Goldbach's Numbers Construction

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Abstract: The internal structure of the natural numbers reveals the relation between the weak and the strong Goldbach's conjectures. The three prime integers structure of the odd integers already contains the two prime integers base of the even integers. An explicit one-to-one correspondence between these two structures, defined as Goldbach's numbers exist. Thus, if the weak Goldbach's conjecture is true, the strong Goldbach's conjecture should be. Hopefully, this will bring a happy end to Goldbach's conjectures problem.

Key words: Goldbach weak and strong conjectures, Harald A. Helfgott, Ivan Matveyevich Vinogradov, Bertrand's postulate, Goldbach's numbers.

The Prussian mathematician Christian Goldbach suggested in 1742 year that the prime numbers are not only multiplication but addition blocks of the natural integers. The statements are known as the weak: "Every odd integer greater than 7 represents the sum of three odd, not necessarily distinct primes", and the strong: " Every even integer greater than 4 represents the sum of two odd, not necessarily distinct primes", Goldbach's conjectures. Until today, no proof of the strong conjecture is offered .

Since then, progress has been made by the work of Russian mathematician Ivan Matveyevich Vinogradov in 1937, and the paper "The Ternary Goldbach Conjecture is True", published in 2014 by Harald A. Helfgott, is the final proof of the weak conjecture. The approach used by Mr. Helfgot rests on the well-established the circle method, the large sieve and exponential sums.

The integer one, regardless of whether someone declares it to be or not to be a prime number has its natural place and role in the natural numbers. In this paper, the integer one is the member of the "odd prime number set" $\Pi = 1, 3, 5, 7, 11, \cdots$, and Goldbach's conjectures are valid for the odd integers greater or equal to three and for the even natural numbers greater or equal to two.

Definition: *All integers which are the three prime $3G$ and two prime $2G$ integers sums are Goldbach's numbers.*

Remark: All odd integers are 3G numbers, and. as it stands now, the even Goldbach's numbers are the subset of all even integers. However, *it seems that it is possible to rebuild all Goldbach's numbers on an already existing a proper Goldbach's integer subset* $\mathbf{N}_m = 1, 2, 3 \cdots, m$ *in* $\mathbb{N}$. Such an attempt will be made in this paper.

Remark: Further, we use $2a$ and $3a$ notation for the integers in the 2G and 3G Goldbach sets. The set of all natural integers is $\mathbb{N}$, the set of all primes is Π, the set of all odd primes smaller or equal to a prime p is Π_p, the sets of all integers smaller or equal to a prime p is $\mathbf{N}^p$, the set of all odd integers smaller or equal to p is N_p, and finally the set of all even integers smaller than the p is N'_p. Corresponding Goldbach's sets are $2\mathrm{G}_p$ and $3\mathrm{G}_p$, and the Goldbach,s set $\mathrm{G}_p = 2\mathrm{G}_p \cup 3\mathrm{G}_p$.

Remark: We specify the following notation, $|x\rangle$ stands for a column and $\langle y|$ for a row vector. The overline of an integer $\overline{z}$ indicates that z belongs to the column vector. The set of all primes is Π, $3a$ are elements of 3G and $2b$ are elements of 2G set. The pairing operation of two integers is $\hat{\wedge} : \hat{\wedge}(\xi, \eta)\xi \wedge \eta \sim \xi + \eta$. The operation $|\mathrm{x}\rangle\langle \mathrm{x}|$ creates two-dimensional objects by pairing objects. For example, the matrix $|\mathrm{x}\rangle\langle \mathrm{Y}|$ is the coupling of the column $\big|\mathrm{x}\big\rangle$ and the row $\big\langle \mathrm{Y}\big|$ vector entries. The $\wedge$ symbol couples the arrays. The projection operation $\downarrow$ of a set A on the set B is $A \downarrow B = A \cap B$. The lift of a set A is $A \uparrow B = A \cap B$. The set operations are used in the standard way and perhaps in a similar meaning. The operations $\oplus$ and $\ominus$ are the general objects addition operation.

