Goldbach's Numbers Construction

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Abstract: Goldbach's numbers, all-natural integers which satisfy Goldbach's conjectures are all odd integers and a subset of the even integers. Naturally, they appear in the proof of Goldbach's conjectures. In this paper, the construction of Goldbach's numbers approach is used to prove Goldbach's conjectures, hopefully, it will bring a happy end.

Key words: Goldbach weak and strong conjectures, Harald A. Helfgott, Ivan Matveyevich Vinogradov, Bertrand's postulate, Goldbach's numbers.

The Russian mathematician Christian Goldbach suggested in 1742 year that the prime numbers are not only multiplication but addition blocks of the natural integers. The statements are known as the weak: "Every odd integer greater than 7 represents by a sum of three odd, not necessarily distinct primes", and the strong: " Every even integer greater than 4 is represented by a sum of two odd, not necessarily distinct primes", Goldbach's conjectures. As of today, no proof of the strong conjecture..

Since then, the progress had been made by the work of Russian mathematician Ivan Matveyevich Vinogradov in 1937, and the paper "The Ternary Goldbach Conjecture is True", published in 2014 by Harald A. Helfgott, is the final proof of the weak conjecture. The approach used by Mr. Helfgot rests on the well-established approach based on the circle method, the large sieve and exponential sums. However, it seems that it is possible to construct all Goldbach's numbers on a proper integer subset, already Goldbach's subset $\mathbb{N}_m$, . Here, Goldbach's numbers are the sums 2G and 3G of two and three prime integers.

Further, we use $2a$ and $3a$ notation for the integers in the 2G and 3G Goldbach sets. The set of all natural integers is $\mathbb{N}$, the set of all primes is Π, the set of all primes smaller or equal to a prime p is Π_p, the sets of all integers smaller or equal to a prime p is $\mathbb{N}_p$, the set of all odd integers smaller or equal to p is N_p, and finally the set of all even integers smaller the p is N'_p. Corresponding Goldbach's sets are $2\mathrm{G}_p$ and $3\mathrm{G}_p$.

Remark: Numerical calculations have proven that both Goldbach's conjectures are true for all integers $n \leq 8.875 \cdot 10^{30}$. Therefore, for each prime number $p : 7 \leq p < 8.875 \cdot 10^{30}$ the integer set $\mathbb{N}_p$, is the Goldbach's set $\mathrm{G}_p = 2\mathrm{G}_p \cup 3\mathrm{G}_p$. We construct the integer sets

$$\mathbb{N}_p^{2p} \quad = p + \mathrm{G}_p, \quad \mathbb{N}_{2p} = \mathbb{N}_p \cup \mathbb{N}_p^{2p}.$$

Since, $\mathrm{G}_p = 1.2, 3, \cdots, p$, $\mathbb{N}_p^{2p} = p+1, p+2, p+3, \cdots, 2p$, so that $\mathbb{N}_{2p} = 1.2, 3, 4, 5, \cdots, p, p+1, p+2, p+3, \cdots, 2p = \mathbb{N}_{2p}$, and all integers in the set $\mathbb{N}_{2p}$ are smaller and equal to the $2p$. The following Corollary gives two conclusions.

Corollary 1. *All odd integers in the set $\mathbb{N}_{2p}$ are the Goldbach $3G_p$ integers and all even integers in that set are sums of the four primes.*

■ The odd integers set $\mathsf{N}_q \subset \mathbb{N}_{2p}$ is Goldbach set $3\mathrm{G}_p$. For, all even numbers in G_p set are $2b = \alpha + \beta$ Goldbach's numbers, and all odd integers in the set $\mathbb{N}_{2p}$ are the projection

$$(p + \mathrm{G}_p)\!\downarrow\!(2\mathbb{N} + 1) \quad = \{p + 2a = p + \alpha + \beta = 3a \in\in 3\mathrm{G}_{2p}\} \subset \quad 3\mathrm{G}_{2p}\,.$$

