Mach’s principle and gravitational shielding

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Abstract. We propose that inertia is the result of the gravitational force of induction of the entire Universe; that is, we understand that the force of inertia is the induction force exerted on a body by its accelerated movement with respect to the Universe, a statement that we call Mach's principle. We calculate this force by applying the weak field theory of General Relativity and using the results of electromagnetic theory. To avoid the divergence of the integral in calculating the induction force, we assume that in the Universe's early stages there was a strong gravitational absorption, which weakened as the cosmic density decreased. Admitting an *ad hoc* absorption function, we demonstrate Mach's principle, verifying that the inertial mass of a body is proportional to the gravitational mass with a proportionality coefficient dependent on cosmic time. Applying this result, we calculate the relative variation of the universal gravitation constant.

1. Introduction

Newton's theory considers the Universe to be homogeneous and infinite, whose bodies are subject to gravity, which depends on the distance between the interacting bodies and is an action at a distance (that is, it propagates instantaneously). Then, the entire Universe does not affect the local bodies since the action of the opposite part counteracts the gravitational action of one part of the Universe since the direction of the force is radial. Only nearby objects (the Sun, planets, etc.), which are irregularly distributed, exert gravitational forces on local bodies.

For what has been said, local bodies are isolated from the Universe; therefore, according to the Newtonian conception, the Universe does not affect the movements of local bodies, although all the objects of the cosmos exert forces on them.

A fundamental concept in Newton's physics is inertia, or the property that bodies have to oppose change in motion (that is, changes in their speed), which means that it is necessary to apply a force to change the speed.

We quantify inertia by inertial mass, which we define as the ratio between the force applied to the body and the acceleration it acquires. For Newtonian physics, mass is an innate and invariable property of bodies, which measures the amount of matter they contain.

In classical physics, we distinguish inertial and non-inertial reference frames. In the first frame, the law of inertia and the law of proportionality between force and acceleration or Newton's law ($\mathbf{F} = m\mathbf{a}$) are fulfilled. Non-inertial frames are accelerated with respect to inertial frames, and Newton's law is not applicable unless additional terms are added to the applied force, which are incorrectly called fictitious or inertial forces (Segura, 2019).

We noted that these fictitious forces are not forces caused by interaction with other bodies or fields; that is, the name force is not appropriate; they are only additional terms with which we maintain the form of Newton's law in non-inertial frames and which are the product of the mass and an acceleration. It is also inappropriate to identify these fictitious forces with the force of inertia, which is a different concept, as we see below.

In order to understand the concept of inertial force, it is necessary to replace Newton's law with the principle of dynamic equilibrium or D'Alembert. In this context, the force of inertia is a real

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force and, therefore, measurable *, which acts on anybody accelerated with respect to an inertial frame and of magnitude $\mathbf{F}_i = -m\mathbf{a}$ and is the same for all reference frames, whether inertial or non-inertial. Then, the law of dynamic equilibrium or D'Alembert states that the sum of all the forces that act on a body (the applied ones and the inertia force) is zero

$$\mathbf{F} + \mathbf{F}_i = 0. \tag{1}$$

We insist that the erroneously called fictitious forces are different from the force of inertia, which acts on all accelerated bodies and is a real force. Add that we apply D'Alembert's law (1) in both inertial and non-inertial frames as long as we measure the acceleration with respect to an inertial frame and that the applied forces only depend on the distances between the interacting bodies.

If the force of inertia, as we have just defined it, is a real force, it must be exerted by another body either by direct action or through the mediation of a field. The law of dynamic equilibrium does not specify the source of the inertial force, which is a doubt resolved by Mach's principle.

Mach's idea consists of accepting D'Alembert's principle and adding that the force of inertia (and therefore the phenomenon of inertia) results from the action of the entire Universe on accelerated bodies. Although Mach did not suggest what type of interaction the inertial force exerted by the Universe is, we are forced to consider that it is gravitational. Mach argues that the force of inertia acts only on bodies that are accelerated with respect to the entire Universe, which is why he assumed that the Universe must intervene in the generation of the force of inertia. Mach did not specify the mechanism by which the gravity of the Universe produces the force of inertia.

We understand Mach's principle as the statement that inertia, and consequently inertial mass, is generated by the gravitational action of the entire Universe (Bondi & Samuel, 1996), (Barbour & Pfister editors, 1995)

Until the 20th century, the scientific community did not realize that classical physics uses two types of mass: inertial mass, which we have previously defined, and gravitational mass, which we should better call gravitational charge, which is the mass that appears in the law of gravitation of Newton and which we assume is an invariable magnitude of the body. There is ample experimental evidence of the validity of the equivalence principle (Segura, 2023a), which states that inertial and gravitational mass are proportional. If we properly choose the universal gravitation constant, we achieve that both masses have the same value, which is why both masses were considered the same concept.

There is a fundamental difference between gravitational and inertial mass, which refers to distinct phenomena. Any theory of inertia must account for the proportionality of the two types of mass.

Although Mach's principle was a known idea since Einstein began his research on gravitation, there was difficulty in specifying a theory that would develop it (for the status of this problem in 1951, see Bondi, 1951, pp. 27-33). (Sciama, 1953) was the first to treat the generation of inertial force with a concrete theory. In his "toy theory", the force of inertia results from the gravitational induction of the Universe, that is, the force of gravitational origin produced by the relative movement of the source. Although Sciama's theory has been widely valued, few authors have followed this idea (Martín *et al*., 2007).

Most of those who have investigated Mach's principle have developed new gravitational theories to replace General Relativity. Brans-Dicke type theories have been the most considered (Brans and Dicke, 1961).

In our research, we adopt Sciama's original concept, which proposes that the force of inertia arises from the gravitational induction created by a body's accelerated motion relative to the Universe. Although they have some flaws, we accept the general theory of Relativity and the lambda-CDM cosmological model. Since the gravitational field at the current cosmic moment is very weak, instead of the complete tensor theory, to find the equation of motion we use the vector field theory that we derived from General Relativity (Segura, 2023b) (Weinberg, 1971, pp. 241-249)

* Newton seems to be referring to the force of inertia when he wrote: «This vis insita, or innate force of matter, is a power of resisting (not driving), by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forward in right line.»

(Chandrasekhar, 1965) (Segura, 2013, pp. 41-54). This simplification, which we do not apply to early cosmic epochs, has the advantage that we extend the theory of electromagnetic induction to gravitation (Panofsky & Phillips, 1929, pp. 297-301).

In the appendix, we briefly present the theory that we have developed (Segura, 2018), from which we obtain the following results:

* The entire Universe is an inertial reference frame. To a certain extent, the Universe is comparable to Newtonian absolute space.
* The force of cosmic induction is proportional to the acceleration of the body on which it acts, not depending on the speed, at least in the classical approximation.
* The force of induction is proportional to the gravitational mass.
* The sense of the induction force is opposite to the sense of the body's acceleration.
* Inertial mass is proportional to gravitational mass.

$$m_i = \chi(t) m_g \tag{2}$$

$\chi(t)$ is the coefficient of inertia, a function that depends on cosmic time.