Remark: Numerical calculations have proven that both Goldbach's conjectures are true for all integers $n \leq 8.875 \cdot 10^{30}$. Therefore, for each prime number $p : 5 \leq p < 8.875 \cdot 10^{30}$ the integer set $\mathbf{N}^p$, is the Goldbach's set $\mathrm{G}_p = 2\mathrm{G}_p \cup 3\mathrm{G}_p \equiv \mathbf{N}$. Moreover, each odd integer in $\mathbf{N}^p$, is the sum of four primes. We construct the integer sets

$$\mathbf{N}_p^{2p} \quad = p + \mathrm{G}_p, \quad \mathbf{N}^{2p} = \mathbf{N}^p \cup \mathbf{N}_p^{2p}.$$

Since, $\mathrm{G}_p = 1.2, 3, \cdots, p$ $\ \mathsf{N}_p^{2p} = p+1, p+2, p+3, \cdots, 2p$, so that $\mathbf{N}_{2p} = 1.2, 3, 4, 5, \cdots, p, p+1, p+2, p+3, \cdots, 2p = \mathbf{N}^{2p}$, and all integers in the set $\mathbf{N}^{2p}$ are smaller and equal to the $2p$. The following Corollary gives two important conclusions.

Corollary 1. *All odd integers in the set $\mathbf{N}^{2p}$ are the Goldbach $3G_p$ numbers, and all even integers in that set are sums of the four primes.*

■ The odd integers set $\mathsf{N}_q \subset \mathbf{N}^{2p}$ is Goldbach set $3\mathrm{G}_p$. For, all even numbers in G_p set are $2b = \alpha + \beta$ Goldbach's numbers, and all odd integers in the set $\mathbf{N}^{2p}$ are the projection

$$(p + \mathrm{G}_p)\downarrow(2\mathbf{N}+1) \quad = \{p + 2a = p + \alpha + \beta = 3a \in\in 3\mathrm{G}_{2p}\} \subset \quad 3\mathrm{G}_{2p}\,.$$

Since $\mathbf{N}^p$ already contains the 3G Goldbach's set, all odd integers in the set $\mathbb{N}_{2p}$ are Goldbach's $3\mathrm{G}_{2p}$ numbers. Further, all odd integers in the G_p set are $3a = \alpha + \beta + \gamma$ integers, so that all even integers in the set $\mathbf{N}^{2p}$ are the projection

$$(p + \mathrm{G}_p)\downarrow 2\mathbf{N} \quad = \{p + 3a = p + \alpha + \beta + \gamma\} \subset \quad 2\mathrm{G}_{2p}\,,$$

However, every even integer in the set $\mathbf{N}^p$ is already the sum of four primes so that every even integer in the set $\mathbf{N}^{2p}$ is the sum of four primes. The main conclusion is that the weak Goldbach's conjecture holds on the integer set $\mathbf{N}^{2p}$. ■

Remark: The $3\mathrm{G}_p$ integers extend from the set $\mathbf{N}^p$ to a larger set $\mathbf{N}^{2p}$, and the even integers there are only the four prime sums. However, if we could show that the even numbers on the extension of the set are the $2\mathrm{G}_p$ numbers, we would have all we need to make the next extension. For, according to Bertrand's postulate, there is a prime $q : p < q < 2p$, the set $\mathbf{N}_q \subset \mathbf{N}^{2p}$ would be complete Goldbach's set G_q, and a new extension $\mathbf{N}_q \to \mathbf{N}_{2q}$, $2q > 2p$ could be done. Further, we would proceed by the mathematical induction to construct all integer sets of Goldbach's numbers.

Such construction is possible. Goldbach's numbers 3G offer plenty of the prime couples their components in the number set 2G. If we could show that there is a one-to-one correspondence between the 3G numbers and a subset of their couple components that are already in 2G set, we would be done. Such construction would equip each number set $\mathbf{N}^{2p'}$ with 2G numbers, making it complete Goldbach's set, the base for the next extension. The construction and its conclusions are carried up through the sequence of the nested integer sets $\mathbf{N}^p \subset \mathbf{N}^q \subset \mathbf{N}^{2p}$, $p < q < 2p < q' \cdots$.

Corollary 2 gives an explicit construction of such correspondence. *We notice that an essential pre-request to construction is that Goldbach's weak conjecture must hold on each such $\mathbf{N}^{2q}$ set.* The rest of the paper presents step-by-step construction of a one-to-one correspondence between the 3G and 2G subsets of the Natural numbers[1]. The construction does not depend on the size of the set, and it applies to any finite set, see reference [4].