Since $\mathbb{N}_p$ already contains the 3G Goldbach's set, all odd integers in the set $\mathbb{N}_{2p}$ are Goldbach's $3\mathrm{G}_{2p}$ numbers. Further, all odd integers in the G_p set are $3a = \alpha + \beta + \gamma$ integers, so that all even integers in the set $\mathbb{N}_{2p}$ are the projection

$$(p + \mathrm{G}_p)\!\downarrow\!2\mathbb{N} \quad = \{p + 3a = p + \alpha + \beta + \gamma \subset \quad 2\mathrm{G}_{2p}\,,$$

However, every even integer in the set $\mathbb{N}_p$ is already the sum of four primes so that every even integer in the set $\mathbb{N}_{2p}$ is the sum of four primes. In conclusion, the weak Goldbach's conjecture holds on the integer set $\mathbb{N}_{2p}$. ■

Corollary 2. *Grim's weak conjecture is true on the set of all integers.*

■ The weak Goldbach's conjectures are true on the set $\mathbb{N}_{2p}$. Further, we proceed by induction on the prime p. According to Bertrand's postulate, there is a prime q between p and $2p$, and all above holds for the sets $\mathrm{G}_q = \mathbb{N}_q$ and $\mathbb{N}_{2q}$. Now, assume that q is an arbitrary prime in the induction sequence. Again, by Bertrand's postulate, there is a prime r between q and $2q$, and the same as above applies. Goldbach's weak conjecture holds on the $\mathbb{N}_{2r}$. Consequently, Goldbach's weak conjecture holds on all set $\mathbb{N}$. ■

In the rest of the paper, we will show that the strong Goldbach's conjecture holds also on all set $\mathbb{N}$, see the paper *"Contribution to Goldbach's Conjectures,"* Number Theory, on the **viXra** site, by Radomir Majkic. However, the proof is from word to word repeated here for the completeness of the paper.

Further underlined truth is that the weak Goldbach's conjecture is true. We specify the following notation, $|x\rangle$ stands for a column and $\langle y|$ for a row vector. The overline of an integer $\overline{z}$ indicates that z belongs to the column vector. The set of all primes is Π, $3a$ are elements of 3G and $2b$ are elements of 2G set. The pairing operation of two integer is $\hat{\wedge} : \hat{\wedge}(\xi, \eta)\xi \wedge \eta \sim \xi + \eta$. The operation $|\mathrm{x}\rangle\langle\mathrm{x}|$ creates two-dimensional objects by pairing objects. For example, the matrix $|\mathrm{x}\rangle\langle\mathrm{Y}|$ is the coupling of the column $\big|\mathrm{x}\big\rangle$ vector and row $\big\langle\mathrm{Y}\big|$ vector entries. The $\wedge$ symbol couples the arrays. The projection operation $\downarrow$ of a set A on the set B is $A \downarrow B = A \cap B$. The lift of a set A is $A \uparrow B = A \cup B$.
The set operations are used in the standard way and perhaps in a similar meaning. The operations $\oplus$ and $\ominus$ are the general objects addition operation.

Definition: *The pairing operation $\hat{\wedge}$ is the "onto complete" if the projection operation $3G \downarrow 2G$ and the lift operation $2G \uparrow 2G$ are onto. The operation is "distinct onto complete" if the onto complete is supported by the all set $2G$.*

Corollary 3. *Cardinal numbers of the sets $3G$ and $2G$ are identical.*

■ The proof, supported by the calculation in the Appendix, is done by construction in the following few logical steps.

