Generalized Newtonian Gravity and Dark Matter

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Abstract

We use dark matter as a mass complement to the central galactic mass in Newton's Universal Law of Gravity. The generalized Newtonian gravity thus obtained puts on theoretical footing many of the observations obtained only experimentally.

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Introduction

Dark matter is a hypothetical construct in theoretical cosmology. It is necessitated by experimental results like the radial velocities in outer regions of galaxies, and gravitational lensing measurements. While dark matter seems necessary to explain these observations, not much else is known about it. Its nature and origin are not known, and its special distribution and temporal evolution are not understood. Any information that is currently believed to be known about dark matter comes, for example, from attempts to understand galaxy formation using models such as the Λ Cold Dark Matter (ΛCDM). The Λ refers to the cosmological constant that Einstein had used in General Relativity, which he had subsequently withdrawn. However, later developments in cosmology, for example the accelerated expansion of the universe, necessitated an additional hypothesis requiring Dark Energy which could be explained in terms of Λ, and the parameter has since been reintroduced. Dark matter and dark energy are assumed to be separate hypothetical entities which are necessitated in theoretical cosmology by two very different phenomena: namely the galactic rotation curves and accelerated expansion of the universe. It is, therefore, assumed that dark matter and dark energy are separate entities, and they exist independently of each other, each with its own properties.

Further research in these two areas of cosmology, namely the dark matter and dark energy, help specify their properties, and the properties for the two are very different. It is understood that dark matter is clumped, has density profiles varying with the radial distance, and it acts as a glue for galaxy formation, anchoring Baryonic matter in it. On the other hand, dark energy is supposed to be a property of the vacuum, and it has a constant density with homogeneous and isotropic distribution. In the ΛCDM model, dark matter and dark energy are separate components of the cosmological constant, Λ.

Despite a large body of experiments, both ground based and space based, and a wealth of observational data, together with phenomenological data fitting techniques, the fact remains that the current understanding about dark matter and dark energy is scant; the two entities remain hypothetical in nature, and model dependent.

Therefore, there is need to seek new theoretical foundations. Research in this paper is in this spirit: it explores a new yet intuitive theory based on incorporating dark matter into Newtonian gravity and thus to generalize it. The theory is simple and intuitive, uses simple mathematics, and presents a theoretical framework for some highly significant facts in cosmology which we know from experimental observations, without a current theoretical basis for them. An example is Hubble's Law which we know observationally, yet we currently have no theoretical foundation for it. We will explain this and some other aspects of cosmology which are observationally studied but no theoretical bases for them currently exist.

There will be observations and theoretical considerations that this theory will struggle with. However, that is in the nature of new theories to impart impetus for further observational and theoretical studies.

Generalization of Newton's Gravity and Some Consequences

Newton's Law of Universal Gravity can be stated as in equation (1) below.

$$F(R) = \frac{GMm}{R^2} \qquad (1)$$

Where F represents the gravitational force exerted on a probe body of mass m by another body of mass M. Here G is a universal constant of gravity, and R is the distance between the two bodies.

This equation is valid if M and m are the only masses involved. This was believed to be so until recently when it was necessary to postulate the existence of dark matter which is a carrier of mass. It is not understood how dark matter is distributed in space. The current theories seem to assert that dark matter clumps, has distance dependent density profiles, acts like a glue in the formation of galaxies, is bound to the ordinary matter, and Baryonic matter is anchored in dark matter so that the dark matter is dragged along the Baryonic matter even when galaxies collide. These are some of the properties of dark matter inferred over time, though there is no existing theory that validates these properties. Our current understanding is largely a postulate and any properties are model dependent.

There is another hypothesized entity, namely dark energy, which is necessitated by the observations under Hubble's Law, that the universe is expanding and doing so in an accelerated manner such that the expansion speed is increasing with increasing distances in the inter galactic space. As is the case with dark matter, dark energy also is assumed to have certain properties, such as dark energy is associated with space or vacuum, its density is constant, and it is uniformly distributed in space, and it is isotropic.

These properties of dark matter and dark energy are common knowledge within the community, as is also the case with respect to the statements we made in the introduction, and we have, therefore, not sited references.

We now depart from the current understanding about the dark matter. In our ansatz, we attribute the same properties to dark matter as are assumed for dark energy. Thus, according to our ansatz, dark matter is a property of the space or vacuum, it has a constant density ρ_0, it is uniformly distributed in space, and it is isotropic.

With this redefinition we have retained no known property of dark matter except that it has a mass attribute that interacts gravitationally. Therefore, we have introduced a new entity which we will call “hamost”. Instead of talking in terms of dark matter, going forward we will use hamost in our further development. For example, instead of dark matter mass, we will talk about mass due to hamost.

Let us now get back to equation (1). In addition to the masses M and m, we now also have mass due to hamost. Let us denote this mass by $m_h(R)$ and we can write Newton’s Law of Gravitation as in equation (2) below.

$$F(R) = \frac{Gm}{R^2}[M + m_h(R)] \qquad (2)$$

How much is this mass due to hamost with mass density ρ_0? If the two bodies are separated by a distance R, then the mass due to hamost is that which is enclosed in a sphere of radius R.

$$m_h(R)=\int_0^R 4\pi R^2 \rho_0 dR \qquad (3)$$

Recall Newton’s Shell Theorem, according to which the gravitational force outside a spherical shell of total mass M is the same as if the total mass M is concentrated at the center of the spherical shell; and the gravitational force at a point within the spherical shell is identically zero. Mass due to hamost behaves as though the entirety of it were concentrated at the galactic center. Due to this effect, mass due to hamost may manifest as lumpy, even though there is no lumpiness in hamost.

