Electron viscosity and electromagnetic radiation for the dynamic deformations of metals

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The electron viscosity is popular in the dynamic deformations of metals, and it was revealed to dominate the related energy dissipation at low temperatures. The free electron model was extensively utilized to investigate the electron viscosity of the related phenomena including electron viscosity of mobile dislocations and the attenuation coefficient of elastic waves at low temperatures. However, the potential energy of the "free" electrons was neglected totally. In this work, the mechanical-electric coupling which contains both the potential energy and kinetic energy of "free" electrons was taken into account. And it was found that the attenuation coefficients of the longitudinal and shear waves of metals at cryogenic temperatures are proportional to the electrical conductivity and the square of angular frequency, which was in accord with experimental observations. The longitudinal and shear waves in a metal was found to induce the electromagnetic radiation whose frequency is the same as the stress wave. In addition, the electron viscosity was discovered to result in a temperature increase over the compression wave front. The temperature increase depends on the strain gradient, and a larger strain gradient may lead to a larger temperature rise during the compression wave front. Furthermore, the electron viscosity of the mobile edge and screw dislocations was obtained in theory. And the order of calculated magnitude in terms of the mechanical-electric coupling strength that can be determined by the attenuation coefficient of the longitudinal and shear wave agrees with experimental results. Overall, the revealed important effects of the electron viscosity for the dynamic deformations of metals were investigated and the obtained findings may aid in understanding the related phenomena deeply.

keywords: electron viscosity; mechanical-electric coupling; electromagnetic radiation; attenuation coefficient; dislocation;

1. Introduction

It was experimentally revealed that the ultrasonic attenuation coefficient of normal state metals at low temperatures is attributed to the free electrons [1], *i.e.*, electron viscosity. As indicated by the experimental observations, the electric attenuation coefficient is linear with the electrical conductivity and the square of the sound frequency [1]. The variations of the ultrasonic attenuation coefficient with the temperature and sound frequency was ever understood in terms of the free electron gas model [2] and the electron-phonon interaction [3]. However, these models neglected the important role of the ionic potential for the electrons in the normal state metal.

Besides, the dislocations are a primary plastic mechanism and the dislocation motion commonly dominates plastic behaviors of the metals. When the dislocation slides in the normal metals, the electrons and phonons will resist the dislocation motion, yielding the electron viscosity and phonon drag [4]. The phonon drag commonly govern the drag coefficient of the mobile dislocations at relatively high temperatures [5, 6, 7]. But the electron viscosity can monitor the drag coefficient of the dislocation sliding at low temperatures [5, 6,]. Owing to the experimental obstacles in measuring the electron drag coefficient of an individual dislocation, the precise experimental determination of the electron drag coefficient is very difficult [5]. Many offers were made to explore the electron viscosity for the dislocation sliding in theory, but some conclusions were still debated, for example, the temperature dependence of electron drag coefficient [4, 5, 6,]. To account for the electron viscosity for the dislocation sliding, the free electron gas model was also utilized [6] which nevertheless did not take the important ionic potential of the electrons into account again. In addition, the debate of the temperature-dependent electron drag coefficient need to be clarified.

In this work, the electron viscosity and electromagnetic radiation for the dynamic deformations of metals were investigated by means of the mechanical-electric coupling (MEC) which considers the ionic potential of the “free” electrons. And the debate of the electron drag coefficient versus temperature was tackled. The structure of this paper was organized as follows. The section 1 offered the introduction. The section 2 briefly introduced the fundamental theory of this work. The section 3 gave the obtained theoretical results and discussions. Wherein, the subsection 3.1 offered the general theory for the electron viscosity and electromagnetic radiation of the deformed metals. The subsection 3.2 discussed the electron viscosity for elastic waves and the involved electromagnetic radiation in metals. The subsection 3.3 touched the electron viscosity during the compression wave front in a metal. The subsection 3.4 focused on the electron viscosity for the mobile dislocations in metals. The section 4 gave the conclusions.

2. Theory

Yuheng Zhang equation should be introduced first, which may be the most important foundation in this work. For any metal, there may usually exist some physical factors such as strain, temperature, doping and so on, which can give birth to alterations of Fermi surface of the material. Analogous to water flowing from a higher position to a lower position, electrons also tend to drift from the regions with higher Fermi surface to the regions with lower Fermi surface regions, thereby inducing an electric field between the regions when the equilibrium state is reached. The physical relation between the formed electrostatic field and the correlated Fermi surface may be described by *Yuheng Zhang equation* [8-12],

$$\nabla E_F = e\vec{E} \tag{1}$$

where e is electron charge, E_F is the electron chemical potential (ECP). The value E_F in this equation may usually be taken relative to the energy of a static electron at infinity without any electromagnetic disturbances and thus it must be negative. Of specially emphasized is that it may not be Fermi energy which is usually encountered in many textbooks and literatures, because Fermi energy for metals only refers to the energy difference between the highest and lowest occupied single-electron states in a non-interacting free electron system at zero temperature and cannot take into account the variations of lowest occupied electron state influenced by some physical factors such as strain, doping and so on. *Yuheng Zhang equation* may be very important in various fields and its applicability should be discussed here. This equation may rigorously hold for systems that must satisfy the following conditions. First, the electron systems must be in an equilibrium state. Second, the electron systems must conform to the number conservation of electrons. Third, the electron must be a point particle and does not exhibit any measureable volume effects, which is the foundation of quantum electrodynamics (QED). *Yuheng Zhang equation* may play an important role in different areas and some significant applications were discussed elsewhere [8-12].

3. Results and discussion

In this work, *Yuheng Zhang equation* would be employed to investigate the dynamic deformations of metals. Based on Equation (1), an important deriving relation may hold in mechanics of materials [9, 11], which is given by

$$\vec{E}_c = \frac{\partial E_F}{e\partial \xi_{ij}} \nabla \xi_{ij} \tag{2}$$

where E_c is the electric field when the electron system reaches equilibrium state, ξ_{ij} are the strain components, the indices are i, j=x, y, z and they obey Einstein summation convention. This equation means that the strains could alter ECP of a material and could generate an electric field in regions with different strains. In the following discussions, the electric field in Equation (2) would be the key in related mechanical phenomena of materials.

3.1 The electron viscosity and electromagnetic radiation of deformed metals

When a metal is deformed, a concomitant electrical current will appear. The general expression of the electrical current is

$$\vec{J}\left(\vec{r},t\right) = \frac{n_e e\vec{p}\left(\vec{r},t\right)}{m_e} \tag{3}$$

where $J(r,t)$, $p(r,t)$ is the position and time dependence of the electrical current and the electron average momentum, respectively, n_e is the electron density, m_e is the electron mass. According to the classical electron dynamics,

$$\frac{d\vec{p}\left(\vec{r},t\right)}{dt} = \left[e\vec{E}\left(\vec{r},t\right) - \nabla E_F\left(\vec{r},t\right)\right] - \frac{\vec{p}\left(\vec{r},t\right)}{\tau_e} \tag{4}$$

where $E(r,t)$ is the real electric field, $E_F(r,t)$ is the position and time dependence of ECP, τ_e is the electron relaxation time and it is built on the relaxation time Approximation. Based on the charge conservation and the Maxwell equations, the relations follow

$$\frac{\partial \rho_e(\vec{r},t)}{\partial t} + \nabla \cdot \vec{J}(\vec{r},t) = 0 \quad (5)$$

$$\varepsilon_0 \varepsilon \nabla \cdot \vec{E}(\vec{r},t) = \rho_e(\vec{r},t) \quad (6)$$

where $\rho_e(r,t)$ is both the position and the time dependent net charge density, ε_0 is vacuum permittivity, ε is the relevant dielectric constant.

