Observational Relativity

—— The Unity of Newton and Einstein

The Second Part:

Gravitationally Observational Relativity (GOR)

—— Spacetime is not Really Curved

Xiaogang Ruan [1*]

1 *Faculty of Information Technology, Beijing University of Technology, Beijing 100124, China.*
* E-mail: adrxg@bjut.edu.cn

Abstract: The Theory of Observational Relativity, the theory of OR for short, is a new discovery and a new theory, which has revealed the root and essence of relativity: All relativistic effects or relativistic phenomena are observational effects and apparent phenomena rather than the objective and true physical reality. In particular, the whole theoretical system of OR has generalized and unified Newton's mechanics and Einstein's theory of relativity, integrating such two great theories in physics into the identical theoretical system under the identical axiom system. The theory of OR is divided into two parts: the theory of inertially observational relativity (IOR); the theory of gravitationally observational relativity (GOR). The theory of GOR takes the three principles of GOR as its axiom system: (1) the principle of GOR equivalence; (2) The Principle of GOR covariance; (3) the principle of the invariance of information-wave speeds. Based on the three principles of GOR, by following or by analogizing the logic of Einstein's general relativity, the author has established the whole theoretical system of GOR, including the field equation of GOR and the motion equation of GOR. GOR's field equation has generalized and unified Einstein's field equation and Newton's field equation (i.e., the Poisson equation form of Newton's law of universal gravitation). GOR's motion equation has generalized and unified Einstein's motion equation and Newton's motion equation (i.e., the second law form of Newton's law of universal gravitation). The theory of GOR has proved an important theorem: **the theorem of Cartesian spacetime** which suggests that the objectively real spacetime could never be curved. So, spacetime is not really curved. Finally, the theoretical system of GOR has generalized and unified Newton's theory of universal gravitation and Einstein's theory of general relativity. It suggests that the theory of GOR is logically consistent not only with Einstein's theory of general relativity but also with Newton's theory of universal gravitation. Such logical consistency and strict correspondence show that the theory of GOR is logically self-consistent, and confirm the logical rationality and theoretical validity of the theory of GOR.

Key Words: general relativity, invariance of light speed, relativistic effects

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Introduction to GOR

The theory of **Observational Relativity** (OR), the theory of OR for short, consists two parts: **the 1st volume of OR: Inertially Observational Relativity** (IOR) generalizes Einstein's theory of special relativity and Newton's theory of inertial mechanics; **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR) generalizes Einstein's theory of general relativity and Newton's theory of universal gravitation.

Before the birth of Einstein's theory of special relativity, people believed in Galileo's doctrine [110] and Newton's laws [81]: space and time are independent of each other, inertial spacetime follows the Galilean transformation and inertial motion follows Newton's mechanics. After the birth of Einstein's theory of special relativity [7], people turned to believe in Einstein's theory: space and time are interdependent of each other, inertial spacetime follows the Lorentz transformation and inertial motion follows Einstein's relativity.

The theory of special relativity was established by Einstein in 1905 based on the hypothesis of the invariance of light speed, However, people do not understand the essence of the invariance of light speed and the relativistic phenomena of spacetime and matter motion in Einstein's theory of special relativity.

The 1st volume of OR: Inertially Observational Relativity (IOR) has established the theory of IOR, generalizing and unifying Galileo and Newton's inertial mechanics and Einstein's special relativity, revealing the root and essence of relativistic phenomena in inertial spacetime and inertial motion.

The theory of IOR provides us with an entirely new understanding of Einstein's theory of special relativity and inertial spacetime.

The theory of IOR tells us that the speed of light is not really invariant.

Now, **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR) will work to establish the theory of GOR, generalizing and unifying Newton's theory of universal gravitation and Einstein's theory of general relativity, revealing the root and essence of the relativistic phenomena of gravitational spacetime and gravitational interaction.

The theory of GOR will provide us with an entirely new understanding of Einstein's theory of general relativity and gravitational spacetime.

The theory of GOR will tell us that spacetime is not really curved.

Einstein's General Relativity and Spacetime Curvature [8]

Before the birth of Einstein's theory of general relativity, people believed in Newton's law of universal gravitation [81]:

All matter bodies in the universe gravitate towards each other due to the effect universal gravitation. It is the universal gravitation that controls the motion of the sun, the moon, and stars in the universe.

In 1915, after ten years of his theory of special relativity, Einstein established the theory of general relativity, extending his relativity theory from inertial spacetime to gravitational spacetime. Einstein's theory of general relativity achieved great success. Based on his theory of general relativity, Einstein made the three famous predictions: (1) gravitational redshift, (2) gravitational deflection, and (3) Mercury's anomalous precession, which were all verified and supported by observations and experiments.

Thus, people turn to believe in Einstein's theory of general relativity.

Both Einstein's theory of general relativity and Newton's theory of universal gravitation are the theories about gravitational interaction, which are the two greatest theoretical systems in the history of human physics.

However, Einstein's understanding of gravity is different from Newton's.

Newton believed that all matter bodies in the universe gravitated towards each other due to the effect universal gravitation, and the motion of all matter bodies in the universe followed the law of universal gravitation [81]. In this way, Newton could explain why the moon revolves around the earth and why the earth revolves around the sun. Einstein believed that matter and energy made spacetime curved, and the curved spacetime made matter bodies move. In this way, Einstein could seemingly also explain why the moon revolves around the earth and why the earth revolves around the sun.

In his theory of general relativity, Einstein geometrized gravitational spacetime and made it become a **curved spacetime**.

It is based on the idea of **spacetime curvature** that Einstein had established the whole theoretical system of his general relativity by means of **curved** Riemannian geometry [111] as the mathematical tool.

However, just as we cannot understand Einstein's invariance of light speed, nor can we understand Einstein's spacetime curvature. Actually, the so-called spacetime curvature is also a relativistic effect, which is the logical consequence of the invariance of light speed. People cannot understand the invariance of light speed as the logical premise, and naturally, cannot understand the spacetime curvature as the logical consequence.

You can feel the effect of the gravitational pull of the earth, but you cannot feel the curvature of spacetime, no matter the curvature of space or the curvature of time.

Anyway, you could not imagine how spacetime to be curved.

Time is just time, without geometric property, which does not matter whether flat or curved. The curved time is beyond human reason. Space is just space, and likewise, it does not matter whether flat or curved. In 3d space, you can construct any manifold, no matter flat or curved.

Nevertheless, the mainstream school of physics still believe in the theory of spacetime curvature, and take it to the extreme:

The accumulation of matter and energy caused spacetime curvature, and spacetime curvature caused the contraction of the universe. The contraction of the universe made matter and energy further accumulated, spacetime further curved, and the universe further contract. Such an evolving process eventually led the universe to contract into a point where spacetime was infinitely curved and the curvature of spacetime was infinite. Thus, with a tremendous crash (no one had heard), a new universe had been born in **the Big Bang**. So, there had been numerous galaxies, the milk way, the solar system, the earth, the moon, and then, our human beings.

According to Hubble, the universe is still expanding today [112].

This is regarded as the evidence of the Big Bang.

From the Principle of general correspondence to GOR [29,30]

The 2nd volume of OR: Gravitationally Observational Relativity (GOR) attempts to establish the theory of General Observational Relativity (GOR) or Gravitationally Observational Relativity (GOR), the theory of GOR for short, extending the theory of OR from the inertial spacetime S_I of IOR theory to the gravitational spacetime S_G of GOR theory, and at the same time, extending Einstein's theory of general relativity from the optical observation agent OA(c) to the general observation agent OA(η).

Einstein's theory of special relativity is the foundation of Einstein's theory of general relativity, and naturally, the logical premises and consequences of Einstein's special relativity, including the invariance of light speed, would be the logical premises of Einstein's general relativity.

Likewise, the theory of IOR is the foundation of the theory of GOR. Therefore, the logical premises and consequences of IOR theory, including the invariance of information-wave speeds, would be the logical premises of GOR theory.

The theory of OR, no matter from the theory of IOR to GOR or from Einstein's theory of general relativity to GOR, has a logical shortcut to follow: to analogize or follow the logic of Einstein's general relativity. In order to pave such a logical shortcut for the theory of GOR, the theory of OR builds a principle based on the Bohr correspondence principle and the principle of relativity: **the Principle of General Correspondence** (GC), the principle of GC for short.

In the history of physics, Bohr's Correspondence Principle has special status [70]. Bohr's correspondence principle elucidates the corresponding relationship between quantum physics and classical physics: if the Planck constant $h \to 0$, then quantum system isomorphically and uniformly converges to classical system. This suggests that quantum mechanics and classical mechanics are isomorphically consistent and therefore logically connected. More importantly, Bohr's correspondence principle has become the important ideological foundation and guiding principle for developing new theoretical systems or models in physics. It is based on his correspondence principle that Bohr developed his atomic structure model [75-77]. Moreover, it is Bohr's correspondence principle that led to the establishment of matrix mechanics [113-115] and promoted the development of quantum theory [116,117].

The theory of OR discovers that (see the theory of IOR and the theory of OR matter waves in the 1st volume of OR) [26,27]: The Planck constant h is actually a parameter of the optical observation system, that is, the energy-frequency ratio of the informons (photons) of the optical agent OA(c), representing the observational resolving-power of OA(c) or the observational uncertainty of OA(c).

