (coupling of Higgs boson presence in SM elementary particles and matter) of the potential field is a reality description, as gravity then clearly is emergent from concentrations of standard model elementary particles coupled to Higgs type bosons, as well is consistent with a gravity forces’ full causality description *by geo-metrics only*, at all quantum short and classical long range distances i.e. in a gravity field *instantly* resulting in a force on local matter/mass, thereby providing as well the link with the classical Einstein tensor-analysis results as well as the conclusion of a *mass* graviton boson.

4.2 Gravitational Potential

A potential at a certain location in space may be used to exchange energy in case the field of potentials has a rate of change in space. The affected energy is related to the property of the field i.e. electrical, gravitational.

A mathematically proper description of a potential field is derived from potential-theory, taken into

a vectorspace description and is started with a *point source*; a further step is illustrated by the introduction of dimensional mass of certain volume and density as realistic source.

For an energy description on quantum level, our interest is thus in the decaying rate of the potential field as a function of vector \underline{r} , having magnitude and direction i.e. the *vector*field partial derivatives in space of a quantum mass source.

The gravitational *potential* Φ of a differentiable scalarfield function of a symmetrical 3D entity with mass M is given by potential theory:

$$\Phi(\underline{r}) = G.M / r \tag{4.1}$$

The description is in magnitudes without sign; the directions become clear in the vector field description. To observe the change of the potential vectorfield in a direction in space, ∇ i.e. the Nabla operator represents the partial differentials in vectorspace: $\nabla = (\delta/\delta x)\underline{i} + (\delta/\delta y)\underline{j} + (\delta/\delta z)\underline{k}$, resulting in the corresponding vectorfield $\nabla.\Phi(\underline{r})$ [19], i.e. acceleration field $\underline{a}_G(\underline{r})$

$$\underline{a}_G(\underline{r}) = \nabla.\Phi(\underline{r}) = \nabla.(G.M/r) = G.M \underline{r} / r^3, \quad \text{re-written as}$$

$$\underline{a}_G(\underline{r}) = G.M. (\underline{r} / r).(1 / r^2), \quad \text{for all } \underline{r} > \mathbf{0} \tag{4.2}$$

With ∇ the Nabla operator, \underline{r} the position vector, \underline{r} / r unity vector, $\Phi(\underline{r}) = G.M/r$, G the gravitational constant, M the (point-)source of the potential field.

Nothing new under the sun, however (4.2) is started in vector space and the derivation is to be formally completed in the vector space, basically by respecting the cause and effect relations: source is the settled field of M providing acceleration values, resulting in the acceleration vector field leading to the force on m in direction of the origin of \underline{r} i.e. in the source. The result is a *vector inner product*.

Equation (4.2) identifies the *gradient* of the *vector field* which physically is the change rate of potential Φ in the direction of vectors \underline{r} . The force \underline{F}_G exerted by the field on a testmass m then is $m.G.M. \underline{r}/r^3 = m.\underline{a}_G(\underline{r})$. The description is finalized in the vector space by facilitating the acceleration $\underline{a}_G(\underline{r})$ vectors as mediator of the gravity force experienced at r in the opposite direction of unit vector \underline{r} / r , by means of the dot (inner) vector product $(1/r^2) \underline{r} / r . \underline{a}_G(\underline{r})$, resulting in *scalar field* values.

Just for the record: a cross product (yielding a vector field) 1.) is not commutative i.e. not suitable for acquiring validity at quantum-scale, and 2.) would require a *non-existing* second component in gravity, e.g. curl for $\delta M(\underline{r})/\delta t = \nabla \times X$, i.e. X as 2nd component and 3.) would yield the zero vector.

The dimensional interpretation of (4.2) is that $(1 / r^2)$ physically relates to a spatial spherical expanding surface reducing the gravitational field density, which accounts for the field decay, and $\underline{r} / r = \underline{r} / |\underline{r}|$ as unit vector determines direction of $\underline{a}_G(\underline{r})$ i.e. in the opposite direction of \underline{r} .

In the end - perhaps surprisingly - by the mathematics formally continued in vector space, the result is a scalar field derived from 2 vectorfields, in contrast with an often assumed vector acceleration or force field. Before moving on with the derivation, it is worth noting that equation (4.2) taken into

vectorspace, in principle has similarity with the *original* summation description of gravity of Newton (!) [..], at the time being considered not to be solvable under generic conditions and mass distributions; the validity of the equation is for $r > 0$, and the equation is *exact* (!) under fully symmetrical conditions.