If the position-dependent ECP is static and does vary with time, the combination of the above equations generates the partial derivative equation

$$\tau_e \frac{\partial^2 \rho_e(\vec{r},t)}{\partial t^2} + \frac{\partial \rho_e(\vec{r},t)}{\partial t} + \frac{\rho_e(\vec{r},t)}{\tau} - \frac{\sigma_0}{e} \nabla^2 E_F(\vec{r}) = 0$$

where the equilibrium time is $\tau = \varepsilon_0 \varepsilon / \sigma_0$, the direct electrical conductivity is $\sigma_0 = n_e e^2 \tau_e / m_e$. Its exact analytical solution may be

$$\rho_e(\vec{r},t) = \frac{\varepsilon_0 \varepsilon \nabla^2 E_F(\vec{r})}{e} \left[1 - e^{-t\left(1-\sqrt{1-4\tau_e/\tau}\right)/2\tau_e} \right]$$

Generally, the ECP not only exhibits the position dependence but also changes with time. To solve the problem, the related physical variables could be written in the following Fourier series.

$$E_F(\vec{r},t) = \sum_{\omega} E_F(\vec{r},\omega) e^{-i\omega t} = \sum_{\vec{q},\omega} E_F(\vec{q},\omega) e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

$$\vec{p}(\vec{r},t) = \sum_{\omega} \vec{p}(\vec{r},\omega) e^{-i\omega t} = \sum_{\vec{q},\omega} \vec{p}(\vec{q},\omega) e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

$$\vec{E}(\vec{r},t) = \sum_{\omega} \vec{E}(\vec{r},\omega) e^{-i\omega t} = \sum_{\vec{q},\omega} E(\vec{q},\omega) e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

$$\rho_e(\vec{r},t) = \sum_{\omega} \rho_e(\vec{r},\omega) e^{-i\omega t} = \sum_{\vec{q},\omega} \rho_e(\vec{q},\omega) e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

where q is wave vector, ω is the angular frequency. As a consequence, the electron momentum is

$$\vec{p}(\vec{q},\omega)=\frac{e\tau_e}{1-i\omega\tau_e}\left[\vec{E}(\vec{q},\omega)-\frac{i\vec{q}E_F(\vec{q},\omega)}{e}\right]$$

And the electrical current is

$$\vec{J}(\vec{r},t)=\sum_{\vec{q},\omega}\frac{\sigma_0}{1-i\omega\tau_e}\left[\vec{E}(\vec{q},\omega)-\frac{i\vec{q}E_F(\vec{q},\omega)}{e}\right]e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

The net charge density can be expressed as follows

$$-i\omega\rho_e(\vec{q},\omega)+\frac{1}{1-i\omega\tau_e}\left[\frac{\rho_e(\vec{q},\omega)}{\tau}-\frac{\sigma_0(i\vec{q})^2}{e}E_F(\vec{q},\omega)\right]=0$$

where the equilibrium time is $\tau=\varepsilon_0\varepsilon/\sigma_0$. In terms of simple calculation, the net charge density is

$$\rho_e(\vec{q},\omega)=\frac{\sigma_0\tau(i\vec{q})^2E_F(\vec{q},\omega)}{e\left[1-i\omega\tau(1-i\omega\tau_e)\right]} \tag{8}$$

And the derivative of the electric field may be

$$i\vec{q}\cdot\vec{E}(\vec{q},\omega)=\frac{(i\vec{q})^2E_F(\vec{q},\omega)}{e\left[1-i\omega\tau(\omega)(1-i\omega\tau_e)\right]}$$

If the Lorentz gauge is used, the related magnetic vector potential and the electric potential obey the following relations [13]

$$\nabla\cdot\vec{A}(\vec{r},t)+\frac{1}{c_m^2}\frac{\partial\varphi(\vec{r},t)}{\partial t}=0 \tag{9}$$

$$\nabla^2\varphi(\vec{r},t)-\frac{\partial\varphi(\vec{r},t)}{\partial t^2}=-\frac{\rho_e(\vec{r},t)}{\varepsilon_0\varepsilon} \tag{10}$$

where $\vec{A}(\vec{r},t)$ is the position and time dependence of the magnetic vector potential, $\varphi(\vec{r},t)$ is the electric potential, the speed is $c_m^2=1/\mu_0\varepsilon_0\varepsilon$. In terms of calculation, the electric potential can be obtained. Performing the Fourier transformation yields

$$(i\vec{q})^2 \varphi(\vec{q},\omega) - \frac{(-i\omega)^2}{c_m^2}\varphi(\vec{q},\omega) = -\frac{(i\vec{q})^2 E_F(\vec{q},\omega)}{e\left[1-i\omega\tau(1-i\omega\tau_e)\right]}$$

The solution is

$$\varphi(\vec{q},\omega) = \frac{-(i\vec{q})^2 E_F(\vec{q},\omega)}{e\left[(i\vec{q})^2 + \omega^2/c_m^2\right]\left[1-i\omega\tau(1-i\omega\tau_e)\right]} \tag{11}$$

The magnetic vector potential can be divided into two parts. One is the parallel component, and the other is the perpendicular component.

$$\vec{A}(\vec{q},\omega) = \vec{A}_\perp(\vec{q},\omega) + \vec{A}_\parallel(\vec{q},\omega)$$

According to the Lorentz gauge, the parallel component conforms to the relation

$$\vec{q}\cdot\vec{A}_\parallel(\vec{q},\omega) = \frac{\omega}{c_m^2}\varphi(\vec{q},\omega)$$

$$\nabla^2\vec{A}(\vec{r},t) - \frac{1}{c_m^2}\frac{\partial^2\vec{A}(\vec{r},t)}{\partial t^2} = -\mu_0\vec{J}(\vec{r},t) \tag{12}$$

$$(i\vec{q})^2\vec{A}(\vec{q},\omega) - \frac{(-i\omega)^2}{c_m^2}\vec{A}(\vec{q},\omega) = -\mu_0\vec{J}(\vec{q},\omega)$$

As a consequence, the solution of the magnetic vector potential is

$$\vec{A}(\vec{q},\omega) = \frac{-\mu_0\vec{J}(\vec{q},\omega)}{\left[(i\vec{q})^2 + \omega^2/c_m^2\right]}$$

The electrical current can also be divided into two parts. One is the parallel component, and the other is the perpendicular component.

$$\vec{J}(\vec{q},\omega) = \vec{J}_\perp(\vec{q},\omega) + \vec{J}_\parallel(\vec{q},\omega)$$

Based on the above equations, the parallel component of the electrical current is

$$\vec{J}_\parallel(\vec{q},\omega) = -\frac{\sigma_0\omega\tau\vec{q}E_F(\vec{q},\omega)}{e\left[1-i\omega\tau(1-i\omega\tau_e)\right]} \tag{13}$$

Here the word "parallel" refers to the physical components parallel to the wave vector, *i.e.*, parallel to the direction of the strain gradient. For the electron motion in the perpendicular direction, there is no driving force accelerating electrons in the direction, so the perpendicular components of the physical variables $J_{\perp}(\vec{q},w)$, $E_{\perp}(\vec{q},w)$, $A_{\perp}(\vec{q},w)$ may be so small that they can be ignored safely.

