A New Approach to Unification Part 1: How a ToE is possible

Jürgen Kässer

As in spite to intense search at present apparently there is no approach leading to a theory including the standard model of particle physics and general relativity, it is discussed whether such a theory is possible at all. In the following is shown that a ToE is possible if the either/or condition of current unification theories for background-dependent or -independent is replaced by a both/and. In this part 1, the foundation of such a theory is presented. In the following parts 2 and 3 particle and gravitational physics are derived from this foundation and in part 4 fundamental open fundamental questions of actual physics are answered by a new interpretation of physical quantities and an outline of a new cosmology is given.

Contents

1 Introduction

With its special and general theory of relativity Einstein has created the foundations of our present understanding of the world. These two theories however are based on incompatible world views what generates the major problem of actual physics.

For the background-dependent theories based on the special relativity theory space and time form an immovable stage, characterized by a metric with constant coefficients, on which the events take place. Changing the point of view in such a theory in general changes the physical equations. For example moving into a rotating system additional forces occur. Particle physics obeys such laws.

If general relativity is the basis of a theory, the equations remain unaltered in most coordinate systems, but the structure of the space-time varies. The metric becomes a variable. These background independent theories appear mainly in the description of gravity effects.

All attempts to describe particle physics in a background-independent or gravity physics in a background-dependent way have failed.

Einstein did see the problem but did not find a way out. So he left us a world seemingly based on incompatible foundations.

Also all approaches seeking a new physical basis, as attempted in string/M theory [1] or loop quantum gravity [2], had to choose between one of the two alternatives. String/M theory is background dependent (there are attempts to make it independent), loop quantum gravity is background independent.

Great disillusionment arose from the fact that after many years of intensive search none of the particles predicted by M-theory is found. Neither the Large Hadron Collider of the CERN, actually the largest particle physics laboratory in the world, was successful with its search for the additional particles proposed by the Constrained Minimal Supersymmetric Standard Model [3] nor the Gran Sasso National Laboratory in Italy or the Jinping Deep-Underground Laboratory in China in their search for WIMPs (Weakly Interacting Massive Particles), fictive particles of Dark Matter related to the additional particles predicted by supersymmetry.[4]

Since, after many decades of research, neither of the attempts can produce tangible results, scientists are seriously discussing the possibility that there can be no theory in which the two worldviews are unified. Following Gödel's incompleteness theorem [5], which shows that it is impossible to find complete and consistent rules for all of mathematics, they argue that there could be a corresponding theorem in physics as well. This would mean that any attempt to construct a comprehensive theory is doomed to failure.[6, 7]

Decades of unsuccessful searching have pushed physics into a deep crisis. The way in which scalar fields such as quintessence, inflation field etc. are introduced is reminiscent of the Ptolemaic world view, in which simply additional epicycles were introduced to adjust calculations to the observations. Also the more and more increasing complexity of mathematics in actual physics reminds us of Ptolemy. Physics is more reminiscent of a mathematical seminar than a description of nature. The incorrect initial assumption is offset by extensive and complex calculations. Only the correct starting point created by Kepler, which was revolutionary at the time, eliminated the huge mathematical effort previously required

In all approaches, the background-independent/-dependent problem is seen as an either/or decision. It seems impossible to imagine a structure that combines both properties.

The way out of the dilemma can only be to transform the either/or into a both/and.

But this presupposes a fundamental change in the world view. What we perceive can no longer be reality as in reality either/or applies. Perception must become the image of a reality that is inaccessible to us. Only the gradation into inaccessible reality and image gives the degree of freedom of both/and. If it is only a matter of depicting the unknown in the best possible way, sometimes a color change may be necessary, i.e. sometimes a background-independent and sometimes a background-dependent description must be used.

Such an inaccessible reality is nothing new. Kant argues that we cannot know the real world, the noumenon as he calls it, but only an image of it according to our limited sensual and intellectual abilities.

How does this new point of view help to find a ToE?

Plato already illustrated the possibility of different realities with his allegory of the cave. In it, he describes the situation of people living chained in a cave all their lives and see nothing but shadows on a wall and hear voices coming from outside. Through this artifice, Plato turns our world into the inaccessible world for the prisoners and shows the possibility of different realities. Plato believes that these people develop a worldview based on shadows because they know nothing else.

However, Plato's assumption about the prisoner's view of the world underestimates the human inventive spirit. A prisoner eager to learn could think about the origin of the shadows. For this purpose he would need hypotheses about the cause of the shadows and regularities, how the shadows on the wall follow from the existence of the causal entities.

