One Theorem complementary to the Fundamental Theorem of Arithmetic.

Juan Elías Millas Vera.

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0- Abstract:

In this paper I want to show a complementary theorem of the Fundamental Theorem of Arithmetic. Using the delta notation (Δ) I was able to deduce a generic formula involving prime numbers and natural numbers.

1- Introduction:

Gauss Theorem of Arithmetic can be expressed in a simple way as follows: Every natural number can be expressed as product of p_i primes with powers in exactly one way:

$$(1)\quad n=\prod_{i=1}^{k}(p_i)^{(\alpha_i)}$$

For α_i belong to Naturals.
We will start here our work.

2- Delta notation tool:

As I shown previously in some papers [1]:

$$(2)\quad \underset{k}{\overset{i=1}{\Delta}} a_i=a_k\div a_{(k-1)}\div\ldots\div a_2\div a_1$$

This is the divisory notation or just Delta notation for operators.

3- Deduction of the formula:

First we start in the FTArith formula:

$$(3)\quad n=\prod_{i=1}^{k}(p_i)^{(\alpha_i)}$$

Now we use the delta notation in both sides:

$$(4)\quad \underset{k}{\overset{i=1}{\Delta}} n_i=\underset{k}{\overset{i=1}{\Delta}}\prod_{i=1}^{k}(p_i)^{(\alpha_i)}$$

We simplify:

$$(5) \quad \underset{k}{\overset{i=1}{\Delta}} n_i = (p_i)^{(\alpha_i)}$$

Because being opposite serial operators we can cancel one with the other,
We now correct powers:

$$(6) \quad \left(\underset{k}{\overset{i=1}{\Delta}} n_i\right)^{\left(\frac{1}{\alpha_i}\right)} = \left((p_i)^{(\alpha_i)}\right)^{\left(\frac{1}{\alpha_i}\right)}$$

Simplify again:

$$(7) \quad p_i = \sqrt[\alpha_i]{\underset{k}{\overset{i=1}{\Delta}} n_i}$$

And now we can state the Theorem:
Theorem: Every prime number can be expressed as α_i root of i-th natural numbers in serial division by other powers of prime numbers inside the root, in infinite different ways. For α_i and i belong to Naturals.

4- Some easy examples:

Being $p, q, r \in Primes$:

$$(8) \quad p = \sqrt[a]{p^a}$$

For $a \geqslant 2$, $p^a \in \mathbb{N}$ but $p^a \notin Primes$.

$$(9) \quad p = \sqrt[a]{p^a \cdot q^b \div q^b}$$

For $a \geqslant 2$, $b \geqslant 2$, $p^a, q^b \in \mathbb{N}$ but $p^a, q^b \notin Primes$.

$$(10) \quad p = \sqrt[a]{p^a \cdot q^b \cdot r^c \div q^b \div r^c}$$

For $a \geqslant 2$, $b \geqslant 2$, $p^a, q^b, r^c \in \mathbb{N}$ but $p^a, q^b, r^c \notin Primes$.

5- Bibliography:

[1] Millas Vera, Juan Elías. Review of the Serial Operators Theory (https://vixra.org/abs/2109.0029)