* All cosmic epochs contribute to the inertial mass of an object with varying degrees of intensity.
* As cosmic time progresses, the coefficient of inertia also increases, which implies that the object's inertial mass is increasing as well, assuming that the object's gravitational mass remains constant.

Our results are consistent when we identify the force of cosmic induction and the force of inertia. However, we encountered a problem while calculating the coefficient of inertia caused by the gravitational effect of the initial stages of the Universe, where we found a divergent integral.

To overcome this difficulty, in the present investigation, we assume the existence of strong gravitational absorption in the primitive stages of the Universe, which dampens the gravitational induction produced by the Big Bang, making the coefficient of inertia finite when we consider the action of the entire cosmos.

2. Gravitational induction force. The problem of distances

There is no possibility of deducing Mach's principle if we consider that gravity obeys Newton's law, which is not a field theory but one of action at a distance. However, suppose we accept that gravity is a field, as is General Relativity or electromagnetism, whose signals propagate at the speed of light. In that case, we must accept the existence of induction forces, which depend on relative motion between the source and the test body. Therefore, any gravitational field theory contemplates induction phenomena, which we can use to explain Mach's principle.

As we have said, induction in electromagnetic theory serves as a guide in our research, but we must distinguish between active and passive induction. Although the difference is insignificant in electromagnetism due to the proximity of the interacting charges, it becomes essential when calculating cosmic distances.

We call active induction when the source of the field moves and passive induction when it is the test body in motion. In our problem, it is the test body that moves accelerated with respect to the entire Universe, that is, passive induction, so we must ask how it is possible that when a change in the motion of a test particle occurs an induction force produced by enormously distant cosmological objects, it is as if the gravitational action propagated instantaneously.

This discrepancy is because the test body does not directly interact with the distant cosmic object. Instead, it interacts with the field created by the distant object when it is in its retarded position (that is when the gravitational interaction originates from the cosmic source).

Two distances appear in gravitoelectromagnetic potentials. One of them, which we have called r_p, is the distance travelled by the gravitational signal from the moment it left the source at time *t* until it reached the point in the field at time $\hat{t}$. There is also the distance with which we calculate the volume element $dV(\hat{t},t)$ or volume of a spherical shell centred at the field point

$$dV' = 4\pi\sigma^2(\hat{t},t)\, d\sigma(\hat{t},t)$$

$\sigma(\hat{t},t)$ is the proper distance where the cosmic objects, whose signals leave at *t,* reach the field point at $\hat{t}$ (reception moment).

The distance r_p in formula (A.7) is what we call photonic distance, that is, the distance travelled by the gravitational signal from time t to $\hat{t}$

$$r_p = c\left(\hat{t} - t\right), \tag{3}$$

(Vetö, 2013) assumes that r_p is the photon flux distance, and for (Martín *et al.*, 2003), it is the retarded proper distance.

σ is the retarded proper distance or proper distance at which an object was, that emitting a signal at retarded time t reaches the observer at time $\hat{t}$. By the Robertson-Walker line element in spherical coordinates

$$ds^2 = c^2 dt^2 - R^2\left(t\right)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

for $k = 0$ we find

$$\sigma\left(\hat{t},t\right) = R\left(t\right) r\left(\hat{t},t\right) = R\left(t\right)\int_t^{\hat{t}} \frac{cdt'}{R\left(t'\right)} \tag{4}$$

R is the scale factor at retarded time, and r is the coordinate distance at retarded time t.

3. Gravitational absorption function

The absorption or shielding of gravity has been a recurring idea. However, experimental measurements have not shown that matter can absorb gravity (Borzeszkowski *et al.*, 2003).

However, the situation may be different when gravity has to pass through areas of very high energy density, such as those that exist in the first moments of the Big Bang, although we do not know much about the characteristics of this absorption.

As we have said, when we calculate the coefficient of inertia (A.10), we find a divergent integral due to the high gravity of the Universe's early stages. To overcome this problem, we now assume that gravity absorption exists but is only perceptible for very high densities. With this hypothesis, we obtain, as we will see later, a finite value for the coefficient of inertia, which means explaining Mach's principle through the gravitational induction of the Universe.

We assume that the absorption function depends exclusively on cosmic time, and we understand it as the decrease registered in the gravitational force. The properties we expect from the absorption function $\mu(\xi)$ are:

* $\mu(0) = 0$ that is, the absorption is total at time $t = 0$.
* There is a time ξ_α after which the absorption is so small that we assume it to be zero; therefore, $\mu\left(t > t_\alpha\right) = 1$.
* The union between the two cosmic phases (when it exists and ceases to exist absorption) must have continuity, that is, $\dot{\mu}\left(t = t_\alpha\right) = 0$.
* Finally, the absorption function must satisfy the condition that the inertia coefficient at $t = t_0$ is 1, t_0 at the current age of the Universe.

Apart from the above, we do not know more about the absorption function in the primitive stages of the Universe. The calculations we make below will be speculative. Although the values we find are only approximations, they will be very useful because they will give us qualitative information about the consequences of Mach's principle.

For the calculations we choose *ad hoc* the function

$$\mu\left(\xi\right) = \sin^b\left(a\xi\right), \tag{5}$$

$\xi = t/t_0$, a and b are coefficients determined with the conditions previously imposed on $\mu\left(\xi\right)$ (Graph 1). With the condition $\mu'\left(\xi_\alpha\right) = 0$ $\left(\xi_\alpha = t_\alpha/t_0\right)$ we find

$$\mu\left(\xi\right) = \sin^b\left(\frac{\pi}{2\xi_\alpha}\xi\right). \tag{6}$$

4. Coefficient of inertia with gravitational shielding

From (A.9) we find the function

$$f\left(\hat{\xi},\xi\right)=\left(\frac{11}{2}\frac{\Omega_M^0}{a^3}+7\frac{\Omega_R^0}{a^4}+\Omega_V^0\right)\frac{H_0^2}{c^2}\frac{\sigma^2\sigma'}{r_p}. \tag{7}$$

By (A.10), we find the coefficient of inertia and, therefore, the inertial mass of a body at any moment. If we apply (7) to the Einstein-de Sitter cosmic model, which we choose for simplicity, we find by (A.11)

$$f\left(\hat{\xi},\xi\right)=66\frac{\frac{2}{3}\hat{\xi}\xi^{-1}-\frac{7}{3}\hat{\xi}^{2/3}\xi^{-2/3}+\frac{8}{3}\hat{\xi}^{1/3}\xi^{-1/3}-1}{\hat{\xi}-\xi}, \tag{8}$$

we find an infinite value by integrating (A.10) with (8). This result is what leads us to assume the existence of gravitational absorption. Adapting the absorption function (6) to (8) and imposing the condition that at the current moment $\hat{\xi}=1$, $m_i=m_g$ or $\chi\left(\xi=1\right)=1$, i.e.