The generalization of Newton’s Law of Gravitation as expressed in equation (2), effectively increases the mass M that was present at the galactic center by an amount $m_h(R)$ due to hamost. Newton's Shell Theorem states that the part of hamost that lies beyond the distance R contributes nothing to equation (3) for the probe body situated at distance R. This effective increase in mass depends upon R so that different galactic bodies at different radii R will see a different amount of effective mass at the center. In the solar system, the earth will experience a smaller solar mass than Jupiter will.

Combining equations (2) and (3) we get.

$$F(R) = Gm[\frac{M}{R^2} + \frac{4}{3}\pi R\rho_0] \qquad (4)$$

Hamost is assumed to be present everywhere in the universe with constant density ρ_0, uniformly distributed and isotropic.

Compared to Newton's Law of Gravitation, the second term in equation (4) adds an additional force coming from the mass due to hamost. Let us parenthetically note that hamost is not necessarily of particle origin; rather we have attributed it to be a property of space or vacuum.

This additional force increases linearly with R, and will asymptotically become dominant over cosmological distances. This additional force arises from the mass due to hamost. It increases linearly with increasing R, and it produces asymptotic acceleration and radial velocities, see equations (5 and 7), that increase linearly with increasing R. This behavior is Hubble's Law.

The asymptotic behavior is determined by the mass due to hamost, which was not included in the original Gravitation Law of Newton.

Consider a galactic scale void in the universe. This void is filled with mass due to hamost. If there is no Baryonic matter in this space, equation (4) holds with M=0. Since the Hubble behavior arises from the term representing mass due to hamost, Hubble's law continues to hold over large voids.

Mass due to hamost manifests through equation (4) with M=0, so that a large void is predicted to exert gravitational influence with mass proportional to the size of the void. This includes gravitational lensing effects. Because of Newton's Shell Theorem, this mass would behave as though concentrated at the center. Such an effect is experimentally observed in A520 study [1] that discovered a large void filled with mass due to hamost in the galaxy cluster center. These observations were initially made with Canada-France-Hawaii Telescope, and were further confirmed using Hubble Wide Field Planetary Camera 2. The observation discovered a giant hamost core where there is little Baryonic matter. While such a core is predicted by equation (4), the observation remains an unexplained challenge for the current theory which requires that galaxies should be anchored to mass due to hamost, even during the shock of a collision.

Such large voids exist elsewhere, and future experiments can test the theory in those situations.

The acceleration A(R) obtains by dividing the force F in equation (4) by mass m.

$$A(R) = G[\frac{M}{R^2} + \frac{4}{3}\pi R\rho_0] \qquad (5)$$

The second term arises from additional mass due to hamost, and it dominates asymptotically. That is why Newton's Law of Gravitation, as originally formulated, is inadequate for application over cosmological distances.

The force in equation (4) is generated by the following potential denoted by U(r).

$$U(R) = G[-\frac{M}{R} + \frac{2}{3}\pi R^2\rho_0] \qquad (6)$$

The second term with positive sign arises from the mass due to hamost.

The radial velocity V(R) obtains by integrating the acceleration in equation (5).

$$V^2(R) = G[-\frac{M}{R} + \frac{2}{3}\pi R^2\rho_0] \qquad (7)$$

Where we have used integration by parts, and have dropped higher order terms in G. We have also used chain rule to obtain $dt = \frac{dt}{dR}dR = \frac{1}{V(R)}dR$ and $\frac{dV(R)}{dR} = \frac{dV(R)}{dt}\frac{dt}{dR} = \frac{A(R)}{V(R)}$.

For large R, the first term in equation (7) is negligible, showing that radial velocity rises linearly with R. This behavior of radial velocity rising linearly with radial distance brings attention to Hubble's Law. As R increases, the mass due to hamost begins to dominate in equation (7). Asymptotically, this behavior of velocity increasing linearly with R persists because hamost is everywhere and governs the asymptotic behavior. This is a formal proof as to the validity of Hubble's Law. It can, however, deviate from the linear behavior because of higher order effects in G. This provides some theoretical guidance for the behavior of Hubble parameter.

The accelerated expansion of the universe is measured by Hubble's Law which infers from our theory. In current theories, the expansion of the universe is attributed to Dark Energy, the need for which is, therefore, obviated. In our theory, there is no dark matter, nor dark energy: there is only hamost.

Equation (7) holds for all locations. An observer in another galaxy will also experience Hubble's Law. This is because of uniformity and isotropy property of hamost.

Hubble parameter can be obtained from equation (7) as the coefficient of R, at asymptotic values of R.

$$H = \sqrt{(\tfrac{2}{3}\pi G\rho_0)} \qquad (8)$$

This can be compared with critical density ρ_c given by the expression

$$H^2 = \tfrac{8}{3}\pi G\rho_c \qquad (9)$$

Expressions (8) and (9) give the ratio $\frac{\rho_0}{\rho_c} = \frac{4}{1}$ which implies a closed universe. Even though Einstein's cosmological constant Λ seems to favor a flat universe, there is some observational evidence also for a closed universe [2].

Conclusions

We have used dark matter density, reformulated as hamost density, to generalize Newton's theory of universal gravitation by adding a term, representing the mass due to hamost, to the mass at the galactic center. This theory gradually deviates from Newton's theory at large distances. For small distances, the theory reduces to Newton's gravitation law.

The theory predicts velocities to rise linearly with distance as expressed in equation (7), which represents a theoretical basis for Hubble's Law. Hubble's Law measures the accelerated expansion of the universe, and our theory explains it, therefore the current explanation of this expansion in terms of dark energy is obviated. In our theory, dark matter and dark energy are unneeded: hamost is enough.

Dark matter density implied by equation (7) favors a closed universe.

The theory predicts that large chunks of hamost in empty voids will manifest as gravitational mass in gravitational experiments such as gravitational lensing.

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