He had to think up the law of shadow casting and then would be able not only to explain the shadows, but also to win conclusions about the happening outside of his point of view.

This shows that the world found in a theory must not have anything common with the world of an observer.[8] It is sufficient if for him the known phenomena result. No theory can achieve more.

Physicists are in the position of the chained prisoner. They can make hypotheses about the properties of a for us inaccessible space and assume regularities, how these properties show up to their experiments. The hypotheses are valid if the effects following from the assumptions match the in nature occurring ones. This finally will confirm the hypotheses.[9]

One can win thus both a ToE and a new world view.

What does ToE mean in a context that allows us choosing the color? ToE then implies that it is possible to derive all of 4d physics from a hypothetically defined noumenon and that there is one procedure how a 4d observer realizes its features.

2 Postulating features of the inaccessible world

Having created the basic possibility for a ToE the search for the properties of the noumenon becomes the decisive second hurdle. At first suitable hypotheses about the inaccessible world must be created. The only thing we know about it is: its structure must be such that it produces the effects we observe.

Trying to find out what these hypotheses could be is not straight forward. There is no chance to think in advance which basic assumptions must be chosen to be able to explain the many different effects. So in the beginning every assumption is possible. Elaborating a theory finally will separate the wheat from the chaff. The only justification to any assumptions is given by measurement. The assumptions are good if they are able to explain the results of experiments and become better the more experiments they can explain.

To start thinking on a new approach an impetus is needed. Since the problems of today's physics originate from the split understanding of space and matter, a starting point to look for features of the noumenon must lie in this area. On the other hand, today's physics describes so many things correctly that it cannot be completely wrong. A decision in the past must have caused today's division.

For the new approach called unique root (UR) this separation marker is the Pauli equation, a modified Schrödinger equation describing the behavior of an electron in a magnetic field. It was an ad hoc formulation to implement spin in non-relativistic quantum mechanics. [10]

The spin there is represented by three (2x2) Pauli matrices. Rotation in the spin-space means changing an electron with spin down in one with spin up or vice versa. This can also be understood as the destruction of a particle and the creation of another one.

If the Pauli equation is viewed in a rotated coordinate system, its eigenvalues as observables must be preserved. This links the Special Orthogonal group SO(3) providing rotation in a three dimensional (3d) Euclidean space and the Special Unitary group SU(2) related to rotations in Hilbert space for 2 particles. The spatial symmetries are called external, the symmetries in the particle space internal.

Although rotations in space and in Hilbert space describe physically fully different features, the mathematical groups describing them are close connected. The two groups are local isomorph, have the same number of generators and are both compact.

By the connection of the groups a close relation between spin, a feature related to matter, and space, an expression of geometry, is generated. This means that there is a relationship between the internal and external symmetries not generated by a nontrivial combination of their generators as is tried by M-theory but by choosing groups with common inherent features.

Generalizing the finding brings us to a first postulate of the new approach.

UR Postulate 1. The group describing spatial rotations and the group describing rotations in a generalized spin or matter space of the inaccessible world are local isomorph, have the same number of generators and are both compact.

It can be understood as a feature of the inaccessible world. But it is much more. It gives the philosophical term a at least mathematical meaning.

We do not introduce the symmetries of the Pauli equation in the postulate, as in a flat four dimensional (4d) space special relativity has to hold. SO(3) and SU(2) symmetries so cannot be the basis of a fundamental theory. But there are other pairs of groups that fulfill postulate 1. The closest in complexity to SO(3) and SU(2) are the groups SO(6) and SU(4).

Historically the Pauli equation was overcome by Dirac's relativistic equation. Rotation in space is replaced by the Lorentz-transformation. This means introducing instead of SO(3) the non-compact Lorentz group SO(3,1) what destroys the close relation between internal and external symmetries.

This does not mean that the Dirac equation is wrong, but it must be deducted from a theory that is subject to postulate 1.

If the ToE based on postulate 1 provides in 4d special relativity this is not a demand but a result.

To give postulate 1 also physical relevance we introduce

UR Postulate 2. The inaccessible world forms a Euclidean six dimensional (6d) space without time. It has a Lagrangian with SU(4) symmetry.

Postulate 2 defines a world compatible with postulate 1. Although the defined spaces are not within the grasp of 4d physics, they can be described in physical terms.

3 Structures following from the postulates

The SO(6) and SU(4) symmetries are so restrictive that they generate a structure in the postulated spaces. Even though the 6d space has no time component in analogy to 4d physics, equations can be set up which formally agree with a Klein-Gordon equation, a Dirac equation and a Lagrangian but satisfy the symmetries SO(6) and SU(4).