$$\chi\left(\hat{\xi}=1\right)=\int_0^{\xi_\alpha}\mu(\xi)f\left(\hat{\xi}=1,\xi\right)d\xi+\int_{\xi_\alpha}^{1}f\left(\hat{\xi}=1,\xi\right)d\xi=1,$$

we find, among other possibilities, the coefficients $\xi_\alpha=0.258$ and $b=8$, with which we can determine the coefficient of inertia at the moment $\hat{\xi}$

$$\chi\left(\hat{\xi}\right)=\int_0^{\xi_\alpha}\mu(\xi)f\left(\hat{\xi},\xi\right)d\xi+\int_{\xi_\alpha}^{\hat{\xi}}f\left(\hat{\xi},\xi\right)d\xi=\int_0^{0.258}\sin^8\left(\frac{\pi}{2\cdot 0.258}\xi\right)f\left(\hat{\xi},\xi\right)d\xi+\int_{0.258}^{\hat{\xi}}f\left(\hat{\xi},\xi\right)d\xi. \tag{9}$$

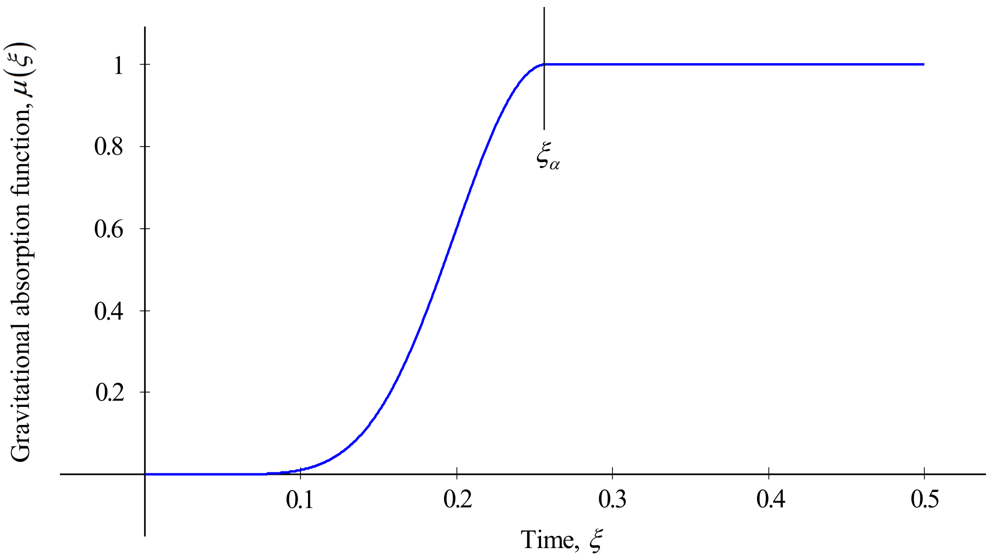
We show the result of the integration in Graph 2 and in Table 1, where we verify that the coefficient of inertia increases with cosmic time. The numerical values are only indicative, both because we do not know the law of gravitational absorption and because of the use of a simplified cosmic model.

In Graph 2, we represent the variation of the inertia coefficient in a Universe filled exclusively with electromagnetic radiation, where we once again verify that $\chi\left(\xi\right)$ is increasing, which means that the inertial mass of the bodies is increasing as long as we consider unalterable gravitational masses.

The variation of the inertial mass is equivalent (dynamically) to the variation of the gravitational constant. The equation of motion in the classical approximation is

$$G_0\frac{m_g m_g'}{r^2}=m_i a=\chi\left(t\right)m_g a\quad\Rightarrow\quad\frac{G_0}{\chi\left(t\right)}\frac{m_g m_g'}{r^2}=m_g a\quad\Rightarrow\quad G=\frac{G_0}{\chi\left(t\right)}, \tag{10}$$

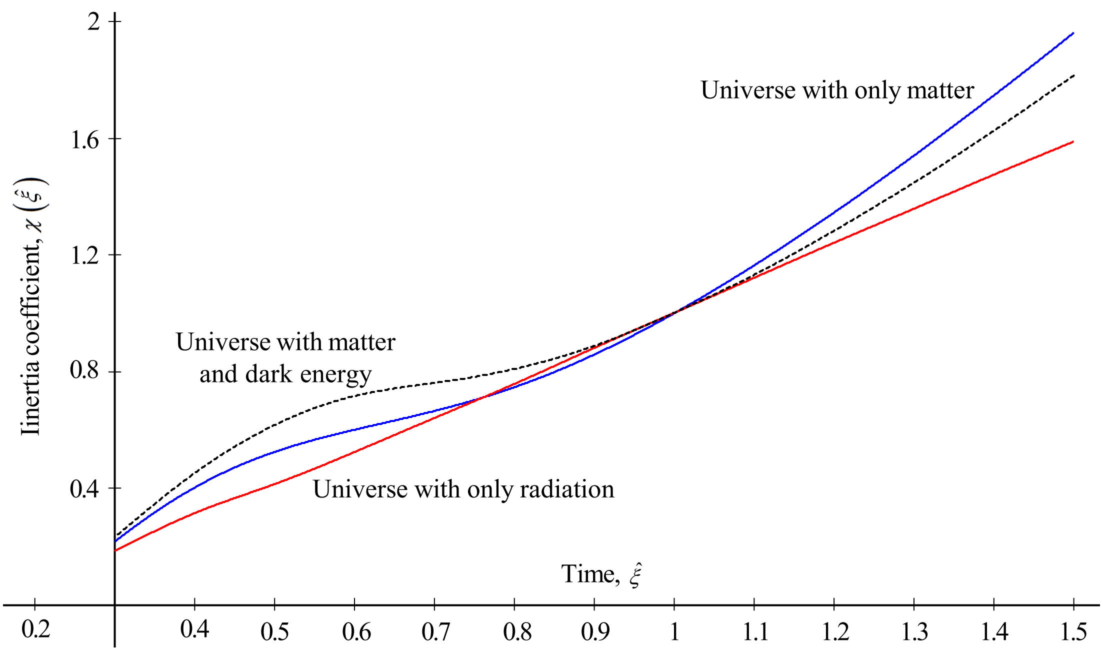
that is, the variation of the inertial mass is equivalent to assuming that G varies with time. From (10)



