SunQM-6s9: Reformulating Schrodinger Equation/Solution to Show Its r-1D Reversed-Diffusion Character for Solar System's {N,n} QM Structure Formation (Drafted in January 2020)

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Abstract

A pre-Sun ball (quantum) collapse process can be intuitively thought as a (mass) reversed diffusion process in r-dimension. It has been noticed for many years (and by many QM scientists) that there is similarity between Schrodinger equation and a diffusion equation. In the current paper, 1) By using Euler formula's complex space (that was defined as the "i-space" in this article), I reformulated the expression of the orbital energy E = K + V to be $K = K_{\theta\varphi} + iK_r$ and $V = V_{\theta\varphi} + iV_r$, and then $E = K + iV^* = (K_{\theta\varphi} + V_r) + i(K_r + V_{\theta\varphi})$; 2) I assumed that a plane-like wave can be used as the solution (i.e., the wave function) of Schrodinger equation (either for an H-atom, or for Solar system), and then separate this 3D plane-like wave into two parts, one in r-1D only, and one in θφ-2D only; 3) Accordingly, (by guessing, not by the mathematical deduction), I separated a traditional spherical 3D Schrodinger equation into two: one in θφ-2D only, and one in r-1D only; 4) While the θφ-2D-only Schrodinger equation is almost same as the traditional one, the r-1D-only Schrodinger equation can be further degenerated into a r-1D diffusion equation with the diffusion constant $D = \frac{\hbar}{2m}$; 5) At a citizen-scientist-level, I confirmed that the θφ-2D plane-like wave function is the solution of the θφ-2D-only Schrodinger equation, and the r-1D plane-like wave function is the solution of the r-1D-only Schrodinger equation as well as the r-1D diffusion equation. In this way, I demonstrated that Schrodinger equation may can be directly degenerated into a diffusion equation (at least in r-dimension). Furthermore, according to this explanation, the other QM scientists' previous result of $D \rightarrow \frac{i\hbar}{2m}$ may can be re-explained as that, the diffusion constant $D = \frac{\hbar}{2m}$ not only radially diffuses in r-1D, but also laterally diffuses in θφ-2D through RF (RotaFusion, or rotation diffusion). If this explanation is correct, then the function of analyzing the E = K + V in an "i-space" maybe is to filter out the RF (the lateral diffusion) in θφ-2D, so that a pure r-1D diffusion can be obtained. Even I am not sure whether this method is correct or not, this deduction certainly opened a new route to reformulate Schrodinger equation and solution. This work may also have created a completely new way to study the relationship between the RF and the energy of the orbital motion. (Note: This article was drafted in Jan. 2020).

Key Words: Quantum mechanics, {N,n} QM, Schrodinger equation, diffusion equation,

Introduction

As wiki "Schrodinger equation" pointed out: "*The Schrodinger equation is a linear partial differential equation that describes the wave function or state function of a quantum-mechanical system. It is a key result in quantum mechanics, and its discovery was a significant landmark in the development of the subject*". The SunQM series papers [1] ~ {16] have shown that the formation of Solar system (as well as each planet) was governed by its {N,n} QM. In SunQM-3 series papers, I studied Solar {N,n} QM structure by using the traditional Schrodinger equation/solution (mostly in Born probability). It showed that the formation of planet's and star's (radial) internal structure is governed by the planet's or star's radial {N,n} QM, the surface mass (atmosphere) movement of Sun, Jupiter, Saturn, and Earth, etc., is governed by Star's (or planet's) θφ-2D QM, and even the formation of either ring structures of a planet, or the belt structures in Solar system, is also governed by the {N,n} QM (the nLL effect). Furthermore, we can even use Schrodinger equation's solution to build a 3D probability density map for a complete Solar system with time-dependent orbital motion. (Note: Also see more SunQM series articles [17] ~ [32] that posted at viXra.org after Jan. 2020, as the more supportive results). Because the Solar system was formed through a series of quantum collapses from a pre-Sun ball, from my "first principle thinking", I believe that the quantum collapse of a pre-Sun ball should can also be described by the Schrodinger equation/solution.

On the other hand, from many other QM scientists' "first principle thinking", it has been a long time thought that Schrodinger equation may be originated from a diffusion equation (simply because it looks like a diffusion equation). Many scientists had tried to prove that [33] ~ [35]. However, all these deductions were so advanced in math that completely beyond my (a citizen scientist's) understanding. In the current paper, I tried to view this problem in a different angle: because we believed that the quantum collapse of a pre-Sun ball is equivalent to a reversed-diffusion of a pre-Sun ball, plus we had shown that this process can be described by the Schrodinger equation/solution (at least for a series of steady state QM states, see SunQM-3s2), then this means that the Schrodinger equation/solution (at least for its r-1D dimension) should directly correlate to a diffusion (or a reversed-diffusion) equation. I started to work on this problem in 2016 (even before I fully understood it), and obtained my own solution in January 2020 (and drafted as this paper). Note: Base on the content, this article was originally grouped with SunQM-4 and SunQM-4s1. Note: In some previous SunQM articles, this article had been cited as either "SunQM-4s3: Schrodinger equation and {N,n} QM", or "SunQM-4s4 …", or "SunQM-6s3: Schrodinger equation and {N,n} QM ... (drafted in January 2020)", or "SunQM-6s10 ...". Note: because this paper was written in January 2020, so the citation of wiki was based on the wiki version of January 2020 and before.

Note: QM means Quantum Mechanics. For {N,n} QM nomenclature as well as the general notes, please see SunQM-1's sections VII & VIII. Note: Microsoft Excel's number format is often used in this paper, for example: x^2 = x^2, 3.4E+12 = 3.4*10^{12} = 3.4×10^{12}, 5.6E-9 = 5.6*10^{-9}. Note: The easiest reading sequence for the (33 posted) SunQM series papers is: SunQM-1, 1s1, 1s2, 1s3, 2, 3, 3s1, 3s2, 3s6, 3s7, 3s8, 3s3, 3s9, 3s4, 3s10, 3s11, 4, 4s1, 4s2, 5, 5s1, 5s2, 7, 6, 6s1, 6s2, 6s3, 6s4, 6s5, 6s6, 6s7, 6s8, and 6s9. Note: for all SunQM series papers, reader should check "SunQM-9s1: Updates and Q/A for SunQM series papers" for the most recent updates and corrections. Note: |n*l*m> means |n,*l*,m> QM state, "nLL" or |nLL> means |n,*l*,m> QM state with *l* = n-1 = L, and *m* = n-1 = L. "nL0" or |nL0> means |n,*l*,m> QM state with *l* = n-1 = L, and m = 0. Note: In the current paper, the cited SunQM series numbers of those pre-posted SunQM papers may not be the final SunQM series numbers (after posting), so, readers may need to match the right SunQM series number (for those pre-posting SunQM papers after they are posted, according to the list of "A series of SunQM papers that I am working on" at the end of current paper) before reading those (pre-posted) citations.

I. A pre-Sun ball quantum collapsing process can be intuitively thought as a mass radial reversed diffusion process (back to a point, not expand from a point)

I-a. A brief review of the known (traditional) Schrodinger equation QM obtained for the H-atom

From QM text books, we learned that the traditional Schrodinger equation has a time-dependent form: (from wiki "Schrodinger equation")

$$i\hbar \frac{\partial}{\partial t}\Psi(r,\theta,\varphi,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(r,\theta,\varphi,t)\right]\Psi(r,\theta,\varphi,t) \qquad \text{eq-1}$$

Under certain physics condition (e.g., plane wave, or hydrogen atom, etc.), eq-1 can be solved by separating the variables so that we can find solutions that are the simple products of

$$\Psi(r,\theta,\varphi,t) = R(r)\ \Theta(\theta)\ \Phi(\varphi)\ T(t) = R(r)\ T(t_r)\ \Theta(\theta)\ T(t_\theta)\ \Phi(\varphi)\ T(t_\varphi) \qquad \text{eq-2}$$

(also see SunQM-6s1's eq-33). Because the φ-1D space is generally rotation-diffused (or RotaFusion, or RF) with the θ-1D space, the function of Θ(θ) is usually not independent of function Φ(φ). Therefore, usually the spherical harmonics function is needed to express the RF between Θ(θ) and Φ(φ), so that the time-independent wave function $\psi(r,\theta,\varphi)$ should usually be represented as

$$\psi(r,\theta,\varphi) = R(r)\ \Theta(\theta)\ \Phi(\varphi) = R_{nl}(r)\ Y_{lm}(\theta,\varphi) \qquad \text{eq-3}$$

Where the *n*, *l*, *m*, are the quantum numbers (for Schrodinger equation and solution).

Note: In this paper, following writings are equivalent: $\Psi(r,\theta,\varphi,t) = \Psi(\vec{\mathbf{r}},t)$, or, $V(r,\theta,\varphi,t) = V(\vec{\mathbf{r}},t)$. Note: For a single particle/planet's circular orbital movement, under the simplest model, we sometimes switch these forms one with another:

$$V(r,\theta,\varphi,t) = V(\vec{\mathbf{r}},t) \xrightarrow{\text{remove t}} V(r,\theta,\varphi) \xrightarrow{V(\theta\varphi)=0} V(r) = V_r$$

I-b. Hypothesis: a "plane-like wave" may can be used to describe a particle/planet if it is in the |nLL> QM state doing circular orbital movement in a central force field

A plane wave shown in eq-4 is one of Schrodinger equation's solutions (under the condition of E = K + V with V ≡ 0):

$$\Psi(\vec{\mathbf{r}},t) = A\ e^{[i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)]} = A\ e^{[i(\vec{\mathbf{p}}\cdot\vec{\mathbf{r}}-E\cdot t)/\hbar]} \qquad \text{eq-4}$$

where the A is the amplitude, $\vec{\mathbf{k}}$ is the wave-number vector, ω is the angular frequency, $\vec{\mathbf{r}}$ is the position vector, and $\vec{\mathbf{p}}$ is momentum vector of the plane wave [36]. From the work in SunQM-3s11 section III-c, and in SunQM-4 section I-c, I believed that eq-4 may can also be used to describe a particle/planet moving in the central force field where E = K + V with $V_r \neq 0$. (Note: strictly to say, an orbital moving particle/planet should be represented by a spherical 3D wave packet rather than a plane wave (like eq-4). However, as a citizen scientist, I may can (inaccurately) simplify a spherical 3D wave packet to have a single wave frequency, that is, a plane wave, in each of φ-1D, θ-1D, and r-1D (see Appendix A). Once we figured out the solution for a plane wave, then we may should be able to figure out the solution for a wave packet by (Fourier) summing many different plane waves with different frequencies, of course, with the help of the professional mathematicians). In eq-4, the origin of the vector $\vec{\mathbf{r}}$ is the same as the origin of the point central force field. We named the eq-4 with V(r) ≠ 0 as the "**plane-like wave**", to distinguish the real plane wave where V ≡ 0.