Definition: *The pairing operation $\hat{\wedge}$ is the "onto complete" if the projection operation $3G \downarrow 2G$ and the lift operation $2G \uparrow 3G$ are onto. The pairing operation is "distinct onto complete" if the onto complete is supported by the all set $2G$.*

Corollary 2. *Cardinal numbers of the sets $3G$ and $2G$ are identical.*

■ The proof, supported by the calculation in the Appendix, is done by construction in the following a few logical steps. Furthermore, it is assumed that Goldbach's weak conjecture is true.[2]

1. *The pairing operation $\hat{\wedge}(3G \downarrow 2G)$ is the distinct onto complete.*

According to the weak Goldbach's conjecture, the 3G set is the 3-primes complete, and for each $3a \in$ 3G

$$\begin{aligned} 3a \quad &= (\xi,\eta,\zeta) = ((\xi,\eta),\zeta) = (\xi,(\eta,\zeta)) = (\eta,(\xi,\zeta)) \;\Rightarrow\; \exists 2b \in \{(\alpha,\beta),\beta,\gamma),(\gamma,\alpha)\} \\ &\forall\, 3a \in \text{3G}\ \exists 2b = (\alpha,\beta) \in \text{2G} \quad \therefore\ \text{3G} \downarrow \text{2G} \subset \text{2G}. \end{aligned}$$

Assume that the lift 2G $\uparrow$ 3G is not onto. Then there is a pair $2b \in$ 2G such that

$$\begin{aligned} &\forall\, 2b \in \text{2G}\ \exists \gamma \in \Pi \therefore \hat{\wedge}(\alpha,\beta,\gamma) = 3a \in \text{3G} \\ \Rightarrow\quad &3a \downarrow 2G = (\alpha,\beta) \in \text{2G} \quad \therefore \text{2G} \uparrow \text{3G}, \end{aligned}$$

contradiction, and $3G \overset{\text{ONTO}}{\longleftrightarrow}$ 2G. The completeness implies that each 3G Goldbach's number is supported by at least one, not necessarily distinct, pair in 3G. To show that 2G are all even integers, we must show that there are sufficiently many mutually distinct couples in the set 2G to support all odd integers. The following part is an explicit construction proof of the existence of the set of distinct even integers supported by distinct prime pairs. Further, the prime number ξ is the family prime of the triplet (ξ,η,ζ), and the prime η is the second prime, the matrix row prime enumerator, and the prime number ζ the matrix column enumerator.

2. *All 3-prime integers of a prime ξ family are supported by the triangular fundamental matrix $\mathcal{B}^{\text{D}}_{\overline{\eta}\Pi}$ of the prime pairs.*

All possible 3-prime integers of a prime η from the prime ξ the family are in the η row vector

$$(\eta,\Pi) \quad = \big\langle|\eta\rangle,\Pi\big| = \big|\eta\big\rangle\big\langle\Pi\big| = \big|\eta\big\rangle\big\langle\overline{\eta};1,3,5,7,\cdots\zeta\cdots\big|$$

of the matrix M1 in the table of matrices in the Appendix. The prime η is coupling to each, one by one prime $\zeta \in \Pi$, the distribution property of the prime η, to form the pair (η,ζ). The collection of all η rows is forming the matrix of the pairs $\big\langle\overline{\eta},\Pi\big|$. Since $\big\langle\overline{\eta},\Pi\big| = \big|\overline{\eta}\big\rangle\big\langle\Pi\big|$ the coupling operation has the multiplication property. While $\big\langle\overline{\eta},\Pi\big|$ is the coupling of the primes the $\big|\overline{\eta}\big\rangle\big\langle\Pi\big|$ is the coupling of the

[1]Radomir Majkic, Contribution to Goldbach's Conjectures, Number Theory, viXra: 2401.0009.

[2]The extension $\mathbf{N}^p \to \mathbf{N}_{2p}$ extends also the validity of the Goldbach's weak conjecture.

arrays. The matrix of the pairs M1 $= \langle \overline{\eta}, \Pi |$ is essential, and will be called the fundamental matrix $\mathcal{B}_{\overline{\eta}\Pi}$ of the pairs.