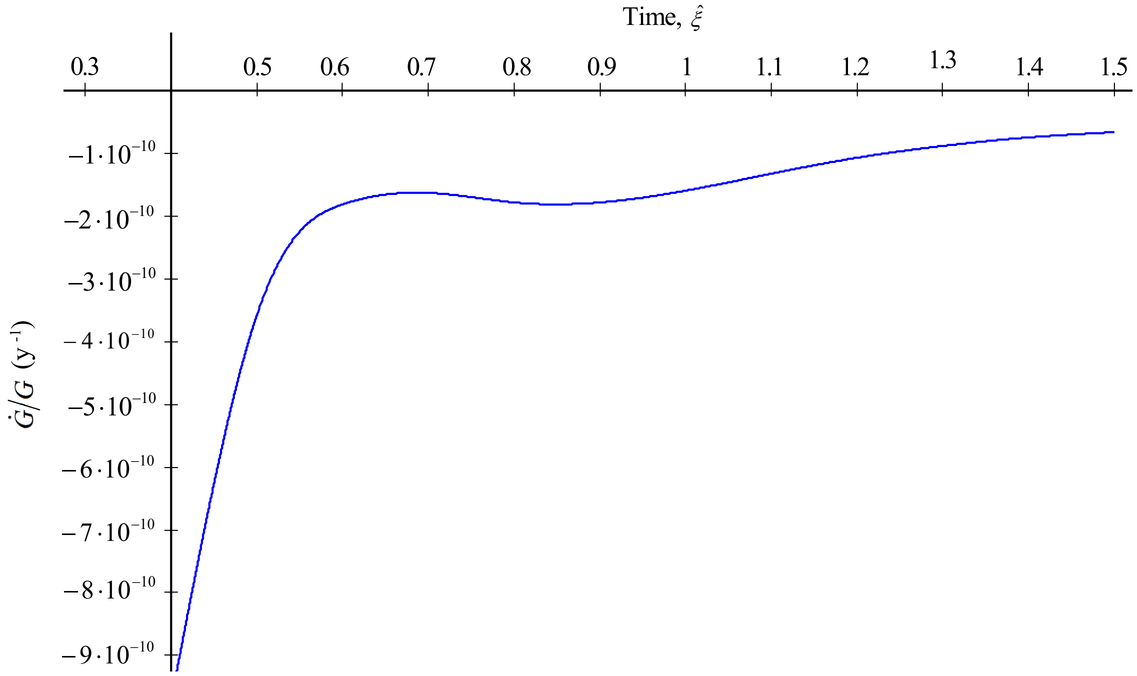
Graph 1.- Example of gravitational absorption function. In the Universe's first moments, its value is very close to 0, indicating great absorption of gravity. Gradually, the absorption function increases, indicating less shielding until the moment ξ_α when the absorption function takes the value 1; that is, absorption ceases to exist. The graph corresponds to the function (6) with the parameters $\xi_\alpha=0.258$ and $b=8$.

$\hat{\xi}$	$\int_0^{0.258} \mu(\xi) f(\hat{\xi}, \xi) d\xi$	$\int_{0.258}^{\hat{\xi}} f(\hat{\xi}=1, \xi) d\xi=1$	$\chi(\hat{\xi})$	$\chi'(\hat{\xi})$
0.3	0.026	0.192	0.217	2.302
0.4	0.178	0.225	0.402	1.521
0.5	0.338	0.187	0.525	0.952
0.6	0.461	0.139	0.601	0.635
0.7	0.542	0.123	0.665	0.696
0.8	0.584	0.163	0.747	0.964
0.9	0.593	0.266	0.859	1.271
1	0.615	0.385	1.000	1.543
1.1	0.662	0.503	1.165	1.740
1.2	0.729	0.617	1.346	1.878
1.3	0.813	0.728	1.541	2.024
1.4	0.912	0.836	1.748	2.085
1.5	1.022	0.941	1.963	2.463

Table 1.- Results of the integration of (9), whose graphic representation is in Graph 2. We verify the importance of the Universe's first moments (second column) in the formation of the inertial mass. The temporal variation of the inertia coefficient (last column) indicates that the inertia, and therefore the inertial mass, is increasing. We have found the results with the Einstein-de Sitter cosmic model.



Graph 2.- In blue, we represent the variation of the inertia coefficient for the Einstein-de Sitter Universe with only matter, using (6) with the coefficients $\xi_\alpha = 0.258$ and $b = 8$. The curve in red is the variation of the inertia coefficient for a Universe with only radiation, in which we have used the absorption function (6) with the coefficients $\xi_\alpha = 0.181$ and $b = 2$. The curve in black with a dashed line corresponds to a Universe with matter and dark energy, with the coefficients $\xi_\alpha = 0.2055$ and $b = 1$ for function (6). In all three cases, the coefficient of inertia increases, indicating that the body's inertial mass increases; this occurs because the Universe observable or causally linked to the body, is increasing, and therefore, the gravity it produces increases. We started drawing the graph at $\hat{\xi} = 0.3$.



Graph 3.- The graph shows the relative variation of the gravitation constant $\dot{G}/G$ measured in years^{-1} for the Einstein-de Sitter model. We verify that G decreases over time because $\dot{G}$ is negative. There was a significant variation of G in the Universe's early stages (not represented in the drawing). The variation was damped until it was practically constant from $\hat{\xi}=0.6$. At the current time $\hat{\xi}=1$, the relative change of G is $-1.6\cdot10^{-10}$ y^{-1}. The decrease of G with time means that the intensity of gravitation decreases under the assumption that $m_i=m_g$; or that the inertial mass increases its value while maintaining the value of G. We have obtained the data for a current age of the Universe of $9.69\cdot10^{10}$ years ($H_0=2.181\cdot10^{-18}$ s^{-1}).

we find

$$\frac{\dot{G}}{G}=-\frac{\dot{\chi}}{\chi^2}=-\frac{1}{t_0}\frac{\chi'(\xi)}{\chi^2(\xi)}. \tag{11}$$

In Graph 3, we represent (11) with respect to ξ for the simplified cosmological model we consider. From Graph 3 we see that the variation of G was very rapid in the first stages of the cosmos, attenuating later. G varies constantly with time, which means a decrease in gravitational force, equivalent to an increase in inertial mass. Currently, $\dot{G}/G=-1.6\cdot10^{-10}\ \mathrm{y}^{-1}$ a value higher than the limit measured by various procedures (Uzan, 2011, p. 77).

We decompose the integral (9)