QM text books showed us that how to test the solutions of Schrodinger equation (see wiki "Schrödinger equation"): applying the eq-4 into the left side of eq-1 gives

$$i\hbar\frac{\partial}{\partial t}\Psi(r,\theta,\varphi,t) = i\hbar\frac{\partial}{\partial t}\left\{A\ e^{[i(\vec{\mathbf{p}}\cdot\vec{\mathbf{r}}-E\cdot t)/\hbar]}\right\} = E\ \Psi(r,\theta,\varphi,t) \qquad \text{eq-5}$$

and applying the eq-4 into the right side of eq-1 gives

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(r,\theta,\varphi,t)\right]\Psi(r,\theta,\varphi,t) = \frac{-\hbar^2}{2m}\nabla^2\left\{A\ e^{[i(\vec{\mathbf{p}}\cdot\vec{\mathbf{r}}-E\cdot t)/\hbar]}\right\} + V\Psi = \left[\frac{p^2}{2m} + V\right]\Psi \qquad \text{eq-6}$$

Then, that the eq-5 equals to eq-6 will give

$$E = \frac{p^2}{2m} + V = K + V \qquad \text{eq-7}$$

or the total energy E of a particle is the sum of kinetic energy K and potential energy V. So a plane-like wave with eq-4 is a solution of Schrodinger equation (at least apparently).

Here I assumed that a "plane-like wave" (in eq-4) can be used to describe a particle (or a planet)'s motion in a central G-(or E-) force field (at least if it is in the |nLL> QM state). (Note: For the discussion of a potential problem of this assumption, see Appendix B).

I-c. The similarity between Schrodinger equation and a diffusion equation

As mentioned before, for a long time, QM scientists noticed that the Schrodinger equation looks like a diffusion equation [33] ~ [35]. In wiki "Heat equation (version of Dec. 20, 2019)" section of "Schrödinger equation for a free particle", "*With a simple division, the Schrödinger equation for a single particle of mass m in the absence of any applied force field can be rewritten in the following way:* $\psi_t = \frac{i\hbar}{2m}\Delta\psi$, *where i is the imaginary unit, ħ is the reduced Planck's constant, and ψ is the wave function of the particle. This equation is formally similar to the particle diffusion equation, …* $D \to \frac{i\hbar}{2m}$ *… this analogy between quantum mechanics and diffusion is a purely formal one. Physically, the evolution of the wave function satisfying Schrödinger's equation might have an origin other than diffusion*". Thus, no clear and convincing result has been made.

In 2013, Takahisa Okino proposed a great point in his paper [35]. Here is his paper's abstract: "*The well-known Schrodinger equation is reasonably derived from the well-known diffusion equation. In the present study, the imaginary time is incorporated into the diffusion equation for understanding of the collision problem between two micro particles. It is revealed that the diffusivity corresponds to the angular momentum operator in quantum theory. The universal diffusivity expression, which is valid in an arbitrary material, will be useful for understanding of diffusion problems*". As a citizen scientist, I am not able to fully understand his deduction (simply because my math level is not good enough). However, Okino's paper is one of the two foundations for the RF (rotational diffusion, or RotaFusion). The second foundation (and the more important one) for the RF is the SunQM-2 Table 6 data's explanation. In SunQM-2s1, I tried to find an independent way (e.g., using the well-known uncertainty principle $[L_x, L_y] = i\hbar L_z$ kind of proof) to show that Schrodinger equation is a RF of the angular momentum unit vector. However, again due to my (a citizen scientist of QM) math level is not good enough, no progress has been made so far. Therefore, SunQM-2s1 has not come out by now, or even may never come out. Thus, the current paper (SunQM-6s9) may can be treated as the substitution for the (most likely dismissed) paper SunQM-2s1.

I-d. A pre-Sun ball quantum collapse process should can be explained as a reversed radial diffusion process

As a citizen scientist of QM, the theories in [33] ~ [35] are beyond my knowledge. So I tried to study this problem with my college level physics and the entry level QM. If we reverse the process of how the pre-Sun nebula collapse into a Sun, clearly it is a mass diffusion process in which the mass from a nearly point center (e.g., the current Sun, or a white dwarf, or even a black hole) is diffused in the r-dimension into a pre-Sun nebula. From the classical physics, we know that this process can be described by a standard 1D diffusion equation (or a 1D heat equation):

$$\frac{\partial}{\partial t} F(r,t) = D\,[\nabla^2 F(r,t)] \qquad \text{eq-8}$$

where F(r,t) is the density function of the diffusing material at 1D location r and at time t, and D is the diffusion coefficient for density function F at location r, and ∇ represents the vector differential operator del (see wiki "Diffusion equation", and see [37]).

Then, let's explore the possible solution of eq-8. Notice that we are only interested in those solutions that can be used to describe the Solar {N,n} QM collapsing dynamics, that is, the solutions that have the similar form as that of eq-4. The first possible solution is

$$F(r,t) = e^{(a \cdot r - b \cdot t)} \qquad \text{eq-9}$$

where a (> 0) and b (> 0) are arbitrary coefficients, and D= $-b/a^2$. Although eq-9 is one possible solution for eq-8, but it does not match to the Solar {N,n} QM physics because it is an exponential increasing curve (which means a mass related quantity increases with r increasing).

Then, a second possible solution is

$$F(r,t) = e^{(-a \cdot r + b \cdot t)} \qquad \text{eq-10}$$

where a (> 0) and b (> 0) are arbitrary coefficients, and D = b/a^2. To prove it, let's input eq-10 into the left side of eq-8, we obtain

$$\frac{\partial}{\partial t}[F(r,t)] = \frac{\partial}{\partial t}\left[e^{(-a \cdot r + b \cdot t)}\right] = b[F(r,t)] \qquad \text{eq-11}$$

and then into the right side of eq-8, we obtain

$$D\,[\nabla^2 F(r,t)] = D\left\{\nabla^2\left[e^{(-a \cdot r + b \cdot t)}\right]\right\} = D\,(-a)^2\,F(r,t) = b\,[F(r,t)] \qquad \text{eq-12}$$

So eq-11 equals to eq-12, thus, eq-10 is also one possible solution of eq-8. Eq-10 not only is one possible solution, but may also be the Solar {N,n} QM physics relevant solution, because it (a mass density related quantity) is an exponential decreasing curve with r. In section II, I will show that only eq-10 (but not eq-9) type of function may be the correct solution of Schrodinger equation for Solar {N,n} QM collapsing dynamics.

II. Reformulating Schrodinger equation/solution to show its r-dimensional reversed-diffusion character for Solar system's {N,n} QM mass collapsing dynamics

II-a. Reformulating an orbital moving particle/planet's momentum p and energy E (including K and V) in Euler formula's i-space.

Based on the general physics, in a (either gravity or electric) central forced {N,n} QM system, a particle/planet's orbital energy has the form of E = K + V, where K is the particle/planet's kinetic energy, V is the particle/planet's potential energy. The establishment of Schrodinger equation is based on the energy conservation. In this section, I used Euler formula's complex space concept (note: here I used the complex number symbol "i" to name it as the "**i-space**") to study the Schrodinger equation's energy conservation process, because (I guessed that) the i-space may allow us to analyze the complicated RF (rotation diffusion, or RotaFusion) phenomenon. In Euler formula

$$e^{i\alpha} = \cos(\alpha) + i\,\sin(\alpha) \qquad \text{eq-13}$$

the property we wanted to use is

$$\left|A\ e^{i\alpha}\right|^2 = |A\cos(\alpha) + i\ A\sin(\alpha)|^2 = [A\cos(\alpha) - i\ A\sin(\alpha)][A\cos(\alpha) + i\ A\sin(\alpha)] = A^2 \qquad \text{eq-14}$$

(or equals to a constant), where α is the angle that rotates value A in the i-space.

Now, let's first define the particle/planet's momentum vector $\vec{\mathbf{p}}$ in i-space as

$$\vec{\mathbf{p}} = \mathbf{p_{\theta\varphi}} + \mathbf{i}\ \mathbf{p_r} \qquad \text{eq-15}$$

then the standard calculation will give us

$$\vec{\mathbf{p}}^2 = p^2 = \left|p_{\theta\varphi} + i\ p_r\right|^2 = \left(p_{\theta\varphi} - i\ p_r\right)\left(p_{\theta\varphi} + i\ p_r\right) = {p_{\theta\varphi}}^2 + \ {p_r}^2 \qquad \text{eq-16}$$

This should satisfy Euler formula because eq-16 satisfies eq-14.

Next, let's define a particle/planet's kinetical energy K in rθφ-3D space as

$$\mathbf{K} = \mathbf{K_{r\theta\varphi}} = \mathbf{K_{\theta\varphi}} + \mathbf{i}\ \mathbf{K_r} \qquad \textbf{eq-17}$$

Notice that in eq-17, the real quantities ($K_{\theta\varphi}$) correlates to the θφ-dimension, and the imaginary quantities (K_r) correlates to the r-dimension. Then

$$\left|K_{r\theta\varphi}\right| = \left|K_{\theta\varphi} + i\ K_r\right| \qquad \text{eq-18}$$

Or, according to eq-7, eq-18 correlates to

$$\left|K_{\theta\varphi} + i\ K_r\right| \rightarrow \frac{p^2}{2m} = \frac{{p_{\theta\varphi}}^2}{2m} + \frac{{p_r}^2}{2m} \qquad \text{eq-19}$$

This should also satisfy Euler formula because eq-19 satisfies eq-13.