Actually, different observation systems or different observation agents (OA(η)

($\eta\in(0,+\infty)$)) have different information-wave speeds ($\eta\in(0,+\infty)$) and different informon energy-frequency ratios ($h_\eta\in(0,+\infty)$)), and therefore, have different observational resolving-power: $\Delta x(\eta)\Delta p(\eta)\geq\hbar_\eta$ (see the principle of measuremental uncertainty in BP-08.3 of Chapter 9 in the 1st volume of OR).

In the theory OR matter waves, the energy-frequency ratio h_η of the informons of OA(η) is the general Planck constant h_η in the general Planck equation $E=h_\eta\eta^2$. With regard to the general Planck constant h_η, the theory of OR matter waves has an extremely important identity, so-called **the identity of general Planck constant** (GPC): $h_\eta\eta\equiv C$ ($C=hc$), the GPC identity for short. The GPC identity $h_\eta\eta=hc$ suggest that the Planck constant h is only a special case of the general Planck constant h_η: $h_\eta=h$ if and only if OA(η) is the optical agent OA(c) ($\eta\to c$).

It should be pointed out that the GPC identity $h_\eta\eta=hc$ is exactly the mathematical formalization of Bohr's correspondence principle: $\eta\to\infty$ as $h_\eta\to 0$, and then the system of quantum mechanics is isomorphically and uniformly transformed into the system of classical mechanics.

The principle of general correspondence reflects the internal connection between different theoretical systems in physics, and at the same time, reflects the internal connection between different observation systems in physics, endowing the Bohr correspondence principle with more universal significance.

According to the theory of OR and the principle of general correspondence, theoretically or logically, all theoretical systems of physics have the intrinsic corresponding relationship, have the common logical premises, and are based on the identical axiom system. Actually, Bohr's correspondence principle, the principle of special relativity or the Galilean invariance, even the principle of general covariance and the principle of equivalence, are all the principles about the corresponding relationships between different theoretical systems of physics.

The principle of general correspondence has generalized Bohr's correspondence principle and the principle of relativity (including the Galilean invariance and the principle of general covariance).

The establishment of the principle of general correspondence contributes to the development of new theories of physics and the unification of old theories of physics, as well as to the test of the logical consistency and self-consistency of the theoretical systems in physics. In particular, the principle of general correspondence has built the logical basis and paved the logical shortcut for the theory of GOR.

Therefore, based on the principle of general correspondence, by analogizing or following Einstein's logic of deducing general relativity from special relativity, the theory of OR could deduce the theory of GOR from the theory of IOR. In this way, the theory of OR could extend the theory of IOR from inertial spacetime to gravitational spacetime, at the same time, could extend Einstein's theory of general relativity from the optical agent OA(c) to the general observation agent OA(η).

Taking advantage of the principle of general correspondence, the whole theoretical system of GOR will be established.

The Theory of GOR [29,30]**:**

the Unification of Newton's Theory of Universal Gravitation and Einstein's Theory of General Relativity

The development history of physics is the history of continuous integration and unification of new and old theories. The integration and unification of new and old theories in physics is the reflection of the development and progress of physics.

The Final Theory of physics [118], or **the Grand Unified Theory** [119], or **the Theory of Everything** [120,121], is the eternal pursuit of physics.

However, our physics seems increasingly fragmented according to Hawking [31].

Newton's theory of universal gravitation [81] and Einstein's theory of general relativity [8] are the two greatest theoretical systems of human physics, the unification of which is undoubtedly of great significance.

In the view of the mainstream school of physics, Newton's theory of universal gravitation and Einstein's theory of general relativity are two independent or separate theoretical systems about gravitation or gravity, and even somewhat contradictory. Moreover, the mainstream school of physics insist that Einstein's theory of general relativity is a better theory of gravity, while Newton's theory of universal gravitation is only a transitional theory of gravity, and at most an approximation of Einstein's general relativity. The mainstream school of physics do not seem to believe that Newton's theory of universal gravitation and Einstein's theory of general relativity need or could be unified.

Actually, both Einstein's theory of general relativity and Newton's theory of universal gravitation are, in Hawking's words, the partial theories of GOR theory, belonging to different observation agents.

In **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR), with the help of the principle of general correspondence, **the theory of GOR** will be constructed on the basis of the invariance of information-wave speeds.

It should be pointed out that the theory of GOR is the gravitational theory of the general observation agent OA(η).

Newton's theory of universal gravitation and Einstein's theory of general relativity are two special cases of GOR theory: Newton's theory of universal gravitation is the gravitational theory of the idealized observation agent OA_∞ ($\eta\to\infty$); Einstein's theory of general relativity is the gravitational theory of the optical observation agent OA(c) ($\eta\to c$).

So, the theory of GOR generalizes and unifies Newton's theory of universal gravitation and Einstein's theory of general relativity.

In the sense of the principle of general correspondence, the theory of GOR has the strict corresponding relationships with both Newton's theory of universal gravitation and Einstein's theory of general relativity. Under the optical agent OA(c): $\eta\to c$ and the GOR gravitational-field equation is strictly reduced to Einstein's gravitational-field equation, and the GOR motion equation (i.e., the GOR geodesic equation) is strictly reduced to Einstein's motion equation (i.e., Einstein's geodesic equation); under the idealized agent OA_∞: $\eta\to\infty$ and the GOR gravitational-field equation is strictly reduced to Newton's gravitational-field equation (i.e., the

Poisson equation form of Newton's universal-gravitation law), and the GOR motion equation is strictly reduced to Newton's motion equation (i.e., the Newton second-law form of Newton's universal-gravitation law).

Such strict corresponding relationships indicate that the theory of GOR is logically consistent not only with Einstein's theory of general relativity but also with Newton's theory of universal gravitation; and moreover, such strict corresponding relationships, from one aspect, confirm the logical self-consistency and theoretical validity of the theory of GOR.

The Theory of GOR [29,30]:
New Theory leads to New Ideas and New Insights

The theory of GOR discovers that, like all the inertial relativistic effects or phenomena in Einstein's theory of special relativity and even that in the theory of IOR, all the gravitational relativistic effects or phenomena in Einstein's theory of general relativity and even that in the theory of GOR are not the objectively physical reality, but are, in essence, the observational effects or apparent phenomena, rooted from the observational locality ($\eta<\infty$) of the realistic observation agents OA(η).

Before the theory of IOR, with regard to Einstein's theory of special relativity [7], we could not understand why the speed of light looks invariant. The theory of IOR has told us that the speed of light is not really invariant. Likewise, before the theory of GOR, with regard to Einstein's theory of general relativity [8], we could not understand why gravitational spacetime looks curved. The theory of GOR will tell us that spacetime is not really curved: the objective and real spacetime, no matter the inertial or the gravitational, never gets curved.

Actually, no matter in the theory of special or general relativity, no matter in the theory of inertial or gravitational relativity, all the relativistic effects or phenomena are observational effects and apparent phenomena rooted from the observational locality ($\eta<\infty$) of the realistic observation agent OA(η) ($\eta\in(0,+\infty)$).

Analogous to the theory of IOR, the theory of GOR will clarify that:

(i) Einstein's theory of general relativity is the theory of the optical observation agent OA(c) ($\eta=c$), in which the gravitational spacetime is the optical image of the objectively physical world presented to us by the optical agent OA(c), not completely objective or real;

(ii) Newton's theory of universal gravitation is the theory of the idealized observation agent OA_∞, in which the gravitational spacetime is the true portrayal of the objectively physical world under the idealized agent OA_∞.

In particular, it is not because Einstein's theory of general relativity is better than Newton's theory of universal gravitation but simply because our observations and experiments mostly rely on the optical observation agent OA(c) that Einstein's theory of general relativity are supported by most observations and experiments.,

So, the theory of GOR discovers that: Newton's theory of universal gravitation is the right theory about gravitational interaction, while Einstein's theory of general relativity is only an approximate theory.

The theory of GOR will provide us with new insight into the gravitational world.

The theory of IOR has already clarified that all matter particles, including photons, have the intrinsic or rest mass of their own. Based on the theory of GOR gravitational redshift, the theory of GOR will make the theoretical prediction of the rest mass, give the theoretical value of the intrinsic mass of photons. Naturally, photons of with different frequencies or energies have different rest masses.

The theory of GOR will reexamine Einstein's three famous scientific predictions, including the gravitational redshift of light and the gravitational deflection of light, as well as the anomalous precession of planetary orbits. The theory of GOR discovers that: the objective and real gravitational world supports Newton rather than Einstein. Actually, Einstein's theoretical predictions based on his theory of general relativity are only observational effects and apparent phenomena presented to us by light and optical observation.

The theory of GOR will reexamine Einstein's predictions of the so-called **Gravitational Waves**. The theory of GOR discovers that Einstein's gravitational wave is actually the information wave of the optical observation agent OA(c).

The theory of GOR will reexamine the theory of **Black Hole** that is based on Einstein's theory of general relativity, and reexamine the theory of **Big Bang** of the universe that is rooted from Hubble's doctrine of **Cosmic Expansion** and based on Einstein's theory of general relativity.