The value of the field as function of r is thus being provided by each *pair* of vectors of the two vectorfields in making up the inner product of $\underline{\nabla} \Phi(r)$ and \underline{r}/r , then yielding a *scalarfield of fieldstrength* $\underline{a} \cdot (\underline{r}/r) = a \cos \pi = -a$, i.e. acceleration values in opposite direction of \underline{r} :

$$\underline{\nabla} \Phi(r) = -\underline{a}_G(r), \quad 4.3$$

$G.M. (\underline{r}/r) \cdot (1/r^2) = (G.M/r^2) \cdot (\underline{r}/r)$, i.e. with unity vector $|\underline{r}/r| = 1$, the field magnitude value $a_G(r)$ is

$$a_G(r) = G.M/r^2 \quad 4.4$$

Two conclusions can be drawn from equation (4.4):

1.) Instead of a direct force, the acceleration field magnitude values $a_G(r)$ act as mediator of the force i.e. a deterministic scalar field of acceleration values expanding in space directly after creation of mass on quantum level. This scalar field is the result of two vectorfields, for each spatial location delivering one pair of vectors $\underline{\nabla} \Phi(r)$ and unit vector \underline{r}/r , in the commutative vector inner product i.e. by the *commutative* property of this product, eq. (4.4) fully *acquires validity* in the descriptions at quantum Planck scale and therewith is mathematically consistent *at all scales*.

As argued, the temporary energy exchange i.e. both the interaction ($M \rightarrow m$) of (external) mass m in the potential field of gravity of M and vice versa ($m \rightarrow M$) relate to motion -changes of mass e.g. acceleration by ‘give and take’ (or ‘take and give’, depending which side you’re on) energy exchange with the potential field, which can be accomplished by *local bosons interacting* with the field i.e. the action of acceleration value resulting in force on quantum scale is immediate. The gravity field then emerges in space as a scalarfield of acceleration fieldstrength values $G.M/r^2$, and therefore the force F_G is a direct effect of cq. caused by coupled scalar bosons interacting with the field; the only scalar boson found to date (2023) is the Higgs boson, and its quantum is postulated to be a *scalar gauge Higgs -type boson*.

The field has the property of expanding in space directly at creation of the source, i.e. the acceleration values of the field emerge and are manifest as the *magnitudes* of the vectorfield i.e. $|\underline{a}(r)|$ and therefore the conclusion arrived at is: the quantum boson of the gravity scalar field is a *mediating Higgs(-type) scalar gauge boson, spin 0*,

(deviating from the classical prediction of spin 2, result of the stress tensor).

The gauge is a function of r and any measurement of the value (or force on mass at this position) in space yields *the predetermined* value, whichever mass (M, m) is applicable as source.

This Higgs-type boson then constitutes the graviton as mediating scalar gauge boson.

The second conclusion is:

2.) Equation (4.4) $a_G(r) = G.M/r^2$ is the field description of a point source with mass M i.e. without dimension, and at first sight yields two *singularities* at $r = 0$.

This is exactly where the dimensionless point source of gravity is defined; i.e. an example of *variable validity violation* (vvv), as the mathematics state $r > 0$, i.e. *a priori* excluding the location of the point source, in fact excluding a most interesting part: the source of gravity.

This illustrates the importance of introduction of dimensions on the Planck scale by substitution of M by specific mass density M_s in a (e.g. spherical) volume. By a tighter definition (§4.3) of the source dimension, singularities *disappear* i.e. meaning that they in fact in physical reality do not exist as validity is $r > 0$ in the straightforward mathematics of the point source.

Abandoning the point source by introducing e.g. a spherical volume $V = 4/3 \pi.r^3$ ($r_{\max} = R$) and specific mass density M_s , the substitutions applied in (4.4) yield for the fieldstrength

$$a_G(r) = 4.\pi.r^3 \cdot G.M_s / 3r^2 = 4.\pi.r.G.M_s/3 \quad \text{for } 0 \leq r \leq R \quad 4.5$$

Valid for $r \geq 0$ inside the source with radius R and yields an expected acceleration $\mathbf{a}_G(\mathbf{r}) = \mathbf{0}$ by gravity of M at $\mathbf{r} = \mathbf{0}$ in the centre of M , increasing *linear* towards the surface at R of the core (see §4.3, *point source*), decaying for $r > R$ by $G.M/r^2$, and thus in principle include the radius of curvature where photons cannot escape i.e. the Schwarzschild radius.

