∞-Cosmoi and Fukaya Categories for Lightcones

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Abstract

We propose some questions about Fukaya categories. Given a class of isomorphisms $0 \sim \tau$, where τ represents the truth value of a particle, and 0 is a 0 object in a Fukaya category, what are its spectral homology theories? This is a variation on the works of P. Seidel and E. Riehl.

Preamble

This document is inspired by both the works of Paul Seidel and the intense collaborations of Verity and Riehl. We propose symplectic cohomology as a possible spectral homology theory, softly answering the question laid out in the abstract.

Notation 0.1. *We write $Pen(Sing(K))$ for the category of pencils with fixed point $Sing(K)$.*

The theme of this manuscript is that modding a lightcone by a quasi-isomorphism to a zero object induces a causal structure on it. The torsion category of [5] acts upon a pencil of Lefshetz hypersurfaces centered about the singularity $\hat{b} \in \mathbb{L}^4$. This yields certain locally privileged geodesics upon which the Ricci iteration

$$Ric_{g_i \to g_{i+1}} : x_t \longmapsto x_{t+1}$$

acts on a polynomial x.

$$\begin{array}{c} \mathscr{B}_0 \\ {\scriptstyle mod\ by\ \sim}\uparrow \\ \mathscr{B}_{\geq 0} \\ {\scriptstyle globalize}\downarrow \\ \mathscr{B} \end{array}$$

The above diagram represents the relationship between the (from top to bottom) reduced, relative, and classical Fukaya category. Our take on it is that a natural

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topology for this lightcone is induced by Riehl's ∞-cosmoi graded over a pencil of hypersurfaces of the lightcone. We let a prime p be realized physically as a particle. Gradations of fields represent gauge fields with weighted slice spectra of valuations, which correspond to quantum mechanical probability distributions.

We let

$$\mathfrak{o}(\alpha) : \alpha \longmapsto \tau \circ \alpha$$

be the canonical valuation for any α. This valuation is given by a flatly embedding tautological line bundle with a characteristic p. We can use Scholze's tilting procedure

$$(q_0)^\flat : 0 \longmapsto p$$

to lift out of the zero object into a short exact sequence, which can be used to triangulate a category $\mathscr{C}$.

We establish once and for all the canonical class

$$\mathscr{C}^\infty := (\infty, \mathbb{C}) \equiv \infty|_{sSets_{Fin}}/\mathbb{C}$$

Seidel, in his legendary lecture on Fukaya categories [1], gives us the following equivalences:

$$\begin{array}{ccccc} \mathbb{C} & \cong & \mathcal{F}(M \setminus B) & \cong & \mathbb{L}^4 \setminus \hat{b} \\ \uparrow & & & & \downarrow \\ [[q]] & \cong & \mathcal{F}_{\geq 0}(M) & \cong & \mathbb{L}^4_{/\ell} \\ \downarrow & & & & \downarrow \\ ((q)) & \cong & F(M) & \cong & \mathbb{L}^4 \end{array}$$

where the right-hand-side is the author's interpretation. We define ℓ to be a curve in an infinite loop space. When embedded into timewise distinct sections of a Minkowski lightcone, these give us a stratification consisting of causal geodesics. We denote the moduli space of all lightcones by

$$\mathscr{C}^\infty \supset \int_\infty^{\infty \to \infty} \mathbb{L}^4$$

here and it is indeed an ∞-cosmos of Emily Riehl. That is to say, every individual lightcone is itself in fact an ∞-category. We, however, prefer to use the term quasi-category, when we are emphasizing the full subcategory whose morphisms are quasi-isomorphisms $\hat{q} \longmapsto q$.

Contents

1 Infinity Cosmoi

Notation 1.1. *We let $\sim$ denote a Seidel quasi-fibration, and $\simeq$ the Riehl isofibration, which we treat as a "strictified" or enriched quasi-fibration. For a solid definition of the former, we refer the reader to [14].*

Let $\mathscr{C}^\infty$ be the category whose objects are lightcones, and whose morphisms are Quillen equivalences. Let $\mathscr{C}^\infty_\sim$ be the same category, but whose morphisms are quasi-fibrations.

