New Exact Solution to Einsteins Field Equation Gives a New Cosmological Model

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Abstract

Haug and Spavieri have recently presented a new exact solution to Einstein's field equations. In this paper, we will explore how this new metric could potentially lead to a new model for the cosmos. In the Friedman model, the cosmological constant $\Lambda = 3\left(\frac{H_0}{c}\right)^2\Omega_\Lambda$ must be introduced ad-hoc in Einstein's field equations or, alternatively, directly into the Friedmann equation. However, a similar constant automatically emerges in our cosmological model directly from Einstein's original 1916 field equations, which initially did not include a cosmological constant. We will analyze this, and it appears that the cosmological constant is little more than an adjustment for the equivalence of the mass-energy of the gravitational field, which is not taken into account in other exact solutions but is addressed in the Haug and Spavieri solution. Our approach seems to indicate that the Hubble sphere can be represented as a black hole, a possibility that has been suggested by multiple authors, but this is a quite different type of black-hole universe that seems to be more friendly than that of a Schwarzschild black-hole.

Key words: Mass-charge, general relativity, cosmological constant, Hubble sphere.

1 Introduction

Einstein [1] published in 1916 his field equation that is the foundation in general relativity and was given as

$$R_{\mu\nu} - Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{1}$$

Already in 1917 Einstein [2] added what today is known as the Cosmological constant to obtain his extended field equation

$$R_{\mu\nu} - Rg_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{2}$$

Einstein proposed that $\Lambda = \frac{2}{r^2}$, where r represents an unknown horizon radius, suggesting a limit to how gravity could be acting. He introduced the cosmological constant to maintain a steady-state universe, a prevailing consensus among leading physicists and cosmologists at that time. However, with Hubble's discovery of galaxies mostly exhibiting redshift and the subsequent interpretation of this phenomenon as indicative of an expanding universe, the Big Bang theory emerged. In response, Einstein had to discard the cosmological constant, and it is even reported that he referred to it as the biggest blunder of his time (see Gamow [3]).

In 1998, two teams of astrophysicists, one led by Saul Perlmutter [4] and another led by Brian Schmidt and Adam Riess [5], found that high-z supernovas could not fit the standard model at that time. The cosmological constant was reintroduced and is now a crucial part of the Λ-CDM model. The value of the cosmological constant is currently assumed to be $\Lambda = 3\left(\frac{H_0}{c}\right)^2\Omega_\Lambda = \frac{3}{r_H^2}\Omega_\Lambda$, where Ω_Λ is the ratio between the energy density due to the cosmological constant and the critical (Friedmann) density of the universe. Observations today indirectly predict omega to be approximately $\Omega \approx 0.6889$.

However, it's essential to bear in mind that much of today's cosmology is based on observations that are, once again, interpreted through a mathematical lens. This mathematical lens involves specific solutions to Einstein's field equation that have been studied for years and have achieved consensus through the development of a comprehensive framework around them. An integral part of today's cosmological model, known as Λ-CDM, involves the Friedmann [6] equations, where one of the central equations is given by:

$$\frac{8\pi\rho + \Lambda c^2}{3} = H_0^2 \tag{3}$$

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric and the Friedmann equations form the foundation of the standard Big Bang cosmological model, including the Λ-CDM model, which assumes the accelerating expansion of space, attributed to dark energy. It is worth mentioning that the Friedmann equation, even without the cosmological constant set equal to zero, can be derived from Newtonian gravity (see [7]) as well as from the Schwarzschild metric.

The cosmological constant Λ is somewhat inserted ad hoc into Einstein's field equation and then calibrated to observations, along with assumptions such as the existence of dark energy, despite no direct observation of dark energy. This perspective may still be valid, but it is also worthwhile to explore alternatives. With a recent exact solution to Einstein's field equation, it becomes natural to inquire about the cosmological model it leads to. This is a question we aim to address, at least to some extent, in this paper.