Thus the related electric field is

$$\vec{E}_{\parallel}\left(\vec{q},\omega\right)=\frac{i\vec{q}E_F\left(\vec{q},\omega\right)}{e\left[1-i\omega\tau\left(1-i\omega\tau_e\right)\right]} \tag{14}$$

The electrical current will induce the electrical energy loss and the electromagnetic radiation. The electrically dissipated power may be

$$P_e=\frac{\sigma_0}{2}\int_V\sum_{\vec{q},\omega}\frac{\left[\omega\tau\vec{q}E_F\left(\vec{q},\omega\right)\right]^*\omega\tau\vec{q}E_F\left(\vec{q},\omega\right)}{e^2\left[\left(1-\omega^2\tau\tau_e\right)^2+\left(\omega\tau\right)^2\right]}dV \tag{15}$$

And the generated electromagnetic radiation outside the metal is

$$\vec{A}_r\left(\vec{r},\omega\right)=\frac{\mu_0}{4\pi}\int_V\frac{\vec{J}\left(\vec{r}',\omega\right)e^{i\vec{k}\cdot\left(\vec{r}'-\vec{r}\right)}}{\left|\vec{r}'-\vec{r}\right|}dV' \tag{16}$$

where k is wave vector of the electromagnetic radiation and it equals $k=\omega/c$, $\overrightarrow{A_r}(\vec{r},\omega)$ is the position dependence of the magnetic vector potential with angular frequency ω. So the position and time dependence of the magnetic vector potential is

$$\vec{A}_r\left(\vec{r},t\right)=\sum_{\omega}\vec{A}_r\left(\vec{r},\omega\right)e^{-i\omega t} \tag{17}$$

The corresponding magnetic field of the radiation can be derived

$$\vec{B}_r\left(\vec{r},t\right)=\nabla\times\vec{A}_r\left(\vec{r},t\right) \tag{18}$$

The above discussion indicates that the metal undergoing the time-dependent deformation will give birth to an electrical current which further induces the electromagnetic radiation. Reversely, the exact detection of the electromagnetic radiation outside the metal may assist one in uncovering the dynamic deformation of the metal.

In general, any complicated dynamic strains of the metal may be regarded as a result of the action of the stress waves. And the strain rate may be expressed as

$$\dot{\xi}_{ij} = \vec{v}_r \cdot \nabla \xi_{ij}$$

where v_r is the velocity of the related stress wave. Concomitant with the propagation of the stress waves in a metal is the electrically dissipated power which can be calculated by substituting the above equation and the equation (2) into the equation (15). In order to illustrate it, some typical cases will be touched in the subsequent sections.

3.2 Electron viscosity for elastic waves in metals

3.2.1 Electron viscosity for the longitudinal elastic waves in metals

The experiments indicated that the attenuation of the elastic waves in a metal at low temperatures can be mainly attributed to the electron viscosity [1, 14, 15]. To explore the effect of the electron viscosity, a longitudinal wave propagating in *x* direction in a metal is considered and could be described by

$$u_l = A_l e^{-\alpha_l x} e^{i(q \cdot x - \omega_q t)}$$

where q is wave vector, ω_q is the wave vector dependence of the angular frequency and it is $\omega_q = qv_l$, v_l is the propagation speed of the longitudinal wave, u_l is the displacement vector, A_l is the vibration amplitude of the longitudinal elastic wave, α_l is the attenuation coefficient for the elastic wave because of the electron viscosity at low temperatures. Based on the definition of the strains [16, 17], the strain components induced by the elastic wave may be given by

$$\xi_{xx} = \left(iq - \alpha_l\right) A e^{-\alpha_l x} e^{i\left(q \cdot x - \omega_q t\right)}$$

Due to the Poisson effect, the normal strain in *y* and *z* direction can be given based on the Poisson's ratio. For a simple cubic lattice, the normal strains are

$$\xi_{yy} = -\upsilon \xi_{xx}$$

$$\xi_{zz} = -\upsilon \xi_{xx}$$

where υ is the Poisson's ratio, ξ_{yy}, ξ_{zz} are the normal strains in *y* direction and *z* direction, respectively. The strain gradient induced by the elastic wave in the metal may be

$$\nabla \xi_{xx} = \left(iq - \alpha_l\right)^2 A e^{-\alpha_l x} e^{i\left(q \cdot x - \omega_q t\right)}$$

The strains may further causes the variations of the ECP which can be expressed as below.

$$E_F\left(x\right) = E_{F0} + \frac{\partial E_F}{\partial \xi_{xx}} \xi_{xx} + \frac{\partial E_F}{\partial \xi_{yy}} \xi_{yy} + \frac{\partial E_F}{\partial \xi_{zz}} \xi_{zz}$$

Because of the symmetry of the simple cubic metal, the above equation may be simplified

$$E_F\left(x\right) = E_{F0} + \frac{\partial E_F}{\partial \xi_V}\left(1 - 2\upsilon\right) \xi_{xx}$$

where ξ_V is the volume strain. So there exists an electric field accompanying the longitudinal elastic waves and this electric field can be obtained according to Equation (2). The related electrical current can be obtained according to Equation (13),

$$\vec{J}\left(\vec{q}\right) = \frac{i\sigma_0 \omega_q \tau \left(iq - \alpha_l\right)^2 \left(1 - 2\upsilon\right) A}{\left[1 - i\omega_q \tau \left(1 - i\omega_q \tau_e\right)\right]} \frac{\partial E_F}{e \partial \xi_V}$$

$$\vec{J}\left(x\right) = \frac{\sigma_0 i\omega_q \tau \left(1 - 2\upsilon\right)}{\left[1 - i\omega_q \tau \left(1 - i\omega_q \tau_e\right)\right]} \frac{\partial E_F}{e \partial \xi_V} \left(iq - \alpha_l\right)^2 A e^{-\alpha_l x} e^{i\left(qx - \omega_q t\right)}$$

The electrical current may lead to the dissipation power per unit volume according to Equation (15).

$$P_e = \frac{\sigma_0}{2}\frac{(\omega_q\tau)^2(1-2\upsilon)^2}{(1-\omega_q^2\tau\tau_e)^2+(\omega_q\tau)^2}\left(\frac{\partial E_F}{e\partial\xi_V}\right)^2\left(q^2+\alpha_l^2\right)^2 A^2 e^{-2\alpha_l x}$$

In general, the relation $\omega_q\tau >> 1$ may be popularly valid for the elastic waves whose frequency usually ranges from 1 MHz to 100 MHz in the related experiments. The electrically dissipated power density may be

$$P_e \approx \frac{\sigma_0}{2}(1-2\upsilon)^2\left(\frac{\partial E_F}{e\partial\xi_V}\right)^2\left(q^2+\alpha_l^2\right)^2 A^2 e^{-2\alpha_l x}$$