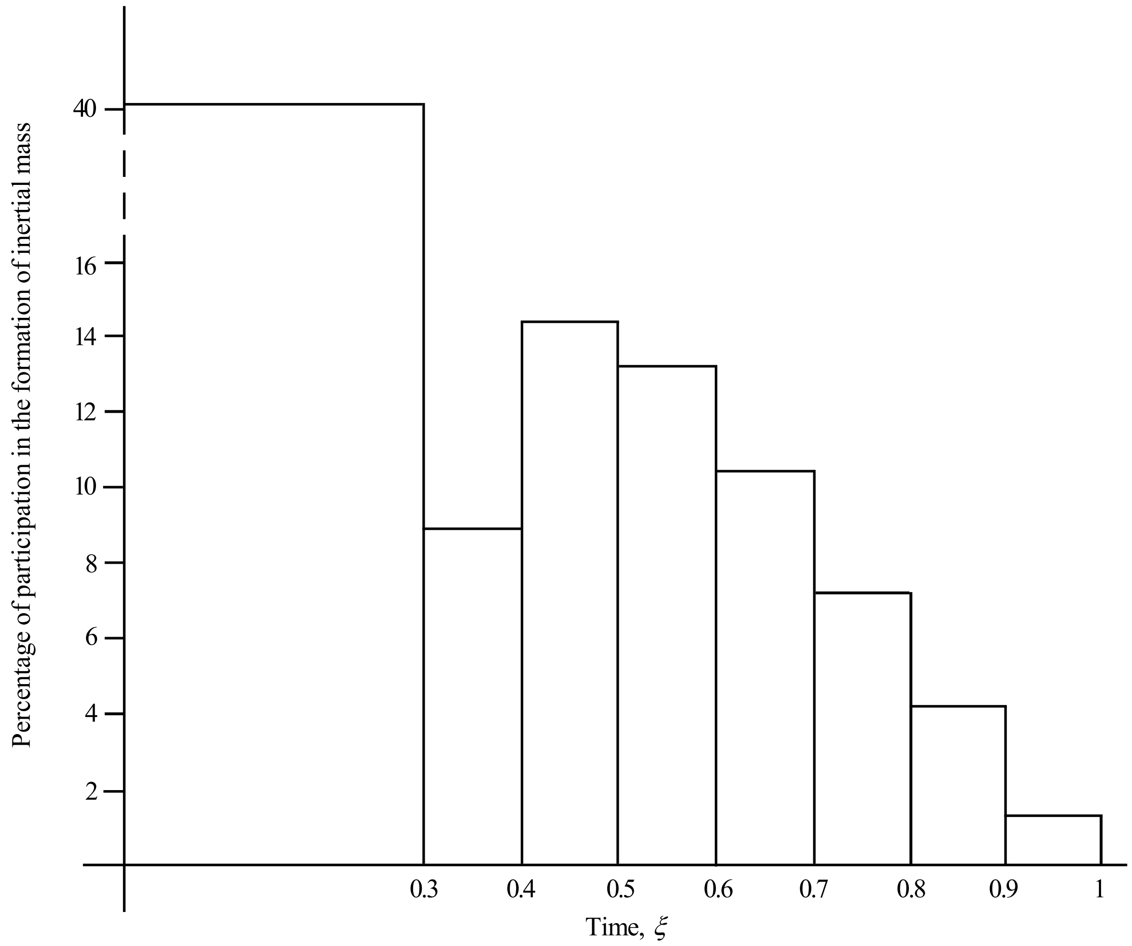
$$\chi(\hat{\xi})=\int_0^{0.258}\mu(\xi)f(\hat{\xi},\xi)d\xi+\int_{0.258}^{0.3}f(\hat{\xi},\xi)d\xi+\int_{0.3}^{0.4}f(\hat{\xi},\xi)d\xi+....+\int_{\hat{\xi}-0.1}^{\hat{\xi}}f(\hat{\xi},\xi)d\xi,$$

in this way, we can know the contribution to the inertial mass of different cosmic epochs. In Graph 4, we represent the contributions to the inertial mass, in per cent, according to the retarded moment. We verified that most of the inertial mass, 41%, corresponds to signals emitted between moments 0 and 0.3. The minor contribution to the inertial mass corresponds to the signals emitted in time close to the current time, that is, gravitational actions that come from nearby bodies.

5. Universe with matter and radiation

Let us consider another simplified cosmological model that, in addition to matter, contains dark energy and has zero scalar curvature. The relative scale factor of this model is (Raine & Thomas, 2001, p.88)

$$a=\sqrt[3]{\frac{1-\Omega_\Lambda^0}{\Omega_\Lambda^0}}\sinh^{2/3}\left(\frac{t}{t_\Lambda}\right)=\sqrt[3]{\frac{1-\Omega_\Lambda^0}{\Omega_\Lambda^0}}\sinh^{2/3}(\alpha\xi) \tag{12}$$



Graph 4.- We show the percentage participation of each cosmic epoch in the generation of the inertial mass of a body for the current moment. The interval between 0 and 0.3 is responsible for 40.9% of the inertial mass. Recent epochs of the Universe contribute to a lower average. For example, 0.9 to 1 only contributes 1.3%. The intervals on the horizontal axis indicate the emission times of the gravitational signals currently arriving at the body.

where

$$t_\Lambda = \frac{2}{c\sqrt{3\Lambda}}; \quad \Omega_\Lambda^0 = \frac{\Lambda c^2}{3H_0^2} \quad \Rightarrow \quad t_\Lambda = \frac{2}{3H_0}\frac{1}{\sqrt{\Omega_\Lambda^0}}; \quad \xi = \frac{t}{t_0}; \quad \alpha = \frac{t_0}{t_\Lambda},$$

Λ is the cosmological constant, Ω_Λ^0 is the vacuum density parameter at the present moment.

We deduce the Hubble constant from (12)

$$H = H_0\sqrt{\Omega_\Lambda^0}\coth(\alpha\xi) \tag{13}$$

when $t = t_0$, $H = H_0$, then from (13) we find

$$\frac{t_0}{t_\Lambda} = \alpha = \operatorname{arcoth}\frac{1}{\sqrt{\Omega_\Lambda^0}} \quad \Rightarrow \quad H_0 t_0 = \frac{2}{3}\frac{1}{\sqrt{\Omega_\Lambda^0}}\operatorname{arcoth}\frac{1}{\sqrt{\Omega_\Lambda^0}}. \tag{14}$$

By definition of the dark energy density parameter

$$\Omega_\Lambda = \Omega_\Lambda^0\frac{H_0^2}{H^2} \quad \Rightarrow \quad \Omega_\Lambda = \tanh^2(\alpha\xi). \tag{15}$$

6. Calculation of the force of inertia in a Universe with dark matter and energy

With the results of the previous section we determine the force of inertia by (A.8)

$$\delta\mathbf{F}_i = \left[\frac{11}{2}\coth^2(\alpha\xi) - \frac{9}{2}\right]\frac{H_0^2}{c^2}\Omega_\Lambda^0 m_g \mathbf{a}\frac{\sigma^2(\hat{\xi},\xi)\sigma'(\hat{\xi},\xi)}{r_p}d\xi =$$

$$=\left[\frac{11}{2}\coth^2(\alpha\xi)-\frac{9}{2}\right](H_0t_0)^2\,\Omega_\Lambda^0 m_g\mathbf{a}\frac{\overline{\dfrac{\sigma^2(\hat{\xi},\xi)}{c^2t_0^2}}\;\overline{\dfrac{\sigma'(\hat{\xi},\xi)}{ct_0}}}{\hat{\xi}-\xi}d\xi, \qquad (16)$$

we have taken into account (13), (15) and

$$\rho_M=\Omega_M\rho_c=(1-\Omega_\Lambda)\rho_c;\quad \rho_\Lambda=\Omega_\Lambda\rho_c;\quad p_\Lambda=-\rho_\Lambda c^2;\quad \rho_c=\frac{3H^2}{8\pi G},$$

ρ_M, ρ_Λ and ρ_c are the energy densities of matter in *t*, vacuum and critical, Ω_M is the density parameter of matter and *H* is the Hubble constant at time *t*.