2 Cosmological model from the mass-charge metric

Recently, Haug and Spavieri [8] derived a novel exact solution to Einstein's field equations. Their solution belongs to the Weyl class of metrics, a category that encompasses well-known metrics such as the Schwarzschild metric, the Reissner-Nordstrom metric, as well as the Kerr and Kerr-Newman metrics. It is crucial to ascertain the potential new predictions that this metric solution may offer. We will show that the new metric may lead to a new cosmological model, where with our solution the cosmological constant emerges directly from Einstein's 1916 field equation.

The new HS metric is given by in S.I. units:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r} + \frac{G^2M^2}{c^4 r^2} + \frac{Q^2}{r^2} + \frac{P^2}{r^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{c^2 r} + \frac{G^2M^2}{c^4 r^2} + \frac{Q^2}{c^2 r^2} + \frac{P^2}{c^2 r^2}\right)^{-1}dr^2 + r^2\Omega^2 \tag{4}$$

We are interested in the case when the charge Q and magnetic moment P are set to 0 ($Q = P = 0$), or alternatively when they cancel each other out. Then, the metric simplifies to

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r} + \frac{G^2M^2}{c^4r^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{c^2 r} + \frac{G^2M^2}{c^4r^2}\right)^{-1}dr^2 + r^2\Omega^2 \quad (5)$$

We can find the singularity of the metric by setting the time component to zero:

$$\left(1 - \frac{2GM}{c^2 r} + \frac{G^2M^2}{c^4r^2}\right) = 0 \quad (6)$$

And solving for r, this gives $r = \frac{GM}{c^2}$, in other words, half the Schwarzschild radius. We will assume the Hubble radius to be equal to $R_H = \frac{c}{H_0} = \frac{GM_u}{c^2}$, where M_u is the equivalent mass of the observable universe (energy plus mass). This would indicate that the observable universe is inside some type of black hole, or at least that the Hubble sphere has properties similar to a black hole. This idea is not new; it goes back to at least Pathria [9] in 1972, and later others, see, for example, [10]. Whether the universe could be, or not, inside a black hole, is an ongoing topic actively discussed even these days [11–14]. Lineweaver and Patel [15] as late as 2023 are again raising the question of whether we could live inside a black hole. But, as they point out, if it occurs inside a Schwarzschild black hole, this does not seem to be possible. However, what about a Haug-Spavieri black hole? As we will soon see, such black holes seem to be more friendly for planets, stars, and thereby even life than a Schwarzschild black hole. Our black-hole universe can also potentially be an expanding black hole falling inside the $R_h = ct$ type of models that also are actively discussed to this day, but under different metrics and assumptions than based on our recent new exact solution to Einsteins field equation, see [16–22].

We will use the time component of our metric to derive what seems to be an equation that describes the Hubble sphere in a new and interesting way. Since we look at the Hubble sphere as a new form of "black hole" defined by the Haug and Spavieri metric, we are interested in the case where the time component is set equal to zero. From this, we get:

$$\begin{aligned}
1 - \frac{2GM}{rc^2} + \frac{G^2M^2}{c^4r^2} &= 0 \\
1 - \frac{2GM}{rc^2} + \frac{G^2M^2}{c^4\left(\frac{GM}{c^2}\right)^2} &= 0 \\
1 - \frac{2GM}{rc^2} + \frac{G^2M^2}{c^4\left(\frac{c^2}{H_0^2}\right)} &= 0 \\
\frac{2GM}{rc^2} - \frac{G^2M^2\frac{H_0^2}{c^2}}{c^4} &= 1 \\
\frac{8\pi GM}{3\frac{4}{3}\pi r^3c^2} - \frac{3\frac{H_0^2}{c^2}}{3} &= \frac{1}{\frac{G^2M^2}{c^4}} \\
\frac{8\pi\rho}{3} - \frac{3\frac{H_0^2}{c^2}c^2}{3} &= \frac{c^2}{\frac{G^2M^2}{c^4}} \\
\frac{8\pi\rho}{3} - \frac{3\frac{H_0^2}{c^2}c^2}{3} &= H_0^2
\end{aligned} \quad (7)$$