These equations are only formal. We do not know what the entities they describe mean or what the equations describe. Since they apply to the inaccessible world, we have no way to test them. For us only their effects in 4d can be measured and interpreted.

The 6d space is Euclidean. This is not changed if curvilinear coordinates adapted to specific symmetries are used. It will be shown that for the 4d observer however it has far-reaching consequences. At first we'll consider Cartesian coordinates adapted to translational symmetry.

3.1 6d Klein-Gordon equation

Fields without spin or single components of a spinor field in 4d are described by a Klein-Gordon equation that usually contains a mass term. By symmetry reasons this however is not allowed (see section 3.3.1). Therefore a Klein-Gordon equation without mass term $\left(-\frac{\partial^2}{\partial t^2} + c^2\Delta_x\right)\psi = 0$ is used as starting point. x represents the spatial coordinates, c speed of light and ψ the 4d wave function.

The related equation in 6d with SO(6) symmetry using summation convention becomes

$$\partial^\alpha \partial_\alpha \phi = 0 \text{ with } \alpha = 1, 2 \ldots 6. \tag{1}$$

ϕ is the 6d wave function.

3.2 6d Dirac equation

Fields with half-integral spin in 4d are described by the Dirac equation. Following in generating an equivalent 6d equation the deduction of the 4d one we introduce a 6d Dirac operator $\gamma^\alpha \partial_\alpha$. The γ^α are the counterparts of the Dirac matrices.

Multiplying two 6d Dirac operators gives $\gamma^\alpha \gamma^\beta \partial_\alpha \partial_\beta$. This is the operator of a 6d Klein-Gordon equation without a mass term if the anticommutator $[\gamma^\alpha, \gamma^\beta]_+ = \gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\delta^{\alpha\beta}$ holds. $\delta^{\alpha\beta}$ is Kronecker's delta.

The 6d Dirac operator then leads us to the 6d Dirac equation sought-after

$$\gamma^\alpha \partial_\alpha \phi = 0. \tag{2}$$

The anticommutator defines for the possible γ^α a Clifford algebra Cl(6) . Following the arguments shown in [11] we find: Any product of the γ^α can be transformed to a few linearly independent elements. Arranged in ascending order we get inclusive the unit element E:
$(E, \gamma^1 \ldots \gamma^6, \gamma^1\gamma^2 \ldots \gamma^5\gamma^6, \ldots, \gamma^1\gamma^2\gamma^3\gamma^4\gamma^5\gamma^6)$.
These $2^8 = 64$ elements can be represented by 8x8 matrices.
Introducing the Pauli spin matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and the two dimensional unit matrix E2 the six γ^α can be chosen as

$$\begin{array}{ll} \gamma^1 = \sigma^1 \otimes E2 \otimes E2 & \gamma^2 = \sigma^3 \otimes \sigma^1 \otimes E2 \\ \gamma^3 = \sigma^3 \otimes \sigma^3 \otimes \sigma^1 & \gamma^4 = \sigma^2 \otimes E2 \otimes E2 \\ \gamma^5 = \sigma^3 \otimes \sigma^2 \otimes E2 & \gamma^6 = \sigma^3 \otimes \sigma^3 \otimes \sigma^2 \end{array}$$

where $\otimes$ means the Kronecker product.

Written with block matrices this means

$$\gamma^1 = \begin{pmatrix} 0 & E4 \\ E4 & 0 \end{pmatrix} \gamma^2 = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix} \gamma^3 = \begin{pmatrix} b & 0 \\ 0 & -b \end{pmatrix}$$

$$\gamma^4 = \begin{pmatrix} 0 & -iE4 \\ iE4 & 0 \end{pmatrix} \gamma^5 = \begin{pmatrix} c & 0 \\ 0 & -c \end{pmatrix} \gamma^6 = \begin{pmatrix} d & 0 \\ 0 & -d \end{pmatrix}$$

with

$$a = \begin{pmatrix} 0 & E2 \\ E2 & 0 \end{pmatrix} b = \begin{pmatrix} E2 & 0 \\ 0 & -E2 \end{pmatrix} c = \begin{pmatrix} 0 & -iE2 \\ iE2 & 0 \end{pmatrix} d = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}.$$

The determinant of all matrices is 1. The trace is 0. So the 64 possible matrices generate the group of special unitary 8x8 matrices SU(8).

The eigenfunctions of the 6d Dirac equation are spinors with 8 components.

A main aspect in UR is that symmetries found in 6d propagate to 4d. Thus the SU(8) symmetry found here is a main argument to explain particle physics. But this we'll see later.