1. *The pairing operation $\hat{\wedge}(3G \downarrow 2G)$ is the distinct onto complete.*

According to the weak Goldbach's conjecture, the 3G set is the 3-primes complete, and for each$3a \in 3\mathrm{G}$

$$\begin{aligned} 3a &= (\xi,\eta,\zeta) = ((\xi,\eta),\zeta) = (\xi,(\eta,\zeta)) = (\eta,(\xi,\zeta)) \;\Rightarrow\; \exists 2b \in \{(\alpha,\beta),\beta,\gamma),(\gamma,\alpha)\} \\ &\forall\, 3a \in 3\mathrm{G}\ \exists 2b = (\alpha,\beta) \in 2\mathrm{G} \quad \therefore\ 3\mathrm{G} \downarrow 2\mathrm{G} \subset 2\mathrm{G}. \end{aligned}$$

Assume that the lift $2\mathrm{G} \uparrow 3\mathrm{G}$ is not onto. Then there is a pair $2b \in 2\mathrm{G}$ such that

$$\begin{aligned} &\forall\, 2b \in 2\mathrm{G}\ \exists \gamma \in \Pi \therefore \hat{\wedge}(\alpha,\beta,\gamma) = 3a \in 3\mathrm{G} \\ \Rightarrow\quad & 3a \downarrow 2G = (\alpha,\beta) \in 2\mathrm{G} \quad \therefore 2\mathrm{G} \uparrow 3\mathrm{G}, \end{aligned}$$

contradiction, and $3G \overset{\text{ONTO}}{\longleftrightarrow} 2\mathrm{G}$. The completeness implies that each 3G Goldbach's number is supported by at least one, not necessarily distinct, pair in 3G. To show that 2G are all even integers, we must show that there are sufficiently many distinct couples in the set 2G to support all odd integers. The following part is an explicit construction proof of the set of distinct odd integers supported by distinct prime pairs. The prime number ξ is the family prime of the triplet (ξ,η,ζ), and the prime η is the matrix row prime enumerator.

2. *All 3-prime integers of a prime ξ family are supported by the triangular fundamental matrix $\mathcal{B}^{\mathrm{D}}_{\overline{\eta}\Pi}$ of the prime pairs.*

All possible 3-prime integers of a prime η from the prime ξ the family are in the η row vector

$$(\eta,\Pi) \quad = \big\langle|\eta\rangle,\Pi\big| = \big|\eta\big\rangle\big\langle\Pi\big| = \big|\eta\big\rangle\big\langle\overline{\eta};1,3,5,7,\cdots\zeta\cdots\big|$$

of the matrix M1 in the table of matrices in the Appendix. The prime η is coupling to each, one by one prime $\zeta \in \Pi$, the distribution property of the prime η, to form the pair (η,ζ). The collection of all η rows is forming the matrix of the pairs $\big\langle\overline{\eta},\Pi\big|$. Since $\big\langle\overline{\eta},\Pi\big| = \big|\overline{\eta}\big\rangle\big\langle\Pi\big|$ the coupling operation has the multiplication property. While $\big\langle\overline{\eta},\Pi\big|$ is the coupling of the primes the $\big|\overline{\eta}\big\rangle\big\langle\Pi\big|$ is the coupling of the arrays. The matrix of the pairs $\mathrm{M1} = \big\langle\overline{\eta},\Pi\big|$ is essential, and will be called the fundamental matrix of the pairs $\mathcal{B}_{\overline{\eta}\Pi}$.

The simple inspection of the matrix $M1$ shows the redundancy of the fundamental matrix, the characteristic of all matrices in the construction. The first case of redundancy is the couple multiplicity due to the matrix's main diagonal symmetry, and the second case is the pair multiplicity based on the pair equivalence. Else two pairs are equivalent if they contribute the same value even integer. The goal is to construct the matrices without multiplicities. The Appendix shows the explicit calculation.