4.3 System sources, singularities and normalization

One of the sources below can be used, as a point-source is a mathematical abstraction causing anomalies.

Point source

Elaborating on conclusion 2.) in §4.2 :

At $r = 0$, the mathematics would be required to provide all values of the fieldstrength between 0 and infinite, i.e. is *undefined* at the location of the source, therefore is excluded a priori by validity of variable r : $r > 0$. This can be solved in two ways, 1.) by including room for the fieldchange e.g.

$0 \leq r \leq r_1$, evolving e.g. with $f(r) = C.r$ $C = \text{constant}$, treated in §5 and eq. (4.5), by including a sphere around $r = 0$ with a mass density in the volume $4\pi r^3/3$, i.e. includes $r = 0$ in the variable validity: $r \geq 0$., or 2.) by definition of the field value at $r = 0$ e.g. by a generalised Dirac function, the Dirac source.

Dirac Source

The Dirac source in system-theory is convenient as it describes the normalized field generation in terms of an ideal pulse (creation) of mass in the natural domain by the Dirac function: $\delta_m(r, t) = 1$ at r , valid at $t_1 = 0$ (i.e. in the ‘big bang’). In the transformed (k, r) frequency domain i.e. the curvature domain, the amplitude at all the frequencies i.e. of curvature yields ‘1’ i.e. an equally *ideal* uniform distribution in the transformed domain. By considering a *non-ideal source* of e.g. a spherical volume around

$r = 0$ instead of a Dirac function at $r = 0$, 1.) room is created for the function and 2.) it is possible to arrive at the non-ideal frequency distribution in terms of curvature in space (i.e. underlying the proposal in this paper).

Singularity

In case of the point source field, the singularities are $v\bar{v}$'s and don't have a physical meaning, and as well can be removed by either one of the two proposed possibilities in §4.3, illustrated in the classical description in §5.

This as well can be shown for e.g. the *Kretschmann invariant* [25], a description with 2nd derivatives of the Einstein tensor metric, where r^6 drops out from the denominator when substitution for M is applied as illustrated in §5 - the singularities disappear.

The foregoing as well shows that by expanding to a realistic source, the distribution in the transformed domain of the non-ideal *i.e. a physical source*, is 'shaped' i.e. shown as *deviation* from a ideal value '1' in the variable *amplitude* (intensity) in occurrence of *curvature* in space trajectories i.e. as direct result of *mass* distribution in space on quantum and classical level.

4.4.1 Geodesic curvature

Using the results of (§4.2) for quantum objects M and m in each other's field, the accelerations are in the field of m : $a_1(r) = G.m/r^2$ with force $F_1 = M \cdot G.m/r^2$ and in the field of M : $a_2(r) = G.M/r^2$ with $F_2 = m \cdot G.M/r^2$. The total force F_G between objects then is

$$F_G = 2G.M.m/r^2 \tag{4.6}$$

Based on the conclusion of validity of §4.2 eq. 4.4, upscaling (4.6) by $M \gg m$ to e.g. a black hole M , and by considering the principles of action and compensation, we explore the motion of m .

For the motion in a field of M , without influences of other sources, the trajectory is to comply with the 'principle of least action', Lagrange [30], Feynman [6].

The actions are two *accelerations*: of gravity and of curved motion, leading to gravity force F_G and in curved motion required centripetal force F_C i.e. F_G acts as the required F_C .

With $F_G = F_C$ in the entire trajectory taken by the particle in the field, gravity force F_G delivers F_C , resulting in zero *acceleration* of the particle in the trajectory: the curved geodesic.

I.e. gravity *acceleration* a_G experienced by m is *compensated* in curved motion of m by the centripetal *acceleration* a_C on m , thereby yielding the 'least (stationary) action' in the movement of a particle in the field, where F_C is perpendicular to the tangent vector \underline{T} of the curved trajectory.

Note that in compensation, the actions of accelerations and forces are physically *present*.