Next, we set $\Lambda = 3\frac{1}{r_H^2} = 3\left(\frac{H_0}{c}\right)^2$, which corresponds to today's Cosmological constant: $\Lambda = 3\frac{1}{r_H^2}\Omega_\Lambda = 3\left(\frac{H_0}{c}\right)^2\Omega_\Lambda$ with $\Omega_\Lambda = 1$. We get

$$\frac{8\pi\rho - \Lambda c^2}{3} = H_0^2 \tag{8}$$

This result is almost identical to the Friedmann equation of the universe, except that we did not need to insert ad hoc the Cosmological constant into Einstein's field equation. Furthermore, this model predicts a negative sign in front of Λ, while the Friedmann model is normally written with a + in front of it. A negative cosmological constant, however, cannot be excluded and is actively debated in a series of papers to this day [23–29]. The fact that the cosmological constant automatically appears in our model, without the need of inserting it ad hoc in the field equation, could potentially be of great importance for its cosmological interpretation. It could mean that the cosmological constant simply represents, or is linked to, the gravitational field energy of the mass inside the observable universe.

Our model also predicts twice the mass of the universe (equivalent mass, as we do not distinguish between energy and mass, just as in the Friedmann model). The mass of the universe in our model is obtained by solving equation 8 with respect to the mass embedded in its density, $\rho = \frac{M_u}{\frac{4}{3}\pi R_H^3} = \frac{3H_0^2}{4\pi G}$, which gives:

$$M_u = \frac{c^3}{GH_0} \tag{9}$$

while the Friedmann critical universe model mass is

$$M_c = \frac{c^3}{2GH_0} \tag{10}$$

This means the Friedmann critical universe model simply does not take into account the gravitational field energy of the mass in the observable universe. That is why the critical Friedmann mass is exactly half of what is predicted from the Haug and Spavieri metric. It could even be that the dark energy is simply gravitational field energy. This would mean the dark energy constitutes exactly 50% of the total energy in the observable universe. This is less than what predicted by the standard model but is basically identical to what is predicted in some alternative cosmological models, like the FSC model, as seen in [30].

3 Similarity with Pathria ad hoc modified Schwarzschild metric

Pathria [9] in his black hole universe model, somewhat modified the Schwarzschild metric to

$$ds^2 = -\left(1 - \frac{2GM}{rc^2} - \frac{1}{3}\Lambda r^2\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 + r^2\Omega^2 \tag{11}$$

Pathria was relying on ad hoc insertion of the Cosmological constant.

The cosmological constant is given by $\Lambda = 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda$ in the case $\Omega_\Lambda = 1$ then we have

$$\Lambda = 3\left(\frac{H_0}{c}\right)^2 = \frac{3}{r_H^2}$$

.

The Haug-Spavieri metric can be written as

$$ds^2 = -\left(1 - \frac{2GM}{rc^2} + \frac{G^2M^2}{c^4r^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2} + \frac{G^2M^2}{c^4r^2}\right)^{-1}dr^2 + r^2\Omega^2 \tag{12}$$

In the special case we are interested in using the metric on the universe mass we have $M = M_u$ and

$$ds^2 = \left(1 - \frac{2GM_u}{rc^2} + \frac{G^2M_u^2}{c^4r^2}\right)c^2dt^2 - \left(1 - \frac{2GM_u}{rc^2} + \frac{G^2M_u^2}{c^4r^2}\right)^{-1}dr^2 + r^2\Omega^2 \tag{13}$$

In addition when the Hubble radius is equal to the horizon radius we have $r = \frac{GM_u}{c^2} = r_H = \frac{c}{H_0}$, bear also in mind that the Cosmological constant is $\Lambda = 3\left(\frac{H_0}{c}\right)^2$ as derived from our metric, this means we can re-write our metric as

$$ds^2 = \left(1 - \frac{2GM_u}{r_Hc^2} + \frac{1}{3}\Lambda r_H^2\right)c^2dt^2 - \left(1 - \frac{2GM_u}{r_Hc^2} + \frac{1}{3}\Lambda r_H^2\right)^{-1}dr^2 \tag{14}$$