Notice that the duplicates of the identical symmetric pairs in the matrix M1 are shaded. The identical pair multiplicity eliminates by the removal of the left lower triangular sub-matrix of the fundamental matrix. Exactly

$$\hat{\mathsf{D}}\mathcal{B}_{\overline{\eta}\Pi} \quad = \hat{\mathsf{D}}\big\langle|\overline{\eta}\rangle,\Pi\big| = \big|\overline{\eta}\big\rangle\big\langle\hat{\mathsf{D}}\Pi\big| = \big|\overline{\eta}\big\rangle\big\langle\Pi^{\mathrm{D}}\big| = \quad \mathcal{B}^{\mathrm{D}}_{\overline{\eta}\Pi},$$

and the reduced fundamental matrix $\mathcal{B}^{\mathrm{D}}_{\overline{\eta}\Pi}$ is the unshaded triangular matrix of the matrix M1 in the table of the matrices in the Appendix. The multiplication property of the coupling induces the reduced upper right triangular prime matrix Π^{D} in the matrix M2 in the Appendix.
The reduction operator $\hat{\mathsf{R}}$ removes the equivalence multiplicity from the matrix M2. A pair (η,ζ) in a current row η cancels with an equivalent pair in any of the previous rows, which is the corresponding ζ prime is canceled in the reduced prime matrix Π^{D}. Exactly

$$\begin{aligned} \hat{\mathsf{R}}\mathcal{B}_{\overline{\eta}\Pi^{\mathrm{D}}} &= \big|\eta\big\rangle\big\langle\hat{\mathsf{R}}\Pi^{\mathrm{D}}\big| = \big|\eta\big\rangle\big\langle\Pi^{\mathrm{DR}}\big| = \quad \mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi} \\ \Pi^{\mathrm{DR}}_{\eta} &= \hat{\mathsf{R}}\Pi^{\mathrm{D}}_{\eta} = \Pi^{\mathrm{D}}_{\eta} \ominus \sum_{1<\eta'<\eta}^{\eta} \Pi^{\mathrm{D}}_{\eta} \cap \Pi^{\mathrm{D}}_{\eta'} \\ \Rightarrow \quad \mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi} &= \bigcup_{\eta} \big|\overline{\eta}\big\rangle\big\langle\Pi^{\mathrm{DR}}\big|. \end{aligned}$$

The matrices M2 and M3 in the Appendix show the calculation. Unshaded entries of the matrices M2 and M3 are the primes and even integers of the unit multiplicities in the reduced matrices of the prime and the even numbers.

Remark: The distinct primes in the matrix M2 and distinct couples in the fundamental matrix M3 are all possible distinct primes of the reduced prime matrix Π^{DR} and all possible couples of the reduced fundamental matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$. By construction, these two matrices are in one-to-one correspondence. Moreover, the fundamental matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$, once created, is unique for all the family representatives ξ.

3. *There are exactly as many distinct prime pairs as there are odd numbers.*

The matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$ is a fundamental matrix unique for all family primsξ. Each single family prime ξ couples to the same fundamental reduced matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$ to create all the family prime ξ odd integers $\mathcal{T}_\xi = \bar{\xi} \wedge \mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$. Since 2-prime integers of the matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$ are distinct by the construction the odd integers $\mathcal{T}_\xi$ are distinct $3\mathrm{G}_\xi$ Goldbach's numbers. While each of the matrices M3.1, M3.2, M3.3, ... is the family of the distinct 3-prime integers their intersections are not empty. Inherited multiplicity of the $3\mathrm{G}_\xi$ numbers eliminates by the family multiplicity reduction operator $\hat{\Psi}$.
Further, the sets $3\mathrm{G}(1), 3\mathrm{G}(3), 3\mathrm{G}(5), \cdots, 3\mathrm{G}(\xi) \cdots$ are distinct 3G families of the odd integers with the intersections $3\mathrm{G}(\xi) \cap_{1<\xi'<\xi} 3\mathrm{G}(\xi') \neq \emptyset$. Then for each family prime ξ

$$\hat{\Psi}(3\mathrm{G}(\xi)) \quad = 3\mathrm{G}(\xi) \ominus \sum_{1<\xi'<\xi} 3\mathrm{G}(\xi) \ominus (3\mathrm{G}(\xi) \cap 3\mathrm{G}(\xi')) = \quad 3\mathrm{G}_\xi \quad \Rightarrow \quad 3\mathrm{G} = \bigcup_\xi 3\mathrm{G}_\xi .$$