It seems that there are two completely different ways to calculate $\left|p_{\theta\varphi} + i\ p_r\right|^2$. If I want to obtain the real valued p_r^2, or to compare eq-18 to p^2, then it should be calculated as eq-16. Therefore, we obtain the traditional result

$$K_{\theta\varphi} = \frac{{p_{\theta\varphi}}^2}{2m} \qquad \text{eq-20}$$

and

$$K_r = \frac{{p_r}^2}{2m} \qquad \text{eq-21}$$

However, if I want to know the relationship between the imaginary valued ip_r and the imaginary valued iK_r, or to compare eq-15 with eq-17, then it may should be calculated as:

$$\left(p_{\theta\varphi} + i\ p_r\right)^2 = \left(p_{\theta\varphi} + i\ p_r\right)\left(p_{\theta\varphi} + i\ p_r\right) = {p_{\theta\varphi}}^2 + 2p_{\theta\varphi}\ i\ p_r + \ (i\ p_r)^2 \qquad \text{eq-22}$$

Dividing eq-22 by (2m), we have

$K = K_{r\theta\varphi} = K_{\theta\varphi} + i\,K_r \rightarrow \frac{p^2}{2m} = \frac{(p_{\theta\varphi}+i\,p_r)^2}{2m} = \frac{p_{\theta\varphi}{}^2+2p_{\theta\varphi}\,i\,p_r+(i\,p_r)^2}{2m}$ eq-23

And then taking away eq-20 $K_{\theta\varphi} = \frac{p_{\theta\varphi}{}^2}{2m}$ from eq-23, we have

$i\,K_r = \frac{2p_{\theta\varphi}\cdot i\,p_r+(i\,p_r)^2}{2m}$ eq-24

Because $p_{\theta\varphi}\cdot p_r \equiv 0$ (due to that they are in orthogonal), then we have

$i\,K_r \xrightarrow{\text{its "real" values in the i−space should equal to}} \frac{(i\,p_r)^2}{2m}$ eq-25

Then, treat either the "i" on the left side and the "i^2" on the right side of eq-25 as the unit, and extract out the value part, we have

$|K_r| = \frac{(\,p_r)^2}{2m}$ eq-26

We will need to use this relationship later on.

Similarly, we can also define

$V = V_{r\theta\varphi} = V_{\theta\varphi} + i\,V_r$ eq-27

and its conjugated form

$V^* = V_{r\theta\varphi}{}^* = V_{\theta\varphi} - i\,V_r$ eq-28

Also notice that in both eq-27 and eq-28, the real quantities ($V_{\theta\varphi}$) apparently correlates to the θφ-2D, and the imaginary quantities (V_r) apparently correlates to the r-1D. I say "apparently", because this situation will be completely changed after the RF (or, after the rotation-diffusion, see eq-31). Both eq-27 and eq-28 should also satisfy Euler formula. Because V has the same physical unit as that of K, it recessively inherits the p^2 property (that satisfy eq-19 or eq-14). Since in a central gravity forced {N,n} QM system, a circular orbital moving particle/planet's potential energy V has

$V_{\theta\varphi} \equiv 0$ (Note: this is valid only for one particle/planet per circular orbit, also see section II-g-5) eq-29

Then from the centrifugal force $F = ma = \frac{mv^2}{r}$ equals to the gravity attraction force $F = \frac{GMm}{r^2}$, (notice that here v is the circular orbital velocity that equals to v_φ, or $v_{\theta\varphi}$), we have $\frac{GMm}{r^2} = \frac{mv_{\theta\varphi}{}^2}{r} \rightarrow \frac{GMm}{r} = mv_{\theta\varphi}{}^2$, or

$V_r = -\frac{GMm}{r} = -mv_{\theta\varphi}{}^2 = -\frac{p_{\theta\varphi}{}^2}{m}$ eq-30

where G is the gravitational constant, M is the G-force center's (or Sun's) mass, m is the orbital moving particle/planet's mass. Thus V_r is directly correlate to $p_{\theta\varphi}{}^2$ in θφ-dimension. Let's recall that in eq-20, $p_{\theta\varphi}{}^2$ correlates to $K_{\theta\varphi}$ in θφ-dimension.

Now, we need to put the equation "E = K + V" into the i-space of Euler's formula. We can do that because the E is a conserved value for an orbital moving particle/planet, and like V, E also recessively inherits p^2 property that satisfy eq-19 and eq-13. However, if we define E = K + iV, then $V_r = -\frac{p_{\theta\varphi}{}^2}{m}$ (eq-30) does not work with $K_{\theta\varphi}$ in θφ-dimension. After many tries, I found that the only way to make V_r correlate to $K_{\theta\varphi}$ in θφ-2D and also to satisfy the new r-1D Schrodinger equation, is to define

$$\mathbf{E = K + iV^* = (K_{\theta\varphi} + i\,K_r) + i(V_{\theta\varphi} - i\,V_r) = (K_{\theta\varphi} + V_r) + i(K_r + V_{\theta\varphi})} \qquad \textbf{eq-31}$$

So the real quantities (which correlates to the θφ-2D) contains $K_{\theta\varphi}$ and V_r, and the imaginary quantities (which correlates to the r-1D) contains K_r and $V_{\theta\varphi}$. In other words, for a central force field bound particle/planet (that doing the circular orbital movement), in the energy's i-space, (because of RF phenomenon), its $K_{\theta\varphi}$ and V_r should be grouped together, while its K_r and $V_{\theta\varphi}$ should be grouped together. Here is a big assumption: I believed that eq-31 is valid for any bound or even unbound particle's energy analysis in a point-central force field.

Here I want to (repeatedly) emphasize a key concept: why I can use the "i-space" to re-formulate E = K + V to be eq-31? It is because: 1) The orbital energy E is conserved, so that E is a fixed value in the orbital motion; 2) Schrodinger equation/solution (that is based on the orbital energy conservation) is believed to be able to describe the orbital motion; 3) In Euler formula eq-13 and eq-14, the A^2 is a fixed value, so it can be perfectly used to represent the conserved orbital energy E (that is also a fixed value).

(Note: The similar idea was also shown in paper SunQM-3s1's section-V, "… *I guess that we can use the orthogonal vector adding method (z^2 = x^2 + y^2) to add them … $E_{nlm}^{(1\theta\varphi)}$ and $E_{nlm}^{(1r)}$* …".)

II-b. Using a plane-like wave function to represent a particle/planet's wave function in Schrodinger equation

As mentioned in section I-b, here we may can use a plane-like (rθφ-3D) wave eq-4 to describe a particle/planet's (nLL QM state) orbital movement in a central gravity force field (note: this is a citizen-scientist-leveled assumption). After bring eq-15 and eq-31 into the plane wave eq-4, we have

$$\Psi(\vec{\mathbf{r}},t) = A\,e^{[i(\vec{p}\cdot\vec{r}-E\cdot t)/\hbar]} = A\,e^{(i\{(p_{\theta\varphi}+i\,p_r)\cdot r-[(K_{\theta\varphi}+V_r)+i(K_r+V_{\theta\varphi})]\cdot t\}/\hbar)} = A\,e^{\{i[p_{\theta\varphi}\cdot r-(K_{\theta\varphi}+V_r)\cdot t]/\hbar\}}e^{\{[-p_r\cdot r+(K_r+V_{\theta\varphi})\cdot t]/\hbar\}}$$

eq-32

We can further separate Ψ(r,θ,φ,t) in eq-32 as a product of two independent wave functions Ψ(r,t) and Ψ(θ,φ,t):

$$\Psi(\theta,\varphi,t) = A_1\,e^{\{i[p_{\theta\varphi}\cdot r-(K_{\theta\varphi}+V_r)\cdot t]/\hbar\}} \qquad \text{eq-33}$$

Note: in eq-33 $p_{\theta\varphi}\cdot r$ should be the two (scalar) values' product, not the two vectors' dot-product, (also see section-V for an alternative manipulation); and

$$\Psi(r,t) = A_2\,e^{\{[-p_r\cdot r+(K_r+V_{\theta\varphi})\cdot t]/\hbar\}} \qquad \text{eq-34}$$

where $V_{\theta\varphi} \equiv 0$. (Note: in eq-34, $p_r \cdot r$ can be either the two (scalar) values' product, or two vectors' dot-product). And

$$\mathbf{\Psi(r,\theta,\varphi,t) = \Psi(r,t)\,\Psi(\theta,\varphi,t)} \qquad \textbf{eq-35}$$

where $A = A_1 \cdot A_2$. Thus, eq-33 looks like a θφ-2D dimensional related wave function, and eq-34 looks like a r-1D dimensional related wave function. In the next few sections, I will prove that eq-33 is a θφ-2D-only wave function (that matches the θφ-2D-only Schrodinger equation), and eq-34 is a r-1D only wave function (that matches the r-1D only Schrodinger equation).

II-c. Reformulate Schrodinger equation (by guessing) to separate r-1D away from θφ-2D

Furthermore, because V_r (in eq-31) ended to be in the θφ-2D dimension, for a circular orbital moving particle/planet, I guessed that the θφ-2D-only Schrodinger equation may became

$$i\hbar\frac{\partial}{\partial t}\Psi(\theta,\varphi,t) = \left[\frac{-\hbar^2}{2m}\nabla_{\theta\varphi}{}^2 + V(r,t)\right]\Psi(\theta,\varphi,t) \qquad \textbf{eq-36}$$

(Note: in eq-36, because Ψ(θ,φ,t) excluded the r-dimension, so we may can re-write $\nabla_{\theta\varphi}^2$ as ∇^2, thus eq-37 is also fine. However, if we don't specify Ψ is in θφ-2D only, then eq-37 is incorrect).

$$i\hbar\frac{\partial}{\partial t}\Psi(\theta,\varphi,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(r,t)\right]\Psi(\theta,\varphi,t) \qquad \text{eq-37}$$

Similarly, I guessed that the r-1D-only Schrodinger equation may become

$$i\hbar\frac{\partial}{\partial t}\Psi(r,t) = \left[\frac{-\hbar^2}{2m}\nabla_r{}^2 + V(\theta,\varphi,t)\right]\Psi(r,t) \qquad \textbf{eq-38}$$

Or,

$$i\hbar\frac{\partial}{\partial t}\Psi(r,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\theta,\varphi,t)\right]\Psi(r,t) \qquad \text{eq-39}$$

(Note: in eq-38, because Ψ(r,t) excluded the θφ-2D dimension, so we can re-write ∇_r^2 as ∇^2, thus eq-39 may be also fine. However, if we don't specify Ψ is in r-1D only, then eq-39 is incorrect). This may can be explained as: due to RF, for a (point-central force field) bound particle/planet (that doing the circular orbital movement), its kinetic energy in θφ-2D ($K_{\theta\varphi}$) is only affected by V_r (but not $V_{\theta\varphi}$) component of V(r,θ,φ) through Schrodinger equation, and its kinetic energy in r-1D (K_r) is only affected by $V_{\theta\varphi}$ (but not V_r) component of V(r,θ,φ) through Schrodinger equation.

Here is another big assumption: I guessed that eq-36 and eq-38 may be valid for any bound or unbound particle's energy analysis in a point-central force field.