Let

$$(\mathscr{C}^\infty_\sim)/\Delta = \mathcal{U}^\infty_\Delta$$

be a uniform space. Then, the fiber spectra of this space is given by smooth motion between stratifications given by the diagonal span

$$\mathcal{X}_y \longleftarrow \Delta \longrightarrow \mathcal{Y}_x$$

We define $\mathcal{X} \bigwedge \mathcal{Y}$ to be $(\mathcal{Y} \bigwedge \mathcal{X})^{-1}$, where

$$(\mathcal{X} \bigwedge \mathcal{Y}) = \Delta^+$$

and

$$(\mathcal{Y} \bigwedge \mathcal{X}) = \Delta^-$$

such that $x \in (\mathcal{X} \bigwedge \mathcal{Y})$ and $y \in (\mathcal{Y} \bigwedge \mathcal{X})$.

We can think of ∞ as a projective point, and particularly, when we are working with the model lightcone, it is the point corresponding to $\mathbb{P} \times \mathbb{A}^1 \times \hat{b}$, where $\hat{b}$ is the singularity of the lightcone. The realiztion of the point

$$||\infty|| \longrightarrow \{*\}$$

is specifically a map out of $\mathscr{C}^\infty_\sim$. This is a particularly nice map when the r.h.s. is treated as a simplicially enriched category with fibrations, cofibrations, and weak equivalences. This allows us to treat our ∞-categories like Cisinski categories, or, at the very least Waldhausen categories.

There is a natural morphism

$$\mathscr{C}^{\infty} \hookrightarrow SH$$

given by the Haine-Pstragowski weight filtration. In order to calculate the spectral information of a particle, we need to select a gauge boson and normalize the weight of each realization to the probabilistic center of its mass spectrum.

At the ∞ level, each fibration of a cosmos is a let binding, which is equivalent to a rank one isomorphism. This is represented by the sampling of a wave function in infinite-dimensional Hilbert space at a specialized point in a Lagrangian submanifold containing the center of a lightcone. The author's preferred ontological description of this phenomenon is that the phenomenological velocity of these particles increases at or around $\hat{b}$.

∞-cosmoi are essentially effective spectral cohomology theories.

Example 1.1. *See the proof of [3, C.1.14].*

∞-cosmoi force us to consider not just any ordinary functor, but a special class of functors called ∞-functors, which are $\leq \infty$-cells.

Notation 1.2. *For an ∞-category $\mathscr{C}^{\infty}$, we will say that the category is an (∞, n)-category if $m \leq n$ for any m-cell in $\mathscr{C}^{\infty}$.*

Definition 1.1. *The cosmic Galois group*

$$Cos_{\mathcal{G}}$$

is given by the map

$$\mathscr{A}|_{\mathbb{K} \subset \mathscr{A} \times \mathcal{G}} \xrightarrow{\sim} \mathbb{K}$$

This is a variation on a theme of Connes [12]. More properly, this is a motivic map from an ∞-cosmos (seen as a "wild" object) to a more "tame" topological stack. This leads to an intriguing duality between, on the one hand, Noohi's topological stacks, and on the other hand, fields, which are considered here to be both mathematical and physical entities. Note that while the cosmic Galois group has actions which are ∞-maps, it is not a group in the ordinary sense in that its inverse are only k-maps for $k \ll \infty$. This leads to a deformation quantization, as explained by Konstevich [13].

2 Fukaya Categories

Fukaya categories are the appropriate category for establishing a connection from ∞-cosmoi to the preferred localization at a bounded point in physical space. That is to say, maps $\mathscr{C}^{\infty} \xrightarrow{\mathscr{B}} \mathbb{L}^4$, specifically to an appropriate Lagrangian submanifold, factor uniquely through one of the three Fukaya categories established in the preamble. These are the reduced, relative, and pure/classical/unreduced categories. We let

$$V/q \nvDash (\{*\} \in V) \sim q$$

denote

$$V \setminus q$$

where

$$V = \{v_0, ..., v_\infty\}$$

is a variety. Seidel himself uses $v_m = Sup(V)$, but we must induce quasi-isomorphism:

$$(v_m \sim v_\infty) \cong ((m \in \mathbb{K}^\circ) \sim \infty)$$

in order for our categories to play nicely enough with one another to constitute an ∞-cosmos.