With $F_C = m.v^2 / r = k \cdot m.v^2 = F_G$ we find

$$2G.M.m/r^2 = m.v^2 / r \tag{4.7a}$$

This equation elegantly reveals where the twain almost secretly meet: mathematically at first sight $m.v^2 / r$ violates the uncertainty relation in quantum perspective by simultaneous exact use of $p = m.v$ and r . The RH part of equation (4.7a) however is in the transformed domain i.e. (k, r) space

domain with $k = 1/r$ and r may be *anywhere* (x, y, z) i.e. undefined spatially on the surface of a sphere with radius r , i.e. indicating that in eq. (4.7a), variable r *physically* acquires a different *meaning* left (field decay) and right (curvature $1/r$). The resulting eqs. (4.7) and (4.8) therefore are valid in the quantum realm. Mathematically it takes the form as in eq. (4.8) with $k(r)$.

K.Schwarzschild [26, eq.(14)] was the first to publish the exact solution and J.Droste put a footnote arriving at his eq. (7), agreeing that it was equal to (14).

In the derivation of J.Droste [21], his *notion* of different *meaning* of r is captured strikingly in his phrase “this r is not the same as occurs in (4)” after his substitution $\xi = a/r$, arriving at eq. (7) in the original 1916 paper, allowing both field decay and curvature in his (and Schwarzschild’s) results to be *combined* mathematically with different *validity of variables* r in 1 equation.

Continuing, in 4.7a, by the curvature $k(r)$ domain i.e. ‘*not the same r as in (4)*’, variables r and r remain separated :

$$2G.M.m/r^2 = m.v^2 / r = k. m. v^2 , \tag{4.7}$$

i.e. in a field of gravity source M , at distance r , the *trajectory* of particles is *deformed with curvature* $k(r)$ from straight:

$$k(r) = 2.G.M / r^2.v^2 \quad r > 0, v > 0, M > 0 \tag{4.8}$$

In (4.7), at first sight the mass of the test particle m drops out (instantly giving rise to interpretations), and as result in (4.8), the curvature of the geodesic trajectory at distance r of a black hole with property M is governed *only* by the *speed* v and *location* r of the particle in the gravity field of M . Outside the field e.g. $r \rightarrow \infty$, $k = 0$, at large distance and/or very weak fields, in practice no curvature is present because of M .

Eq. (4.8) as well illustrates ad hoc the principle of multi-path summations of R. Feynman [29] by the resulting least-action trajectories of mass quantum particles in the field of M : the curvature at r depends on the speed v of the particles, and a *myriad of trajectories* thus exist for quanta which are ‘*least action*’ i.e. eq. (4.8) includes the set of trajectories escaping the source e.g. particles in the ‘*lensing*’ property of strong gravitation.

In the field of a large enough M e.g. the black hole, the minimum value of curvature is reached at $v = c$, with in case m is a photon, curvature (4.8) yields by substitutions $r = r_s$, $1/r_s = \kappa_s$ and $v = c$:

$$\kappa_s = 1/r_s = c^2 / 2G.M \tag{4.9}$$

i.e. the Schwarzschild curvature/radius.

Then in case of r_s i.e. a massive dimensional source instead of a point source, validity of r is $r > r_s$. For treatment of singularities see §4.2, §4.3 and §5.

The obvious conclusion: eq. (4.9) is identical to eq. (5.1). The latter is derived from the classical

Einstein tensor equations in general relativity of a symmetrical spherical core and shows definitely where *'the twain meet'*.

4.4.2 Conclusions

A. Einstein concluded that the presence of concentrated mass e.g. of massive objects (black holes, stars, planets etc.) caused *'space'*-curvature i.e. deforming space and thereby trajectories.

In the treatment in §4.4.1, mass of the object under study at first sight disappears in (4.7), and therefore support for a possible conclusion of *space-curvature* seems obvious *without a mass property* showing up in this equation. Because of the assumed space deformation therefore the thought of gravity not being a fundamental force may have settled.