Did the universe really experience the big bang 13.8 billion years ago?

Human beings need to reacquaint the objective world from the perspective of the theory of OR, and reshape human being's view of nature.

10 Preliminary for GOR

The theory of GOR, so-called **Gravitationally Observational Relativity** (GOR), attempts to extend the theory of IOR, so-called **Inertially Observational Relativity** (IOR), from inertial spacetime to gravitational spacetime, and meanwhile, to extend Einstein's theory of general relativity from the optical observation agent OA(c) to the general observation agent OA(η). So, the theory of GOR could be interpreted as the theory of **General Observational Relativity** (GOR).

The basic concepts and definitions of IOR theory, as well as the basic logical premises and consequences of IOR theory, will become the logical foundation of GOR theory. The fundamental problem of Einstein's general relativity will become the fundamental problem of GOR theory.

The theory of OR, so-called **Observational Relativity** (OR), in **the 1st volume of OR: Inertially Observational Relativity**, has already clarified: "Human cognition or understanding of the objective world not only depends on observation, but also is restricted by observation." All theoretical systems or spacetime models of physics, including Newton's theory of universal gravitation and Einstein's general relativity, have without exception been branded with the marks of observation.

The spacetimes in all the theoretical systems or spacetime models of physics are the observational spacetimes, rather than the objective spacetime.

The observational spacetime of IOR theory is inertial spacetime, and can be called **the IOR inertial spacetime**. **The 1st volume of OR: Inertially Observational Relativity** has defined the concept of **Observation Agent** (OA) for the theory of IOR, endowing observation with the definite role and status in observational inertial-spacetime.

The observational spacetime of GOR theory is gravitational spacetime, and can be called **the GOR gravitational spacetime**. **The 2nd volume of OR: Gravitationally Observational Relativity** will extend the concept of **Observation Agent** (OA) from the IOR inertial spacetime to the GOR gravitational spacetime, endowing observation with the definite role and status in observational gravitational-spacetime, revealing the linkage of Newton's theory of universal gravitation and Einstein's theory of general relativity to their specific observation systems. Finally, the theory of GOR will be established to clarify the root and essence of gravitational relativistic effects or phenomena.

The theory of GOR, as the gravitational theory of the general observation agent OA(η) ($\eta \in (0,+\infty)$), will generalize and unify Newton's theory of universal gravitation and Einstein's theory of general relativity.

10.1 The Concepts and Principles from IOR to GOR

Naturally, the theory of IOR is the foundation of the theory of GOR.

In **the 2nd volume of OR: Gravitationally Observational Relativity**, the theory of GOR follows the basic ideas, principles, and concepts of IOR theory in **the**

1st volume of OR: Inertially Observational Relativity. Actually, these basic ideas, principles, and concepts are also the basic ideas, principles, and concepts of OR theory including IOR and GOR).

In particular, the logical consequences of IOR theory might become the logical starting point of GOR theory.

The following terms in the theory of IOR, including the concepts, definitions, principles, theorems, and their corollaries, remain valid in the theory of GOR and might become the logical foundation of GOR theory.

10.1.1 Terms

The terms (see Sec. 1.1.3 **Related Terminology**) in **the 1st volume of OR: Inertially Observational Relativity** is the basic concept of OR or IOR theory, related to **Observation**. Those terms or concepts will continue to be employed in **the 2nd volume of OR: Gravitationally Observational Relativity**.

S/N	Terms	Simple Interpretations
01	Observation Agent: OA(η)	observation system
02	Information Wave	matter waves for transmitting information
03	Informon	matter particles composing information waves
04	Information-Wave Speed: η	the speed of OA(η) transmitting information
05	Idealized Agent: OA_∞	the observation agent OA(η) as $\eta\to\infty$
06	Optical Agent: OA(c)	the observation agent OA(η) as $\eta\to c$
07	Spacetime Information	space and time information of observed objects
08	Free Spacetime: S_F	with no matter interactions in it
09	Intrinsic Spacetime: $X^{4d}{}_\infty$	objective and real spacetime
10	Observed Spacetime: $X^{4d}(\eta)$	observational spacetime observed by OA(η)
11	Intrinsic Quantity: U_o=U_∞	objective and real physical quantity
12	Observed Quantity: $U(\eta)$	observational quantity observed by OA(η)
13	Observational Locality: $\eta<\infty$	OA(η) needs time to transmit information.

10.1.2 Definitions

The theory of GOR will follow or extend the following definitions in **the 1st volume of OR: Inertially Observational Relativity**.

S/N	Definitions	Simple Interpretations
Def. 1.1	Observation Agent: OA(η)	extended from the IOR inertial spacetime to the GOR gravitational spacetime
Def. 1.1	Observed Spacetime: $X^{4d}(\eta)$	1d time: $x^0=\eta t$; and 3d sapce: (x^1,x^2,x^3)
Def. 1.2	Physical Quantity	distinguishing between U_o and $U(\eta)$
Def. 1.2	Intrinsic Quantity: U_o=U_∞	U_o defined in the free spacetime S_F or U_∞ defined in the idealized spacetime $X^{4d}{}_\infty$

Def. 1.2	Observed Quantity: $U(\eta)$	defined in the observational spacetime $X^{4d}(\eta)$
Def. 2.1	The Cosmic Speed: Λ	the speed as matter-wave frequency $f_\eta \to \infty$
Def. 2.2	Time	distinguishing between τ and $t(\eta)$
Def. 2.2	Intrinsic Time: $\tau = t_\infty$	objective and real time, defined in the free spacetime S_F or idealized spacetime $X^{4d}{}_\infty$
Def. 2.2	Observed Time: $t(\eta)$	defined in the observational spacetime $X^{4d}(\eta)$
Def. 2.3	Standard Clock: T_o	at rest in the free spacetime S_F or at rest in the idealized spacetime $X^{4d}{}_\infty$

10.1.3 Principles and Theorems

The theory of GOR will follow the following principles and theorems as well as their corollaries in **the 1st volume of OR: Inertially Observational Relativity**.

Principles and Theorems	Simple Interpretations
Physical Observability	All physical quantities are observable: the observed values are definite and finite.
Corol. 2.1 of the Cosmic Speed Λ	The Λ is the ultimate speed of the universe and could not be exceeded in observation.
Corol. 2.2 of the Cosmic Speed Λ	The Λ is invariant or the same relative all inertial observers.
The Invariance of TFR	The ratio of the observed time dt to the observed frequency f is an invariant: $\mathrm{d}t/f = \mathrm{d}\tau/f_o$.
The Invariance of IWS	The speed η of OA(η) is invariant or the same relative to all inertial observers.
Corol. 3.1 of the Invariance of IWS	The cosmic speed Λ is actually the information-wave speed η of OA(η).
Corol. 3.2 of the Invariance of IWS	the information-wave speed η of OA(η) could not be exceeded in inertial observation.
Corol. 3.3 of the Invariance of IWS	The invariance of light speed holds true only if OA(η) is the optical agent OA(c).

where, TFR is the abbreviation for **Time-Frequency Ratio**, and IWS is the abbreviation for **Information-Wave Speeds**.

The invariance of time-frequency ratio will play a special role in exploring the problem of gravitational redshift. In particular, in deducing the theory of GOR, the theorem of the invariance of information-wave speeds will be accorded the title of principle, that is, **the principle of the invariance of information-wave speeds**, and being employed as the logical starting point of the theory of GOR.

10.2 The Scene of GOR Gravitational Spacetime

In order to relate and analyze the theory of GOR, to deduce the GOR model of gravitational spacetime, the theory of OR needs to first define the gravitational spacetime in GOR theory and set the scene of GOR gravitational spacetime.

Naturally, the most familiar scene is the gravitational scene set by Newton's in the law of universal gravitation [81]: the two-body system (M,m) in celestial two-body problem, where M and m are two matter particles (mass points), representing two celestial bodies, for example, the sun and the earth or the earth and the moon. In Newton's law of universal gravitation $F=GMm/r^2$, M and m are equal: M gravitates to m, and the gravitational force is $F=F_{Mm}$; m gravitates to M, and the gravitational force is $F=F_{mM}$. According to Newton's third law of motion, F_{Mm} and F_{mM} are action and reaction, equal in size and opposite in direction.

In Einstein's theory of general relativity, the gravitational scenes set by both Einstein [8] and Schwarzschild [80] are similar to the gravitational scene in Newton's law of universal gravitation, still the celestial two-body system (M,m). However, the difference is that, in Einstein's theory of general relativity, as depicted in Fig. 10.1, the large celestial body M makes its surrounding spacetime curved, and the curved spacetime makes small celestial body m circle the large M.

In Einstein's theory of general relativity, the effect of universal gravitation has been geometrized by the doctrine of spacetime curvature.

Actually, gravitational spacetime is not real curved.

But anyway, as a mathematical model, Einstein's doctrine of spacetime curvature could yet be mathematically regarded as a formalized method for the effect of universal gravitation.