The Goldbach's set 3G rests on the collection of the distinct prime pairs by the construction, it is distinct onto complete, and the number of the distinct 3G integers is the same as the number of the distinct pairs in the set 2G, or the sets 3G and 2G are distinct onto complete with respect to the pairing operation. The matrix M4 in the Appendix shows the calculation. ■

Corollary 4. *If the weak Goldbach's conjecture is true the strong Goldbach's conjecture is.*

■ All above is obtained under condition that the weak Goldbach's conjecture is true. All possible Goldbach's numbers 3G are supported by the fundamental matrix $\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}$ and $3\mathrm{G} = |\overline{\Pi}\rangle\langle\mathcal{B}^{\mathrm{DR}}_{\overline{\eta}\Pi}|$. According to the first part of Corollary 1 sets 3G and 2G are "onto complete" with respect to the pairing operation, and according to Corollary 2 they are the "distinct onto complete" with respect to the same operation. Thus, the Goldbach's numbers 2G, 3G and the odd $2\mathbb{N}+1$ and even$2\mathbb{N}$ integers have the same cardinal numbers. Consequently, the strong Goldbach's conjecture is true.
In conclusion Goldbach's conjectures are true. ■

APPENDIX

The following table is the collection of the matrices supporting the construction of the all 3G Goldbach's integers on the set of all 2G Goldbac;s integers to show the one-to-one correspondence between two sets, all under condition that the weak Goldbah's conjecture is true.

Table 1. Construction of the 3G Integers

MATRIX M1: Fundamental Matrix $\mathcal{B}_{\overline{\Pi}\Pi}$

$\forall\,\xi$	$\zeta \rightarrow$	**1**	**3**	**5**	**7**	**11**	**13**	**17**	**19**	**23**	$\cdots$
	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)	(η,ζ)		
	1	(1,1)	(1,3)	(1,5)	(1,7)	(1,11)	(1,13)	(1,17)	(1,19)	(1,23)	$\cdots$
	3	(3,1)	(3,3)	(3,5)	(3,7)	(3,11)	(3,13)	(3,17)	(3,19)	(3,23)	$\cdots$
	5	(5,1)	(5,3)	(5,5)	(5,7)	(5,11)	(5,13)	(5,17)	(5,19)	(5,23)	$\cdots$
	7	(7,1)	(7,3)	(7,5)	(7,7)	(7,11)	(7,13)	(7,17)	(7,19)	(7,23)	$\cdots$
	11	(11,1	(11,3)	(11,5)	(11,7)	(11,11)	(11,13)	(11,17)	(11,19)	(11,23)	$\cdots$
	13	13,1)	(13,3)	(13,5)	(13,7)	(13,11)	(13,13)	(13,17)	(13,19)	(17,23)	$\cdots$
	17	(17,1)	(17,3)	(15,5)	(17,7)	(17,11)	(17,13)	(17,17)	(17,19)	(19,23)	$\cdots$
	19	(19,1)	(19,3)	(17,5)	(19,7)	(19,511	(19,13)	(19,17)	(19,19)	(23,23)	$\cdots$

MATRIX M2: Diagonal Symmetric Primes $\Pi^{\mathrm{D}} = \hat{\mathsf{D}}\,\Pi$

$\forall\,\xi$	(η,ξ)	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	
	$(1,\xi)$	1	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(3,\xi)$		3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(5,\xi)$			5	7	11	13	17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(7,\xi)$				7	11	13	17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(11,\xi)$					11	13	17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(13,\xi)$						13	17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(17,\xi)$							17	19	23	29	31	37	41	43	47	53	$\cdots$
	$(19,\xi)$								19	23	29	31	37	41	43	47	53	$\cdots$