The simple inspection of the matrix $M1$ shows the redundancy of the fundamental matrix, the characteristic of all matrices in the construction. The first case of redundancy is the couple multiplicity due to the matrix's main diagonal symmetry, and the second case is the pair multiplicity based on the pair equivalence. Else two pairs are equivalent if they contribute the same value even integer. The goal is to construct the matrices without multiplicities. The Appendix shows the explicit calculation.

Notice that the duplicates of the identical symmetric pairs in the matrix M1 are shaded. The identical pair multiplicity eliminates by the removal of the left lower triangular sub-matrix of the fundamental matrix. Exactly

$$\hat{\mathsf{D}}\mathcal{B}_{\overline{\eta}\Pi} \quad = \hat{\mathsf{D}}\langle|\overline{\eta}\rangle, \Pi| = |\overline{\eta}\rangle\langle\hat{\mathsf{D}}\Pi| = |\overline{\eta}\rangle\langle\Pi^{\mathrm{D}}| = \quad \mathcal{B}^{\mathrm{D}}_{\overline{\eta}\Pi},$$

and the reduced fundamental matrix $\mathcal{B}^{\mathrm{D}}_{\overline{\eta}\Pi}$ is the unshaded triangular matrix of the matrix M1 in the table of the matrices in the Appendix. The multiplication property of the coupling induces the reduced upper right triangular prime matrix Π^{D} in the matrix M2 in the Appendix.

The reduction operator $\hat{\mathsf{R}}$ removes the equivalence multiplicity from the matrix M2. A pair (η, ζ) in a current row η cancels with an equivalent pair in any of the previous rows, which is the corresponding ζ prime is canceled in the reduced prime matrix Π^{D}. Exactly

$$\begin{aligned} \hat{\mathsf{R}}\mathcal{B}_{\overline{\eta}\Pi^{\mathrm{D}}} \quad &= |\eta\rangle\langle\hat{\mathsf{R}}\Pi^{\mathrm{D}}| = |\eta\rangle\langle\Pi^{\mathrm{DR}}| = \quad \mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi} \\ \Pi^{\mathrm{DR}}_{\eta} \quad &= \hat{\mathsf{R}}\Pi^{\mathrm{D}}_{\eta} = \Pi^{\mathrm{D}}_{\eta} \ominus \sum_{1<\eta'<\eta}^{\eta} \Pi^{\mathrm{D}}_{\eta} \cap \Pi^{\mathrm{D}}_{\eta'} \\ \Rightarrow \quad \mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi} \quad &= \bigcup_{\eta} |\overline{\eta}\rangle\langle\Pi^{\mathrm{DR}}|. \end{aligned}$$

The matrices M2 and M3 in the Appendix show the calculation. Unshaded entries of the matrices M2 and M3 are the primes and even integers of the unit multiplicities in the reduced matrices of the prime and the even numbers for each family prime ξ.

Remark: The distinct primes in the matrix M2 and distinct couples in the fundamental matrix M3 are all possible distinct primes of the reduced prime matrix Π^{DR} and all possible couples of the reduced fundamental matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$. By construction, these two matrices are in one-to-one correspondence. Moreover, the fundamental matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$, once created, is unique for all the family representatives ξ.

3. *There are exactly as many distinct prime pairs as there are odd numbers.*

The matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$ is a fundamental matrix unique for all family prims ξ, that is each of all family prime numbers ξ couples to the single fundamental reduced matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$ to create all the family prime ξ odd integers $\mathcal{T}_{\xi} = \bar{\xi} \wedge \mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$. Since 2-prime integers of the matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$ are distinct by the construction the odd integers $\mathcal{T}_{\xi}$ of a family ξ are distinct $3\mathrm{G}_{\xi}$ Goldbach's numbers.

While each of the matrices M3.1, M3.2 M3.3, … is the family of the distinct 3-prime integers, their intersections are not empty. Inherited multiplicity of the $3\mathrm{G}_{\xi}$ numbers eliminates by the family multiplicity reduction operator $\hat{\Psi}$.