The basic logic of GOR theory lies in the principle of general correspondence: by analogizing or following the logic of Einstein's deducing the theory of general relativity, the theory of OR could deduce the theory of GOR, and the scene of GOR gravitational spacetime could be set as depicted in Fig. 10.1.

The GOR Gravitational Scene: Let (M,m) be the two-body system in celestial two-body problem, M the large celestial body whose mass is distributed in spherical symmetry and forms the spherically symmetric gravitational-spacetime with the center at M, m the small celestial body which is idealized as a mass point and circles the large celestial body M in the gravitational spacetime of M.

The scene of GOR gravitational spacetime depicted in Fig. 10.1 will be applied to the deduction of the theory of GOR, including the determination of the space and time of GOR gravitational spacetime, the derivation and calibration of the GOR gravitational-field equation and the GOR motion equation, and the interpretation of Einstein's famous scientific predictions.

As depicted in Fig. 10.1, we agree that if no special instructions:

(i) P stands for the observed object, in the celestial two-body system (M,m), the small celestial body m is the observed object P.

(ii) $O(T,X,Y,Z)$ is the coordinate system of GOR gravitational spacetime, and O is the mass center of the large celestial body M.

(iii) $O_o(T_o,X_o,Y_o,Z_o)$ is the intrinsic coordinate system of P, belonging to the free spacetime S_F, in which P is at rest, O_o not only is the coordinate origin but also represents the intrinsic observer of P, and according to Def. 1.2, the observational (observed) time t_o is exactly the standard time or the intrinsic time (proper time) τ.

(iv) O_P is the observer of P at its coordinate point in the GOR gravitational spacetime $O(T,X,Y,Z)$, whose observational (observed) time is $t_P=t(r)$, depending on the coordinate position of P (or the small celestial body m) in the GOR gravitational spacetime $O(T,X,Y,Z)$.

(v) In the GOR gravitational spacetime $O(T,X,Y,Z)$, different observers O_A and O_B may have different spacetime metrics at the coordinate points A and B: $g_{\mu\nu}(r_A)\neq g_{\mu\nu}(r_B)\ (r_A\neq r_B)$;

(vi) Unless otherwise specified, the theory of GOR adopts Einstein's summation convention: English letters $(i,j,k,\ldots)$ for the indices of 3d space (x^1,x^2,x^3) with the summation range of {1,2,3}; Greek letters $(\lambda,\mu,\nu,\ldots)$ for the indices of 4d spacetime (x^0,x^1,x^2,x^3) with the summation range of {0,1,2,3}.

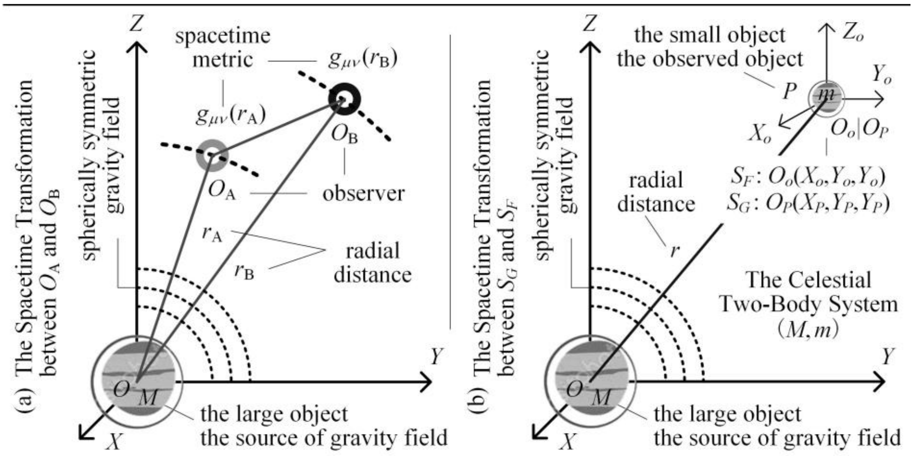

Figure 10.1 The Scene of GOR Gravitational Spacetime: the Celestial Two-Body System (*M,m*). (a) The Spacetime Transformation between the Observers O_A and O_B: the points A and B of the gravitational spacetime S_G have different spacetime metrics $g_{\mu\nu}(r_A)$ and $g_{\mu\nu}(r_B)$, and therefore, the observational spacetimes of O_A and O_B are different, which need to be transformed. (b) The Spacetime Transformation between the gravitational spacetimes S_G and the Free Spacetime S_F: according to Def. 1.2, the physical quantities of the observed object P at rest in free spacetime S_F are the objective and real physical quantities of P, but the observed physical quantities of P observed by observers in the gravitational spacetime S_G are different from the objective and real physical quantities of P, which need to be transformed.

The gravitational spacetime in the theory of GOR, like that in Einstein's theory of general relativity, is also **curved**. Therefore, as depicted in Fig. 10.1 ((a) and (b)), there are two types of spacetime transformations in the GOR gravitational spacetime $O(T,X,Y,Z)$.

(a) The Spacetime Transformation between O_A and O_B

As depicted in Fig. 10.1(a), like in the gravitational spacetime of Einstein's theory of general relativity, the points A and B in the gravitational spacetime

$O(T,X,Y,Z)$ of the theory of GOR may have different spacetime metrics $g_{\mu\nu}(r_A)$ and $g_{\mu\nu}(r_B)$. Therefore, the observers O_A at A and O_B at B may have different observational spacetimes, including different observed times t_A and t_B, which may need to be transformed between O_A and O_B.

(b) The Spacetime Transformation between S_G and S_F

As depicted in Fig. 10.1(b), the observed object P or the small celestial body m originally has its intrinsic spacetime O_o, belonging to the free spacetime S_F. Suppose that P and its intrinsic observer O_o are relatively stationary in the free spacetime S_F. According to Def. 1.2, the observed time t_o of O_o is the proper time τ. However, when P or m is in the GOR gravitational spacetime S_G, the intrinsic observer O_o is turned into the gravitational observer O_P, and the observed time t_P of O_P is just the observational time that may not necessarily be the objective and real time τ. In order to get the proper time τ, it is necessary to transform the observational time t_P of O_P into the intrinsic time τ of O_o (that was called **the standard time** by Einstein).

As a matter of fact, as long as let O_A be O_P and $r_B \to \infty$, then Fig. 10.1(a) and Fig. 10.1(b) would be equivalent or the same.

10.3 Observation Agents for GOR

Definition 1.1 of Chapter 1 in **the 1st volume of OR: Inertially Observational Relativity** defines a new concept: **Observation Agent**, which can be regarded as the core concept of OR theory, and is the connotation of the concept of **Observation** in the theory of **observational relativity** (OR).

The general observation agent OA(η) in Def. 1.1 is the coordinate framework of 4d spacetime (see Eq. (1.2)), which is actually the generalization of Minkowski 4d spacetime (see Eq. (1.1)).

The theory of IOR extends the coordinate framework of Minkowski 4d spacetime (Eq. (1.1)) from the optical agent OA(c) to the general observation agent OA(η) (Eq. (1.2)). Now, the theory of GOR attempts to extend the observation agent of IOR theory from inertial spacetime to gravitational spacetime.

10.3.1 The Coordinate Framework of 4d Spacetime in Einstein's Theory of General Relativity

Minkowski spacetime [50,51], or the coordinate framework of Minkowski 4d spacetime, is a formalized tool developed by Minkowski specifically for Einstein's theory of special relativity. Minkowski spacetime is a 4d differentiable manifold with the spacetime metric $g_{\mu\nu}$ (μ,ν=0,1,2,3) and the spacetime line-element ds, and therefore is a metric spacetime. Naturally, as the formalized tool of Einstein's theory of special relativity, Minkowski spacetime $X^{4d}(c)$ in Eq. (1.1) is inertial spacetime with the metric $g_{\mu\nu}=\eta_{\mu\nu}=\text{diag}(+1,-1,-1,-1)$, known as **Minkowski metric**, which is a constant metric. So, Minkowski spacetime is not curved but flat.

At first, Einstein did not believe that the coordinate framework of Minkowski 4d spacetime was necessary or significant for his theory of relativity. However, after embarking on the construction of general relativity, Einstein gradually realized that his theory of general relativity seemed to have to be built on the coordinate framework of 4d spacetime created by Minkowski. Thus, he extended Minkowski spacetime from the inertial spacetime of special relativity to the gravitational spacetime of general relativity, and highly praised Minkowski spacetime.

Thus, the coordinate framework of Minkowski 4d spacetime in Eq. (1.1) of Chapter 1 needs redefining for gravitational spacetime and writing as follows:

$$\mathrm{OA}(c) \triangleq \left\{ \begin{array}{l} X^{4\mathrm{d}}(c): \left\{ \begin{array}{l} x^0 = ct; \\ x^1 = x,\ x^2 = y,\ x^3 = z \end{array} \right\} \\ \mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^{\mu} \mathrm{d}x^{\nu} \quad \left(g_{\mu\nu} = g_{\mu\nu}\left(x^{\alpha}, c\right)\right) \end{array} \right\} \tag{10.1}$$

where OA(c) still represents the optical observation agent that employs light or electromagnetic interaction as the observation medium, $X^{4\mathrm{d}}(c)$ still represents the observational spacetime of OA(c) (x^0 the 1d time and (x^1,x^2,x^3) the 3d space that can be represented by the Cartesian coordinates (x,y,z)), $g_{\mu\nu}$ is the spacetime metric of $X^{4\mathrm{d}}(c)$, and ds is the spacetime line-element of $X^{4\mathrm{d}}(c)$.