The energy density of the elastic waves including both the elastic energy and the kinetic energy may be

$$W_l = G_Y\left(q^2+\alpha_l^2\right)A^2 e^{-2\alpha_l x}$$

where G_Y is the Young's modulus, W_l is the energy density of the longitudinal elastic wave. The energy of the longitudinal elastic wave may be attenuated by the electrically dissipated power

$$\frac{dW_l}{dt} = -P_e$$

In the case that $q >> \alpha_l$, the attenuation coefficient may be derived

$$\alpha_l \approx \frac{\sigma_0(1-2\upsilon)^2}{4G_Y v_l}\left(\frac{\partial E_F}{e\partial\xi_V}\right)^2 q^2 \qquad (18)$$

According to this relation, the electron viscosity-induced attenuation of a longitudinal elastic wave depends on several important factors such as electrical conductivity σ_0, angular frequency ω_q and normal strain-induced mechanical-electric coupling (MEC) strength. As is shown, the attenuation coefficient is proportional to the electrical conductivity and a higher electrical conductivity at lower temperatures usually result in a larger attenuation coefficient, which was verified by ultrasonic damping experiments at low temperatures [1, 14]. Also shown by the relation, the attenuation due to electron viscosity may be proportional to square of angular frequency, and the attenuation may be much more serious for elastic waves with shorter wavelength and higher frequency than that with longer wavelength and lower frequency. This point was also confirmed well by related experiments [1]. On the other hand, the attenuation coefficient may also be monitored by the MEC strength $|\partial E_F/e\partial\xi_V|$, and a higher MEC may lead to a much more notable electron damping effect for longitudinal elastic waves in metals.

Inversely, the precise measurement of the attenuation coefficient for a longitudinal wave at low temperatures may help to determine the normal strain-induced MEC which is an important characteristic parameter for the metal,

$$\left|\frac{\partial E_F}{e\partial\xi_V}\right| \approx \frac{2v_l}{(1-2\upsilon)\,\omega_q}\left(\frac{G_Y v_l \alpha_l}{\sigma_0}\right)^{1/2} \tag{19}$$

The accurate value of MEC is very important, and it may further assist people in understanding the electron viscosity of mobile edge dislocations at low temperatures, which will be discussed later.

Table 1 The physical parameters of some typical metals.

metals / Parameters	Al	Sn	Cu	Pb
Electrical conductivity (*S/m*)	1×10^{10} (30K) [5]	1×10^{10} (10K) [1]	2.5×10^{9} (40K) [1]	1.67×10^{9} (10K) [1]
Young's modulus (*Pa*)	7×10^{10}	5×10^{10}	13×10^{10}	1.6×10^{10}
Longitudinal wave velocity (*m/s*)	6.42×10^{3}	3.48×10^{3}	4.76×10^{3}	2.35×10^{3}
Longitudinal wave	0.54×10^{6}	10.3×10^{6}	47.8×10^{6} [15]	26.5×10^{6} [1]

frequency (*Hz*)	[14]	[1]		
Attenuation Coefficient (*DB/cm*)	0.0032 [14]	0.605 [1]	0.213 [15]	1.34 [1]
Normal strain MEC $\left\|\partial E_F / e\partial\xi_V\right\|(V)$	0.35	0.13	0.078	0.14
Electrical conductivity (*S/m*)	2×10^{10} (1K) [5]	1×10^{10} (10K) [1]	2.5×10^{9} (40K) [1]	1.67×10^{9} (10K) [1]
Shear modulus (*Pa*)	2.6×10^{10}	1.8×10^{10}	4.8×10^{10}	0.56×10^{10}
Shear wave velocity (*m/s*)	3.04×10^{3}	$1.9\times10^{3}s$	2.325×10^{3}	0.7×10^{3}
Shear wave frequency (*Hz*)	34.4×10^{6} [18]	20×10^{6} [1]	22.5×10^{6} [1]	10.1×10^{6} [1]
Attenuation Coefficient (*DB/cm*)	10.7 [18]	0.82 [1]	0.417 [1]	2.35 [1]
Shear strain MEC $\left\|\partial E_F / e\partial\xi_S\right\|$ (*mV*)	9.8	2.7	7.6	2.8

3.2.2 Electromagnetic waves radiated by the longitudinal elastic wave

According to the discussion in section 3.2.1, when a longitudinal elastic wave propagates in the metal, the yielding current density is

$$\vec{J}(x,t) = \vec{e}_x J(q) e^{-\alpha_l x} e^{i(qx-\omega_q t)} \tag{20}$$

where the parameter $J(q)$ is

$$J(q) = \frac{\sigma_0 i\omega_q \tau(1-2\upsilon)}{\left[1-i\omega_q\tau\left(1-i\omega_q\tau_e\right)\right]} \frac{\partial E_F}{e\partial\xi_{xx}} (iq-\alpha_l)^2 A \tag{21}$$

The electrical current will radiate the electromagnetic waves and the magnetic vector potential is [13]

$$\vec{A}_r(\vec{r},t) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{r}',t) e^{i\vec{k}\cdot(\vec{r}'-\vec{r})}}{\left|\vec{r}'-\vec{r}\right|} dV' \tag{22}$$

where $k=\omega_q/c$ is the wave vector of the radiated electromagnetic wave, c is the light speed. In the far-field regions, it may be simplified to be

$$\vec{A}_r(\vec{r},t) \approx \frac{\mu_0 e^{i\vec{k}\cdot\vec{r}}}{4\pi r}\int_V \vec{J}(\vec{r}',t)\,dV'$$

Insertion of the expression of electrical current may result in the following equation

$$\vec{A}_r(\vec{r},t) = -\vec{e}_x \frac{\mu_0 e^{i(\vec{k}\cdot\vec{r}-\omega_q t)}}{4\pi r}\frac{J(q)}{(iq-\alpha_l)} S\left(1-e^{iqL-\alpha_l L}\right) \tag{23}$$

where S is the cross-section area of the metal, L is the length of the metal. As a result, the corresponding magnetic field is

$$\vec{B}_r(\vec{r},t) = \frac{\mu_0 e^{i(\vec{k}\cdot\vec{r}-\omega_q t)}}{4\pi r}\frac{J(q)S}{(iq-\alpha_l)}\left(1-e^{iqL-\alpha_l L}\right)\left[\vec{e}_y\left(ik_z r-\frac{z}{r}\right)-\vec{e}_z\left(ik_y r-\frac{y}{r}\right)\right] \tag{24}$$

Meanwhile the electric field component of the electromagnetic wave can be obtained by the relation

$$\vec{E}_r(\vec{r},t) = \frac{ic}{k}\nabla\times\vec{B}_r(\vec{r},t)$$

Based on the expressions, the radiated electromagnetic waves sensitively depends on the physical properties of the longitudinal elastic wave. First, the frequency of the radiated electromagnetic waves is the same as the frequency of the longitudinal elastic wave. Second, the magnitude of the radiated electromagnetic waves rests with the wave length of the longitudinal elastic wave. A more intensive electromagnetic wave will be radiated by the longitudinal elastic wave with shorter wave length. Third, the magnitude of the radiated electromagnetic waves is also dependent on the MEC of the metal. If the metal exhibits a big MEC, the magnitude of the electromagnetic waves radiated by the longitudinal elastic wave will also be large.