This can be seen as the extreme case of our metric when we are dealing with the observable universe having a horizon equal to the black-hole horizon, similar to what Pathria assumed in his paper. However, his metric was somewhat improvised, as it requires the insertion of the Cosmological constant ad hoc. In contrast, our cosmological constant, $\Lambda = 3\left(\frac{H_0}{c}\right)^2$, comes straight out from working with the limit of the metric in this new mass-charge metric. This should not necessarily be interpreted as us living inside a black hole Hubble sphere, but rather that there could be an information limit horizon determining how far signals and gravitational waves can travel based on the density of the universe; this could be the Hubble radius.

It is, however, important to be aware that our metric does not have a cosmological constant to start with. It is in the special case when our metric is applied to a black hole and analyzed from the event horizon or very close to the event horizon, that the radius r equals the event horizon, $r = r_h$. For any specific black hole, the event horizon is a constant, at least if the black hole is not growing, or radiating, over a relatively short time period. The Hubble sphere fits many of the mathematical properties of a black hole, as also pointed out by other authors. The Hubble sphere has a given radius equal to the Hubble radius when considered as a black hole. In other metrics like the Schwarzschild metric, there are no terms that resemble the cosmological constant in the metric itself. However, in our metric, when applied to a black hole, then $r = r_H$, and the term in the metric $G^2M_u^2/(c^2r_H^2)$ can be written as $\frac{G^2M^2}{c^2r_H^2} = \frac{3}{r_H^2}\frac{1}{3}r_H$, and the first part $\frac{3}{r_H^2} = 3\frac{H_0^2}{c^2}$ is then identical to the cosmological constant in the standard Friedmann model when $\Omega_\Lambda = 1$, just as shown in our metric written in the form 14. This means we get a new interpretation of the cosmological constant. Since other metrics miss the $\frac{G^2M^2}{c^2r^2}$ term, they must compensate for this by ad-hoc insertion of a cosmological constant in the field equation itself or alternatively later into the metric. In weak gravitational fields like for the Earth or the sun the term $\frac{G^2M^2}{c^2r^2}$ is insignificant for predictions compared to observations. In this case, as no cosmological constant is needed, the Schwarzschild metric is sufficient for describing the gravitational field of the Earth or the Sun. However, when it comes to black holes, this term is very significant, and if one use metrics where it is not included, one will need to adjust for it by other means in order to fit observation predictions. The insertion ad hoc of the cosmological constant directly in the field equation seems to be the alternative. However this should be carefully studied by multiple researchers over time to reach the optimal interpretation to be verified carefully with observations, etc.

Therefore, it is more correct to say that our metric gives a potential new interpretation of the cosmological constant, rather than say that it introduces a cosmological constant. Actually, each black hole will then have a different Λ, which is simply related to 3 divided by the black hole horizon radius squared. That other metrics, like the Schwarzschild metric, lack the term $\frac{G^2M^2}{c^2r^2}$ could be one of the reasons why one needs to insert ad hoc a similar term through a cosmological constant to get the model to work better. Then it is naturally no longer a Schwarzschild metric, but closer to our metric when dealing with a black hole.

There are multiple differences between this black hole universe and a Schwarzschild black hole universe or a Pathria black hole universe. In a Schwarzschild black hole, the escape velocity $v_e = \sqrt{\frac{2GM_c}{r}}$ is always above c inside the black hole. This indicates, as in a standard black hole, that all the mass will end up in the central singularity, resulting in an uninhabitable and very harsh universe. In the Haug-Spavieri metric (when $Q = P = 0$), we get an escape velocity of $v_e = \sqrt{\frac{2GM_u}{r} - \frac{G^2M_u^2}{c^2r^2}}$. Here, the escape velocity never goes above c, indicating that matter could be evenly distributed inside a Haug-Spavieri black hole. In other words, it is a livable black-hole Hubble sphere.