However, derived from fundamental potential theory mathematics of fields (e.g. as for the electrical field) and with validity on the Planck scale and upwards, eq. (4.8) illustrates that curvature is governed by *object property speed* v ($0 < v < c$) and *location* r in a gravity field of M i.e. the geodesic trajectory is curved, however *space properties* i.e. described by dimensions and coordinate systems, are not deformed i.e. do *not act as cause* of curvature e.g. by being stretched, compressed i.e. or otherwise locally deformed. A deformation, without a causality relation with dimension and coordinates obviously is not supported when the curvature does not consist of coordinate variables (x, y, z) directly in a deviating dimensional relation. If a causality relation would exist, at least one of the dimension coordinates would be somehow related to curvature e.g. mass. Furthermore, the argument that testmass m disappears (4.7) and *thus* curvature /deformation does not depend on the mass of the object i.e. gravity, is *false*, as m dropped out in (4.7) because of *compensation* of acceleration in the stationary action principle causing zero acceleration in the geodesics, whereby *physical properties remain present* (i.e. do not disappear or 'evaporate' in contrast with some singularities), shifting the gravity related property of the testmass m to *energy* mediated by mass m of the particle i.e. by rewriting (4.8):

$$k(r) = G.M.m / \frac{1}{2} m.v^2.r^2 = G.M.m / E_m .r^2 ,$$

Factually the result is fully *independent* of dimensions, which leads to the conclusion: *'space' is not deformed due to presence of matter.*

The model of a rubber sheet with massive objects *visualizing* gravity by deformations of space, therefore in fact turns out to be a truly *bad* example.

Matter may be considered a kind of *aggregation state* of energy ($E = m.c^2$) e.g. 'condensed as mass' at still high energy levels during the 'big bang' forming massive elementary particles, as status quo result of (spontaneous) symmetry breaking in an environment of continually lower energy density levels in an expanding universe.

When space is not deformed, and with a mathematically proper definition of 4D spacetime with time as 4th, truly *orthogonal* component (instead of parameter axis) in an *identical* relation with (as) the other 3 dimensions, it therefore is not very likely that *time* in that case would be affected in *any way* by matter, *not being related to a true property* of a particle or energy.

Deformation is for geodesic trajectories by *cause and effect relations* between field and (elementary) particle; trajectories are straight without gravity, and inside a gravity field depend on

particle properties – i.e. similar as in an electric potential field and particles with electrical charge. Consequently, photons of different wavelengths i.e. energy, by identical speed of c m/s, are affected showing identical deformed trajectories at a distance r_1 from a massive object M , causing the lensing property of light. Therefore light and e.g. radio signals of objects, can reveal these objects in projection, when behind stars and e.g. black holes.

5. The Schwarzschild radius, singularity

The classical description is given by the solution of the Einstein tensor equations related to gravity. The Schwarzschild radius is *the characteristic* for a black hole, and is unique in the sense that such massive object only has an horizon hiding anything captured behind it.

The exact solution for the Schwarzschild radius r_s from the Einstein equations is [25, 26]

$$r_s = 2GM/c^2 \quad (5.1)$$

in which G is the gravitational constant, M the mass of the black hole.

The value r_s is the radius where the curvature caused by gravity prevents photons to escape the black hole and are captured. If the massive body constitutes of one type of particle to e.g describe a neutron star, r_s is the maximum radius (i.e. minimum curvature) where neutrons cannot escape. The mass M is again substituted with $m_{sn} \cdot V$ and speed $c = v$, the equation then shows

$$r_s = 2G \cdot m_{sn} \cdot V / v^2 \quad \text{for all } r_s > 0 \quad (5.2)$$

In case the neutron star is an almost perfect symmetrical sphere, substitution of $V = 4/3 \cdot \pi \cdot r^3$ yields

$$r_s = (2G \cdot m_{sn} \cdot 4/3 \cdot \pi \cdot r^3) / v^2 \quad \text{or}$$

$$r_s / r = r^2 \cdot (2G \cdot m_{sn} \cdot 4/3 \cdot \pi) / v^2 \quad \text{for all } r \geq 0, \text{ i.e.}$$

$$\kappa(r) = r^2 \cdot (2G \cdot m_{sn} \cdot 4/3 \cdot \pi) / r_s \cdot v^2 \quad (5.3)$$

with m_{sn} = neutron specific mass (kg/m³), r the radius of the star and particle speed v (m/s).

At $r = r_s$ this yields for curvature $\kappa_s = 1/ r_s$

$$\kappa_s = r_s \cdot (2G \cdot m_{sn} \cdot 4/3 \cdot \pi) / v^2 \quad (5.4)$$

Eq. (5.4) shows that at $v = c$ m/s, curvature reaches the smallest value i.e. for photons in the outer part of a massive black hole.