Applying (14) in (16), we find

$$\delta\mathbf{F}_i=f(\hat{\xi},\xi)m_g\mathbf{a}\,d\xi;\quad f(\hat{\xi},\xi)=\frac{4}{9}\operatorname{arcoth}^2\left(\frac{1}{\sqrt{\Omega_\Lambda^0}}\right)\left[\frac{11}{2}\coth^2(\alpha\xi)-\frac{9}{2}\right]\frac{\overline{\dfrac{\sigma^2(\hat{\xi},\xi)}{c^2t_0^2}}\;\overline{\dfrac{\sigma'(\hat{\xi},\xi)}{ct_0}}}{\hat{\xi}-\xi}, \qquad (17)$$

using (A.14) we can now do the integration of the function $f(\hat{\xi},\xi)$.

We assume that σ is the same function as the Einstein-de Sitter Universe to simplify the calculations; this is justified because the cosmic factors for both cosmological models are very similar, at least for reception times not much higher than $\hat{\xi}=1$.

We do the integration for a model with void density parameter 0.685 and with (6) as the absorption function. Requiring that the coefficient of inertia is unity for the current moment, we find, for example, the parameters $\xi_\alpha=0.2055$ and $b=1$ for (6). Once again, we indicate that the absorption function (6) is an approximation; therefore, the values we find will only serve to reach some qualitative conclusions.

We show the results of the integration of $\mu(\xi)f(\hat{\xi},\xi)$ in Graph 2 (dashed black line), and we verify that the inertia coefficient increases with cosmic time, as in the models with only matter or radiation.

Graph 5 represents $\dot{G}/G$. The shape of the curve is similar to that found for matter only (Graph 3). However, the values found are one order of magnitude smaller and, therefore, closer to the limit values found experimentally.

7. Conclusions

We understand Mach's principle as the statement that the inertia of a body is the result of the gravitational action of the Universe as a whole. More specifically, the force of inertia is the force of gravitational induction produced by the accelerated movement of a body in relation to the Universe.

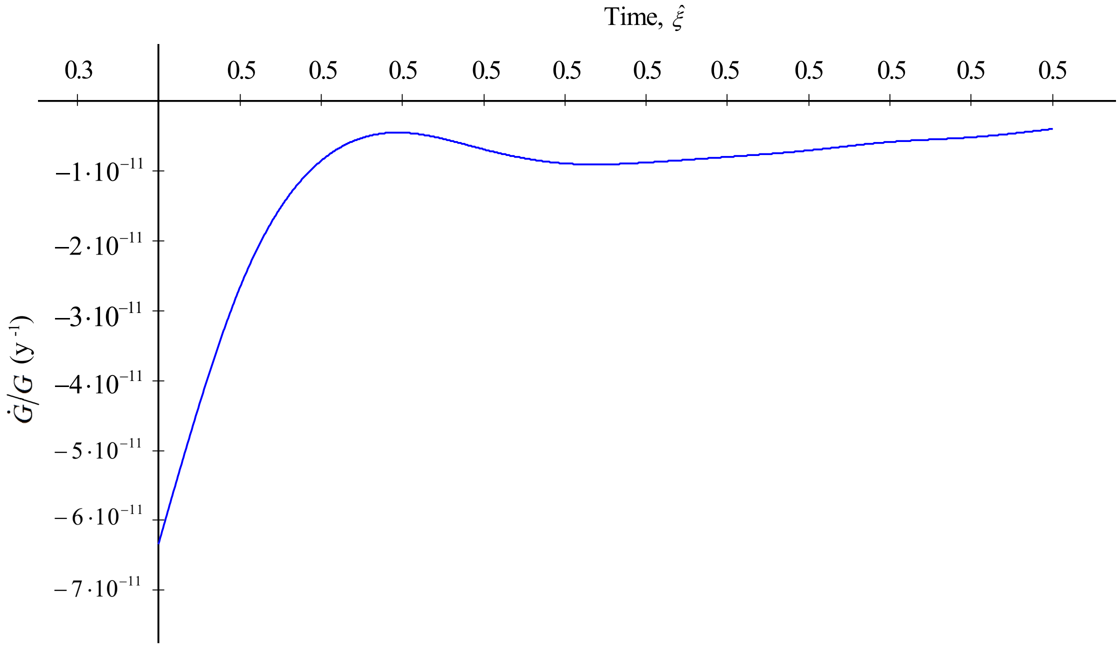
We calculate the induction force on an accelerated body and verify that it has the properties required by the laws of classical physics: it depends exclusively on the acceleration, is proportional to it and has the opposite sense. That is, the inertial mass is proportional to the gravitational mass, and its proportionality coefficient is dependent on cosmic time.

However, when we calculate the induction force produced by the entire Universe, we obtain a divergence due to the high gravity originating in the early stages of the cosmos.

To avoid this problem, we assume that there is a shielding or gravitational absorption, which must be very high at the beginning of the Universe but disappears when the Universe has expanded sufficiently. Speculatively, we have chosen an absorption function, exclusively dependent on time, that meets some minimum requirements but is an arbitrary hypothesis. However, although the specific values we obtain are incorrect, the results are interesting because they provide qualitative information related to Mach's principle.

Of the various conclusions we obtain from our reasoning, we limit ourselves to calculating the variation of the universal gravitation constant for several simplified cosmic models. Verifying that *G* decreases over time, there is a great variation in the first cosmic stages, to stabilize when the Universe's energy density decreases.

There are two other significant consequences of Mach's principle that we did not analyze in this research but that we now mention: the variation of the atomic emission frequency and the variation of the nuclear binding energy, both produced by the variation of the inertial mass.



Graph 5.- Relative variation of the gravitation constant $\dot{G}/G$ measured in year^{-1} for a Universe containing dark matter and energy. The curve has the same shape as that corresponding to a Universe with only matter, but it is one order of magnitude smaller. The relative variation of *G* for this cosmic model for the current moment is $8\cdot 10^{-11}\ \mathrm{y}^{-1}$.

The emission frequencies of an atom depend on the inertial mass of the electrons. Therefore, these frequencies are not unalterable but depend on the moment of their emission. Since the radiation that now reaches us originated in the past when the inertial mass was smaller, the frequencies we observe are lower than the frequencies of equal radiation produced today; this means that there would be shift of spectral lines even if there were no cosmic expansion. Alternatively, the redshift that we measure corresponds partly to the expansion of the Universe and partly to the variation in inertial mass. Therefore, the variation of inertial mass has important consequences in cosmological theory.

The rest energy of a body depends on its inertial mass (Segura, 2023a); if this varies as required by Mach's principle, its rest energy will also vary and, consequently, the energy released in nuclear processes. Therefore, the nuclear binding energy of a chemical element varies with time, in the sense that it increases as the inertial mass increases, which would produce noticeable modifications in stellar evolution.

Appendix

Below, we present a summary, with some corrections, of our article "Mach's Principle: the origin of the inertial mass (II)" (Segura, 2018).