Unlike the OA(c) in Eq. (1.1) of Chapter 1, the observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c) in Eq. (10.1) is not inertial spacetime but gravitational spacetime. Actually, Eq. (10.1) generalizes Eq. (1.1): in Einstein's theory of special relativity, the metric $g_{\mu\nu}$ of the observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c) is the Minkowski metric $\eta_{\mu\nu}$=diag(+1,–1,–1,–1), independent of the spacetime coordinates of $X^{4\mathrm{d}}(c)$, and is a special case of Eq. (10.1); in Einstein's theory of general relativity, the metric $g_{\mu\nu}=g_{\mu\nu}(x^{\alpha},c)$ of observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c) depends on the spacetime coordinates x^{α} (α=0,1,2,3) of $X^{4\mathrm{d}}(c)$.

So, the observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c) in Einstein's theory of general relativity looks or appears to be somewhat **curved**.

In Einstein's theory of general relativity, the metric of the observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c) not only depends on the spacetime coordinate x^{α} (α=0,1,2,3) of $X^{4\mathrm{d}}(c)$, but also is related to the speed c of light: $g_{\mu\nu}=g_{\mu\nu}(x^{\alpha},c)$. Actually, the coordinate framework of 4d spacetime in Einstein's theory of general relativity (Eq. (10.1)) is still the optical observation system.

10.3.2 The Gravitational Observation Agent in GOR

In his theory of general relativity, Einstein extended the coordinate framework of Minkowski 4d spacetime from inertial spacetime $X^{4\mathrm{d}}(c)$ (Eq. (1.1): $g_{\mu\nu}=\eta_{\mu\nu}$) to gravitational spacetime $X^{4\mathrm{d}}(c)$ (Eq. (10.1): $g_{\mu\nu}=g_{\mu\nu}(x^{\alpha},c)$).

Now, the theory of GOR needs to extend the general observation agent OA(η) from the inertial spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η) (Eq. (1.2): $g_{\mu\nu}=\eta_{\mu\nu}$) in the theory of IOR to the gravitational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η) (Eq. (10.2): $g_{\mu\nu}=g_{\mu\nu}(x^{\alpha},\eta)$) in the theory of GOR, and meanwhile, extend the optical observation agent OA(c) (Eq. (10.1)) in Einstein's theory of general relativity to the general observation agent OA(η) (Eq. (10.2)) in the theory of GOR.

As stated repeatedly by the theory of OR, in theory, all the forms of matter motion could be employed as observation media to transmit observed information for observers [26-30]. Different observation media mean different observation systems or different observation agents: the eye is a type of observation agent, employing light as the observation medium; the ear is another type of observation agent, employing sound as the observation medium. Human perception or observation of the objective world including gravitational spacetime needs to take advantage of different types of observation media or different types of observation agents.

Analogous to the coordinate framework OA(c) (Eq. (10.1)) of Minkowski 4d spacetime in Einstein's theory of general relativity, by substituting the information-wave speed η for the speed light c in Eq. (10.1), the general observation agent OA(η) in the theory of GOR can be defined as follows.

Definition 10.1 (Observation Agent): An observation system employing a specific observation medium to transmit observed information for observers is referred to as an observation agent and denoted as OA(η), which in the gravitational spacetime of GOR is defined a metric spacetime as

$$\mathrm{OA}(\eta) \triangleq \left\{ \begin{array}{l} X^{4\mathrm{d}}(\eta): \left\{ \begin{array}{l} x^0 = \eta t; \\ x^1 = x,\, x^2 = y,\, x^3 = z \end{array} \right\} \\ \mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu \quad \left(g_{\mu\nu} = g_{\mu\nu}\left(x^\alpha, \eta\right)\right) \end{array} \right\} \tag{10.2}$$

where the observation medium of OA(η) can be any form of matter motion or any matter wave, η is the information-wave speed of OA(η), i.e., the speed of the observation medium transmitting observed information; $X^{4\mathrm{d}}(\eta)$ represents the 4d gravitational spacetime observed by OA(η), x^0 is the 1d time, and (x^1,x^2,x^3) is the 3d space that can adopt the Cartesian coordinate (x,y,z); ds is the spacetime line-element of $X^{4\mathrm{d}}(\eta)$, $g_{\mu\nu}=g_{\mu\nu}(x^\alpha,\eta)$ is the spacetime metric of $X^{4\mathrm{d}}(\eta)$, depending on the spacetime coordinate x^α (α=0,1,2,3) of $X^{4\mathrm{d}}(\eta)$.

Unlike the OA(η) in Eq. (1.2) of Chapter 1, the observational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η) in Eq. (10.2) is gravitational spacetime not inertial spacetime. Actually, the general observation agent OA(η) defined in Eq. (10.2) has the more general meaning, which generalizes the inertial observation agent defined in Eq. (1.2): in the theory of IOR, the spacetime metric $g_{\mu\nu}$ of the observational spacetime $X^{4\mathrm{d}}(\eta)$ is the Minkowski metric $\eta_{\mu\nu}$=diag(+1,–1,–1,–1), does not depend on the spacetime coordinate of $X^{4\mathrm{d}}(\eta)$ and can be regarded as a special case of Eq. (10.2); in the theory of GOR, the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(x^\alpha,\eta)$ of the observational spacetime $X^{4\mathrm{d}}(\eta)$ depends on the spacetime coordinate x^α (α=0,1,2,3) of $X^{4\mathrm{d}}(\eta)$. Therefore, similar to the optical observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c) in Einstein's theory of general relativity, the general observational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η) in the theory of GOR also looks or appears to be somewhat **curved**.

However, according to Eq. (10.2) in Def. 10.1 of GOR theory, the so-called spacetime curvature is not the objectively physical reality.

As a matter of fact, in the theory of GOR, the metric $g_{\mu\nu}=g_{\mu\nu}(x^\alpha,\eta)$ of the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the observation agent OA(η) depends not only on the spacetime coordinate x^α (α=0,1,2,3) of $X^{4d}(\eta)$, but also on the information-wave speed η of OA(η): the observational spacetimes of different observation agents would be curved to different degrees and have different curvatures. This suggests that the so-called **spacetime curvature** relies on the observation agent OA(η) or on the information-wave speed η of OA(η), in other words, depends on observation.

So, no matter in the theory of GOR or in Einstein's theory of general relativity, the so-called **spacetime curvature** is only a sort of observational effect or apparent phenomenon, rather than the objective physical reality.

The theorem of Cartesian spacetime in the theory of GOR will prove that: $g_{\mu\nu}(\eta)\rightarrow\eta_{\mu\nu}$ as $\eta\rightarrow\infty$. So, the objectively real spacetime would never be curved.

Obviously, the general observation agent OA(η) defined in Def. 10.1 generalizes the coordinate framework OA(c) of Minkowski 4d spacetime extended for Einstein's theory of general relativity: if $\eta\rightarrow c$, then the general agent OA(η) in Eq. (10.2) is exactly the optical agent OA(c) in Eq. (10.1).

Just as Einstein's theory of general relativity could be built on the 4d spacetime coordinate-framework of the optical observation agent OA(c) defined in Eq. (10.1), the theory of GOR, or the theory of **Gravitationally Observational Relativity** (GOR), could be built on the 4d spacetime coordinate-framework of the general observation agent OA(η) defined in Eq. (10.2)

According to the principles of physical observability, regardless of the inertial spacetime $X^{4d}(\eta)$ ($g_{\mu\nu}=\eta_{\mu\nu}$) or the gravitational spacetime $X^{4d}(\eta)$ ($g_{\mu\nu}=g_{\mu\nu}(x^\alpha,\eta)$), any realistic observation agent OA(η) has the observational locality ($\eta<\infty$).

In essence, all relativistic effects, including gravitational relativistic effects, are rooted from the observational locality of observation agents.

11 GOR Logic and the Principle of General Correspondence

In **the 1st volume of OR: Inertially Observational Relativity**, based on the more basic axiom system than that of Einstein's special relativity, starting from the most basic logical premise (the definition of time in Def. 2.2) [26,27], the theory of OR has derived the general Lorentz transformation, not only generalized and unified the Galilean transformation and the Lorentz transformation, but also revealed the corresponding relationship of isomorphic consistency between different theoretical systems in physics, linking the principle of correspondence with the principle of relativity, endowing Bohr's correspondence principle with more general and universal significance.

Based on the general Lorentz transformation, the theory of IOR, i.e., the theory of **Inertially Observational Relativity** (IOR), has been established.

In the theory of IOR, even Newton's inertial mechanics and Einstein's special relativity are also isomorphically consistent. So, Newton's theory of inertial mechanics and Einstein's theory of special relativity have been generalized and unified by the theory of IOR into the same theoretical system under the same axiom system. The theory of IOR reflects the intrinsic corresponding relationship between the theoretical systems of different observation agents.