Starting from the Einstein results ($r_s > R$), property M is substituted by specific (average) mass M_s and volume V and thus relates to the mass *inside* the physical structure. This volume V introduces by substitution in (5.2) the factor r^3 .

The conclusion is that equation (5.3) for massive objects, shows that with $r \rightarrow 0$ i.e. in the centre, curvature $\kappa(r) \rightarrow 0$, (supporting the validity conclusion 2. of (4.4), §4.3).

For a black hole or planet that does not constitute of one type of particle, an average density $m_{\text{Sav}} =$ average specific mass (kg/m^3) may be introduced e.g. in a spherical layered structure with each layer a different specific density, with the same perhaps surprising effect for a black hole, the conclusion is:

The *intriguing naked singularity* in a black hole in the classical Einstein derived descriptions **disappears** by a change to *mass density* as a property, illustrating the impact of exclusion of *dimensions* e.g. in interpretations and all related literature.

From the equation (5.3) the conclusion is that in case of limit $r \rightarrow 0$, *the curvature* disappears and a (virtual) trajectory is straight, i.e. in a black hole with a higher specific mass than a neutron star and a concentration of mass towards the centre, the *curvature* caused by gravitation at $r = 0$ equals 0 as well, which gives every reason to conclude that a *naked singularity does not exist* in the centre, in contrast with current interpretations, proposed ‘wormholes’ and assumed ‘warped space’ in the centre of black holes.

From both the quantum scale and the classical derivation abstracted *point source* as well as *reality sources*, the solution of zero curvature and thus zero acceleration due to gravity at $r = 0$, *no* singularity can be assumed - this **fits** fully in what in fact is to be expected from a symmetrical model of an extreme massive large object, as the attracting forces of gravity in the centre are *fully compensated* by symmetry, however *don't* disappear: mass in the centre is attracted from all sides equally (resulting in tremendous pressure, decaying towards the surface), therefore the *curvature of the (virtual) geodesic* drops from maximum at the physical surface towards zero curvature in the centre of the object .

6. Postscript

This is my 3rd paper of the triptych, published in papers [17, 20, 24] in quantum mechanics, a quest researching an *entirely different* (from usual wave function model) *mathematical description* to arrive at a deterministic and complete theory in quantum mechanics. The results fit seamlessly with classical physics. The wave-function model managed to serve since its inception as *essential* however limp support of the successful theory, i.e. despite severe shortcomings in collapses and probability results, as well causing a myriad of papers in *interpretations* - several discords are treated in the 2nd paper – and, despite extensive attention over 100 years in the physics community, showing limited *advances* in the development and understanding of the theory.

Exploring the subject, I remember a discussion with a Nobel Laureate with whom I shared my very first thoughts. After an encouraging reply I soon found out that I developed quite different insights: abandoning the usual wave-function Hamiltonian + matrix mathematics and instead turning to the integral transformed frequency domain and Green's functions + transformation based system-theory, while strictly maintaining Abelian group mathematics. This approach in the 1st paper resulted in the model of the double slit-experiment with deterministic and exact results on quantum scale; with by-catch: a *one-slit experiment* with *modulated source* showing *double-slit* experiment

results i.e. the proof of energy amplitude-modulation instead of the look-alike however fundamentally different (and impossible at the laboratory energy level) *interference* of photons. The (inverse location) frequency domain description in curvature exceptionally well suits quantum behaviour by exclusion of direct addressing individual quanta with variables, as well as by providing validity of variable r throughout the entire range in gravity, by the a priori mathematical exception of the location of a point-source at $r = 0$; two solutions in this 3rd paper are provided to solve the issue of singularities and to cover full validity of the variable.

The QM triptych papers give a comprehensive description of quantum mechanics due to the setup along cause and effect relations in nature. For a full understanding, background in the presented deviating mathematics in [17] is required; in [20] several interpretational discords are treated almost without mathematics, and [24] connects classical (A. Einstein) and quantum physics in the transformed (k, r) i.e. curvature frequency domain with exact, identical results, however with *deviating* from Einstein's classical physics *conclusions*.

Readers may find out that the theory as presented is *not at all counter-intuitive* as it supports a *deterministic* however *invisible* (undetectable i.e. directly unmeasurable) evolvment at *quantum* level of *natural* events in and at the scale of 'the universe'.

A.V, Herrebrugh, April 2022 - January 2024

Netherlands

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