Axiom 2.1. $\mathbb{L}^4$ *is a holomorphic space.*

This makes sense to prescribe, as we would often want to map our polynomials to harmonic functions. It is particularly appealing to think of $\hat{b}$ (our ∞-point) as a solenoid in potential well whose outbound ∞-maps are harmonic. This paints the classical picture of the distribution of tension on an ordinary spring, reducing the canonical worldline of a particle to a simple harmonic oscillator. This interpretation comes at a price. Namely, we exclude rigid objects, which are resistant to canonical deformation of any kind. This excludes, say the etale cohomology theory from entering into the picture, which limits the number of tools available for this level of analysis. This is a double-edged sword . We can think of Seidel's unreduced Fukaya category as the moduli space of all topoi in which the blow-up at ∞ exists. It is most convenient to work with non-etale points for this category, and to instead work with etale points for the relative one, and Nisnevich points for the reduced category.

3 Lightcones

We present the following (informal) definition for a light-cone, which is appropriate even for grade school students:

Definition 3.1. *Suppose we have two ice cream cones, both touching at their sharpest points. At the base of one of the cones, we have all of the places that the light we see could have came from. At the base of the other, we have all of the places that light could travel to. At the center, we have the present.*

We denote by $\mathbb{L}^\bullet$ a $\bullet$-dimensional lightcone. In the case of 1 time dimension and 3 spacial dimensions, we obtain an orbifold $\mathbb{L}^4 \cong \mathbb{L}^{1+3}$, where the time dimension is a one-dimensional superalgebra. Denote by $\mathbb{L}^4_-$ the past of the lightcone and $\mathbb{L}^4_+$ the future.

Theorem 3.1. *All maps* $\mathbb{L}^4_- \longmapsto \mathbb{L}^4_+$ *factor uniquely through a frame at* $\hat{b}$.

Proof. Construct a net $n = \vec{p}_{ijk}$ lying in the worldline of a particle. We define this net to be a sequence:

$$(n \longrightarrow \infty) \in \mathbb{N}^+$$

which factors uniquely through an irreducible character $\aleph$ in the superalgebra of $\mathbb{L}^4_{\pm}$. $\square$

Remark 3.1. *We can strictify this theorem by require the net to be Cauchy. This requires all of the L-packets to lift to an integrable rational point, or quantum ((q)).*

It is clear that the appropriate field for $\mathbb{L}^4$ is the reduced Fukaya category $\mathbb{C}$, which is the projective image of Emmersons's energy field $\mathbb{E}$. This describes the generic gauge fibers induced by an arbitrary Newtonian force field $\mathbb{F}$ acting on a smooth manifold M. Due to relativistic effects, the mass of the quantum may appear contracted to zero as it approaches $\hat{b}$.

That is to say, we are actually thinking about each quantum as a collection of smooth fibers $\Gamma_\Sigma : \gamma \longrightarrow \delta$, where

$$\delta = \frac{1}{2}\gamma \pm \frac{1}{2}$$

This gives a description of a string at the tree level, regardless of either the orientation of the string, or its openness or closure status. This allows the particle to fulfill the Emmerson-Heisenberg Compatibility condition:

$$\Delta_\alpha \cdot \Delta_\beta \geq \frac{1}{2}|\langle|\hat{\alpha}, \hat{\beta}|\rangle$$

where Δ is the Heisenberg uncertainty of a variable pertaining to the particle. Keep in mind that if the loop ℓ_q is closed, then the particle q is the closure of the loop.

Setting γ to be Planck's reduced constant, we obtain either a 0 or 1 measurement of γ, which represents a realization of a truth value in an arbitrary force field. By normalizing the mass of the particle to its truth value, we obtain either a creation or annihilation event.

Set

$$H_n(\mathbb{L}^4_{\pm}) \simeq (0 \sim \infty)^n$$

to be causally embedded in the lightcone. This is an arity n map, or n-cell, from an unmeasured value to a phase space whose uncertainty is zero.

The first Chern class of this n-cell is invertible, resulting in both a positive and a negative value for $\tau(q)$, which makes it a 1-vector.

3.1 String and Spin Levels

We endow the worldline of a particle $\mathfrak{W}_p$ with a group $\mathfrak{string}$ which encodes the energetic information of p.

At the string level, it is most appropriate to work with $\mathbb{E}$, while at the spin level, it is more convenient to work with $\mathbb{R}$. This is because of the relative restrictions afforded to each. We gain a map:

$$\mathbb{E}^2|_{\mathbb{C}} \longmapsto \mathbb{R}^1|_{\mathbb{Q}}$$

where each $\mathbb{Q}$ is mapped to some $\mathbb{N}$. This basically treats each "good reduction" of a fermionic action like a bosonic one. In fact, the space of these good reduction is where we find Emmerson's energy constant

$$\hbar_{\mathbb{E}} = Quant_{\mathbb{E}}(\mathbb{R})$$

where $Quant_{\mathbb{E}}(\mathbb{R}) = B(O) \otimes_{\mathbb{R}} \mathbb{E}$.