Question: Are eq-36 and eq-38 joint equations (meaning we can't solve one equation without solving the second equation simultaneously)? In eq-36, can we replace Ψ(θ,φ,t) by $Y_{lm}(\theta,\varphi)$ $T(t_{\theta\varphi})$? And in eq-38, can we replace Ψ(r,t) by $R_{nl}(r)$ $T(t_r)$? I don't know the answer at this time.

II-d. To prove that θφ-2D plane-like wave function eq-33 is the solution of the θφ-2D Schrodinger equation

To prove it is correct, let's input θφ-2D plane-like wave function eq-33 into θφ-2D Schrodinger equation eq-36, then we have the left side (of eq-36) as

$$i\hbar\frac{\partial}{\partial t}\Psi(\theta,\varphi,t) = A_1 i\hbar\frac{\partial}{\partial t}e^{\{i[p_{\theta\varphi}\cdot r-(K_{\theta\varphi}+V_r)\cdot t]/\hbar\}} = \left(K_{\theta\varphi}+V_r\right)\Psi(\theta,\varphi,t) \qquad \text{eq-40}$$

and the right side (of eq-36) as

$$\left[\frac{-\hbar^2}{2m}\nabla_{\theta\varphi}{}^2 + V(r,t)\right]\Psi(\theta,\varphi,t) = A_1\left[\frac{-\hbar^2}{2m}\nabla_{\theta\varphi}{}^2 + V(r,t)\right]e^{\{i[p_{\theta\varphi}\cdot r-(K_{\theta\varphi}+V_r)\cdot t]/\hbar\}} = \left[\frac{p_{\theta\varphi}{}^2}{2m} + V_r\right]\Psi(\theta,\varphi,t) \qquad \text{eq-41}$$

So, the left side equals to the right side (after using eq-20), and thus eq-33 is the correct solution of eq-36. Furthermore, (at a citizen-scientist-level), because eq-36 is one of the (dissociable) sub-equations of eq-1 (in θφ-2D dimension), then eq-33 should be one of the correct solutions of eq-1 (in θφ-2D dimension).

II-e. To prove that the r-1D plane-like wave function eq-34 is the solution of the r-1D Schrodinger equation

Let's input r-1D plane-like wave function (eq-34) into r-1D Schrodinger equation (eq-38), then we have the left side (of eq-38) as

$$i\hbar \frac{\partial}{\partial t}\Psi(r,t) = A_2\, i\hbar \frac{\partial}{\partial t} e^{\{[-p_r \cdot r + (K_r + V_{\theta\varphi})\cdot t]/\hbar\}} = i\left(K_r + V_{\theta\varphi}\right)\Psi(r,t) \qquad \text{eq-42}$$

and the right side as

$$\left[\frac{-\hbar^2}{2m}\nabla_r{}^2 + V(\theta,\varphi,t)\right]\Psi(r,t) = A_2\left[\frac{-\hbar^2}{2m}\nabla_r{}^2 + V(\theta,\varphi,t)\right] e^{\{[-p_r \cdot r + (K_r + V_{\theta\varphi})\cdot t]/\hbar\}} = \left[\frac{-(-p_r)^2}{2m} + V(\theta,\varphi,t)\right]\Psi(r,t) = \left[\frac{(ip_r)^2}{2m} + V_{\theta\varphi}\right]\Psi(r,t) \qquad \text{eq-43}$$

Now we want to show that (the "real" values in the i-space of the) eq-42 (i.e., $K_r+V_{\theta\varphi}$ at the right side of eq-42) equals to (the real values of the right side of) eq-43 (i.e., $\frac{(p_r)^2}{2m} + V_{\theta\varphi}$). Because $V_{\theta\varphi} \equiv 0$ (also see section II-g-5), it forced that $iV_{\theta\varphi} = 0 = V_{\theta\varphi}$. Also by using eq-25 (to treat either the "i" or the "i^2" as the unit), we see that eq-42 does "equal" to eq-43 (at the citizen scientist level). Therefore eq-34 is the correct solutions of eq-38. Furthermore, because eq-38 is one of the (dissociable) sub-equation of eq-1 (in r-1D), then eq-34 should be one of the correct solutions of eq-1 (in r-1D).

II-f. Degenerate the r-1D Schrodinger equation to be a diffusion equation

Let's applying $V_{\theta\varphi} \equiv 0$ to both eq-42 and eq-43, and because eq-42 "equals" to eq-43, we have

$$i\hbar \frac{\partial}{\partial t}\Psi(r,t) = iK_r\,\Psi(r,t) \xrightarrow{\text{its "real" values in the i-space should equal to}} \frac{-\hbar^2}{2m}\nabla_r{}^2\,\Psi(r,t) = \frac{(ip_r)^2}{2m}\Psi(r,t) \qquad \text{eq-44}$$

This is in Euler formula's i-space, or the complex space. If only comparing the results in the complex space, we can move their quantities from i-space into a new real space for comparison. To do this, we need to remove the i from the two left items, and remove $i^2 = -1$ from the two right items from eq-44 (note: this is a citizen-scientist-leveled assumption). Then, we have

$$\hbar \frac{\partial}{\partial t}\Psi(r,t) = K_r\,\Psi(r,t) \xrightarrow{\text{equals to}} \frac{\hbar^2}{2m}\nabla_r{}^2\,\Psi(r,t) = \frac{(p_r)^2}{2m}\Psi(r,t) \qquad \text{eq-45}$$

or, after simplifying,

$$\frac{\partial}{\partial t}\Psi(r,t) = \frac{\hbar}{2m}\nabla_r{}^2\,\Psi(r,t) \qquad \text{eq-46}$$

or just shows the operators

$$\frac{\partial}{\partial t} = \frac{\hbar}{2m}\nabla_r{}^2 \qquad \text{eq-47}$$

Immediately we know that eq-47 is a r-1D diffusion equation (or heat equation) with diffuse constant

$$D = \frac{\hbar}{2m} \qquad \text{eq-48}$$

Note: only when $V_{\theta\varphi} \equiv 0$, eq-38 can be degenerated to be a diffusion equation as eq-46.

To prove that r-1D plane-like wave function (eq-34) is the solution of the diffusion equation (eq-47), let's input eq-34 to the left side of eq-47:

$$\frac{\partial}{\partial t}\Psi(r,t) = A_2\ \frac{\partial}{\partial t} e^{\{[-p_r \cdot r + (K_r + V_{\theta\varphi})\cdot t]/\hbar\}} = \frac{(K_r + V_{\theta\varphi})}{\hbar}\ \Psi(r,t) = \frac{K_r}{\hbar}\ \Psi(r,t) \qquad \text{eq-49}$$

and then input eq-34 to the right side of eq-47:

$$\frac{\hbar}{2m}\nabla^2\ \Psi(r,t) = A_2 \frac{\hbar}{2m}\nabla^2 e^{\{[-p_r \cdot r + (K_r + V_{\theta\varphi})\cdot t]/\hbar\}} = \frac{p_r{}^2}{2m\hbar}\Psi(r,t) \qquad \text{eq-50}$$

Because eq-26 showed $|K_r| = \frac{(p_r)^2}{2m}$, then eq-49 does equal to eq-50, and thus eq-34 is the solution of eq-47. Therefore, eq-34 is the solution not only for the r-1D Schrodinger equation (eq-38), but also for the r-1D diffusion equation (eq-47).

II-g. More discussions (on section II)

1) The meaning of the diffusion coefficient $D = \frac{\hbar}{2m}$ in eq-48. Let's reformulate it to

$$D = \frac{\hbar}{2m} = \frac{1}{4\pi}\left(\frac{h}{m}\right) \qquad \text{eq-51}$$

h/m divided by 4π means a physical quantity (of h/m) radiate and spread in all 4π directions (evenly, or on average) from a point center. It may can be explained as: it is the r-dimensional wave function $\Psi(r,t) = A_2\ e^{\{[-p_r \cdot r + (K_r + V_{\theta\varphi})\cdot t]/\hbar\}}$ (see eq-34, where $V_{\theta\varphi} \equiv 0$) that radiate and spread in all 4π directions from a point center.

In SunQM-2's Table 1 and Table 3, I showed that in the Solar {N,n//6} QM, H = h/m is a (the quantum number n dependent) pseudo (Planck) constant across all planets, and even across both the gravitational QM and the electric QM, although I was not able to give out the physics meaning of H. By re-writing eq-51 into eq-52, maybe I can explain H as a particle/planet's r-dimensional (reversed) diffusion constant per radial spreading.

$$D = \frac{\hbar}{2m} = \frac{H}{4\pi} \qquad \text{eq-52}$$

where H = h/m, and h is the Planck's constant, m is a scaler factor (in the mass unit kg, see SunQM-2).

(Note: Similarly, for the uncertainty principle $\sigma_p\ \sigma_x \geq \hbar/2$, we may can re-write it as $\sigma_p\ \sigma_x \geq h/(4\pi)$, and then we may also can re-explain it as that the uncertainty of a RF rotation in unit of h that radial spreading over all 4π directions from a point center).

2) Furthermore, according to the above explanation, the other QM scientists' previous result (see wiki "Heat equation") of $D \to \frac{i\hbar}{2m}$ may can be re-explained as that, the diffusion constant $D = \frac{\hbar}{2m}$ not only radially diffuses in r-1D, but also laterally diffuses in θφ-2D through RF (RotaFusion, or rotation diffusion), (because in paper SunQM-2, "*I believe that the complex number sign "i" of "iħ" in equations (or operators) reflects the real-time RF movement*"). If this explanation is correct, then the function of eq-31, eq-17, and eq-27 (i.e., to analyze the E = K + V in an "i-space") maybe is to filter out the RF (the lateral diffusion) in θφ-2D, so that a pure r-1D diffusion can be obtained.

3) Notice that the traditional Schrodinger equation (eq-1) is equivalent to the θφ-2D-only Schrodinger equation (eq-36), and the r-1D-only Schrodinger equation (eq-38) is more or less deviated from the traditional Schrodinger equation (eq-1). The result (in eq-36 and eq-38) that a particle/planet's kinetic energy in θφ-2D only affected by V_r, and a particle/planet's kinetic energy in r-1D (K_r) only affected by $V_{θφ}$, may can only be explained by the RF process. So the importance of this article is that it opened a completely new way to study the relationship between the rotation-diffusion (RF) and the energy of the orbital movement, even though I am not sure how correct this method is.

4) Notice that in the diffusion equation eq-47, $\nabla_r^{\ 2}$ only applies on the r-1D in a rθφ-3D dimension's central force (G-, or E-, or others) formed bound QM state. Also notice that eq-47 may only valid under the condition of $V_{θφ} \equiv 0$.