3.3 6d Lagrange density

Gauge theories are based on the idea that the Lagrangian has to be invariant under a local symmetry. This is to avoid physically not reasonable results of quantum mechanics demanding quantities being constant all over the universe. The Lagrangians of the Dirac or Klein-Gordon equation alone are not invariant. Introducing however an additional force field that by itself is also not invariant, compensating of the symmetry breaking terms of the two fields is possible. So the Lagrangian as a whole becomes invariant under the specific symmetry transformation.

The Standard Model of particle physics uses several Lagrangians with different symmetries. Their different boson fields generate the non-gravitational forces. Demanding invariance of the Lagrangian under U(1), SU(2) or SU(3) gives the electromagnetic, weak or strong force.

The Lagrangian of UR has an essential difference to those used in 4d. The demand of local isomorphism relates internal and external symmetries. So gauge invariance in UR means invariance of the Lagrangian under SU(4) symmetry and any ambiguity in choosing inner symmetries is eliminated.

As a template to formulate the 6d Lagrangian the Lagrangian of weak or strong force (as described e.g. in [12, 13]) can be used. According to the deduction of these 4d Lagrangians in which interaction between two resp. three fields is considered the 6d Lagrangian of UR with its SU(4) symmetry calls for introducing the interaction of four entities, each of them obeying the 6d Klein-Gordon or Dirac equation.

To formulate the Lagrangian of UR at first the Lagrangian $\mathcal{L}6_{D1}$ of a single 6d Dirac equation is introduced

$$\mathcal{L}6_{D1} = \phi^{+}\gamma^{\alpha}\partial_{\alpha}\phi. \tag{3}$$

Here ϕ^{+} is the adjoint spinor[†].

To generate a Lagrangian of the shape necessary for SU(4) symmetry four Lagrangians $\mathcal{L}6_{D1}$ are added to

$$\mathcal{L}6_{D4} = \phi_1^{+}\gamma^{\alpha}\partial_{\alpha}\phi_1 + \phi_2^{+}\gamma^{\alpha}\partial_{\alpha}\phi_2 + \ldots + \phi_4^{+}\gamma^{\alpha}\partial_{\alpha}\phi_4.$$

As the Dirac matrices in the Lagrangians introduced in 4d do not influence the unitary structures also the 6d Dirac matrices are assumed not to influence the SU(4) structure. Combining the four spinors ϕ_i to $\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T$ (T means transposed), defining

$$\gamma^{\alpha}\partial_{\alpha}\Phi = (\gamma^{\alpha}\partial_{\alpha}\phi_1, \gamma^{\alpha}\partial_{\alpha}\phi_2, \gamma^{\alpha}\partial_{\alpha}\phi_3, \gamma^{\alpha}\partial_{\alpha}\phi_4)^T$$

and

$$\Phi^{+}\Phi = \phi_1^{+}\phi_1 + \phi_2^{+}\phi_2 + \ldots + \phi_4^{+}\phi_4$$

the Lagrangian with four components can be written as

$$\mathcal{L}6_{D4} = \Phi^{+}\gamma^{\alpha}\partial_{\alpha}\Phi. \tag{4}$$

Neglecting the influence of spin simplifies the expression. Then a Lagrangian $\mathcal{L}6_{KG4}$ combining four Lagrangians $\mathcal{L}6_{KG1}$ of the single 6d Klein-Gordon equation

$$\mathcal{L}6_{KG1} = (\partial_{\alpha}\phi)^{+}(\partial^{\alpha}\phi) \tag{5}$$

can be constructed. Combining again four fields this time obeying the Klein-Gordon equation to $\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T$ it follows

$$\mathcal{L}6_{KG4} = (\partial_{\alpha}\Phi)^{+}(\partial^{\alpha}\Phi). \tag{6}$$

In analogy to the usage in 4d we call Φ a wave function although it describes neither waves nor particles but states.

That the so found Lagrangians are invariant under SU(4) transformations an appropriate boson field has to be introduced. Assuming for the interaction of the fields the in 4d usual minimal coupling and adding a term $\mathcal{L}6_B$ that allows describing boson fields also in absence of fermion fields we get

$$\mathcal{L}6_D = \Phi^{+}\gamma^{\alpha}(\partial_{\alpha} - igA_{\alpha})\Phi - \mathcal{L}_B \tag{7}$$

[†]In an adjoint matrix A^{+} compared to A columns and rows are interchanged and the elements are replaced by their conjugate complex ones. The product of a Dirac spinor ψ and its adjoint ψ^{+} in 4d however produces not a Lorentz invariant Lagrangian. To overcome this problem in 4d dimensions a modified adjoint Dirac spinor $\bar{\psi} = \psi^{+}\gamma^{0}$ is introduced with $\gamma^{0} = \begin{pmatrix} E2 & 0 \\ 0 & -E2 \end{pmatrix}$.