Remark 3.2. *By its design, $Quant_{\mathbb{E}}(\mathbb{R})$ is constructible. One nice candidate for its fabric is a sheaf of Abelian groups. In some sense, it is the Abelianization of the group $B(O)$. For $O < 3$ this is trivial.*

We have $Pshv(\hbar\mathbb{E}) \cong \mathscr{A}$, where $\mathscr{A}$ is an ∞ cosmos. The sheafification is an exit path

$$\mathcal{EP}_{Shv} : QCoh(\mathscr{A}) \longrightarrow Coh(\mathcal{A})$$

where $\mathcal{A}$ is a Lie algebra. Note that, since the range of the morphism is real, the algebra fails to be non-trivially super. Recall from the work of Sati and Schreiber that $Spin(\mathcal{A})$ is a real Lie group whence the path groupoid over $Rep(\mathcal{A})$ is smooth.

4 Associators and Binors

From [8]:

"The binor calculus forms the underpinning of the Penrose theory of spin networks."

Let $\mathscr{S}^n : s \xrightarrow{\bullet^n} t$ be an n-cell in an ∞-groupoid. We will let the closure of $\mathscr{S}^n$ to be $n = (\Delta \sim q)$. Here, Δ is the associator for

$$\begin{array}{ccc} s & \xrightarrow{1} & s' \\ {\scriptstyle 1}\downarrow & \searrow^{2} & \downarrow{\scriptstyle 1} \\ s' & \xrightarrow{1} & t \end{array}$$

We define the map

$$\mathbb{1} : \mathscr{S}^{n>1} \longrightarrow \mathscr{S}^{1\vee 0}$$

which is a binary classifier sending every $n > 1$ to a binary tree.

Definition 4.1. *A binor, $B_{0|1}$ is the smallest full subcategory of $B_{p|q}$.*

Proposition 4.1. *Any collection of binors*

$$\bigcup_{0}^{1} B_{0|1} = \mathscr{B}$$

is replete and has auto-equivalences defining a reflexive subtopos for every $1 \in B_{0|1}$.

Axiom 4.1.

$$((1 \setminus 0) \vee (0 \setminus 1) = \frac{1}{2}P(x)$$

A property of binors is that they are binary classifiers which map

$$\tau \longrightarrow \{0\}to\{1\}$$

for all $(\tau(x) \cong (\tau \sim x))$ and for all (x). Here, x is weakly equivalent to P for all $P(x) : x \longrightarrow x$.

Definition 4.2. *An associator, $\mathfrak{a}$, is the smallest completion of $\mathbb{K} = \mathbb{A} \cup \mathbb{B}$ such that*

$$\mathbb{K}^\circ = sup\mathbb{A} \vee sup\mathbb{B}$$

when $(sup(a) \cong sup(b))$, making $\mathbb{A} \vee \mathbb{B}$ transitivite for all $a \in \mathbb{A}$ and $b \in \mathbb{B}$.

Proposition 4.2. *An associator $\mathfrak{a}$ is equal to $A \wedge B$ for groups A and B of quasi-fibrations.*

Proposition 4.3.

$$\forall \mathfrak{a} \quad \exists a_i \in \mathfrak{a} \xrightarrow{j} a_j \in \mathfrak{a} \xrightarrow{k} a_k \in \mathfrak{a}$$

making $\mathfrak{a}$ a 3-cycle.

Call the above series $a(ijk)$. Write

$$Hol_p^{a(ijk)}$$

for the holonomy groupoid of a parton p. Every action of a flow tensor on $Hol_p^{a(ijk)}$ may be written $\lambda(p)$, where λ is an eigenfuctor in $End(p)$.

Proposition 4.4. *Every map $\mathscr{S}^n \xrightarrow{B_{0|1}} \mathscr{S}^{0\vee 1}$ has a unique decomposition*

$$\lambda(p) \cdot \lambda(p)'$$

where

$$p' = \dot{p}$$

is the mirror image of p.