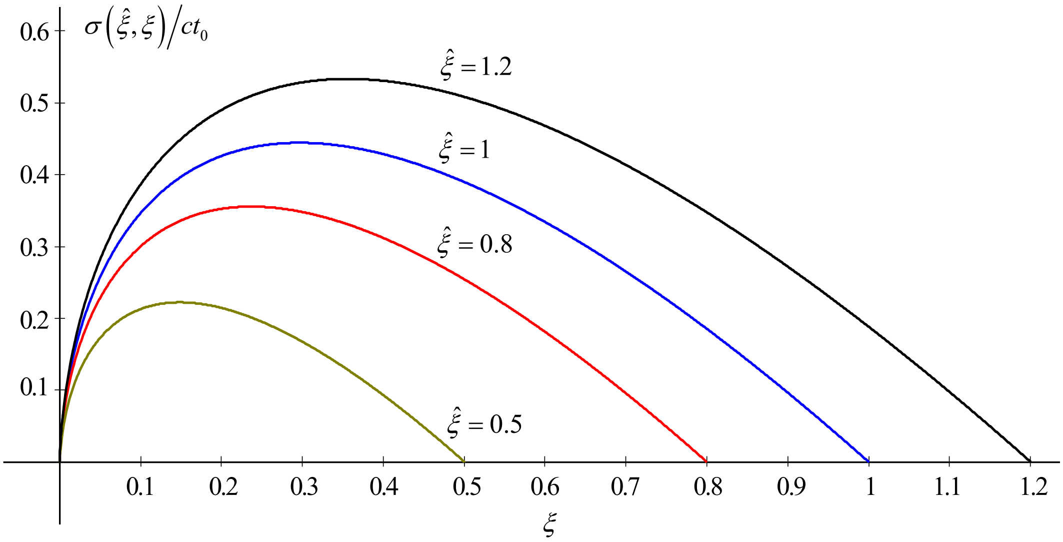
A.1.- Equation of motion

The equation of motion of a free particle in a gravitational field is the geodesic equation

$$\frac{d^2x^r}{d\tau^2}+\Gamma^r_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau}=0, \tag{A.1}$$

τ is the proper time of the particle, and x^r is its coordinates. Christoffel symbols appear for three reasons: by using curvilinear coordinates instead of Cartesian coordinates, by the acceleration of the reference frame, or by the presence of a gravitational field. We calculate the Christoffel symbols for Cartesian coordinates and an inertial reference frame, finding the second order in the inverse of *c* (Segura, 2013, pp. 41-54)

$$\frac{d\mathbf{v}}{dt}=-\nabla\phi-4\frac{\partial\mathbf{A}}{\partial t}+4\mathbf{v}\wedge\left(\nabla\wedge\mathbf{A}\right)-\nabla\left(\frac{2\phi^2}{c^2}+\frac{\psi}{c^2}\right)+3\frac{\mathbf{v}}{c^2}\frac{\partial\phi}{\partial t}+4\frac{\mathbf{v}}{c^2}\left(\mathbf{v}\cdot\nabla\right)\phi-\frac{v^2}{c^2}\nabla\phi, \tag{A.2}$$



Graph 6.- The graphs are the retarded proper distance functions corresponding to various reception instants for the Einstein-de Sitter Universe. That is, the distance of the source that emits the signal at time ξ and reaches the body at time $\hat{\xi}$. We observe that the curves have a maximum, which corresponds to the maximum proper distance that is causally linked to the body at time $\hat{\xi}$.
The graphs show that the derivative σ is positive until it reaches the maximum and then negative. In the calculations in the text, $d\sigma$ corresponds to the width of a spherical shell, which must be a positive distance, so we take its absolute value.

ϕ is the Newtonian scalar potential, $\mathbf{A}$ is the vector potential, and ψ is the inductive scalar potential. (A.2) is a function of distance and coordinated time, non-measurable magnitudes. Following an idea from (Brans, 1962) we express (A.2) in terms of the proper distance and the proper time (Segura, 2023b) *

$$\frac{d^2\boldsymbol{\sigma}}{d\tau^2}=\frac{d\mathbf{w}}{d\tau}=-\nabla'\phi-4\frac{\partial\mathbf{A}}{\partial\tau}+4\mathbf{w}\wedge\left(\nabla'\wedge\mathbf{A}\right)-\frac{1}{c^2}\nabla'\psi+\frac{\mathbf{w}}{c^2}\frac{\partial\phi}{\partial\tau}+2\frac{\mathbf{w}}{c^2}\left(\mathbf{w}\cdot\nabla'\right)\phi-\frac{w^2}{c^2}\nabla'\phi,$$

σ is the proper distance, ∇' is a derivation with respect to the components of the proper distance, and $\mathbf{w}$ is the proper velocity. If we depreciate the speed w of the particle with respect to c, we find

$$\frac{d\mathbf{w}}{d\tau}=-\nabla'\phi-4\frac{\partial\mathbf{A}}{\partial\tau}, \tag{A.3}$$

we do not consider $\nabla\wedge\mathbf{A}$, it is of one lower order than (A.3) with respect to the inverse of c (Panofsky & Phillips, 1929, p. 300).

A.2.- Cosmic induction force

From (A.3), we deduce that the gravitational force acting on a body of gravitational mass m_g in a weak gravitational field is

$$\mathbf{F}=m_g\left(-\nabla'\phi-4\frac{\partial\mathbf{A}}{\partial\tau}\right). \tag{A.4}$$

If the body is at rest relative to the Universe, there are no inductive terms, that is, components of ϕ and $\mathbf{A}$ dependent on the relative velocity. Therefore, only the Newtonian force would exist, which, as we have seen, is nullified when it is calculated for the entire Universe, as long as the cosmological principle is applicable. Later, we verified that the force (A.4) was also zero if the body had uniform and rectilinear movement.

Suppose that the particle is accelerated with respect to the Universe; then there will be relative motion and therefore, ϕ and $\mathbf{A}$ will have inductive terms. This inductive force is what we

* Some authors omit factor 4 in the second member but place it in the definition of $\mathbf{A}$. In any case, this factor is a differentiating element concerning the Lorentz force of electromagnetism.

identify with the force of inertia.

The equation of motion (A.1) is valid using the constant G_0 (its usual value); that is, we assume that the inertial mass is equal to the gravitational mass. Now, if m_g and m_i were proportional with a proportionality factor χ, then the constant would have to be changed to $G = G_0/\chi$.

Since the constant *G* appears in the gravitational potentials which we derive from Christoffel's symbols, we will obtain

$$\frac{d^2x^r}{d\tau^2} + \frac{m_g}{m_i}\Gamma^r_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau} = 0 \quad \Rightarrow \quad m_i\frac{d\mathbf{w}}{d\tau} = m_g\left(-\nabla'\phi - 4\frac{\partial \mathbf{A}}{\partial \tau}\right). \tag{A.5}$$

We consider (A.5) valid even if the proportionality between m_i and m_g is time-dependent because, during the time involved in the movement of a particle, we can consider the relationship between m_i and m_g constant.