3.2.3 Electron viscosity for the shear elastic waves in metals

As was indicated by the well-designed experiments, the attenuation of the shear elastic waves in a metal at low temperatures also mainly originates from the electron viscosity [1]. In order to explore the involved electron viscosity, a shear elastic wave propagating in x direction and vibrating in z direction can be considered and described by

$$u_s = A_s e^{-\alpha_s x} e^{i(q\cdot x-\omega_q t)}$$

where α_s is the attenuation coefficient for the shear elastic wave because of the electron viscosity at low temperatures, ω_q is the corresponding angular frequency and it is $\omega_q = qv_s$, v_s is the propagation speed of the shear wave, u_s is the displacement vector in z direction, A_s is the vibration amplitude of the shear elastic wave. Thus, the related shear strain is

$$\xi_{zx} = (iq-\alpha_s) A_s e^{-\alpha_s x} e^{i(q\cdot x-\omega_q t)}$$

The ECP can be written as

$$E_F(x) = E_{F0} + \frac{2\partial E_F}{\partial \xi_{zx}} \xi_{zx}$$

Analogous to the discussion in section 3.2.2, the induced electrical current density can be obtained

$$\vec{J}(x,t) = \vec{e}_x \frac{i\sigma_0 \omega_q \tau (iq-\alpha_l)^2}{\left[1-i\omega_q \tau (1-i\omega_q \tau_e)\right]} \frac{2\partial E_F}{e\partial \xi_{zx}} A_s e^{-\alpha_s x} e^{i(q\cdot x-\omega_q t)} \qquad (25)$$

So the electrically dissipated power density can be derived

$$P_e = \frac{2\sigma_0 (\omega_q \tau)^2}{(1-\omega_q^2 \tau \tau_e)^2 + (\omega_q \tau)^2} \left(\frac{\partial E_F}{e\partial \xi_{zx}}\right)^2 (q^2+\alpha_s^2)^2 A_s^2 e^{-2\alpha_s x} \qquad (26)$$

The energy density of the shear elastic wave including both the elastic energy and the kinetic energy is

$$W_s = 2G_s\left(q^2 + \alpha_s^2\right)A_s^2 e^{-2\alpha_s x} \tag{27}$$

where G_s is the shear modulus of the metal. In analogy with the discussion for the longitudinal elastic wave, the low-temperature attenuation coefficient owing to the electron viscosity can be given by

$$\alpha_s = \frac{P_e}{2v_s W_s} \tag{28}$$

The relation $\omega_q\tau$>>1 may hold right for the elastic waves whose frequency usually ranges from 1 MHz to 100 MHz in the related experiments. Furthermore, the relations may be applicable $\partial E_F/\partial\xi_{zx} = \partial E_F/\partial\xi_{yz} = \partial E_F/\partial\xi_{xy}$ for the metals with the simple cubic symmetry. As a result, substitution of the equation (26) and equation (27) into the equation (28) generates

$$\alpha_s \approx \frac{\sigma_0}{G_s v_s}\left(\frac{\partial E_F}{e\partial\xi_S}\right)^2\left(q^2 + \alpha_s^2\right)$$

where ξ_S signifies the shear strains. For the ultrasonic wave utilized in the experiments, the relation usually fulfills q>>α_S. Thus, the attenuation coefficient for the shear elastic wave is

$$\alpha_s \approx \frac{\sigma_0}{G_s v_s^3}\left(\frac{\partial E_F}{e\partial\xi_S}\right)^2\omega_q^2 \tag{29}$$

Analogous to the case of the longitudinal elastic wave, it is indicated that the attenuation coefficient highly depends on the electrical conductivity, the angular frequency and the shear strain-induced MEC. More specifically, the attenuation coefficient is proportional to the square of angular frequency as well, which was confirmed by experiments in single crystalline metal at low temperatures [1]. Besides, the attenuation coefficient at low temperatures varies linearly with the electrical conductivity. The electrical conductivity of the metal generally increases with the decrease of temperature and thereby can cause a more visible attenuation for the shear elastic wave, as was observed in experiments [1]. Furthermore, the attenuation coefficient presents the dependence of shear strain-induced MEC. In general, the shear strain-induced MEC may be weaker than the normal strain-induced MEC, so the attenuation coefficient of the shear elastic wave might be much smaller than that for the longitudinal elastic waves with the same frequency.

The accurate measurement of attenuation coefficients of shear elastic waves at low temperatures is important and could also help people obtain the shear strain-induced MEC,

$$\left|\frac{\partial E_F}{e\partial \xi_S}\right| = \frac{v_s}{\omega_q}\left(\frac{\alpha_s G_s v_s}{\sigma_0}\right)^{1/2} \tag{30}$$

To estimate the magnitude of the shear strain-induced MEC, the shear strain-induced MEC can be calculated by means of the above equation, and the typical values can be listed in Table 1. As is shown, the shear strain-induced MEC is much smaller than the normal strain-induced MEC. The estimation of the shear strain-induced MEC is not only significant for realizing the attenuation coefficient but also vital for understanding the electron viscosity of the screw dislocation which will be addressed in the followings.

3.2.4 Electromagnetic waves radiated by the shear elastic wave

Like the case of the longitudinal elastic wave, the propagation of the shear elastic wave can also radiate the electromagnetic wave. Based on the equation (22), the magnetic wave vector of the radiated electromagnetic wave in the far-field regions is given by

$$\vec{A}_r(\vec{r},t) \approx \frac{\mu_0 e^{i\vec{k}\cdot\vec{r}}}{4\pi r} \int_V \vec{J}(\vec{r}', t)\, dV'$$

where the electrical current density is

$$\vec{J}(x, t) = \vec{e}_x J(q) e^{-\alpha_s x} e^{i(q\cdot x - \omega_q t)}$$

where the parameter is

$$J(q) = \frac{i\sigma_0 \omega_q \tau (iq - \alpha_l)^2}{\left[1 - i\omega_q \tau \left(1 - i\omega_q \tau_e\right)\right]} \frac{2\partial E_F}{e\partial \xi_{zx}} A_s$$

Thus, the magnetic vector potential of the radiated electromagnetic wave in the far-field regions follows

$$\vec{A}_r(\vec{r},t) = -\vec{e}_x \frac{\mu_0 e^{i(\vec{k}\cdot\vec{r} - \omega_q t)}}{4\pi r} \frac{J(q)}{(iq - \alpha_s)} S\left(1 - e^{iqL - \alpha_s L}\right) \tag{31}$$

where S is the cross section of the metal, L is the length of the metal. The magnetic field of the radiated electromagnetic wave can be obtained

$$\vec{B}_r(\vec{r},t) = \nabla \times \vec{A}_r(\vec{r},t)$$

where $B_r(r, t)$ is the magnetic field component of the radiated electromagnetic wave. The electric field component of the radiated electromagnetic wave is

$$\vec{E}_r(\vec{r},t) = \frac{ic}{k} \nabla \times \vec{B}_r(\vec{r},t)$$

where $E_r(r, t)$ is the electric field component of the radiated electromagnetic wave.

As is shown, the electromagnetic wave radiated by the shear elastic wave is sensitively dependent on the physical properties of the shear elastic wave. The frequency of the radiated electromagnetic wave is the same as the shear elastic wave. The magnitude of radiated electromagnetic wave depends on the shear strain-induced MEC and the wave vector. If the metal displays a small shear strain-induced MEC or the wave vector is small, the radiated electromagnetic wave will be weak.