It is based on the corresponding relationship of isomorphic consistency between different theoretical systems in physics that this chapter will establish a principle: **the Principle of General Correspondence**, which would provide a logical bridge or a logical shortcut for the theory of GOR.

The principle of general correspondence will play an important role in the logical deduction of GOR theory.

11.1 Bohr's Correspondence Principle

In 1920, Bohr officially established the principle of correspondence [71], which later became known as **the Bohr correspondence principle**.

However, the basic thought of Bohr's correspondence principle could be traced back to the establishment of Bohr's atomic theory and Bohr's atomic model in 1913 [75-77]. It was Based on the basic thought of the principle of correspondence, Bohr established his theory and model of hydrogen atom.

Actually, the basic thought of the principle of correspondence could also be traced back to the establishment of Planck's blackbody radiation law or blackbody radiation formula in 1900 [14].

The Basic Thought of Bohr's Correspondence Principle: There must be some intrinsic linkage or corresponding relationship between quantum mechanics and classical mechanics. Under certain conditions, the two theories could be transformed into each other.

There are various interpretations for Bohr's correspondence principle, in which the most common are two limit expressions:

(i) the limit correspondence of Bohr's quantum number (energy level): $n \to \infty$;

(ii) the limit correspondence of Planck's constant: $h \to 0$.

11.1.1 The Limit Correspondence of Bohr's Quantum Number: $n \to \infty$

The Principle of Bohr's Correspondence (in the form of Bohr' quantum number): Let n be the atomic energy level (the principal quantum number). As $n \to \infty$, quantum models converge to classical models, and quantum physical quantities converge to classical physical quantities, that is,

$$\lim_{n \to \infty} \{\text{Quantum Quantity}\} = \{\text{Classical Quantity}\} \tag{11.1}$$

The quantization of energy is the watershed between quantum physics and classical physics: (i) the continuity of energy represents classical physics; (ii) the discretization of energy represents quantum physics. Bohr believed that, in the case of large quantum numbers, the energies of electrons and atoms tend to be continuous. So, quantum physical models must converge to classical physical models as $n \to \infty$.

On the one hand, based on classical physics, suppose that electrons move in a circle around the atomic nucleus, then according to Newton's laws and Coulomb's law, one could derive the relational formula between the speed v and orbital radius r of an electron:

$$\frac{KZe^2}{r^2} = \frac{m_e v^2}{r} \quad \text{or} \quad v = \sqrt{\frac{KZe^2}{m_e r}} \tag{11.2}$$

where is $K=1/4\pi\varepsilon_0$ is Coulomb's constant, ε_0 the vacuum permittivity, Z the atomic number, and m_e the electron mass.

Taking $T_C=2\pi r/v$ as the period of electron orbit, the frequency f_C of electromagnetic (EM) radiation in the classical case can be calculated as follows:

$$f_C = \frac{1}{T_C} = \frac{v}{2\pi r} = \frac{1}{2\pi r}\sqrt{\frac{KZe^2}{m_e r}} \tag{11.3}$$

where f_C is the classical frequency of EM radiation.

Equation (11.3) suggests that, according to the calculation of classical physics, the EM radiation frequency of atoms could take continuous values.

On the other hand, experiments show that the EM radiation frequency of atoms can only take some discrete values. In 1889, Rydberg established the following empirical formula for the spectral lines of hydrogen atom based on experiments:

$$\frac{1}{\lambda_Q} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \quad \begin{pmatrix} n' = 1, 2, 3, \cdots; \\ n = n'+1, n'+2, n'+3, \cdots \end{pmatrix} \tag{11.4}$$

where λ_Q was the wavelength of the spectral lines, and R was the Rydberg constant that needed to be determined by experiment.

In 1911, Rutherford proposed a planetary model of atomic structure, in which electrons revolve around the atomic nucleus, just like the planets revolving around the sun. Rutherford's model has serious defect: according to the classical theory of electromagnetic radiation, the electrons in Rutherford's model would ceaselessly lose energy due to the energy radiation of electromagnetic waves, and eventually collapse to the nucleus. Rutherford could not explain the problem.

In 1913, Bohr established the Bohr model of atomic structure on the basis of the Rutherford model, in which Bohr set up three important postulates [75-77].

(i) The Condition of Stationary State

An atom could only exist stably in discrete energy states (or energy levels), that is, the so-called **stationary state**; while an electron in the atom could only move in a circle around the nucleus at a specific energy level n with the discrete energy E_n:

$$E_n = -\frac{Rhc}{n^2} Z^2 \quad (n = 1, 2, 3, \cdots) \tag{11.5}$$

Equation (11.5) implies that an electron in the atom can orbit stably at a specific energy level with specific energy.

(ii) The Condition of Quantum Transition.

An electron in the atom can transfer between different energy levels by radiating energy or absorbing energy. As the electron transfer from the high-energy level n to the low-energy level n', it would release a photon with certain energy ΔE and frequency f_Q; on the contrary, as the electron absorbs a photon with certain energy ΔE and frequency f_Q, it would transfer from the low-energy level n' to the high-energy level n. The energy ΔE and the frequency f_Q of the released photon or the absorbed photon satisfy

$$\Delta E = E_n - E_{n'} = hf_Q \quad \text{or} \quad f_Q = \frac{\Delta E}{h} = \frac{E_n - E_{n'}}{h} \tag{11.6}$$

Let $n'=n-1$ and $n>>1$ (i.e., n is a large quantum number). Then, according to the condition of stationary state (Eq. (11.5)) and the condition of quantum transition (Eq. (11.6)), we have that

$$\begin{cases} \Delta E = RhcZ^2 \left(\dfrac{1}{(n-1)^2} - \dfrac{1}{n^2} \right) = 2RhcZ^2 \dfrac{1}{n^3} \\ f_Q = \dfrac{\Delta E}{h} = 2RcZ^2 \dfrac{1}{n^3} \end{cases} \tag{11.7}$$

where f_Q is the quantum frequency of EM radiation.

Equation (11.7) suggests that, as $n \to \infty$, $\Delta E \to 0$: the energy of an electron in the atom would tend to be continuous between adjacent energy levels.

In such a case, according to the basic thought of Bohr's correspondence principle, the quantum physical quantities would converge to the corresponding classical physical quantities: $f_Q=f_C$, $r_n=r$, and $v_n=v$. In the case of a hydrogen atom (Z=1), from Eqs. (11.2-3) and Eq. (11.7), we have the orbital radius r_n and the speed

v_n of the electron in the energy level n:

$$v_n = \sqrt{\frac{Ke^2}{m_e r_n}} \quad \text{and} \quad r_n = \sqrt[3]{\frac{Ke^2}{4^2\pi^2R^2c^2m_e}}n^2 \tag{11.8}$$

(iii) The Quantization of Angular Momentum.

In order to finally derive the theoretical model of atomic structure, Bohr made the quantization of the angular momentum L of electrons:

$$L = m_e v_n r_n = n\frac{h}{2\pi} = n\hbar \quad \left(n = 1, 2, 3, \cdots; \hbar = \frac{h}{2\pi}\right) \tag{11.9}$$

From the orbital speed $v_n=\sqrt{(Ke^2/m_e r_n)}$ of the electron in the energy level n (Eq. (11.8)) and the quantization of angular momentum (Eq. (11.9)), Bohr could deduce another expression of the orbital radius r_n:

$$m_e v_n r_n = \sqrt{Ke^2 m_e r_n} = \frac{nh}{2\pi} \quad \text{or} \quad r_n = \frac{n^2h^2}{4\pi^2Ke^2m_e} \tag{11.10}$$

By the contrast of Eq. (11.10) and Eq. (11.8), Bohr obtained the theoretical value of the Rydberg constant R:

$$R = \frac{2\pi^2K^2e^4m_e}{h^3c} = \frac{m_e e^4}{8\varepsilon_0^2h^3c} \quad \left(K = \frac{1}{4\pi\varepsilon_0}\right) \tag{11.11}$$

Now, the Rydberg formula is no longer an empirical formula, but a theoretical model on the basis of the Bohr theory of atomic structure; the Rydberg constant R is no longer an empirical value, but a theoretical one from the Bohr model of atomic structure. It is thus clear that Bohr's correspondence principle played the important role in the establishment of the Bohr theory and model of atomic structure.

However, it is worth noting that Eq. (11.7) is only a case of the large quantum numbers; $f_Q=f_C$ is not the limit correspondence of $n\to\infty$. Actually, there is no the corresponding relationship of isomorphic consistency between the classical frequency f_C in Eq. (11.3) and the quantum frequency f_Q in Eq. (11.7): f_Q cannot isomorphically and uniformly be transformed into f_C through $n\to\infty$.

As Shomar pointed out [31], the case of large quantum numbers or $n\to\infty$ do not always represent the classical physical systems. This seemingly implies that large quantum numbers $n>>1$or even the limit correspondence of quantum number $n\to\infty$ lacks of generality and universal significance.