5) Actually, $V_{θφ} \equiv 0$ has two very different meanings: First, it may mean that there is only one particle/planet in one n orbit (or in one n shell); Second, it may also mean that all mass in this n shell is evenly distributed, so that for any particle inside this n shell, its $V_{θφ}$ potential is unable to differentiate from that of any other particle in the θφ-2D space. It is obvious that, when using the r-1D-only Schrodinger equation (eq-38) to describe the pre-Sun ball's collapse (or the reversed diffusion), $V_{θφ} \equiv 0$ must means that all mass in this n shell is evenly distributed. (See Appendix C for more discussion).

6) Notice that the r-1D Schrodinger equation's solution eq-34 has the same form as that of eq-10 (but not the eq-9).

7) Just like that Schrodinger (wave) equation was deduced from nowhere (except it was based on the matter wave) but it works, eq-17, eq-27, and eq-31 may also can be said that they were deduced from nowhere (except they were based on the RF of the Schrodinger equation), and they worked. Here I need to acknowledge following three authors because their work had inspired me to adapt Euler's formula for the analysis in this paper:
7a) Takahisa Okino. From his paper [35], I first time knew that "i" can added to any physical variable (like the time variable in his paper) "at your will" for a deduction (even though I don't fully understand his deduction due to my math level is not high enough). Also notice that in his paper, Okino also obtained $D = \frac{\hbar}{2m}$ (before my deduction, but in a completely different method).
7b) Yingtao Yang (杨映涛). From his paper [38], I first time knew that for a deduction, a real quantity physical parameter can be analyzed as a complex quantity "at your will" (like the equation-11 in his paper).
7c) "3Blue1Brown" (or Grand Sanderson?), his (series) online video courses "Understanding $e^{i\pi}$ in 3.14 minutes" [39] educated me many new things on Euler's formula (beyond what I had leaned in my college).
7d) So, in my work (in the current paper), I added the "i-space" at my will, and defined eq-17, eq-27, and eq-31 at my will. However, because I did not fully understand either Okino's or Yang's deduction, I also did not fully understand my own deduction (even though it seems works). I guessed that it must have something to do with RF.
7e) Here is the point that I want to emphasize: by using many big assumptions, I know that I have mixed some concepts (intentionally or I was forced to), so that the whole deduction in this paper becomes a citizen-scientist-leveled work. While my research work on SunQM-2s1 is nearly abandoned, the research work on the current paper (SunQM-6s9) does open a completely new road to explore the relationship between the RF orbital motion and the orbital energy. Eq-31, eq-17, and eq-28 must have some deep physical meaning that we will explore later.

8) So, although Schrodinger equation/solution was originally used to describe the r-dimensional equilibrium of diffusion and reversed-diffusion for an H-atom (and/or for the Solar system), it may can also be used as either a diffusion equation only (to describe a neutron star explosion, also see Appendix D); or as a reversed-diffusion equation only (to describe the quantum collapse process of a pre-Sun ball during the star formation).

III. Modeling of a reversed-diffusion process (that based on the Schrodinger equation's solution) matches to a pre-Sun ball's collapsing process (semi-quantitatively)

(Note: This is a citizen-scientist-leveled work). In this section, I did a simple (and rough) modeling to see that, whether (the r-1D-only Schrodinger equation solution of) eq-34 is a physics meaningful solution for a pre-Sun ball's {N,n} QM collapsing dynamics. In the Bohr-QM, we know that there is a relationship of $v_n = v_1/n$, thus

$$p = p_n = mv_n = m\frac{v_1}{n} = \frac{p_1}{n} \qquad \text{eq-53}$$

and thus

$$K = K_n = \frac{{p_n}^2}{2m} = \left(\frac{{p_1}^2}{n^2}\right)\frac{1}{2m} \qquad \text{eq-54}$$

Although both eq-53 and eq-54 were originally obtained from the Bohr model, they are valid for all the nLL QM states (or, only in φ-1D dimension under the Schrodinger equation/solution). Now I make a big assumption that they are also valid in rθφ-3D dimension. This is equivalent to say that they are also (assumed to be) valid in r-1D dimension. Bring eq-53 and eq-54 into eq-34 (with $V_{\theta\varphi}$=0), we have the probability density (notice that it correlates to the mass density)

$$|\Psi(r,t)|^2 \propto \left|e^{[(-p_r\cdot r+K_r\cdot t)/\hbar]}\right|^2 = \left\{e^{\left[\left(-\frac{p_1}{n}\cdot r+\frac{{p_1}^2}{n^2\,2m}\cdot t\right)/\hbar\right]}\right\}^2 = \left\{e^{\left[-\frac{p_1}{\hbar}\cdot\frac{r}{n}+\left(\frac{p_1}{\hbar}\right)^2\left(\frac{\hbar}{2m}\right)\cdot\frac{t}{n^2}\right]}\right\}^2 \qquad \text{eq-55}$$

Then, I did a semi-quantitative modeling calculation for eq-55 by choosing $\frac{p_1}{\hbar}$ = 0.5, and $\frac{\hbar}{2m}$ = 1. The calculated $|\Psi(r,t)|^2$ was shown in Table 1, and also plotted in a 3D plot as shown in Figure 1. For a mass collapsing (or mass reversed-diffusion) process, we know that as time passed, the mass is more concentrated to the center. From the general QM knowledge, we know that the closer to the center (i.e., the smaller the r is), the smaller the quantum number n will be. However, we don't know the exact mathematical relationship between r, n, and radial distribution of the mass. So in the calculation of Table 1, I made a big assumption: as the mass moving closer to center upon the time t, the n decrease upon t in a simplified relation as shown in column 1 vs. column 3. In the calculation, I also assumed that the value of mass density equals to the value of probability density $|\Psi(r,t)|^2$. From Table 1 and Figure 1, we can see that as the time increasing, the original (almost) evenly distributed mass density (in the range of $0 \le r \le 8$, at t = 0) become more concentrated at the center, while the total mass in the whole space (see column 13 of Table 1) kept almost the same. So this semi-quantitative modeling does illustrate that eq-34 is Schrodinger equation's one solution which is physically meaningful for Solar {N,n} QM's mass collapsing dynamics.

More discussions (on Table 1 and Figure 1).

1) As I mentioned before, only when $V_{\theta\varphi} \equiv 0$, the r-1D-only Schrodinger equation (eq-38) can be degenerated to be a diffusion equation (eq-46). In the case of the quantum collapsing of a pre-Sun ball, if the mass is evenly distributed in the same n shell (of the pre-Sun ball), it will also have the effect of $V_{\theta\varphi} \equiv 0$. Thus, we may can directly use the diffusion equation eq-46 to calculate the reverse-diffusion process for the quantum collapse of a pre-Sun ball.

2) The column 13 calculates (a kind of) total mass at each time point during the collapsing (or reversed-diffusion) process. It was calculated as the sum of the mass density multiplies the volume for each spherical shell. For example, for t = 0,

$$\sum_{r=0}^{r=7}\frac{4}{3}\pi[(r_{r+1}{}^3 - r_r{}^3)D_r] = \frac{4}{3}\pi[(r_1{}^3 - r_0{}^3)\times 1.00 + (r_2{}^3 - r_1{}^3)\times 0.90 + \cdots + (r_8{}^3 - r_7{}^3)\times 0.50] = 1257$$

where D_r is the density (of probability, or mass) in the range from r_n to r_{n+1}. Notice that we can adjust n value for each time point t (or vice versa) to make column 13's values become more constant. But due to both column 1's assumption and column 13's calculations were (kind of) over simplified, this whole model is only a semi-quantitative modeling.

3) This modeling showed that in the r-1D-only wave function eq-34, for the function of $e^{\{[-p_r\cdot r+E\cdot t]/\hbar\}}$, when E > 0 it is a reversed-diffusion (or mass collapsing), but when E < 0 it is a true diffusion. Because E < 0 is not relevant to a physical meaningful solution for Solar {N,n} QM's mass collapsing dynamical process, I do not interesting to discuss it here.

Table 1. Modeling calculation of eq-55 (equivalent to eq-34, or eq-10) with $p_1/\hbar = 0.5$, and $\hbar/(2m) = 1$.

	Mass Density = ↘		r= →									
n=			0	1	2	3	4	5	6	7	8	Σ(mass)
10	t= ↓	0	1.00	0.90	0.82	0.74	0.67	0.61	0.55	0.50	0.45	1257
9		2	1.01	0.91	0.81	0.73	0.65	0.58	0.52	0.47	0.42	1201
8		6	1.05	0.92	0.82	0.72	0.64	0.56	0.50	0.44	0.39	1157
7		8	1.09	0.94	0.82	0.71	0.61	0.53	0.46	0.40	0.35	1093
6		10	1.15	0.97	0.82	0.70	0.59	0.50	0.42	0.36	0.30	1026
5		12	1.27	1.04	0.85	0.70	0.57	0.47	0.38	0.31	0.26	962
4		14	1.55	1.21	0.94	0.73	0.57	0.44	0.35	0.27	0.21	919
3		16	2.43	1.74	1.25	0.89	0.64	0.46	0.33	0.24	0.17	980
2		18	9.49	5.75	3.49	2.12	1.28	0.78	0.47	0.29	0.17	1893

The "Mass Density" in the table is actually the (un-normalized) probability density (that the mass density is correlated to).

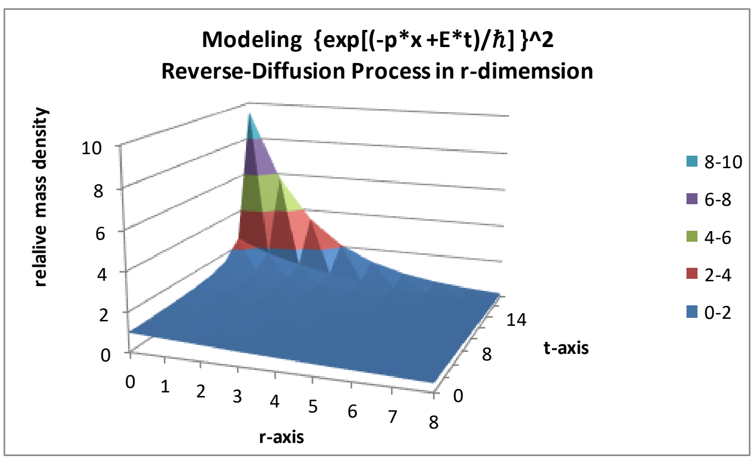

Figure 1. To model a time-dependent, r-1D-only probability (or mass) density change (shown in eq-55) in a reversed-diffusion process. Note: Microsoft Excel's 3D plot was used here.