Proof. Write

$$Cent(p) = \frac{d(\lambda(p), \lambda(p)')}{2}$$

Our proof is reduced to showing that the map

$$\mathcal{S}^n \xmapsto{f} Cent(p)$$

is a binor. This follows from letting $f|_{0|1} \cong B_{0|1}$ be the restriction taking the map to its decomposition.

That this decomposition is unique is obvious given a canonical choice of local coordinates and a neighborhood system on which our distance function is evaluated. □

4.1 Recollection

A collection of smooth fibers admits a description as a gauge field with a principle ultrafilter. In our case, that ultrafilter is a binor which is strictified by an associator. We use the formula

$$\int_0^1 d(\tau_x, x)$$

to induce a triangular foliation on our field

$$\mathbb{F} \star \mathbb{G}_m$$

For a sheaf S_τ of truth values, we define a "kink" to be a twist on a nerve:

$$Dehn(\mathcal{N}^{\aleph})$$

where $\aleph$ is a dominant character, where by

Definition 4.3. *dominant we mean a countably infinite character.*

Recall that Seidel [10] defined a strong equivalence between the Dehn twist and the Picard-Lefschetz monodromy map.

A kink acts on the tangent bundle of all paths

$$T_{\gamma\delta} : \gamma \in \aleph \longrightarrow \delta \in \aleph \simeq (\gamma \longrightarrow \delta) \in \aleph$$

which produces a reduction in the glide along $\aleph$.

Axiom 4.2. *A countably infinite character always contains its cardinality:*

$$\aleph \supseteq card(\aleph)$$

$T_{\gamma\delta}$ defines an orientable micro-surface, or in other words a surface upon which the microlocal analysis of sheaves may be performed on. There is a proliferation of pyknotic objects in this regime, where by regime we mean:

Definition 4.4. *A regime is a topological space containing the homotopy type of an object.*

A property of a regime is that it contains all of the characters which $\aleph$ is transcendental over, but not necessarily algebraic over.

Definition 4.5. *There is an isofibration between the number of kinks of a system, and the third Betti number of the system. We will write*

$$K_n(S) \cong B_3(S)$$

These are sections where the values of a gauge field are trivially measured to be $\{\emptyset\} \vee 0$.

Axiom 4.3. *In characteristic $\ell \neq p$:*

$$S \hookrightarrow \mathscr{S}^n$$

Axiom 4.4. *The space*

$$B\mathscr{S}^n$$

is homeomorphic to

$$BS$$

This homeomorphism is actually acting on presheaves of polynomial rings, which may be Taylor expanded into sheaves. This is analogous to the embedding

$$\mathscr{C}at \overset{\hookrightarrow}{\sim} \mathscr{Q}\mathscr{C}at$$

as pursued in Verity and Riehl's classic text on ∞-categories.[1] [3] Recall that if morphisms (w, f) are weak equivalences, than the third must also be by the 2-out-of-3 property.[2]

See C.1.13 in [ibid] for a formal treatment of the gluing condition we are using. We operate using the convention that $\mathscr{Q}\mathscr{C}ats$ are profinite extensions of $\mathscr{C}$ats. They are in some sense the moduli space of lightcones, as discussed in [Sect. 3]. Our relationships are encapsulated by the following diagram:

$$\begin{array}{ccc} q & \xhookrightarrow{\ \flat\ } & \mathscr{Q}\mathscr{C}at \\ {\scriptstyle \delta}\downarrow & & \downarrow{\scriptstyle \delta} \\ n & \xhookrightarrow[\ \flat\ ]{} & \mathscr{C}at \end{array}$$

where δ is the discretization of functor, and where $\flat$ is Scholze's canonical tilting functor.[3] We re-appropriate this functor here for a strictly $qfpp$ morphism, or a canonical flat *embedding*:

$$\bullet^\flat : \bullet \in \mathfrak{char}_0(R) \longrightarrow \bullet \in \mathfrak{char}_p(R)$$

for a prime $p \neq \ell$. Here, $qfpp$ stands for "quasi-flat pre-projective."

We have a diagram

$$\begin{array}{ccc} q_0 & \longrightarrow & q_\infty \\ \downarrow & & \downarrow \\ \dot{q}_0 & \longrightarrow & \dot{q}_\infty \end{array}$$

factoring through $p : o \longrightarrow ... \longrightarrow (n < \infty)$. The bottom leg here is the transcendent morphism of the mirror image of a homologous pair (q_0, q_∞).