The comparison between the electromagnetic wave radiated by the longitudinal elastic wave and that radiated by shear elastic wave is interesting. They both arise from the motions of the free electrons during the propagation of the elastic waves in the metal. Their magnitude both show the similar dependence of the wave vector and the MEC. However, the normal strain-induced MEC is usually much larger than the shear strain-induced MEC, as was indicated in previous sections. As a result, the magnitude of the electromagnetic wave radiated by the longitudinal elastic wave may be much larger than that radiated by the shear elastic wave. In other words, the detected electromagnetic wave may be commonly dominated by the longitudinal elastic waves in the metal, but the contribution from the shear elastic wave may be subsidiary.

3.3 Electron viscosity during the compression wave front in a metal

When one dimensional compression wave propagates stably in a metal, a dramatic strain popularly exists during the compression wave front, which is illustrated in Figure 1.

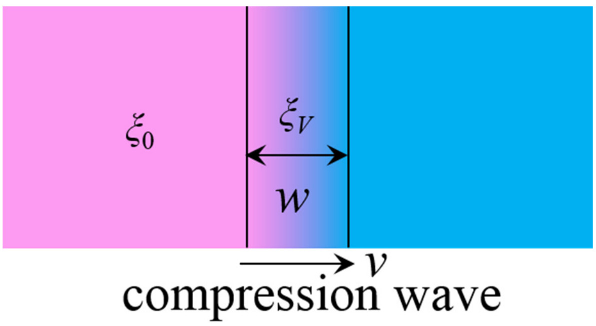

Figure 1 Schematic diagram of the metal under the compression wave.

The strains in the metal can be described by

$$\xi(x-vt)=\begin{cases}\xi_0;\ x-vt<0;\\ \xi_V(x-vt);\ 0<x-vt<w;\\ 0;\ x-vt>w;\end{cases}$$

where v is the propagation speed of the compression wave, w is the width of the compression wave front, ξ_0 is the compressive strain of the metal after the compression wave, ξ_V is the volume strain and it is $\xi_V=\xi_{xx}+\xi_{yy}+\xi_{zz}$. Supposing that the metal is simple cubic, the time and position-dependent ECP can be written as

$$\nabla E_F(x)=\frac{\partial E_F}{\partial \xi_V}\nabla\xi_V(x-vt)$$

Performing the Fourier transformation yields

$$qE_F(q)=\frac{\partial E_F}{\partial \xi_V}q\xi_V(q)$$

The employment of the equation (15) can give the electrically dissipated power during the compression wave front,

$$P_e=\frac{\sigma_0}{2}\int_V\sum_{\vec{q}}\frac{\left[\omega_q\tau\vec{q}E_F(q)\right]^*\omega_q\tau\vec{q}E_F(q)}{e^2\left[\left(1-\omega_q^2\tau\tau_e\right)^2+\left(\omega_q\tau\right)^2\right]}dV$$

where the angular frequency is $\omega_q=qv$. Substitution of the expression of ECP into the above equation will yield

$$P_e=\frac{\sigma_0}{2}\left(\frac{\partial E_F}{e\partial\xi_V}\right)^2\int_V\sum_{\vec{q}}\frac{\left(\omega_q\tau\right)^2\left[q\xi_V(q)\right]^2}{\left[\left(1-\omega_q^2\tau\tau_e\right)^2+\left(\omega_q\tau\right)^2\right]}dV$$

In general, the relations may persist $\omega_q\tau$ >>1. As a result, the electrically dissipated power can be simplified as

$$P_e\approx\frac{\sigma_0 S}{2}\left(\frac{\partial E_F}{e\partial\xi_V}\right)^2\int_0^w\left[\nabla\xi_V(x-vt)\right]^2dx \qquad (32)$$

Consequently, the temperature increase due to the electrically dissipated energy can be obtained

$$\Delta T_e = \frac{\sigma_0}{2vC_V}\left(\frac{\partial E_F}{e\partial \xi_V}\right)^2 \int_0^w \left[\nabla \xi_V\left(x-vt\right)\right]^2 dx \tag{33}$$

where C_V is the heat capacity of the metal per unit volume, ΔT_e is the temperature increase because of the electrically dissipated energy. The second derivative of the strain over space is supposed to be very small and could be ignored over the compression wave front. Thus, the temperature increase can be simplified to be

$$\Delta T_e = \frac{\sigma_0}{2vC_V}\left(\frac{\partial E_F}{e\partial \xi_V}\right)^2 \frac{\xi_0^2}{w} \tag{34}$$

The uplift of the temperature heavily depends on the MEC $|\partial E_F/e\partial\xi_V|$, the compressive strain and the strain gradient within the compressive wave front. A larger strain gradient, in another word, a bigger strain rate will lead to a higher increase of the temperature for the metal. Therefore, the shock waves which can cause the largest strain gradient and strain rate will result in the largest temperature rise. To estimate the temperature increase due to the electrically dissipated energy within the compression wave front, the typical values for the physical variables in the following table can be taken. The uplift of the temperature is about 40 K. Thus the temperature increase induced by the electrically dissipated energy in the metal may be noticeable when the strain gradient fulfills the condition $|\nabla\xi_V| > 10^8/m$ or the strain rate fulfills $|\dot{\xi}_V| > 10^{11}/s$.

It should be noted that in the above analysis the temperature dependence of the electrical conductivity and the heat capacity within the wave front was not taken into account. The simple treatment is rational if the temperature increase is not very high.

Nevertheless, if the temperature increase can cause the remarkable alterations of the physical parameters, the variation of the electrical conductivity and the heat capacity with temperature must be considered in the wave front.

Here the conventional method of the temperature calculation for the shock-loading metal is briefly commented. The temperature of shock-loaded metal was commonly calculated by the Grüneisen ratio and the Grüneisen equation of state [16, 19]. However, the conventional method totally neglected the important electron viscosity across the shock wave front. Within the shock wave front, it was revealed that the electron transferring occurs and thereby a notable electrical current emerges, as was consistent with the observed electromotive force for the metals under shock loading [9, 20, 21]. The transfer of electrons and the related electrical current may not influence the Grüneisen equation of state that builds on the stable state of the material and thus they were not considered seriously before. But they can indeed cause the energy dissipation and thereby enables to induce a temperature rise which should be added in the calculations of the temperature of the metal after shock loading.

Table 2 The physical parameters for a typical metal under the compression waves.