Exactly, the sets $3\mathrm{G}(1), 3\mathrm{G}(3), 3\mathrm{G}(5), \cdots, 3\mathrm{G}(\xi)\cdots$ are distinct 3G families of the odd integers with the intersections $3\mathrm{G}(\xi) \cap_{1<\xi'<\xi} 3\mathrm{G}(\xi') \neq \emptyset$. The operator $\hat{\Psi}$ eliminates all 3-prime integers from 3G which appear in the previous families, that is

$$\hat{\Psi}(3\mathrm{G}(\xi)) \quad = 3\mathrm{G}(\xi) \ominus \sum_{1<\xi'<\xi} 3\mathrm{G}(\xi) \ominus (3\mathrm{G}(\xi) \cap 3\mathrm{G}(\xi')) = \quad 3\mathrm{G}_{\xi} \quad \Rightarrow \quad 3\mathrm{G} = \bigcup_{\xi} 3\mathrm{G}_{\xi}.$$

The Goldbach's set 3G rests on the collection of the distinct prime pairs by the construction, and the number of the distinct 3G integers is the same as the number of the distinct pairs in the set 2G. Equivalently, the sets 3G and 2G are distinct onto complete with respect to pairing operation. Thus, the sets 3G and 2G have the the same cardinal numbers. The matrix M4 in the Appendix shows the calculation. ■

Corollary 3. *All natural integers are Goldbach numbers.*

■ The conclusion of Corollary 2 does not depend on the size of the integer set, and Corollary 2 applies to the set $\mathbf{N}^{2p}$. Thus, there is a one-to-one correspondence between $3G_{2p}$ and $2G_{2p}$ sets, and the set $\mathbf{N}^{2p}$ is complete Goldbach sets. According to the Bertrands postulate, must exist a prime $q : p < q < 2p$, and the set $\mathbf{N}_q$ is also the set of Goldbach's integers, and it satisfies all conditions to extend to the integer set $\mathbf{N}_{2q}$.
The mathematical induction assumption is that the extensions consecutively construct to an arbitrary prime level q. By Bertrand's postulate, there exists a prime $r : q < r < 2q$ and an extension to the integer set $\mathbf{N}_{2r}$ is done to construct the next complete Goldbach's set. Since the prime r is arbitrary, all natural numbers are Goldbach's integers. ■

Hopefully, this will bring a happy end to Goldbach's conjectures problem.

APPENDIX

The following table is the collection of the matrices supporting the construction of all 3G and 2G Goldbac's integers to show the one-to-one correspondence between two sets. The construction base is a proper subset $\mathbf{N}_p = \{1, 2, 3, \cdots, p\} \subset\in \mathbb{N}$, p is a prime number, on which Goldbah's conjectures hold.

Table 1. Construction of the 3G Integers

		MATRIX M1: Fundamental Matrix $\mathcal{B}_{\overline{\Pi}\Pi}$								
$\forall\xi$	$\zeta \rightarrow$	**1**	**3**	**5**	**7**	**11**	**13**	**17**	**19**	**23**	$\cdots$
	ξ	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	
	1	(1,1)	(1,3)	(1,5)	(1,7)	(1,11)	(1,13)	(1,17)	(1,19)	(1,23)	$\cdots$
	3	(3,1)	(3,3)	(3,5)	(3,7)	(3,11)	(3,13)	(3,17)	(3,19)	(3,23)	$\cdots$
	5	(5,1)	(5,3)	(5,5)	(5,7)	(5,11)	(5,13)	(5,17)	(5,19)	(5,23)	$\cdots$
	7	(7,1)	(7,3)	(7,5)	(7,7)	(7,11)	(7,13)	(7,17)	(7,19)	(7,23)	$\cdots$
	11	(11,1	(11,3)	(11,5)	(11,7)	(11,11)	(11,13)	(11,17)	(11,19)	(11,23)	$\cdots$
	13	13,1)	(13,3)	(13,5)	(13,7)	(13,11)	(13,13)	(13,17)	(13,19)	(17,23)	$\cdots$
	17	(17,1)	(17,3)	(15,5)	(17,7)	(17,11)	(17,13)	(17,17)	(17,19)	(19,23)	$\cdots$
	19	(19,1)	(19,3)	(17,5)	(19,7)	(19,511	(19,13)	(19,17)	(19,19)	(23,23)	$\cdots$