[1] We also refer the reader to this book, pg. 443 for my favorite definition of "categories with fibrant objects." This book also seems to have some profound references to say about homotopy type theory.

[2] In a later note, we will use this property to define a transformation from an acyclic group to a cyclic group of order 3, which corresponds to a spin of 2/3.

[3] See [4]

Set

$$p_i\delta_i(x):$$
$$q \xrightarrow{\sim} \hat{\Sigma}\hat{q}$$

to be an iso-fibration sending q to its suspension category of Quillen equivalences. We have

$$Maps(\cdot,\cdot) \equiv Yon([[q]])$$

then we impose our localization $p_i\delta_i(x)$:

$$\begin{array}{ccccc} & & \overset{\simeq}{\frown} & & \\ q & \xrightarrow{\sim} & \dot{\hat{\Sigma}}\hat{q} & \xrightarrow{\sim} & \Sigma\hat{\Sigma}\hat{q} \\ \downarrow{\cdot} & & \vdots & & {\cdot}\downarrow \\ \dot{q} & \xrightarrow{\sim} & \dot{\Sigma}_q & \xrightarrow{\sim} & \dot{\Sigma}\Sigma_q \\ & & \underset{\simeq}{\smile} & & \end{array}$$

giving us

$$\begin{array}{ccc} Maps(\cdot,\cdot) & \equiv & Yon([[q]]) \\ & \searrow \quad \Longrightarrow \quad \swarrow & \\ & \dot{q} & \end{array}$$

Thereby establishing a Yoneda principle for Fukaya categories.

5 Galois Connections

We define a special category of connections, the *Galois connections* $Gal_\Gamma : \tau(x) \longmapsto x$ whose projection is the identity on a quasi-fibration $(x \sim p)$. These will be of special importance to us in the case when τ is given by the classical evaluation map

$$ev_0 = \mathfrak{o}(x) \longrightarrow |x|$$

where $|x|$ is the absolute value realization of x. This is, in some sense, a second quantization of a collection of quasi-quanta in a potential well equipped with a gradient, where by "gradient' we mean a curvature of the gauge field over p.

We think ev_0 as a map

$$R^\times \longrightarrow r$$

of a ring to its most representative unit.

Definition 5.1. *We define the module r-mod to be the module of highest weight, such that the map*

$$(H_n(r) \longrightarrow H_{n+1}(r' \sim r)) \xrightarrow{\sim} \mathfrak{Walls}$$

is an isofibration.

A map from R to r-mod is essentially a projection out of a moduli space Mod_R onto an instanton:

$$Mod_R \twoheadrightarrow (\hat{i} \sim \hat{b} \in \Box_{\mathbb{E}})$$

The notion of a Galois connection is in some sense dual to the notion of forcing. For a connection

$$Gal_\Gamma : \mathscr{A} \longrightarrow \mathscr{B}$$

write

$$\mathscr{A} \dashv \mathscr{B}$$

for the adjunction formed by "de-sheafifying" the space $\mathscr{B}_{et}$. A Galois connection has the following property:

Property 5.1. *$G_\Gamma : x \xrightarrow{\sim} y$ consists of a series of successive isofibrations:*

$$x_0 \simeq ... \simeq (x_\infty \wedge y_\infty) \simeq ... \simeq y_0$$

whose center is an $(\infty, 2)$-groupoid.

Notation 5.1. *By $x \xrightarrow{\sim} y$, we mean that $x \simeq y$ is a valid isofibration.*

Remark 5.1. *We refer the reader to [9] for an account of a connection*

$$G_\Gamma : \mathscr{A} \longmapsto \mathcal{B}^d \cong M$$

This paper also introduces the the wrapped Fukaya category, *Fuk_{Wr} of Fukaya categories tensored with branes wrapped over a kinked point p_k.*

where $\mathcal{B}^d$ is a brane of topological dimension d, meaning that it is homeomorphic to $\mathbb{R}^d$. This is an instantiation of the Lurie-Riehl straightening of a modulated category $\mathscr{C} - mod$. The unstraightening of this space is defined to be the hyperbolic blow-up of the space's rational points:

$$B^{\uparrow}(Rat(Spec(\mathcal{R})))$$

This operator admits a left adjoint,

$$B_{\downarrow}(Rat(Spec(\mathcal{R})))$$

which sends the group-like objects in Fuk_{Wr} to point-like analogues in the reduced Fukaya category $\mathscr{B}_0$.