11.1.2 The Limit Correspondence of Planck's Constant: $h\to 0$

The Principle of Bohr's Correspondence (in the form of Planck' constant): As Planck's constant $h\to 0$, quantum models converge to classical models, and quantum physical quantities converge to classical physical quantities, that is,

$$\lim_{h\to 0}\{\text{Quantum Quantity}\} = \{\text{Classical Quantity}\} \tag{11.12}$$

The quantization of energy originates from Planck equation: $E=hf$; $h\neq0$ implies the discretization of energy and represents quantum physics. Naturally, if $h=0$ then the energy E tends to be continuous. However, it is worth noting that $h=0$ and $n\to\infty$ may not have the same or equivalent effect. In the view of Shomar [122], the Planck constant $h\to0$ must represent classical physics, but the Bohr quantum number $n\to\infty$ may not represent classical physics.

In 1900, Planck theoretically derived the Planck law of blackbody radiation from his quantum hypothesis $E=hf$ [14]. Planck's formula of blackbody radiation

$$u(f,T)=\frac{8\pi hf^3}{c^3}\frac{1}{e^{hf/kT}-1} \tag{11.13}$$

is extremely consistent with the experiment of blackbody radiation, and can be regarded as the integration of Rayleigh-Jeans law [12,13] and Wien approximation [11]. More importantly, the quantum hypothesis $E=hf$ as the prerequisite of Planck's law of blackbody radiation marks the birth of quantum physics [15,16].

Planck realized that there existed the corresponding relationship between his law of blackbody radiation and Rayleigh-Jeans law: as $h\to0$, the Planck formula of quantum blackbody radiation (Eq. (11.13)) converges to (or is isomorphically and uniformly transformed into) the Rayleigh-Jeans formula of classical blackbody radiation:

$$\lim_{h\to0}u(f,T)=\lim_{h\to0}\frac{8\pi hf^3}{c^3}\frac{1}{e^{hf/kT}-1}=\frac{8\pi kTf^2}{c^3} \tag{11.14}$$

where k is the Boltzmann constant, T the temperature, and c the speed of light.

Equation (11.14) suggests that the Planck formula of blackbody radiation and the Rayleigh-Jeans formula of blackbody radiation have the same form and structure, in other words, have the corresponding relationship of isomorphic consistency, and are isomorphically consistent.

Planck formalized such a corresponding relationship of isomorphic consistency as follows (see Shomar's literature [122]):

$$\lim_{h\to0}\{\text{Quantum Physics}\}=\{\text{Classical Physics}\} \tag{11.15}$$

Equation (11.15) suggests that, unlike the limit correspondence of Bohr's quantum number $n\to\infty$, the limit correspondence of Planck' constant $h\to0$ is not only reflected in the level of physical quantities, but also in the level of physical concepts and the laws of physics. The corresponding relationship between different theoretical systems in physics lies not only in the convergence of physical quantities and the consistency of results, but also in the convergence of form or structure, and the consistency of concepts or logic.

11.2 The Principle of Relativity and the Principle of Correspondence

As we all know, in addition to the hypothesis of the invariance of light speed, Einstein's theory of special relativity also has an important logical prerequisite: the principle of relativity.

The principle of relativity is one involving the symmetry property of spacetime [123]: relativity is a specific form of symmetry.

The Basic thought of Relativity Principle: Spacetime is symmetrical, so all observers are equal or have equal rights, that is, no one is more superior than another, a law of physics must be the same or have the same form in all reference frames.

The principle of relativity was first explicitly stated by Galileo, i.e., the so-called **Galilean invariance** [110,124]: all the inertial observers of classical mechanics are equal or have equal rights, a law of classical mechanics must be the same or have the same form in all inertial reference frames.

In Einstein's theory of special relativity [7], the Galilean invariance was extended by Einstein from classical mechanics to general physics, that is, so-called **the special principle of relativity**. Later, in Einstein's theory of general relativity [8], **the special principle of relativity** was extended by Einstein from inertial reference frames to general reference frames, that is, so-called **the general principle of relativity** or **the principle of general covariance**.

Thus, the principle of relativity has gained more general significance.

Actually, the principle of relativity is also a form of correspondence principle, which clarifies the intrinsic linkage or corresponding relationship of the physical laws in different reference frames, and requires the physical models to have the same form or structure in different reference frames. In other words, the principle of relativity requires that the physical models in different reference frames are isomorphic, and have the isomorphic consistency, or have the corresponding relationship of isomorphic consistency. Therefore, based on the principle of relativity, the physical models of different reference frames could isomorphically and uniformly be transformed into each other.

Based on the principle of relativity, by exchanging the corresponding physical quantities of different observers or different reference frames, one could fulfil the corresponding transformation between the physical models in the way of isomorphically-consistent correspondence. Such transformation is only that between the corresponding physical quantities, while the forms and structures of physical laws or physical models remain unchanged.

The Galilean transformation is a physical model of spacetime transformation between different inertial observers, which clarifies the principle of relativity, or in other words, clarifies the isomorphic consistency of physical laws or physical models between different inertial observers or different inertial reference frames:

$$\begin{matrix} O' \to O: & O \to O': \\ x = x' - v't' & x' = x - vt \\ y = y' & y' = y \\ z = z' & z' = z \\ t = t' & t' = t \end{matrix} \quad \begin{pmatrix} v = -v' \\ \Gamma_\infty \equiv 1 \end{pmatrix} \tag{11.16}$$

where *t*, *x*, *y*, *z*, *v* are the observed physical quantities of the inertial observer *O*, and *t*′, *x*′, *y*′, *z*′, *v*′are the observed physical quantities of the inertial observer *O*′; *v* is the speed at which O' moves along the x-axis of O relative to O, and v' is the speed at which O moves along the x'-axis of O' relative to O'.

Equation (11.16) shows that the spacetime transformations $O \to O'$ and $O' \to O$ have exactly the same form and structure, and are symmetrical and isomorphic. According to the principle of relativity, one could directly obtain the $O' \to O$ by exchanging the corresponding physical quantities between O and O' in the $O \to O'$ ($t \rightleftarrows t'$, $x \rightleftarrows x'$, $y \rightleftarrows y'$, $z \rightleftarrows z'$, $v \rightleftarrows v'$), and vice versa. This is not only the embodiment or reflection of the principle of relativity, but also that of the principle of correspondence.

As stated in Sec. 4.2 of Chapter 4 in **the 1st volume of OR: Inertially Observational Relativity**, in Einstein's three-step deduction of special relativity, the second step is based on the principle of relativity to make the corresponding transformation between the observational (or observed) spacetimes of the inertial observers O and O' in the way of isomorphically-consistent correspondence, in which the principle of relativity actually plays the role of the principle of correspondence. In this way, not only the spacetime models $O \to O'$ and $O' \to O$ are isomorphically consistent, but also the Lorentz transformation and the Galilean transformation are isomorphically consistent.

It is thus clear that the principle of relativity implies that the physical laws or physical models of different reference systems and different observers should be or must be isomorphically consistent.

The core thought of the principle of relativity is: **All observers are equal**.

And the theory of OR will further clarify: **All observation agents are equal**.

11.3 OR and the Principle of Correspondence

One physical world, one logical system.

As the formalized models of the identical physical world, all the theoretical systems of physics must follow the identical axiom system or the common logical premises. Therefore, there must exist the intrinsic linkage or corresponding relationship between each other: all the theoretical systems in physics must be logically consistent. Such intrinsic linkage and corresponding relationship are the embodiment of the logical consistency of different theoretical systems in physics.

The theory of OR has revealed the intrinsic linkage or corresponding relationship between different theoretical systems (including between Einstein's relativity theory and classical mechanics, between Einstein's relativity theory and quantum mechanics, as well as, between quantum mechanics and classical mechanics), and in particular, has revealed the intrinsic linkage or corresponding relationship between different observation systems (including between the general observation agent OA(η) and the optical agent OA(c), as well as, between the general observation agent OA(η) and the idealized agent OA_∞), endowing Bohr's correspondence principle with more general and universal significance.

11.3.1 The Theory of OR Matter Waves and The General Planck Constant h_η

In the limit sense of Planck's constant $h \to 0$, Bohr's correspondence principle is one on the corresponding relationship between the quantum physics of the optical agent OA(c) and the classical physics of the idealized agent OA_∞: $h \neq 0$ represents quantum physics, $h=0$ represents classical physics, and $h \to 0$ implies that quantum physics converges to classical physics. The actual value of Planck constant is determined by the experiment of blackbody radiation: $h=6.626\times10^{-34}$ J·s.

So, what does 6.626×10^{-34} J·s $= h > 0$ mean?

According to the theory of OR [26,27], according to the theory of OR matter waves in Chapter 6 of **the 1st volume of OR: Inertially Observational Relativity**, Planck constant h is just one of the parameters of the optical observation system, representing the observational resolving-power of the optical agent OA(c) or the observational uncertainty of the optical agent OA(c).

Planck constant h came from Planck equation $E=hf$ that represented the energy of a single photon and was originally the quantum hypothesis introduced by Planck in 1900 for his law of blackbody radiation [14]. Later, de Broglie extended Planck equation $E=hf$ from photons to all matter particles, derived the de Broglie relation $\lambda=h/p$, and established de Broglie's theory of matter waves [17-19]. However, it worth noting that de Broglie's theory of matter waves is just one theory of the optical observation agent OA(c), in which Planck equation $E=hf$ is only a hypothetical premise of Planck's law, and de Broglie's generalization of $E=hf$ is only speculation, rather than theoretical and logical consequence.