4 Maximum mass of spheres inside the Hubble sphere

The escape velocity in the Haug-Spavieri metric (when $Q = P = 0$) is given by

$$v_e = \sqrt{\frac{2GM}{r} - \frac{G^2M^2}{c^2r^2}} \tag{15}$$

If we now set $v_e = xc$ where x is simply a scaling factor of c and solve equation 15 for r then we get

$$r = \frac{GM}{x^2c^2}(1 - \sqrt{1 - x^2}) \tag{16}$$

Where $x > 1$, the radius will be imaginary. We interpret an imaginary radius of a gravitational object here as it does not describe a physical object. Therefore, the solution makes sense only when $x \leq 1$. This corresponds to cases where the escape velocity is always less than or equal to c, making sense. We can compare this to the Schwarzschild metric, where the escape velocity is given by $v_e = \sqrt{\frac{2GM}{r}}$. Setting up the equation $xc = \sqrt{\frac{2GM}{r}}$ and solving for r gives:

$$r = \frac{2GM}{x^2c^2} \tag{17}$$

Here, even for x higher than 1, the radius is real. This corresponds to a Schwarzschild black hole having an escape velocity above c inside the black hole. This has not been interpreted as anything moving faster than the speed of light, but it has implications for the fact that in a Schwarzschild black hole, all the mass can be packed inside the central singularity. This is not the case in the Haug-Spavieri black hole. In the Haug-Spavieri metric x can maximum take the value of 1 and this will constrain on the maximum mass (and Mass density) anywhere also inside the black hole, we must have (where $x \leq 1$):

$$v_e = c = \sqrt{\frac{2GM_i}{xr_h} - \frac{G^2M^2}{c^2x^2r_h^2}} \tag{18}$$

where r_h is the black-hole horizon, $r_H = \frac{GM_{BH}}{c^2}$, and M_i is the mass inside a sphere inside the black hole with a radius xr_h. Solved for M_i this gives

$$M_i = xM_{BH} \tag{19}$$

This simply means that for any sphere with radius xr_h, where $x \leq 1$ inside the Haug-Spavieri black hole (with the same center) then the mass can maximum be xM_{BH}. For the full black hole $x = 1$ and then $M_i = M_{BH}$. In the special case x is set so that $x = \frac{l_p}{R_H}$, that is for a Planck length radius sphere at the center of the Haug-Spavieri black hole then the mass inside this radius is

$$M_i = xM_u = \frac{l_p}{R_H}M_u = \frac{l_p}{\frac{c}{H_0}}\frac{c^3}{GH_0} = \frac{l_p c^2}{G} \tag{20}$$

and since the Planck [31, 32] length is given by $l_p = \sqrt{\frac{G\hbar}{c^3}}$ we can replace this into the equation above and we get

$$M_i = \frac{\sqrt{\frac{G\hbar}{c^3}}c^2}{G} = \sqrt{\frac{\hbar c}{G}} = m_p \tag{21}$$

It follows that in the central Planck volume of such a black hole, the mass can never be more than the Planck mass, and the maximum density is the Planck mass density, $\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$. This has several possible interpretations. First of all it seemingly resolving the challenge with a central singularity as it would then be impossible to pack all the mass of the whole black whole in the center as this would lead to imaginary radius. One interpretation would mean that the universe, if it started with a big bang, surprisingly began within a Planck mass black hole. This interpretation we think also is unlikely to represent reality. Alternatively, it could mean that the universe did not start in a singularity with zero spatial volume, but that the mass of the universe, $M_u = \frac{c^3}{GH_0}$, was initially packed into $\frac{M_u}{m_p}$ number of Planck mass particles with a radius of l_p. In other words that the universe mass was quantized into Planck mass micro black holes packed together. This would lead to the idea that just before the Big Bang, the whole universe was inside a sphere of about the volume of a proton; see [33]. This in our view seems more realistic than the universe started in a zero spatial volume singularity. A third alternative hypothesis for interpretation is that every point in the universe can be seen as a Planck mass particle in a state of quantum fluctuation with an information horizon equal to the Haug-Spavieri metric black hole horizon, $r_h = \frac{GM_u}{c^2}$.