Axiom 5.1.

$$((B_{\downarrow}(-) \circ B^{\uparrow}(-) \vee (B_{\uparrow}(-) \circ B_{\downarrow}(-)) = Id_{-}$$

This axiom is isomorphic to the statement that restricting and then unrestricting is the same thing as the zero action: doing nothing. This is to say:

$$X_{b_j} = B^{\uparrow}_{\downarrow}(b_i)$$

for a fiber spectrum centered about an etale fiber $j = |i|_{et}$.

Recall that

$$X_{b_j} = Id_{(X,b)}$$

is an equivalence between the objects $1 \in \mathbb{1}$ and $1 \in \mathbb{2}$. Thus, the result of applying the identity functor to X_{b_j} is to obtain an isofibration

$$\mathbb{1} \xrightarrow{\sim} \mathbb{2}$$

which factors uniquely through an idempotent of lowest weight.

Theorem 5.1. *The map*

$$\mathbb{1} \xrightarrow{\sim} \mathbb{2}$$

factors uniquely as a map

$$((0 \sim \infty) \sim \hat{1}) \longrightarrow 1$$

where $\hat{1}$ is a unit in $R^{\times} \subseteq (\mathbb{1}, \mathbb{2})$.

Proof. Using Emmerson's modified axiom of choice, we let $\mathfrak{w}_0$ be an element of lowest weight in the set of units of the ring R. Assume that this is the least upper bound in $\mathbb{1}$. Then, we can identify the quasi-isomorphism

$$\hat{1} \simeq \mathfrak{w}_0$$

with a net in $\{\mathbb{1}, \mathbb{2}\}$ such that all morphisms $0 \longrightarrow ... \longrightarrow (0 \sim 1)$ factor uniquely through the quasi-isomorphism. □

6 Fermionic Structures

Fermionic structures strictify non-commutative structures by enforcing a "strong" version of anti-commutativity. For a fermionic structure $\mathcal{S}_{\mathcal{F}erm}$ on a brane, we have:

$$\mathcal{S}_{\mathcal{F}erm} = ker(\mathcal{S}) \setminus \mathcal{A}_{Ab}$$

Write

$$(H_{ijk})\lambda(x) \cong \lambda((H_i)x \cap (H_j)x \cap (H_k)x)$$

where x is the both the center of the algebra H, as well as the determinant of some submatrix of $SO(3)$. Here, λ is the eigenfunctor given in [Prop. 4.4].

Definition 6.1. *A fermionic structure on a brane $\mathcal{B}^d$ is a pencil of hypersurfaces*

$$Pen(\lambda(x))$$

centered about $\lambda(x)$ which are topological realizations of sheaves of anti-commutative functors.

The functors of our sheaves need not obey the strict law of composition; that is to say

$$g \circ f \text{ does not necessarily } = gf$$

$$\forall \{f, g\} : * \longrightarrow \nabla_*(x)$$

Remark 6.1. *The structure given by the map isomorphism at the beginning of this section is bosonic whence the intersections in the right-hand-side commute, and fermionic elsewise.*

A big question is whether the suspension spectrum of a particle is fermionic or bosonic. We propose that, even in the bosonic case, a particle p may admit a suspension spectrum $\Sigma^\infty(p)$ which is fermionic. This means we have the classical relationship:

$$(Sp(p))R(Sp^-(p) \oplus Sp^+(p))$$

which lifts to an equivalence when the prescribed fiber of p is the one given by the tautological line bundle of a maximal atlas of the Picard functor $Pic(p)$. Viz.:

Proposition 6.1.

$$Taut_{\mathcal{A}}(Pic(p)) \text{ is the uniformizer for } p$$

Proof. Trivial. □

If the ∞-suspension of an instanton's worldsheet is fermionic, then the trace

$$Tr_{\mathfrak{s}} = \{\mathfrak{s} \times \Psi(\hat{i}) | \mathfrak{s} \subset SO(3)\}$$

of the matrix representing its wave-function cannot be reduced to a unit. Namely, the reduction

$$Tr_{\mathfrak{s}} \xrightarrow{*} \mathfrak{h}$$

is obstructed from lifting to an equivalence. This means that the cofibrant objects of $Tr_{\mathfrak{s}}$ have no weak equivalences to objects in the category $\mathbb{1} \times fdHilb$, yielding chaotic fiber spectra and thus chaotic holonomy for bundle gerbes over p.