A.3.- Gravitoelectromagnetic potentials

By application of the Liénard-Wiechert potentials of electromagnetism (Panofskhy & Philips, 1929, pp. 286-289), we find the gravitoelectromagnetic potentials

$$\phi = -G\int\frac{\left[\rho + 3p/c^2\right]}{s}dV; \quad \psi = -2G\int\frac{\left[\rho + p/c^2\right]}{s}dV; \quad A = -\frac{G}{c^2}\int\frac{\left[\left(\rho + p/c^2\right)\mathbf{u}\right]}{s}dV, \tag{A.6}$$

We use the components of the energy-momentum tensor deduced from the linearized theory of General Relativity (Segura, 2019). The bracket in the integrals indicates that we take the retarded values, that is, those corresponding to the time of emission; $\mathbf{u}$ is the relative velocity of the source with respect to the body; we define *s* by $s = r - \mathbf{r}\cdot\mathbf{u}/c$; $\mathbf{r}$ is the position vector of the body with respect to the source; *s* coincides with the relative distance *r* when the speed *u* is very small compared to *c*; ρ is the density of all forms of energy and *p* is the pressure caused by radiation and dark energy.

A.3.- Induction force

We consider that the body of gravitational mass m_g has speed $-\mathbf{u}$ and acceleration $-\mathbf{a}$ with respect to the Universe. We calculate the induction force acting on the body by (A.6) and (A.5).

To integrate the induction force we divide the Universe into spherical shells of negligible thickness *dr*, centered on the body. The position vector of an element of the spherical shell with respect to the body in spherical coordinates is

$$-\mathbf{r} = -r\sin\theta\cos\varphi\mathbf{i} - r\sin\theta\sin\varphi\mathbf{j} - r\cos\theta\mathbf{k}.$$

To calculate the potentials of a spherical shell, the terms of interest are

$$\nabla'\delta\phi = -G\int\left[\rho + 3p/c^2\right]\nabla'\left(\frac{1}{s}\right)dV; \quad \frac{\partial\delta\mathbf{A}}{\partial\tau} = -\frac{G}{c^2}\int\left[\rho + p/c^2\right]\frac{\partial}{\partial\tau}\left(\frac{\mathbf{u}}{s}\right)dV,$$

$\delta\phi$ and $\delta\mathbf{A}$ are the potentials produced by the spherical shell, and the limit of the integral is the volume of the shell.

We use the results found in electromagnetism (Panofsky & Phillips, 1929, pp. 297-300), we neglect the terms $1/r^2$ with respect to $1/r$, since *r* is a cosmic dimension and, we consider $u << c$, then the integration is

$$\delta\mathbf{F} = \frac{4\pi}{3}\frac{G}{c^2}\left[11\rho + 9p/c^2\right]m_g\mathbf{a}\frac{\sigma^2 d\sigma}{r_p}, \tag{A.7}$$

$\delta\mathbf{F}$ is the induction force or force of inertia produced by the spherical shell that is at the retarded proper distance σ. Note that by our previous definition, $-\mathbf{a}$ is the acceleration of the body with respect to the Universe. Therefore, we verify that the force of inertia is opposite to the acceleration of the body. We integrate (A.7) for the entire Universe to find the total inertial force.

A.4.- Cosmic parameters

Using the cosmic density parameters, from (A.7) we obtain

$$\delta\mathbf{F} = \left(\frac{11}{2}\frac{\Omega_M^0}{a^3} + 14\frac{\Omega_R^0}{a^4} + \Omega_V^0\right)\frac{H_0^2}{c^2}m_g\mathbf{a}\frac{\sigma^2\sigma'}{r_p}d\xi \tag{A.8}$$

Ω_M^0, Ω_R^0, Ω_Λ^0 are the density parameters of matter, radiation and vacuum at the current moment t_0; $a = R/R_0$, *R* and R_0 are the cosmic scale factors at time *t* and at the current time t_0; H_0 is the

Hubble constant at the current time; $\sigma' = d\sigma/d\xi$ and r_p is the photonic distance.

A.5.- Coefficient of inertia

The force of inertia on the particle of gravitational mass m_g at time $\hat{\xi}$ is

$$\mathbf{F}_i = \int_0^{\hat{\xi}} \left(\frac{11}{2} \frac{\Omega_M^0}{a^3} + 14 \frac{\Omega_R^0}{a^4} + \Omega_V^0 \right) \frac{H_0^2}{c^2} m_g \mathbf{a} \frac{\sigma^2\left(\hat{\xi},\xi\right)\sigma'\left(\hat{\xi}',\zeta\right)}{r_p\left(\hat{\xi},\xi\right)} d\xi = \left[\int_0^{\hat{\xi}} f\left(\hat{\xi},\xi\right) d\xi \right] m_g \mathbf{a}. \quad \text{(A.9)}$$

Since the acceleration of the body is $-\mathbf{a}$ and the force of inertia is $-m_i\left(-\mathbf{a}\right) = m_i \mathbf{a}$, then the coefficient of inertia (2) is by (A.9)

$$\chi\left(\hat{\xi}\right) = \int_0^{\hat{\xi}} f\left(\hat{\xi},\xi\right) d\xi. \quad \text{(A.10)}$$

A.6.- Application to the Einstein-de Sitter cosmic model

We apply (A.10) to the Einstein-de Sitter Universe, characterized by $\Omega_M^0 = 1$ and null the other cosmic parameters. Using the Friedman equation, we find the scale factor

$$\left(\frac{da}{d\xi}\right)^2 = \left(H_0 t_0\right)^2 \frac{1}{a} \quad \Rightarrow \quad a = \left(\frac{3}{2} H_0 t_0 \xi\right)^{2/3}$$

when $t = t_0$ results $R = R_0$ therefore $H_0 t_0 = 2/3$ and

$$a = \xi^{2/3}.$$

A.7.- Calculation of distances

The coordinate distance is defined by

$$r\left(\hat{\xi},\xi\right) = \int_t^{\hat{t}} \frac{c\,dt'}{R\left(t'\right)} = \frac{3ct_0}{R_0}\left(\hat{\xi}^{1/3} - \xi^{1/3}\right),$$

t is the moment of emission or retarded moment and $\hat{\xi}$ is the moment of reception.

The retarded proper distance is

$$\sigma\left(\hat{\xi},\xi\right) = R\left(\xi\right) r\left(\hat{\xi},\xi\right) = 3ct_0\left(\hat{\xi}^{1/3}\xi^{2/3} - \xi\right)$$

and

$$\sigma'\left(\hat{\xi},\xi\right) = 3ct_0\left(\frac{2}{3}\hat{\xi}^{1/3}\xi^{-1/3} - 1\right),$$

and therefore

$$\sigma^2\left(\hat{\xi},\xi\right)\sigma'\left(\hat{\xi},\xi\right) = 27c^3t_0^3\left(\frac{2}{3}\hat{\xi}\xi - \frac{7}{3}\hat{\xi}^{2/3}\xi^{4/3} + \frac{8}{3}\hat{\xi}^{1/3}\xi^{5/3} - \xi^2\right). \quad \text{(A.11)}$$

We mention that, as we see in Graph 5, the slope of the σ curve is initially positive and then negative. However, we will always take the absolute value, since $d\sigma$ corresponds to the thickness of a spherical shell, which must be positive. Applying these results in (A.9), we find a divergent integral due to the first states of the Big Bang. For this reason, we establish the gravitational absorption hypothesis, as explained in the main text.

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