Parameters	ξ_0	w (nm)	$\lvert\partial E_F/e\partial\xi_V\rvert^2$ (V^2)	σ_0 (S/m)	v (m/s)	C_V (J/Km^3)
Value	–0.1	10	0.1	10^7	4000	3×10^6

3.4 Electron viscosity for the dislocation sliding in metals

3.4.1 *Electron viscosity for the sliding of an edge dislocation*

For simplicity, an individual straight edge dislocation sliding in x direction in a simple cubic lattice is considered. By means of the known stress field [17, 22], strain field, the normal volume strain is

$$\xi_V(x,y) = -\frac{(1-2\upsilon)b_e}{4\pi(1-\upsilon)}\frac{2y}{(x-v_{ed}t)^2+y^2} \tag{35}$$

where v_{ed} is the sliding speed of the edge dislocation. The volume strain rate arising from the slipping of the edge dislocation is given by

$$\dot{\xi}_V(x,y) = \vec{v}_{ed}\cdot\nabla\xi_V(x,y) \tag{36}$$

On the other hand, the strain rate may be understood as a result of the action of the stress waves emitted by the mobile edge dislocation.

$$\dot{\xi}_V(x,y) = \vec{v}_l\cdot\nabla\xi_V \tag{37}$$

where v_l is the speed of the longitudinal wave, $\nabla\xi_V$ is the volume strain gradient during the emitted stress wave. Here the variation of the longitudinal speed with the strain is ignored for simplicity. Therefore, the volume strain gradient of the stress wave arising from the slipping of the edge dislocation is

$$\nabla\xi_V = \frac{v_{ed}}{v_l}\frac{\partial\xi_V(x,y)}{\partial x}$$

During the propagation of the stress waves emitted by the mobile dislocation, the electrically dissipated power will be induced. Based on the equation (32), the electrically dissipated power is

$$P_e = \frac{\sigma_0 L_d}{2}\left(\frac{\partial E_F}{e\partial\xi_V}\right)^2\left(\frac{v_{ed}}{v_l}\right)^2\int\left[\frac{\partial\xi_V(x,y)}{\partial x}\right]^2 dxdy \tag{38}$$

Insertion of the equation (35) gives

$$P_e = \frac{\sigma_0 L_d}{16\pi v_l^2}\left(\frac{\partial E_F}{e\partial\xi_V}\right)^2\frac{(1-2\upsilon)^2}{(1-\upsilon)^2}\frac{b_e^2}{r_{e0}^2}v_{ed}^2$$

where r_{e0} is the cutoff radius of the edge dislocation and it may be relevant with the burgers vector.

As is indicated, the electrically dissipated power caused by the dislocation slipping in a metal depends on the direct electrical conductivity, the slipping speed of the dislocation, the MEC and the magnitude of the Burgers vector. A higher electrical conductivity of the metal at low temperatures will yield a larger dissipated power, and a stronger MEC for the metal may also lead to a larger dissipated power. Since the dissipated power is shown to be proportional to the square of slipping speed of the edge dislocation, the electron viscous force for the dislocation slipping may be proportional to the slipping speed. Thereby, the related electron viscous force per unit dislocation length can be obtained

$$f_e = \frac{\sigma_0}{16\pi v_l^2}\left(\frac{\partial E_F}{e\partial \xi_V}\right)^2 \frac{(1-2\upsilon)^2}{(1-\upsilon)^2}\frac{b_e^2}{r_{e0}^2} v_{ed} \tag{39}$$

The corresponding electron viscous coefficient may be

$$B_e = \frac{\sigma_0}{16\pi v_l^2}\left(\frac{\partial E_F}{e\partial \xi_V}\right)^2 \frac{(1-2\upsilon)^2}{(1-\upsilon)^2}\frac{b_e^2}{r_{e0}^2} \tag{40}$$

As is shown, the electron viscous coefficient may depend on several important factors such as the magnitude of Burgers vector, the electrical conductivity and the MEC. It is proportional to the square of the Burgers vector and a big Burgers vector may correspondingly result in a large electron viscous coefficient. The electron viscous coefficient also sensitively depends on the temperature and a very notable value will emerge at low temperatures, because the electrical conductivity of the metal increases dramatically as the temperature decreases. But another important dragging source of the edge dislocation, *i.e.*, phonon damping mechanism, was found to decrease with the temperature decreasing [5, 23]. As a result, the electron viscosity will exceed the phonon damping mechanism at low temperatures, and the dominant viscosity mechanism of the mobile edge dislocation in a metal is the electron viscosity at low temperatures. As is demonstrated, the electron viscous coefficient may also depend on the MEC obviously, and it is proportional to the square of the normal strain-induced MEC. Therefore, a huge value of the normal strain-induced MEC will lead to a great electron viscous coefficient for the mobile edge dislocation.

Table 3 The physical parameters for some metals and the calculated electron viscous coefficient of an edge dislocation.

metals / Parameters	Al	Sn	Cu	Pb
Electrical conductivity σ_0 (S/m)	2×10^8 (100 K) [24]	7×10^8 (20K) [1]	2.5×10^9 (40K) [1]	1.67×10^9 (10K) [1]
Longitudinal wave velocity v_l (m/s)	6.42×10^3	3.48×10^3	4.76×10^3	2.35×10^3
Poisson ratio υ	0.35	0.36	0.34	0.44
MEC $\|\partial E_F/\partial \xi_V\|$ (V)	0.35	0.13	0.078	0.14
Theoretical B_e ($N\cdot s/m^2$)	4.4×10^{-3}	7×10^{-3}	5.5×10^{-3}	9.4×10^{-3}

To estimate the magnitude of the electron viscous coefficient for the edge dislocation in a metal, the typically physical parameters of some metals can be listed in the following table. According to the normal strain-induced MEC inferred from the attenuation coefficient shown in Table 1, the corresponding electron viscous coefficient can be calculated in terms of the equation (40). It is difficult to perform the comparison between the theoretical results and the experimental results, because the precisely experimental measurement of the electron viscous coefficient of an individual edge dislocation is very challenging and still lacking.

Furthermore, one may note that when the electrical conductivity and the normal strain-induced MEC will vary, the electron viscous coefficient may behave the same as the attenuation coefficient at low temperatures. Therefore, an interesting relation may exist between them

$$\frac{G_Y v_l}{4\pi(1-\upsilon)^2}\frac{\alpha_l/\omega_q^2}{B_e}=\frac{r_{e0}^2}{b_e^2} \tag{41}$$

The relation may reveal that the ratio of the attenuation coefficient over the square of the angular frequency of the longitudinal elastic wave is proportional to the electron viscous coefficient of the edge dislocation at low temperatures. Thereby the measurement of the attenuation coefficient of the longitudinal elastic wave can assist one in determining the electron viscous coefficient of involved edge dislocation at cryogenic temperatures.

3.4.2 *Electron viscosity for the sliding of a screw dislocation*

This section will move to the case of the screw dislocation sliding in a simple cubic lattice. The straight screw dislocation is assumed to be in line with *z* direction and slide in *x* direction with the speed v_{sd}. The strain fields of the screw dislocation is given by [17, 22]

$$\xi_{zx}(x,y)=\xi_{xz}(x,y)=-\frac{b_s}{4\pi}\frac{y}{(x-v_{sd}t)^2+y^2}$$

$$\xi_{zy}(x,y)=\xi_{yz}(x,y)=\frac{b_s}{4\pi}\frac{x-v_d t}{(x-v_{sd}t)^2+y^2}$$

The gradient of ECP can be given by

$$\nabla E_F(x,y)=\frac{2\partial E_F}{\partial\xi_{xz}}\nabla\xi_{xz}(x,y)+\frac{2\partial E_F}{\partial\xi_{yz}}\nabla\xi_{yz}(x,y)$$

Owing to the symmetry of the simple cubic lattice, the shear strain components dependence of the ECP may be identical. Hence, the above equation can be written as

$$\nabla E_F(x,y)=\frac{2\partial E_F}{\partial\xi_S}\left[\nabla\xi_{zx}(x,y)+\nabla\xi_{yz}(x,y)\right] \tag{42}$$

where ξ_S is the shear strain.