IV. The formation process of Solar system may can be thought as three consecutive reversed-diffusion processes in the dimension of r-1D, θ-1D, and φ-1D

According to {N,n} QM, in the formation of Solar system (from a pre-Sun nebula), the mass's collapsing process may can be separated into three consecutive reversed-diffusion process:

Step-1, mass collapsing (or reversed-diffusion) in r-1D space. During this step, > 99% of mass quantum collapsed from {N,n=1..5}o orbit space into {N-1,n=1..5}o orbit space (see SunQM-1s1 and SunQM-3s1).

Step-2, mass collapsing (or reversed-diffusion) in θ-dimension to form pre-Sun disk (or disc?). During this step, the leftover < 1 % mass in {N,n=1..5}o orbital spherical space collapsed in θ-1D space from $0 \le \theta \le \pi$ to $\theta \approx \pi/2$, or from m = -*l*, …, +*l*, to

m = +*l* = n-1. This process is accompanied by a *l*=0,.., n-1 to *l*=n-1 mass collapse (or reversed-diffusion) in Δr-dimension to form several rings/belts (from one disk). The combination of these two processes in step-2 forms the nLL QM effect (which is caused by the self-spinning of pre-Sun ball, see SunQM-3s1, SunQM-3s2, SunQM-3s4, and SunQM-3s10).
Step-3, started from a ring/belt, mass collapsed (or reversed-diffusion, or accretion) in φ-1D to form a planet (see SunQM-3s10 and SunQM-4s1).

Step-1 makes the r-1D-only Schrodinger equation (eq-38) degenerated into a (reversed) diffusion equation (eq-47). Because of this, step-2 would also have made the θ-1D-only Schrodinger equation (hidden in eq-36) to degenerate into a (reversed) diffusion equation, if θ-1D dimension had not been in RF with φ-dimension. Similarly, step-3 would also have made the φ-1D-only Schrodinger equation (hidden in eq-36) to degenerate into a (reversed) diffusion equation, if φ-dimension had not been in RF with θ-dimension. Notice that without a strong self-spin of pre-Sun ball, the step-2 will not happen, and most likely step-3 will also not happen too. So a spin-less star or galaxy will only have step-1 (the r-dimensional reversed diffusion).

In this way, I may have demonstrated that Schrodinger equation is originated from the diffusion equation. It can be degenerated into a (reversed) diffusion equation (at least in r-dimension), and it accurately describes the three consecutive reversed-diffusion processes in r-1D, θ-1D, and φ-1D during Solar system formation. Therefore, this work showed that diffusion (or reverse diffusion, or the gravitational collapsing) is one of many nature attributes of quantum mechanics. Some of the other known nature attributes of QM are: the particle-wave duality, uncertainty principle, rotation diffusion (or RotaFusion, or RF), Simultaneous-Multi-Eigen-Description (SMED), etc.

In SunQM-3s11, I had mentioned that since {N,n} QM structure covers from quark {-17,1} to the Virgo super cluster {10,1} with good consistency (see SunQM-1s2's Table 1), and Schrodinger equation/solution has accurately described the Solar system from {-2,1} to {5,1} (see SunQM-3s11, and SunQM-4), as well as the atom system from {-15,1//6} to {-11,1//6} (see SunQM-1s2 Table 1), I believed that the whole universe can be described by Schrodinger equation and solution, and a single quark can also be described by Schrodinger equation and solution (also see the later work in SunQM-7's Table 1). Here I also believed that the further modification of both Schrodinger equation/solution and some traditional QM's rules (like the Born's probability rule, see SunQM-4) is needed before we can use Schrodinger equation/solution to describe either our universe, or a single quark, or anything in between.

Also, I hope that in one of the future SunQM series papers, the diffusion equation (eq-47) will be used to explore the reversed-diffusion dynamics (or at least the kinetics) of Solar {N,n} QM structure collapsing process.

V. Can we reformulate Schrodinger equation's solution to show its RF character?

In eq-33, vector $p_{\theta\varphi}$ is mostly perpendicular to the vector r (if the vector $\vec{\mathbf{r}}$ is only in r dimension), so that I have to assume that $p_{\theta\varphi} \cdot r$ in eq-33 is two (scalar) values' product. Here I showed an alternative way to make $p_{\theta\varphi} \cdot r \neq 0$, i.e., to have these two vectors ($p_{\theta\varphi}$ and r) to be in cross production:

$$\vec{\mathbf{r}} \times \vec{\mathbf{p}}_{\theta\varphi} = \vec{\mathbf{L}} \qquad \text{eq-56}$$

where $\vec{\mathbf{L}}$ is the angular momentum vector of a particle/planet's orbital motion. Notice that these three vectors are mutual orthogonal. Then we reformulated θφ-2D dimension's Schrodinger equation's solution eq-33 as

$$\Psi(\theta, \varphi, t) = A_1\, e^{\{i[(\vec{\mathbf{r}}\times\vec{\mathbf{p}}_{\theta\varphi})\cdot\vec{\mathbf{L}}_{unit} - (K_{\theta\varphi} + V_r)\cdot t]/\hbar\}} \qquad \text{eq-57}$$

where $\vec{\mathbf{L}}_{unit}$ is the unit vector of vector $\vec{\mathbf{L}}$, so that the dot-product of these two vectors becomes a scalar value. I believe that eq-57 may have shown the RF character of Schrodinger equation/solution (in some way). (Note: eq-57 was purely from my guess). I may will further explore this issue in the future, I also hope that eq-57 can inspire other scientists to explore this issue. (Note: This topic was originally presented in SunQM-2s1. Since the article SunQM-2s1 is (mostly) dismissed, I moved the topic to here). (Note: Here I opened this idea to others, because my 10-years closed-door research phase is going to end in ~7 months (in summer 2024), and I am unable to finish this work by that time).

VI. The Cartesian xyz-coordinate described 3D space is only a sub-space of a spherical rθφ-coordinate described 3D space, thus xyz-3D space may not give a complete description for a point-central field's 3D space

During the {N,n} QM earlier development (2016 ~ 2018), I started to believe that the Cartesian xyz-coordinate described 3D space (in which z-axis, x-axis and y-axis are perpendicular to each other) maybe cannot give a complete description for a point-central field's 3D space. In a xy-2D plane space, the y-axis is fully orthogonal to x-axis. However, in a xyz-3D coordinate space, for anyone of x, y, z-axis, the other two axes may be not (fully) orthogonal with each other. (For example, in a point-central field, relative to z-axis, even though both x-axis and y-axis are fully orthogonal relative to z-axis, the apparent perpendicular between x-axis and y-axis maybe does not mean that x-axis and y-axis are fully orthogonal). I believe that in a point-central field, for z-axis, the only full-orthogonal dimension is the circles (or the circling space) of either exp(+iφ) or exp(-iφ) in xy-plane (i.e., Euler's formula in 2D), or the two circular axes of x+iy and x-iy. If we define exp(+iφ) relative to +z axis is the right-hand rule (see SunQM-6's Fig-1a), then exp(-iφ) relative to +z axis must be the left-hand rule. The right-handed electromagnetic field told us that our world is dominated by the right-hand rule (not the left-hand rule). Once the matter waves interfered with each other, exp(+iφ) and exp(-iφ) become exp(+imφ) and exp(-imφ), (because under the spin-frame, some energy-degenerated states differentiated). On the other hand, in a rθφ-3D space, because r-axis points to all the 4π solid angle, the circular axis of exp(+iφ) become RF. This means, a spherical rθφ-coordinate-described 3D-space not only can characterize the 3D position in the xyz-3D space, but also can characterize the RF in a 3D -space.

In the {N,n} QM physics, we are living in a physical world that is dominated by the point-central fields (mass field, force field, energy field, etc.). The 3D space of the point-central fields not only contains the 3D position character, but also contains the RF character. I believe that the xyz-coordinate only describe the 3D position without the RF, so, I believe that the xyz-coordinate described 3D space is only a sub-space of rθφ-coordinate-described 3D space. Thus, I believe that the xyz-3D space may not give a complete description for a point-central field's 3D space.

(Note: This was part of SunQM-2s1. I put it here because I am not able to work out the major math proof in SunQM-2s1, so that the paper can't be finished so far).

VII. A 3D Euler's formula is expected to be naturally in rotation diffusion (or RotaFusion, or RF)

This hypothesis was part of the work in SunQM-2s1. No significant progress has been made so far. (Note: since SunQM-2s1 is (most likely) dismissed, I moved this topic to here). (Note: Here I opened this idea to others, because my 10-years closed-door research phase is going to end in ~7 months (in summer 2024), and I am unable to finish this work by that time).

Summary and Conclusion

With the help of RF concept, I am able to separate a traditional rθφ-3D Schrodinger equation

$$i\hbar \frac{\partial}{\partial t}\Psi(r,\theta,\varphi,t) = \left[\frac{-\hbar^2}{2m}{\nabla_{r\theta\varphi}}^2 + V_{r\theta\varphi,t}\right]\Psi(r,\theta,\varphi,t) \qquad \text{(copied from eq-1)}$$

into two parts, one in θφ-2D only, and one in r-1D only:

$$i\hbar\frac{\partial}{\partial t}\Psi(\theta,\varphi,t) = \left[\frac{-\hbar^2}{2m}\nabla_{\theta\varphi}{}^2 + V_r\right]\Psi(\theta,\varphi,t) \quad \text{(copied from eq-36)}$$

$$i\hbar\frac{\partial}{\partial t}\Psi(r,t) = \left[\frac{-\hbar^2}{2m}\nabla_r{}^2 + V_{\theta\varphi}\right]\Psi(r,t) \quad \text{(copied from eq-38)}$$

where

$$\Psi(r,\theta,\varphi,t) = \Psi(r,t)\,\Psi(\theta,\varphi,t) \quad \text{(copied from eq-35)}$$

$$K = K_{r\theta\varphi} = K_{\theta\varphi} + i\,K_r \quad \text{(copied from eq-17)}$$

$$V = V_{r\theta\varphi} = V_{\theta\varphi} + i\,V_r \quad \text{(copied from eq-27)}$$

$$E = K + iV^* = \left(K_{\theta\varphi} + V_r\right) + i\left(K_r + V_{\theta\varphi}\right) \quad \text{(copied from eq-31)}$$

I named the combination of eq-36 and eq-38 as the "**newly differentiated Schrodinger equation**", and named the combination of eq-36, eq-38, eq-35, eq-17, eq-27 and eq-31 as the "**newly differentiated Schrodinger equation group**". The newly differentiated r-1D-only equation can be degenerated into a diffusion equation with the diffusion constant $D = \frac{\hbar}{2m}$. Therefore, I may have demonstrated that Schrodinger equation can be directly degenerated into a diffusion equation (at least in r-dimension).