Table 1: Generators of SU(4)

$$\hat{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \hat{\lambda}_2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \hat{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\lambda}_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \hat{\lambda}_5 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \hat{\lambda}_6 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\lambda}_7 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \hat{\lambda}_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \hat{\lambda}_9 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\lambda}_{10} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \hat{\lambda}_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \hat{\lambda}_{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$\hat{\lambda}_{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \hat{\lambda}_{14} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \hat{\lambda}_{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

or

$$\mathcal{L}6_{KG} = ((\partial_\alpha - igA_\alpha)\,\Phi)^+ \,((\partial^\alpha - igA_\alpha)\,\Phi) - \mathcal{L}6_B. \tag{8}$$

g is a coupling factor defining the strength of interaction.
Following the comprehensive description of SU(4) in [14] we find: The A_α representing the boson fields are Hermitian 4x4-matrices that can be described by the 15 generators $\hat{\lambda}_k$ of SU(4) (as cited in table 1) and an according number of Yang-Mills fields $W_{\alpha k}$ as $A_\alpha = \sum_{k=1}^{15} \hat{\lambda}_k W_{\alpha k}$.

There are different possibilities to chose the $\hat{\lambda}_k$.

The free boson field part $\mathcal{L}6_B$ is given by

$$\mathcal{L}6_B = -\frac{1}{4}\sum_{i=1}^{15} W_{\alpha\beta i} W_i^{\alpha\beta} \quad \text{with} \tag{9}$$

$$W_{\alpha\beta i} = \partial_\alpha W_{\beta i} - \partial_\beta W_{\alpha i} + 2g \sum_{j,k=1}^{15} f_{ijk} W_\alpha^j W_\beta^k. \tag{10}$$

The f_{ijk} are the structure constants defined by $[\hat{\lambda}_i, \hat{\lambda}_j]_- = 2if_{ijk}\hat{\lambda}_k$ of SU(4). They are fully antisymmetric.

The factor 1/4 in equation (9) is convention.

Ordering the indices in ascending order there are 29 non-zero structure constants as cited in table 2.

3.3.1 Symmetry of Lagrangian and particle mass

Only Lagrangians with U(1) symmetry can be build on the basis of Klein-Gordon or Dirac equations with a mass term. Lagrangians describing interaction between at least two entities, i.e. with SU(2), SU(3) or SU(4) symmetry cannot have a mass term because it would destroy its symmetry. [15]

Table 2: Non-zero structure constants of SU(4)

i	j	k	f_{ijk}	i	j	k	f_{ijk}	i	j	k	f_{ijk}	i	j	k	f_{ijk}
1	2	3	1	1	4	7	1/2	1	5	6	-1/2	1	9	12	1/2
1	10	11	-1/2	2	4	6	1/2	2	5	7	1/2	2	9	11	1/2
2	10	12	1/2	3	4	5	1/2	3	6	7	-1/2	3	9	10	1/2
3	11	12	-1/2	4	5	8	$\sqrt{3/2}$	4	9	14	1/2	4	10	13	-1/2
5	9	13	1/2	5	10	14	1/2	6	7	8	$\sqrt{3}/2$	6	11	14	1/2
6	12	13	-1/2	7	11	13	1/2	7	12	14	1/2	8	9	10	$1/2\sqrt{3}$
8	11	12	$1/2\sqrt{3}$	8	13	14	$-1/\sqrt{3}$	9	10	15	$\sqrt{2/3}$	11	12	15	$\sqrt{2/3}$
13	14	15	$\sqrt{2/3}$												

4 How the 6d structures are perceived by a 4d observer

4.1 Time, the insurmountable hurdle

Enabling a connection of the timeless inaccessible world with our time-bound reality the emergence of an offshoot is necessary. Once and anywhere in the Euclidean 6d space something must have happend that evolved into a spacetime.

How this could have taken place will be described later. Here we limit ourselves to a postulate and some remarks in anticipation of the presentation in chapter 5 of part 4 of the series.

UR Postulate 3. There exists a 6d space with time, the assigned spacetime. Time herein can be understood as being defined by replacing one variable of the metric of the timeless 6d space by its imaginary. Our 4d spacetime is a submanifold of the assigned spacetime. From a cosmological standpoint it is the final stage of a development from a space with Euclidean to one with a pseudo-Euclidean metric, when the transit variables have settled to time and 5d spatial coordinates.