7 Ergodicity

In this section we give some examples of ergodic systems.

We define

$$X_{b_i} = \sum_{i=0}^{\infty} (x \sim b_i)$$

to be an ergodic system whence the probability density of X is given by the fiber spectrum b_i as parameterized by $i = spec(b)$. For a Fukaya category $\mathscr{B}_\bullet$, this means roughly that every symmetry about the preferred locus is Lebesgue integrable by the function

$$f(l_{\mathbb{K}}) : \int_0^{(2^{dim(l)-1})\pi} \dot{k} \in \dot{\mathbb{K}}$$

where

$$Pen(\dot{k}) = Pen(Sing(\mathbb{K} \setminus l_{\mathbb{K}}))$$

Proposition 7.1. *There is an equivalence*

$$X_{b_i} = Avg(Id_X)$$

given by the etale map

$$|Id_X|_{et} : X \xrightarrow{\sim} X_{b_i}$$

Example 7.1. *Let $X \in Sp$ and let $B(X)$ be a canonical basis for X. Remove a single point from X like so:*

$$X \setminus x_n$$

Then, we have an equivalence

$$\sum_{n=0}^{\infty} x_n \cong \int_0^{\infty} \partial(x) \qquad \forall x \in X$$

Example 7.2. *Let $\mathscr{C}$ be a symmetric monoidal category. Then, the collection of arrows given by*

$$\sum_{f=Id_{c\in\mathscr{C}}}^{f=\mathcal{L}} f : c \longrightarrow ... \longrightarrow c$$

where $\mathcal{L}$ is an infinite long exact series with a singular pole at zero, corresponds precisely to the ∞-cell sending the zero object to itself, which is the identity functor.

Example 7.3. *Let p be a particle in a phase space P. The phase space is said to be "ergodic" if it is homogenous, and if*

$$\lim_{t\to\infty} p_t = Avg(p)$$

such that the location of the particle, after an infinite amount of time, is identical to the most probable location for the particle.

Example 7.4. *Assign to every number in a field $\mathbb{K}$ a weight $\mathfrak{w}_k$. We say a process is ergodic over $\mathbb{K}$ if, for a filtration over $\mathbb{K}$, the weight of each number is equal to the realization of the number. In the case of counting, every element is given a canonical evaluation $\mathfrak{o}(n) : \mathfrak{w}_n \longmapsto n$, and we obtain an operad $\mathfrak{w}_n \odot_{+1} \mathfrak{w}_{Id_e}$.*

Example 7.5. *Let $\mathfrak{D}$ be a dendrite. If*

$$\sum_{n=0}^{\infty} \mathcal{N}_{\mathfrak{D}}^{n}$$

is an ∞-map from $\mathfrak{D}$ to its center, then the dendrite is ergodic.

Proposition 7.2. *The reduced Fukaya category supports more ergodic systems than the relative Fukaya category.*

To see that this is true, note that the map

$$\mathscr{B}_{\geq 0} \curvearrowright \mathscr{B}_0$$

induces a formal specialization

$$(n \in \mathbb{N}) \rightsquigarrow l_{\mathbb{K}}$$

Such that the fiber spectrum X_n is sent to its period-zero instantiation in $\mathbb{K}$. This is by no means a trivial statement. Indeed, one could use this knowledge to construct a highly stable model of ∞-categories over $\mathscr{B}_0$, where each object is a complete Segal space.

Essentially, one lets $\Pi_\infty(\mathscr{B})$ denote the set of all homotopies and their types in the complete, unreduced Fukaya category, and one obtains a map

$$\Pi_\infty(\mathscr{B})|_0 : \mathscr{B} \longmapsto Id_{0 \in \mathscr{B}}$$

Here,

$$Id_0 = \pi_0(\hat{b})$$

and, since this restriction can be applied anywhere on a given topological space (say, a lightcone), we obtain a relativistic choice of preferred locus, and thus the relativity of simultaneity. That is to say, any zero object in the category of light-cones may be used to construct some light-cone in which it is the privileged frame. We then obtain a frame bundle consisting of Lefschetz hypersurfaces, which are topologically equivalent to geodesics, which in $\mathbb{L}^\bullet$ are, in the nicest case, conic sections.

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