Acknowledgements (of all SunQM series articles):

Many thanks to: all the (related) experimental scientists who produced the (related) experimental data, all the (related) theoretical scientists who generated all kinds of theories (that become the foundation of {N,n//q} QM theory), the (related) text book authors who wrote down all results into a systematic knowledge, the (related) popular science writers who simplified the complicated modern physics results into a easily understandable text, the (related) Wikipedia writers who presented the knowledge in a easily accessible way, the (related) online (video/animated) course writers/programmers who presented the abstract knowledge in an intuitive and visually understandable way. Also thanks to NASA and ESA for opening some basic scientific data to the public, so that citizen scientists (like me) can use it. Also thanks to the online preprinting serve vixra.org to let me to post out my original SunQM series research articles.

Special thanks to: Fudan university, theoretical physics (class of 1978, and all teachers), it had made my quantum mechanics study (at the undergraduate level) become possible. Also thanks to Chen-Ning Yang and Tsung-Dao Lee, they made me to dream to be a theoretical physicist when I was eighteen. Also thanks to Shoucheng Zhang (张首晟, Physics Prof. at Stanford Univ., my classmate at Fudan Univ. in 1978) who had helped me to introduce the {N,n} QM theory to the scientific community in 2018.

Also thanks to a group of citizen scientists for the interesting, encouraging, inspiring, and useful (online) discussions: "职老" (https://bbs.creaders.net/rainbow/bbsviewer.php?trd_id=1079728), "MingChen99" (https://bbs.creaders.net/tea/bbsviewer.php?trd_id=1384562), "zhf" (https://bbs.creaders.net/tea/bbsviewer.php?trd_id=1319754), Yingtao Yang (https://bbs.creaders.net/education/bbsviewer.php?trd_id=1135143), "tda" (https://bbs.creaders.net/education/bbsviewer.php?trd_id=1157045), etc.

Also thanks to: Takahisa Okino (Correlation between Diffusion Equation and Schrödinger Equation. Journal of Modern Physics, 2013, 4, 612-615), Phil Scherrer (Prof. in Stanford University, who explained WSO data to me (in email, see SunQM-3s9)), Jing Chen (https://www.researchgate.net/publication/332351262_A_generalization_of_quantum_theory), etc. Note: if I missed anyone in the current acknowledgements, I will try to add them in the SunQM-9s1's acknowledgements.

Reference: (Those citations obtained after January 2020 were listed in grey, because this paper was drafted in January 2020)

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Note: A series of SunQM papers that I am working on:
SunQM-6s10: {N,n} QM Field Theory Development On the S/RFs-force … (drafted in May. 2023).
SunQM-4s4: More explanations on non-Born probability (NBP)'s positive precession in {N,n}QM.
SunQM-7s1: Relativity and non-linear {N,n} QM
SunQM-9s1: Addendums, Updates and Q/A for SunQM series papers.

Note: Major QM books, data sources, software I used for SunQM series papers study:
Douglas C. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. 2009.
David J. Griffiths, Introduction to Quantum Mechanics, 2nd ed., 2015.
Stephen T. Thornton & Andrew Rex, Modern Physics for Scientists and Engineers, 3rd ed. 2006.
John S. Townsend, A Modern Approach to Quantum Mechanics, 2nd ed., 2012.
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(Free) online math calculation software: WolframAlpha (https://www.wolframalpha.com/)
(Free) online spherical 3D plot software: MathStudio (http://mathstud.io/)
(Free) offline math calculation software: R
Microsoft Excel, Power Point, Word.
Public TV's space science related programs: PBS-NOVA, BBC-documentary, National Geographic-documentary, etc.
Journal: Scientific American.

Note: I am still looking for endorsers to post all my SunQM papers (including the future papers) to arXiv.org. Thank you in advance!

Note: With my 33 of SunQM papers that have been posted out so far, I believe that the framework of the {N,n} QM has been fully established. It is clear now that the {N,n} QM description is suitable not only for the mass field, but also for the force field (or the energy field, etc.). Thus, my (10 years of closed-door) research phase on the {N,n} QM will end in less than one year (most likely in the summer of 2024). After that, I will re-write the SunQM papers (~ 35 of them) in the form of a text book. So far, my identity (for the {N,n} QM development) is: a former lecturer of Fudan University, and a current citizen scientist of California.

Appendix A. For a Schrodinger equation's wave function, to show it contains the plane wave character in each dimension of φ-1D, θ-1D, and r-1D

(Note: This is a citizen-scientist-leveled explanation). Let's try to use |10,9,5> QM state wave function
$\psi(r,\theta,\varphi) \propto R(n=10, l=9)\, Y(l=9, m=5)$
as the example. First, according to wiki "Table of spherical harmonics",

$\text{Y}(9,5) = \frac{-3}{256}\sqrt{\frac{2717}{\pi}} \cdot e^{5i\varphi} \cdot \sin^5\theta \cdot (85\cos^4\theta - 30\cos^2\theta + 1).$

Figure 2a showed a plot of Y(9,5) in φ-1D, i.e., Y(9,5) $\propto e^{5i\varphi}$, and it is a standard plane wave in cosine (blue, real, Re[Y(9,5)]) or sine (orange, imaginary, Im[Y(9,5)]). Figure 2b showed a plot of Y(9,5) in θ-1D, i.e.,
$\text{Y}(9,5) \propto \sin^5\theta \cdot (85\cos^4\theta - 30\cos^2\theta + 1)$,
and it is a deformed sine curve, so it still contains a plane wave character. As a citizen scientist, I am unable to plot the r-1D radial wave function ψ(r) $\propto {a_0}^{3/2}$R(10,9). However, according to the plots of ${a_0}^{3/2}$R(3,0), ${a_0}^{3/2}$R(3,1) and ${a_0}^{3/2}$R(3,2) in a QM text book [40], I guessed that it should be something similar as that in Figure 2c (notice that it is not for n=10), that is also a deformed sine curve, and also contains a plane wave character. Then, after combining all φ-1D, θ-1D, and r-1D plane-like waves into to a spherical 3D wave ball, it become something like Figure 2d. (Notice that Figure 2d was built by mimicking wiki "Helioseismology" figure "Illustration of a solar pressure mode ... The surface shows the corresponding spherical harmonic. The interior shows the radial displacement …"). At the surface of this spherical 3D wave ball (see the left half of Figure 2d), it contains 5 complete sine waves in φ-1D (from φ = 0 to φ = 2π), and 5 complete sine waves in θ-1D (from θ = 0 to φ θ = 2π). At the interior of this spherical 3D wave ball (see the right half of Figure 2d), it may contain n number of sine waves in r-1D (from r = 0 to r = $r_{surface}$). Here I need to emphasize that Figure 2d does not depict a solution of Schrodinger equation (because lacking of the imaginary part, it lost the RF character). Figure 2d can only be used to illustrate that a spherical 3D wave packet (that is the solution of Schrodinger equation) contains the plane-like wave character in each of φ-1D, θ-1D, and r-1D space.

Note: The single sharp peak (in pink color) in Figure 2a (or Figure 2b, or Figure 2c) is used to illustrate that, after many different mode (or frequency) of plane-like waves are (Fourier) summed, the resulted wave packet may show a single peak in either φ-1D, θ-1D, and r-1D, or even in rθφ-3D simultaneously. This is exactly as what had been shown in SunQM-3s11's Table-1: after the quantum collapse, the < 1% leftover mass in the pre-Sun ball's n shell spherical space first formed a disc, and then the disc was dissected into several rings, and then (each ring) accreted into a single planet.

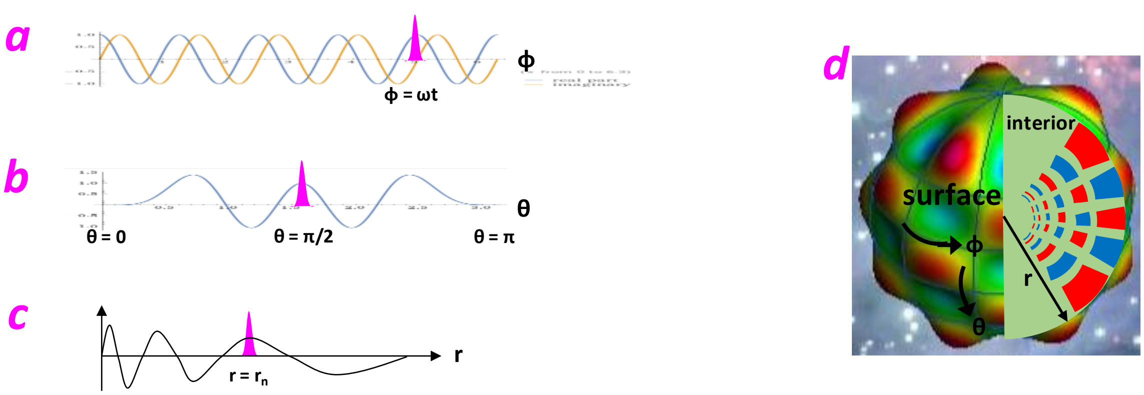

Figure 2a. Plot of Y(9,5) $\propto e^{5i\varphi}$ function in φ-1D (by using WolframAlpha).
Figure 2b. Plot of Y(9,5) $\propto \sin^5\theta \cdot (85\cos^4\theta - 30\cos^2\theta + 1)$ function in θ-1D (by using WolframAlpha).
Figure 2c. Illustration of ψ(r) $\propto {a_0}^{3/2}\text{R}(n,l)$ curve in r-1D.
Figure 2d. Illustration the part of a wave function $\psi(\text{r},\theta,\varphi) \propto {a_0}^{3/2}\text{R}(n, l = 9)\ \text{Y}(l = 9, m = 5)$ that containing the plane-like wave character in each of φ-1D, θ-1D, and r-1D space. The background of the spherical harmonic Re[Y(9,5)] surface was plotted by using the (free) online plotter at "http://icgem.gfz-potsdam.de/vis3d/tutorial".

Appendix B. An unsolved problem of using plane-like wave to be the wave function for an orbital moving planet

Moved to a future SunQM paper. (Note: It was first mentioned in SunQM-4's eq-21 explanation).