Introducing an imaginary coordinate here is not a mathematical trick as it is sometimes used in particle physics to better perform certain arithmetic operations. This is done only by practical reasons as then mathematics becomes easier. At the end of calculation time is reintroduced. With UR things are fully different. As our world is a subspace of the assigned spacetime for us the imaginary of the original coordinate as time is reality, whereas in the 6d Euclidean space the original coordinate stands for a (for us) unknown "reality".

How are the 6d Euclidean space and the assigned spacetime related?

Calling the coordinates in the Euclidean space $x6_e$ and in the assigned pseudo Euclidean $x6_p$ we get for $n = 2, 3...6$ that $x6_e^n = x6_p^n$ and $x6_e^1 = -i\tau$. Trajectories in the pseudo Euclidean space can be parameterized by τ as $x6_p^n = f^n(\tau)$. Viewed from the Euclidean space this means $x6_p^n = f^n(ix6_e^1)$. So the assigned spacetime is not a part of the Euclidean space but of its complex extension.

At first in a surrounding of a fixed point $P_0\left(x6_0^1, ...x6_0^6\right)$ in the 6d Euclidian space an early form of the assigned spacetime emerged with coordinates describing a continous transition from Euclidean to pseudo Euclidean metric.

What triggered the process is unclear.

Characteristic for the temporal content of the variable dimensions is its continuous growth. After the early assigned spacetime was created it grew continually and a bubble expanded into the complex extension of the timeless 6d space. This means that formation of the assigned spacetime is linked to formation of an entity becoming time.

The so introduced time offers a way out of the dilemma of background dependent and independent theories by shifting the decision into 6d spacetime. In the settled assigned spacetime a 6d Lorentz transformation holds. As there is no mass there is also no gravity.

The task will be to show that from these prerequisites in 4d gravity follows.

The postulated time is common in the whole assigned spacetime and in our universe. We will find it as „barycentric coordinate time“.

4.2 Transfer of symmetries

Symmetries of the 6d Euclidean space will have an influence on structures in the assigned spacetime and in our 4d world. How this works is given by

UR Postulate 4. Symmetries of the 6d Euclidean space are transferred to the assigned spacetime and to the 4d spacetime.

Transferred means that the symmetry of the Euclidean space is transmitted to the remaining spatial coordinates of the assigned spacetime and of the 4d spacetime. The unitary symmetries eventually are destroyed by the transition to 4d, but its subgroups and its constituents will be found.

In order to enable the transfer, coordinates cannot be chosen arbitrarily. They must be adapted to a given symmetry. Adapted stands for a bijective map (up to a set of measure zero that produces no effect in the action integral) relating the parameters used to define the coordinates and all points of the space. Specifying one coordinate gives a unique subspace.†

If the Euclidean space is invariant under translations there, in the assigned and in the 4d spacetime Cartesian coordinates are the adapted ones. The coordinate systems of the 6d Euclidean space and the assigned spacetime can be chosen linearly related $T = \tau,\ u = x6^2,\ v = x6^3,\ x^1 = x6^4,\ x^2 = x6^5$ and $x^3 = x6^6$. u and v are the two coordinates not occurring in 4d.

If the Euclidean space has at least locally another symmetry (e.g. invariance under spherical rotations) for the spatial part curvilinear coordinates adapted to the resulting five dimensional symmetry – that is the symmetry that holds for the subspace of the Euclidean space without the one coordinate that is replaced by the time-like one – must be used. A Cartesian type coordinate for time completes the metric.

How the transition works is explained in detail later for spherical symmetry.

The transition to 4d generates a new symmetry achieved by omitting the u, v-coordinates. The three remaining spatial coordinates are adapted to the original symmetry.

Specifying the method how the symmetry must be transferred, the symmetry of the 6d Euclidean space defines the coordinates in the assigned spacetime and finally the coordinates to use in 4d.

4.2.1 Emphasized coordinates

The coordinates being deducted by transposing the symmetry of the 6d Euclidean space to the assigned spacetime and from there to the 4d spacetime are emphasized. They generate a view on the 4d spacetime from the 6d Euclidean space perspective. As the coordinates in the Euclidean space are fixed also these emphasized coordinates are fixed. They do not depend on the situation in the 4d spacetime

The emphasized coordinates so overcome the problem of background dependent or independent coordinates in 4d physics. They can be used for both, particle and gravity physics.

The bubble of the 4d space expands but the emphasized coordinates give a fix network.