MATRIX M2: Diagonal Symmetric Primes $\Pi^{\mathrm{D}} = \hat{\mathsf{D}}\,\Pi$

$\forall\,\xi$	(η,ξ)	ζ=1	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(1,\xi)$	1	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(3,\xi)$		3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(5,\xi)$			5	7	11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(7,\xi)$				7	11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(11,\xi)$					11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(13,\xi)$						13	17	19	23	29	31	37	41	43	47	53	⋯
	$(17,\xi)$							17	19	23	29	31	37	41	43	47	53	⋯
	$(19,\xi)$								19	23	29	31	37	41	43	47	53	⋯

MATRIX M3: Unique Matrix $2\mathrm{G} = \big|\overline{\Pi}\big\rangle\big\langle\Pi^{\mathrm{DR}}\big|$ of All Distinct Prime Couples

$\forall\,\xi$	(η,ξ)	ζ=1	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(1,\xi)$	2	4	6	8	12	14	18	20	24	30	32	38	42	44	48	54	⋯
	$(3,\xi)$				10		16		22	26		34	40		46	50	56	⋯
	$(5,\xi)$									28		36				52	58	⋯
	$(7,\xi)$																	⋯
	$(11,\xi)$																	⋯
	$(13,\xi)$																	⋯
	$(17,\xi)$																	⋯
	$(19,\xi)$																	⋯

MATRIX M3.1: Distinct $3\mathrm{G}(1) = 1 + \big|\overline{\Pi}\big\rangle\big\langle\Pi^{\mathrm{DR}}\big|$ Integers for $\xi = 1$

$\xi = \mathbf{1}$	(η,ξ)	ζ=1	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(1,\xi)$	3	5	7	9	13	15	19	21	25	31	33	39	43	45	49	55	⋯
	$(3,\xi)$				11		17		23	27		35	41		47	51	57	⋯
	$(5,\xi)$									29		37				53	59	⋯
	$(7,\xi)$																	⋯
	$(11,\xi)$																	⋯
	$(13,\xi)$																	⋯
	$(17,\xi)$																	⋯
	$(19,\xi)$																	⋯

MATRIX M3.2: Distinct $3\mathrm{G}(3) = 3 + \big|\overline{\Pi}\big\rangle\big\langle\Pi^{\mathrm{DR}}\big|$ Integers for $\xi = 3$

$\forall\,\xi = \mathbf{3}$	(η,ξ)	ζ=1	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	⋯
	$(1,\xi)$	5	7	9	11	15	17	21	23	27	33	35	41	45	47	51	57	⋯
	$(3,\xi)$				13		19		25	29		37	43		49	53	59	⋯
	$(5,\xi)$									31		39				55	61	⋯
	$(7,\xi)$																	⋯
	$(11,\xi)$																	⋯
	$(13,\xi)$																	⋯
	$(17,\xi)$																	⋯
	$(19,\xi)$																	⋯

MATRIX M3.3:Distinct $3G(5) = 5 + \big|\overline{\Pi}\big\rangle\big\langle\Pi^{\text{DR}}\big|$ Integers for $\xi = 5$

$\xi = \mathbf{5}$	(η, ξ)	ζ=1	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(1, \xi)$	7	9	11	13	17	19	23	25	29	35	37	43	47	49	53	59	$\cdots$
	$(3, \xi)$				15		21		27	31		39	45		51	55	61	$\cdots$
	$(5, \xi)$									33		41				57	63	$\cdots$
	$(7, \xi)$																	$\cdots$
	$(11, \xi)$																	$\cdots$
	$(13, \xi)$																	$\cdots$
	$(17, \xi)$																	$\cdots$
	$(19, \xi)$																	$\cdots$

MATRIX M4: All Distinct $3G = 3G(1) \ominus \sum_{1<\xi'<\xi} 3G(\xi) \ominus [3G(\xi) \cap 3G(\xi')]$ Integers

$\xi = \mathbf{1}$	(η, ξ)	ζ=1	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(1, \xi)$	3	5	7	9	13	15	19	21	25	31	33	39	43	45	49	55	$\cdots$
	$(3, \xi)$				11		17		23	27		35	41		47	51	57	$\cdots$
	$(5, \xi)$									29		37				53	59	$\cdots$
	$(7, \xi)$																	$\cdots$
	$(11, \xi)$																	$\cdots$
	$(13, \xi)$																	$\cdots$
	$(17, \xi)$																	$\cdots$
	$(19, \xi)$																	$\cdots$

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