Appendix C. The apparent $V_{θφ} < 0$ (or $V_{θφ} > 0$) may transform its exertion space from θφ-2D space to r-1D space, and cause the V_r change

In SunQM-6s6's section-II, "Using Schrodinger equation for H-atom (with $V_{θφ} > 0$) to explain the Z > 1 atom's ground state electron configuration (without using the "penetrating" theory)", I apparently treated the same n shell electrons to have $V_{θφ} > 0$ in θφ-2D space, so it appeared to conflict with the current paper section II-g-5 that "all mass in this n shell is evenly distributed" will get $V_{θφ} \equiv 0$. However, after the following more accurate explanation, there should be no confliction. Although all electrons in the same n shell exert repelling force to each other (so that they apparently have $V_{θφ} > 0$), at the equilibriums state, this repelling force cannot (effectively) change the distance between the two electrons in the θφ-2D space within the same n shell (also see SunQM-6s7's Fig-2b, suppose all electrons in the same n shell are in "steady" state), and thus, it causes the effective $V_{θφ} \equiv 0$ between these same n shell electrons. However, the same n shell electrons' repelling force caused $V_{θφ} > 0$ does transform its exertion space from the θφ-2D space to the r-1D space, and causing some counter-balance on the original V_r (that come from the Coulomb force between nucleus and electrons in r-1D space). This is what the SunQM-6s7's Table-2 calculated for (i.e., the $V_{θφ} > 0$ caused V_r change). (Note: This is one of many examples of the dynamic space transformation: a θφ-2D space repelling force transformed into a r-1D space V_r change. See more examples of dynamic space transformation at SunQM-6s8's section I-a: transformation from φ-1D to r-1D; or at SunQM-6s7's section VII-c: transformation either from φ-1D to θ-1D, or from nL0 mode to nLL mode).

Similarly, in SunQM-6s6's section-III, "Using the same Schrodinger equation for H-atom (but with $V_{θφ} < 0$) to (semi-quantitatively) explain the pre-Sun ball's quantum orbital energy level configuration", the apparent $V_{θφ} < 0$ is also transformed its dynamic space from θφ-2D space to exert to the r-1D space, and become the V_r change, while the effective $V_{θφ} \equiv 0$ (because effectively it does not push or pull the evenly distributed objects in the θφ-2D space in the same n shell).

Note: For the repulsive force formed effective $V_{θφ} \equiv 0$, all electrons (that is evenly distributed in this n shell) are in a stable steady state, meaning when you move one electron away from its steady state position, it will move back automatically. However, for the attractive force formed effective $V_{θφ} \equiv 0$, all mass (that is evenly distributed in this n shell) is in a unstable "steady" state, meaning when you move one object away from its "steady" state position, the whole "steady" state will collapse, and all evenly distributed mass in this n shell will be accreted into a single position in this n shell (as illustrated in SunQM-4s1's Fig-6). (Also see my related unsuccessful work in SunQM-4s1's section IV-b).

Again, this is the initial explanation in the "global fitting" of {N,n} QM. If this explanation cause more problem in the future, then more adjustments will be made.

Appendix D. The exploded matter from a supernova may expand quantumly "super-shell by super-shell" in the "worm-crawling style"

(Note: This part should go with SunQM-1s1). In the early SunQM papers, I guessed that during the Solar system formation, the pre-Sun ball collapsed quantumly super-shell by super-shell (from N = 5, to N = 4, 3, 2, 1, 0 super shell), or, from {6,1} size to {5,1}, {4,1}, …down to {0,2} size, (see SunQM-1s1's section-I, and it may should not be a continues collapse as described in the classical physics). Besides that, I also guessed that the fusion process of QM {N,n} structure inside the Sun may also have a quantized expansion dynamics (see SunQM-1s1's section X). Soon after that, I further guessed that in the supernova explosion, the exploded matter may also expand quantumly (i.e., super-shell by super-shell), from N = 0, to N = 1, 2, 3, 4, 5 (as shown in Figure 3a). In Figure 3, notice that in the t_1 phase, almost all (explored) matter is concentrated and constrained in the N super shell (so it can be treated as a stable QM state); in the t_2 phase, all (explored) matter spread in both N and N+1 super shells (with the lower mass density, so it can be treated as an unstable transitional QM state), and in the t_3 phase, all matter goes back to the concentrated state but constrained in the N+1 super shell (so it is another stable QM state). This quantum expansion dynamics may can be illustrated by a worm-crawling process (see Figure 3b). Crab Nebula has a diameter of 11 light years (data from wiki "Crab Nebula"). Using the above hypothesis, we may can determine whether the current Crab Nebula is in a stable QM state or not (if we have the accurate r-1D mass distribution curve of Crab Nebula).

This "**worm-crawling style expansion**" process may also can be used to explain that why the r-1D mass distribution of the current Asteroid belt is perfectly distributed (or constrained) in the {1,8//6}o orbital shell space (see SunQM-1s1's Figure 5). In SunQM-3s11's section-X-7, Asteroid belt was described as the (dried) "ring stain" of the expanding ice-evap-line. Immediately after this "ring stain" dried out (at ~4 billion years ago? a purely guessed time), it had very low probability that all the Asteroid belt's mass was perfectly constrained in the {1,8//6}o orbital shell space only. It was more likely that the r-1D mass distribution spanned not only mainly in the {1,8//6}o orbital shell space, but also minorly in either the {1,7//6}o or {1,9//6}o orbital shell space (Note: this is not a stable QM state, it is an unstable transitional QM state between the two stable QM states). Then, I believe that it was the "worm-crawling style expansion" QM force that either pushed the minority mass in the {1,7//6}o orbital shell to move forward into the {1,8//6}o orbital shell, or pulled the minority mass in the {1,9//6}o orbital shell to move backward into the {1,8//6}o orbital shell (during the last ~4 billion years), so that currently all Asteroid belt's mass was perfectly squeezed in the {1,8//6}o orbital shell space, and thus become a stable QM state.

Similarly, this "worm-crawling style expansion" process may also can be used to explain the r-1D mass distribution of the Kuiper belt (that is currently distributed in the {2,6//6}o orbital shell space, see SunQM-3s10's Fig-2a). In SunQM-3s11's section-X-7, Kuiper Belt (and/or the "cold-KBO", meaning the cold population of Kuiper Belt Objects) was described as the (wet) "ring stain" of the expanding methane-evap-line. In the next few billion years, as the expanding of the methane-evap-line (in the Solar system), Kuiper Belt (as the wet "ring stain") will expand correspondingly (from {2,6//6}o orbital shell to {2,7//6}o orbital shell, and then to {2,8//6}o orbital shell, etc.) with the "worm-crawling style". During the expansion, when this wet "ring stain" dried out (i.e., all methane component in the Kuiper belt's matter evaporated), most likely it will span two n orbital shell spaces (and let's assume that the majority mass is in {2,7//6}o orbital shell space). Then, the "worm-crawling style expansion" QM force will either pushed the minority mass in the {2,6//6}o orbital shell forward into the {2,7//6}o orbital shell space, or pull the minority mass in the {2,8//6}o orbital shell backward into the {2,7//6}o orbital shell space, so that the whole Kuiper Belt's mass will be perfectly constrained in the {2,7//6}o orbital shell space, and to become a stable QM state.

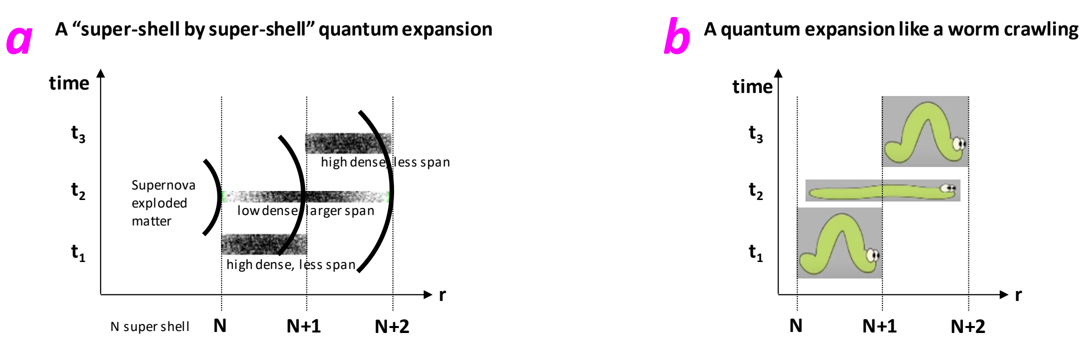

Figure 3a. To illustrate a "super-shell by super-shell" of step-by-step quantum expansion for the exploded matter of a supernova.
Figure 3b. A Crawling Worm, copied and modified according to: https://www.youtube.com/watch?v=UVNpc9tM2LQ. Original Author: OpenToonzTraining, Copyright: Unknown.

Appendix E. Some of my thoughts that related to this topic, and I may work on it in the future

1) Can we use eq-31, $E = K + iV^* = (K_{\theta\varphi} + V_r) + i(K_r + V_{\theta\varphi})$, to explain SunQM-3s1's nLL quantum collapse caused n*l*0 bipolar outflow?

2) To estimate the total time for the quantum collapse from {6,1} to {0,2} for the Solar system formation. Use eq-52 to determine the half-life time t(1/2) for each super-shell collapsing (i.e., {N,1//6} to {N-1,n//6} from N=6 to N=1), because the real H values are known in SunQM-3's Table-1b.

3) A photon propagation may can be treated as a true diffusion process.

4) In the formula **E = K + iV***, K and V belongs to two different bases, so "i" rotates a value to two different basis. The energy conservation links these two values together. Same thing is shown in the text book "Linear Algebra and its application" by Lay, 2006, 3rd ed. pp360, Example 3, Eigen-eq, vector of 2-basis, one for electric current, one for voltage. Ome's law links these two values together. Similarly, mentioned in wiki "Probability amplitude", section "A basic example", (https://en.wikipedia.org/wiki/Probability_amplitude), a photon in 1/3 of H state and 2/3 of V state, will have superposition QM state of |HV> = sqrt(1/3) |H> -i sqrt(2/3) |V> . Notice that the "i" reflects that |V> space is out of |H> space, but the value is RF(ed) out of |H>space into |V> space. However, for the double-slid experiment explanation $|\Psi|^2 = |\Psi 1 + \Psi 2|^2 = |\Psi 1|^2 + |\Psi 2|^2 + 2 |\Psi 1| |\Psi 2| \cos(\varphi 1 - \varphi 2)$, why we do not use "-i"?

5) To figure out all eight planets' elliptical orbit's n' for |nlm> = nLL QM state, for example, Mercury maybe at n'=3*6^2=108, or |108,107,107> QM state, Venus maybe at n'=4*6^3, Earth maybe at n'=5*6^3, etc.