The sliding of the screw dislocation will continuously emit the shear waves which can result in the electrically dissipated power. Using the equation (38), the electrically dissipated power can be calculated as

$$P_e=\frac{\sigma_0 L_d}{2}\left(\frac{2\partial E_F}{e\partial\xi_S}\right)^2\left(\frac{v_{sd}}{v_s}\right)^2\int\left[\frac{\partial\xi_{xz}(x,y)}{\partial x}+\frac{\partial\xi_{yz}(x,y)}{\partial x}\right]^2 dxdy$$

Through simple calculations, the electrically dissipated power for the sliding of the screw dislocation per unit length follows

$$P_e=\frac{\sigma_0}{8\pi v_s^2}\left(\frac{\partial E_F}{e\partial\xi_S}\right)^2\frac{b_s^2}{r_{s0}^2}v_{sd}^2 \tag{43}$$

where r_{s0} is the cutoff radius of the screw dislocation. Hence, the electron viscous force for the sliding of per unit length of the screw dislocation can be given by

$$f_e=\frac{\sigma_0}{8\pi v_s^2}\left(\frac{\partial E_F}{e\partial\xi_S}\right)^2\frac{b_s^2}{b_{s0}^2}v_{sd}$$

Like the edge dislocation, the electron viscous force is also proportional to the sliding speed of the screw dislocation. The related electron viscous coefficient is

$$B_e=\frac{\sigma_0}{8\pi v_s^2}\left(\frac{\partial E_F}{e\partial\xi_S}\right)^2\frac{b_s^2}{r_{s0}^2} \tag{44}$$

In analogy with the edge dislocation, the electron viscous coefficient of a screw dislocation is dependent of the Burgers vector, the electrical conductivity and the shear strain-induced MEC. It presents the sensitive temperature dependence and increases with the decrease of temperature, because the electrical conductivity of metals usually rises as the temperature decreases. The uncovered proportionality between the electron viscous coefficient and the electrical conductivity in equation (44) is in agreement with the dislocation damping constant due to the electron viscosity [6]. As is shown, the electron viscous coefficient of a screw dislocation rests with the square of the shear strain-induced MEC. Different from the case of the edge dislocation, the shear strain-induced MEC is much weaker than the normal stain-induced MEC. So the electron viscous coefficient of a screw dislocation can be anticipated to exhibit a smaller magnitude than that of an edge dislocation at the same temperature.

To prove the theory, a comparison between the theoretical results and experimental results is necessary. However, the direct experimental means of measuring the electron viscous coefficient of an individual dislocation is not yet available. The related measurements cannot establish the accurate value of B_e but was customarily utilized to demonstrate the existence of the electron viscosity and estimate the order of the magnitude of B_e [6]. In order to estimate the order of the magnitude of B_e given by the equation (44), the typical parameters can be substituted into the equation and the cutoff radius takes the value $3b_s/4$ which was ever ordinarily employed [5, 7]. The obtained electron viscous coefficient for a mobile screw dislocation and the involved physical parameters were listed in Table 4. It is observed that the order of the magnitude of the electron viscous coefficient may agree with the estimated value according to the experimental results. Despite that the comparison was rough, the agreement between the theoretical result and the experimental result may still prove the validity of the theory in the work.

Analogous to the case of the edge dislocation, the relation between the attenuation coefficient of shear waves and the electron viscous coefficient of the screw dislocation may be offered by

$$\frac{G_s v_s}{8\pi}\frac{\alpha_s/\omega_q^2}{B_e}=\frac{r_{s0}^2}{b_s^2} \tag{45}$$

It shows that the ratio of the attenuation coefficient of the shear elastic wave over the square of the angular frequency may be proportional to the electron viscous coefficient of the screw dislocation. And the relation can be utilized to obtain the electron viscous coefficient of a screw dislocation by means of the precise measurement of the attenuation coefficient of the related shear elastic wave at cryogenic temperatures.

Table 4 The physical parameters for some metals and the calculated electron viscous coefficient of a screw dislocation.

metals / Parameters	Al	Sn	Cu	Pb
Electrical conductivity σ_0 (S/m)	2×10^8 (100 K) [24]	1×10^{10} (10K) [1]	2.5×10^9 (40K) [1]	1.67×10^9 (10K) [1]
Shear wave velocity v_s (m/s)	3.04×10^3	1.9×10^3	2.325×10^3	0.7×10^3
Poisson ratio υ	0.35	0.36	0.34	0.44
Shear strain MEC $\lvert\partial E_F/\partial\xi_s\rvert$ (mV)	9.8	2.7	7.6	2.8
Theoretical B_e ($N{\cdot}s/m^2$)	1.8×10^{-5}	1.6×10^{-4}	2.5×10^{-4}	4.8×10^{-5}
Experimental B_e($N{\cdot}s/m^2$)	3.8×10^{-5} [5]	–	–	–

The electron viscosity of both the dislocations and the compression waves as well as the attenuation of involved elastic waves at low temperatures can be attributed to the same physical origin, *i.e*, the mechanical-electric coupling effect of the metals, which was referred to as *Yuheng Zhang* effect [8]. It is the mechanical strains that alter the ECP and thereby generate the electrical current, consequently yielding the electrically dissipated power. The series of waves were assumed to be adiabatic in the preceding theoretical treatments. The assumption may be rational, because the propagation of stress waves may be much faster than the thermal transport processes. In addition, the electrical conductivity was treated as a constant despite the variations of both the local strains and the local temperatures during the wave front. It should be especially noted that the electron viscosity may only dominate the mechanical processes at low temperatures, however, it will be subsidiary at the room and even higher temperatures. It is because the phonon viscosity will be dramatically enhanced as the temperature increases, and consequently plays a major role at room temperatures and higher [5, 6, 23].

The free electron model was ever utilized to account for the attenuation coefficient of elastic waves in the metals at cryogenic temperatures [2, 3]. The “free” electrons are unavoidably subject to the attractive potential energy of positive ions in the metal. When the metal is deformed, not only the Fermi energy but also the attractive potential energy of the “free” electrons varies. Nevertheless, the important role of the attractive potential energy of the “free” electrons was totally neglected by the conventional free electron model. Contrarily, the ECP and the MEC include both the Fermi energy and the attractive potential energy. Thereby, the employment of them to explain the related attenuation coefficient and the electron viscosity may be more reasonable.

4. Conclusion

In summary, the electron viscosity in the mechanical phenomena of metals was studied by means of the mechanical-electric coupling (MEC). It was indicated that the electron viscosity dominates the attenuation coefficient of the longitudinal and shear waves in the metals at low temperatures. The induced attenuation coefficient is proportional to electrical conductivity, the square of the angular frequency and the MEC, which was consistent with experimental observations. Besides, the longitudinal and shear waves can cause the electromagnetic radiation whose frequency is identical to the vibration frequency of the waves. Moreover, the electron viscosity can result in a temperature rise in the compression wave front and a large strain gradient in the compression wave front may lead to a dramatic increase of the temperature. The electron viscosity of the sliding dislocations was obtained by considering the electrically dissipated power of emitted waves by mobile dislocations. According to the MEC determined by the attenuation coefficient, the magnitude of the electron viscosity coefficient of the mobile dislocations can be calculated and the obtained results was in agreement with the experimental results. In a word, using the MEC of the metals, the important electron viscosity for the dynamic deformations of metals were revealed in the work and the findings may find important applications in the related areas.

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