†If there is for example a spherical symmetry it is mandatory to use spherical coordinates $[r, \phi, \theta]$. This ensures that each point of the space is addressed unambiguously by the radius and two angles. If e.g. the radius is fix, the unique surface of the sphere is defined. Choosing instead Cartesian coordinates $[x, y, z]$ the sphere is given by $x^2 + y^2 + z^2 = R^2$. If x^2 is given six different cases must be distinguished to define a point.

The map of spherical coordinates for $\theta = 0$ is not bijective. But as this means just a one dimensional set, it does not influence the action integral.

4.3 Transfer of physical content

In a last step, we have to find out, how the equations of the assigned spacetime map into our world, how the relation between 6d and 4d physics is established.

The argumentation to derive this relation uses the action integral $S = \int (dx6)^6 \mathcal{L}6(x6^\alpha)$ (where $(dx6)^6$ is the volume element and $\mathcal{L}6$ the Lagrangian in 6d).

Hamilton's principle allows deducting all of 6d physics from this equation. Depicting this physics of the inaccessible world in the best possible way is goal of 4d physics. The procedure can be understood as the counterpart to the prisoner's realization of the rules of casting shadows in Plato's allegory. It is achieved by the best possible replication of the 6d by a 4d action integral over a proper 4d Lagrangian.

With $x6^\alpha = x6^\alpha(T, u, v, x)$ (x stands for x^1, x^2, x^3) and the Jacobi determinant $J(T, u, v, x)$ using Fubini's theorem the action integral S can be written as

$$S = \int dx^3 dT \int dudvJ(T, u, v, x)\mathcal{L}6(T, u, v, x).$$

In general the boundaries of the inner integral are depending on the variables of the outer one. But this is not the case using the assigned coordinates.

The splitting of the integral at first has no physical meaning but it provides a rule how to accomplish the transition from physics in the assigned to physics in the 4d spacetime. It turns out that the inner integral

$$\hat{\mathcal{L}4} = \int dudvJ(T, u, v, x)\mathcal{L}6(T, u, v, x). \tag{11}$$

can be related to a 4d Lagrangian. We call it "noninterpretable Lagrangian".

To demonstrate what "related" means action integral and Hamilton's principle are to be analyzed in a bit more detail.

1. Physics in the assigned spacetime is defined by the equations of motion deducted by variation of the action integrals based on the 6d Dirac or Klein-Gordon equation.
2. Solving an equation of motion and implementing its solution in the inner part of the action integral this is converted from a functional in a common integral over a function. The integral $\hat{\mathcal{L}4}$ becomes a function of the 6d coordinates T and x. We call it the "6d transfer function". Implementing the 6d transfer function in the 6d action integral gives its correct, extreme value.
3. In 4d the same procedure holds. We can set up a 4d Lagrangian and a 4d action integral, can solve the equation of motion and implement the solution in the 4d Lagrangian. The integrand of the 4d action integral then is given as a function depending on the four 4d coordinates. We call it "4d transfer function".
4. The connection between 6d and 4d is achieved by finding a 4d Lagrangian whose 4d transfer function formally equals the 6d transfer function. The value of the 6d action integral then is not changed if the 4d transfer function is implemented instead of the 6d transfer function. Introducing the 4d transfer function instead of the 6d one in the 6d action integral is allowed as their 4d variables replace the original 6d ones in an integral (dummy variables).
5. It remains to show that the 4d transfer function is defined by a 4d Lagrangian. As the 6d action integral is extreme for the full function solving the equation of motion this holds also for the function after the integration in the inner integral. This means that the outer integral can be understood as a 4d action integral over a 4d Lagrangian, as this is also extremal.

This derivation is also valid for symmetries other than translational when the adapted coordinates are used.

The correct 4d transferfunctions implemented in the 6d action integral gives the right result. This holds however only for the action integral in its form as an integral over functions it holds not for the functional action, as for a 4d observer a variation with regard to u and v is not possible.

We can only evaluate averages over the u, v area. This generates an irremovable deficit of information in 4d physics. But it is the best possible adaption.

Let us confirm the procedure.

UR Postulate 5. 4d physics describes 6d physics best, if the 4d transfer function formally equals the 6d transfer function.

With these five postulates defining properties of the inaccessible world and the procedures how a 4d observer can recognize them the basics for the derivation of the 4d physics are fixed. For cosmology one more postulate is necessary.

Aim in deducting 4d physics is finding the correct 4d transfer function. In general this means a nonlinear integral transform. Nonlinear because the 6d wave functions and Yang-Mills fields in $\hat{\mathcal{L}}4$ and the 4d wave functions and Yang-Mills fields in 4d Lagrangian $\mathcal{L}4$ occur each at least quadratic.