MATRIX M3: Unique Matrix $2\mathrm{G} = \left|\overline{\Pi}\right\rangle\left\langle\Pi^{\mathrm{DR}}\right|$ of All Distinct Prime Couples

$\forall\,\xi$	(η,ξ)	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	
	$(1,\xi)$	2	4	6	8	12	14	18	20	24	30	32	38	42	44	48	54	$\cdots$
	$(3,\xi)$				10		16		22	26		34	40		46	50	56	$\cdots$
	$(5,\xi)$									28		36				52	58	$\cdots$
	$(7,\xi)$																	$\cdots$
	$(11,\xi)$																	$\cdots$
	$(13,\xi)$																	$\cdots$
	$(17,\xi)$																	$\cdots$
	$(19,\xi)$																	$\cdots$

MATRIX M3.1: Distinct $3\mathbf{G}(1) = 1 + \left\langle\Pi^{\mathbf{dr}}\right|$ Integers for $\xi = 1$

$\xi = \mathbf{1}$	(η,ξ)	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	
	$(1,\xi)$	3	5	7	9	13	15	19	21	25	31	33	39	43	45	49	55	$\cdots$
	$(3,\xi)$				11		17		23	27		35	41		47	51	57	$\cdots$
	$(5,\xi)$									29		37				53	59	$\cdots$
	$(7,\xi)$																	$\cdots$
	$(11,\xi)$																	$\cdots$
	$(13,\xi)$																	$\cdots$
	$(17,\xi)$																	$\cdots$
	$(19,\xi)$																	$\cdots$

MATRIX M3.2: Distinct $3G(3) = 3 + \langle \Pi^{DR} \rvert$ Integers for $\xi = 3$

$\forall\, \xi = \mathbf{3}$	(η, ξ)	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	
	$(1, \xi)$	5	7	9	11	15	17	21	23	27	33	35	41	45	47	51	57	$\cdots$
	$(3, \xi)$				13		19		25	29		37	43		49	53	59	$\cdots$
	$(5, \xi)$									31		39				55	61	$\cdots$
	$(7, \xi)$																	$\cdots$
	$(11, \xi)$																	$\cdots$
	$(13, \xi)$																	$\cdots$
	$(17, \xi)$																	$\cdots$
	$(19, \xi)$																	$\cdots$

MATRIX M3.3:Distinct $3G(5) = 5 + \langle \Pi^{DR} \rvert$ Integers for $\xi = 5$

$\xi = \mathbf{5}$	(η, ξ)	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	
	$(1, \xi)$	7	9	11	13	17	19	23	25	29	35	37	43	47	49	53	59	$\cdots$
	$(3, \xi)$				15		21		27	31		39	45		51	55	61	$\cdots$
	$(5, \xi)$									33		41				57	63	$\cdots$
	$(7, \xi)$																	$\cdots$
	$(11, \xi)$																	$\cdots$
	$(13, \xi)$																	$\cdots$
	$(17, \xi)$																	$\cdots$
	$(19, \xi)$																	$\cdots$

MATRIX M4: All Distinct $3G = 3G(1) \ominus \sum_{1<\xi'<\xi} 3G(\xi) \ominus [3G(\xi) \cap 3G(\xi')]$ Integers

$\xi = \mathbf{1}$	(η, ξ)	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	ζ	
	$(1, \xi)$	3	5	7	9	13	15	19	21	25	31	33	39	43	45	49	55	$\cdots$
	$(3, \xi)$				11		17		23	27		35	41		47	51	57	$\cdots$
	$(5, \xi)$									29		37				53	59	$\cdots$
	$(7, \xi)$																	$\cdots$
	$(11, \xi)$																	$\cdots$
	$(13, \xi)$																	$\cdots$
	$(17, \xi)$																	$\cdots$
	$(19, \xi)$																	$\cdots$

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