5 Speculative suggestion on a possible solution to the vacuum catastrophe

So far our paper is based on pure derivations, but even then there could be opening for different interpretations. In this section we would like to offer a more speculative suggestion for a possible solution to the vacuum catastrophe. In addition The vacuum energy, based on observational estimates, is about 5.35×10^{-10} J/m^3, as reported by the Planck Collaboration [34]. On the other hand, quantum field theory predicts that the vacuum energy fluctuation density in the universe is essentially $\rho = \frac{m_p}{\frac{4}{3}\pi l_p^3} \approx 1.23 \times 10^{96}$ kg/m^3. This is more than 120 orders of magnitude higher than observed [35, 36]. The enormous discrepancy between observed and predicted values is known as the vacuum catastrophe, which is still considered an unsolved problem in cosmology. However, since the Hubble sphere fits a black hole solution, and energy dispersion is strongly related to entropy, tentatively, we can take the Planck mass density and simply divide it by the black hole entropy of the Hubble sphere. A good approximation even for our universe would be the Bekenstein-Hawking entropy (based on the Schwarzschild metric), given by

$$S_{BH} = \frac{k_B A}{4l_p^2} = \frac{k_B \pi R_H^2}{l_p^2} \tag{22}$$

where k_B is the Boltzmann constant. If we simply divide the Planck mass density (the quantum field vacuum field energy) by the entropy of the Hubble sphere, we get

$$\frac{\frac{m_p c^2}{\frac{4}{3}\pi l p^3}}{S_{BH}} = \frac{\frac{m_p c^2}{\frac{4}{3}\pi l p^3}}{\frac{k_B \pi R_H^2}{l_p^2}} \approx 3.45 \times 10^{13}\ k/m^3 \tag{23}$$

Based on a Hubble parameter given recently by Kelly et al. [37] of $_{H0} = 66.6^{+4.1}_{-3.3}$ we get a one STD of with a one standard deviation of of $3.12 \times 10^{13}\ k/m^3$ to $3.89 \times 10^{13}\ k/m^3$.

This, we must multiply by the Boltzmann constant to convert it into energy density, and then we obtain $4.77 \times 10^{-10}\ J/m^3$, which is very close to the estimate vacuum energy based on observations value of $5.35 \times 10^{-10}\ J/m^3$. There might be a small adjustment to our prediction here, as we have relied on the Schwarzschild solution by Hawking for the entropy, this likely need slight modification in our new metric. Even if this last part of our paper related to the vacuum catastrophe is more speculative than the rest of the paper, we think it is worth considering it.

6 Conclusion

We have presented a new cosmological model emerging from the Haug-Spavieri metric. There could potentially be multiple cosmological models and interpretations arising from this metric, but the one we describe, is at least one such model. Interestingly, this metric leads to the automatic derivation of a cosmological constant.

We obtain an equation very similar to the Friedmann model, but with the opposite sign of the cosmological constant. This cosmological constant takes a value of $\Lambda = \frac{3H^2}{c^2}$, corresponding to the cosmological constant in the standard model with $\Omega_\Lambda = 1$. However, this new cosmological model predicts twice as high energy density in the Hubble sphere as in the critical Friedmann solution; nevertheless, half of this energy is gravitational field energy. This could therefore potentially explain what the missing dark energy is.

Only ongoing research by multiple researchers over time can likely determine whether this new cosmological model could potentially make more sense than the Λ-CDM model, or need substantial improvements in order to fit observations.

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Declarations

Competing interests

The authors have no competing interests to report.

Availability of data and materials

The study contains no new data.