The transition destroys the perfect symmetries of the 6d Euclidian space with one force, no mass and no gravitation and generates the complicated structures of our universe.

5 Necessity of a 6d Euclidean space

It seems that neglecting the presented arguments leading to the SO(6)/ SU(4) symmetries and postulating instead equation (7) and (8) formulated in the assigned spacetime could also serve as a starting point for UR. But it is not only the transit between space and spacetime generating cosmology, there is also some evidence that the 6d Euclidean space is essential to understand 4d quantum physics.

This is to be illustrated with a simple example: The Klein-Gordon equation (1) is in the Euclidean space an elliptic Laplace equation. For its solution the maximum-minimum principle holds effecting that if ϕ is constant on the boundary of a domain it is also constant inside the domain.

In the assigned spacetime and in our 4d world the Klein-Gordon equation becomes a hyperbolic wave equation. Its solution consists of oscillating functions and looks completely different. Especially it is not constant inside a domain if it is constant on its boundary. A drum with the membrane clamped at the edge can serve as an example.

As the equation is real its solution e.g. in Cartesian coordinates should be expressible with two real trigonometric functions and two integration constants.

History of quantum physics shows that everything is done to transform the oscillating functions appearing as a solution in the assigned spacetime into the constant solution of the Euclidean space.

Using Euler's formula the two real trigonometric functions found as solutions can be converted in two complex exponential functions. Now the solution is the superposition of two plane waves $\phi = Ae^{i(\omega t+kx)} + Be^{-i(\omega t+kx)}$, ($A$, B amplitudes, ω frequency, k wave number). Using the correct (complex) values of A and B the expression still is real.

As $|e^{i(\omega t+kx)}| = 1$ for all x and t introducing the modulus of the wave function seems suitable to achieve the goal. Problem is that the complete solution needs two terms. To be successful one wave must be eliminated. This was achieved by introducing differential operators of first order for the spatial coordinates and demanding that the solution of the wave equation must also be a solution of these operators.

The commutativity of the second and first order operators usually used to argue this procedure however cannot justify omitting one of the two solutions of the Klein-Gordon equation. So it is an additional demand.

The so introduced operators, finally identified as describing momentum, introduce mass into 4d physics.

For the temporal coordinate this strategy is not possible because of the explicit direction of time. To avoid a negative time direction instead the differentiation between particles and antiparticles was introduced.

Taking just one wave function describing a particle or antiparticle moving to the left or the right mandatory generates a complex solution for the real differential equation but allows a domain with constant modulus.

This indicates that basic features of 4d physics result from the necessity of adapting it to the Euclidean space.

References

[1] Katrin Becker, Melanie Becker, John H. Schwarz. *String Theory and M-Theory A Modern Introduction*, Cambridge University Press (2007)

[2] C. Rovelli. *Zakopane lectures on loop gravity* (2011) or arXiv:1102.3660

[3] A. Strumia. *The fine-tuning price of the early LHC*, J. High Energy Phys. 04, p. 73 or arXiv:1101.2195 [hep-ph]

[4] Analysis of the XENON100 Dark Matter Search Data arxiv.org/pdf/1207.3458.pdf

[5] Solomon Feferman. *The nature and significance of Gödel's incompleteness theorems* https://math.stanford.edu/feferman/papers/Godel-IAS.pdf (2006)

[6] Stanley Jaki. *The Relevance of Physics* The University of Chicago Press, Cicago (1966)

[7] Dieter Hattrup. *Der Traum von der Weltformel oder Warum das Universum schweigt*, Herder Verlag Freiburg etc. (2006)

[8] Adam Becker. *What Is Real?*, Basic Books Group (2018)

[9] Wolfgang Pauli. *Physik und Erkenntnistheorie*, Vieweg, Braunschweig etc (1984)

[10] J. Kässer. *A new approach to a structure unifying forces and particles* Int. J. Mod. Phys. E 21, 1250064 (2012)

[11] D. H. Sattinger, O. L. Weaver. *Lie Groups and Algebras with Applications to Physics, Geometry and Mechanics*, Springer Verlag New York etc. (1986) p. 182 f

[12] Mark Srednicki. ibid. p. 416 f

[13] Eckhard Rebhan. *Theoretische Physik II*, ibid S. 934 f

[14] Walter Greiner, Berndt Müller. *Quantenmechanik Symmetrien*, Verlag Harry Deutsch, 4th print. (2005), p. 372f

[15] Eckhard Rebhan. *Theoretische Physik II*, Spektrum Akademischer Verlag Heidelberg etc. (2005) p.936