The theory of IOR, including the theory of OR matter waves, is the product of logic and theory, based on the definition of time and the invariance of time-frequency ratio, which has generalized both Einstein's theory of special relativity and de Broglie's theory of matter waves. The theory of OR matter waves is one theory of the general observation agent OA(η), has generalized the de Broglie matter-wave theory of the optical agent OA(c), possessing the corresponding relationship of isomorphic consistency with de Broglie's theory of matter waves, in which there are two important quantum relations, one is the general Planck equation (Eq. (6.16)) and the other is the general de Broglie relation (Eq. (6.19)):

The General Planck Equation: $E(\eta) = h_\eta f(\eta) \quad \left(h_\eta = h(\eta)\right)$

The General de Broglie Relation: $p(\eta) = \dfrac{h_\eta}{\lambda(\eta)} \quad \left(\lambda(\eta) f(\eta) = \eta\right)$

where $h_\eta=h(\eta)$ is the Planck constant of OA(η), or **the general Planck constant**; $f(\eta)$, $\lambda(\eta)$, $E(\eta)$ and $p(\eta)$ are the physical quantities of the observed object P measured by OA(η): respectively, the observed frequency, wavelength, energy and momentum of P as a matter wave.

The theory of OR matter waves has generalized de Broglie' theory of matter waves, which extends de Broglie's theory of matter-wave from the optical agent OA(c) to the general observation agent OA(η). According to the theory of OR matter waves, a specific observation agent OA(η) has its specific Planck constant h_η, and therefore, has its specific observational resolving-power and observational uncertainty. However, the quantum laws or quantum models of different observation agents have the exactly same form and structure, and hence are isomorphically consistent. So, according to the logic of the principle of relativity: **All observation agents are equal or have equal rights**.

Thus, quantum models of different observation agents could isomorphically and uniformly be transformed into each other.

Let OA(η_1) and OA(η_2) be two observation agents, then it follows that:

$$\begin{cases} \lim\limits_{\eta_1 \to \eta_2} \left\{ E(\eta_1) = h_{\eta 1} f(\eta_1) \right\} = \left\{ E(\eta_2) = h_{\eta 2} f(\eta_2) \right\} \\ \lim\limits_{\eta_1 \to \eta_2} \left\{ p(\eta_1) = \dfrac{h_{\eta 1}}{\lambda(\eta_1)} \right\} = \left\{ p(\eta_2) = \dfrac{h_{\eta 2}}{\lambda(\eta_2)} \right\} \end{cases} \tag{11.17}$$

According to the general Planck equation $E=h_\eta f$ of OR theory, the value of the general Planck constant h_η depends on the observation agent OA(η): different observation agents have different Planck constants.

This is an important discovery!

Naturally, if $\eta \to c$, then OA(η)$\to$OA(c), the general Planck constant h_η is exactly the Planck constant h, the general Planck equation $E(\eta)=h_\eta f(\eta)$ is exactly the Planck equation $E=hf$, and the general de Broglie relation $\lambda(\eta)=h_\eta/p(\eta)$ is exactly the de Broglie relation $\lambda=h/p$. It is thus clear that Planck equation $E=hf$ and de Broglie relation $\lambda=h/p$ are the quantum models of the optical observation agent OA(c), and only valid under the optical agent OA(c).

In particular, if $\eta \to \infty$, then OA(η)$\to$OA$_\infty$, the quantum models of the general observation agent OA(η) converge to the classical models of the idealized observation agent OA$_\infty$, the quantum energy $E(\eta)$ tends to be continuous, and therefore, the general Planck constant $h_\eta \to 0$; conversely, $h_\eta \to \infty$ as $\eta \to 0$.

According to the general Planck equation $E(\eta)=h_\eta f(\eta)$ and the statement or analysis of the limit correspondence of Planck constant $h_\eta \to 0$ in Sec. 11.1.2, Bohr's correspondence principle is actually one principle on the corresponding relationship between the optical observation agent OA(c) and the idealized observation agent OA$_\infty$, involving two types of physical systems or two types of observation agents:

(i) The quantum system of the optical agent OA(c): $\eta \to c$, OA(η)=OA(c), $h_\eta=h$, the information-wave speed $\eta=c$;

(ii) The classical system of the idealized agent OA$_\infty$: $\eta \to \infty$, OA(η)=OA$_\infty$, $h_\eta=0$, the information-wave speed $\eta=\infty$.

In summary, it can be concluded that: $\boldsymbol{h_\eta \to 0}$ **iff** $\boldsymbol{\eta \to \infty}$; $\boldsymbol{h_\eta \to h}$ **iff** $\boldsymbol{\eta \to c}$.

Under the principle of simplicity, the most concise formula for $\boldsymbol{h_\eta \to 0}$ **iff** $\boldsymbol{\eta \to \infty}$ is the inverse proportion formula: $xy=k$, in which k is the constant of proportionality.

So, the theory of OR develops an important identity, so-called **the identity of general Planck constant** (the GPC identity for short, see Sec. 6.7 in Chapter 6):

The GPC Identity (Eq. (6.31)): $h_\eta \eta = C \;\; (C \equiv hc) \quad \begin{cases} h_\eta \to 0 \text{ iff } \eta \to \infty \\ h_\eta \to h \text{ iff } \eta \to c \end{cases}$

where $C \equiv hc$ determined by $\boldsymbol{h_\eta \to h}$ **iff** $\boldsymbol{\eta \to c}$; the general Planck constant $h_\eta \to 0$ implies that the quantum model converges to the classical model.

The GPC identity $h_\eta \eta = hc$ in Eq. (6.31) of the theory of OR matter waves suggests that quantum models of different observation agents are isomorphic, have the corresponding relationship of isomorphic consistency, or in other words, have the same form and structure. Bohr's correspondence principle in the limit sense of Planck constant (Eq. (11.15)) implies that the optical observation agent OA(c) and the idealized observation agent OA_∞ are equal. Then the GPC identity $h_\eta \eta = hc$ implies that: **All observation agents are equal or have equal rights**.

It should be pointed out that the GPC identity $h_\eta \eta = hc$ is exactly the formalized expression of **the principle of general correspondence** that generalizes Bohr's correspondence principle.

11.3.2 The Corresponding Relationship between OR Spacetime Transformations

Bohr' correspondence principle reflects the intrinsic linkage or corresponding relationship between quantum physics and classical physics. Likewise, in terms of the requirement of logical consistency, there must also be the intrinsic linkage or corresponding relationship between the Galilean transformation and the Lorentz transformation, as well as that between Einstein's theory of relativity and Newton's theory of mechanics.

The mainstream school of physics believe that there is the approximate corresponding relationship between the Lorentz transformation and the Galilean transformation: at a lower speed ($v \ll c$), the Lorentz factor $\gamma = \Gamma(c) = 1/\sqrt{(1-v^2/c^2)}$ (≈ 1) is approximate to the Galilean factor Γ_∞ ($\equiv 1$), the Lorentz transformation is approximate to the Galilean transformation. On these grounds, the mainstream school of physics believe that there is the logical consistency between the Lorentz transformation and the Galilean transformation, as well as between Einstein's theory of relativity and Newton's theory of mechanics. The mainstream school of physics further conclude that the Lorentz transformation is better, and the Galilean transformation is only an approximation; Einstein's theory of relativity is better, and Newton's theory of mechanics is only an approximation.

Actually, there is no the directly corresponding relationship between the Lorentz transformation and the Galilean transformation.

In Sec. 4.3 **The General Lorentz Transformation** (GLT) of Chapter 4, the theory of OR, from the logical premises and the logical route different from that of Einstein's theory of special relativity, has theoretically deduced the transformation of IOR spacetime in differential form (Eq. (4.16)), which has the more general and universal significance than that in algebraic form [26,27]. Then, by setting the initial conditions: $x=x'=0$, $y=y'=0$, and $z=z'=0$ at $t=t'=0$, one can integrate the transformation of IOR spacetime in differential form (Eq. (4.16)) and obtain the transformation of IOR spacetime in algebraic form as follows:

The GLT (eq. (4.18)):
$$\begin{array}{ll} O' \to O: & O \to O': \\ x = \Gamma\left(x' + vt'\right) & x' = \Gamma\left(x - vt\right) \\ y = y' & y' = y \\ z = z' & z' = z \\ t = \Gamma\left(t' + \dfrac{vx'}{\eta^2}\right) & t' = \Gamma\left(t - \dfrac{vx}{\eta^2}\right) \end{array} \qquad \left(\Gamma = \frac{1}{\sqrt{1 - v^2/\eta^2}}\right)$$

This is so-called **the general Lorentz transformation**.

Obviously, the transformation of IOR spacetime in Eq. (4.18) has the exactly same form and structure as the Lorentz transformation, generalizing the Lorentz transformation. Therefore, it is referred to as **the general Lorentz transformation** in the theory of OR. Actually, as stated in the theory of IOR, both the Galilean transformation and the Lorentz transformation are only special cases of the general Lorentz transformation: the Lorentz transformation is the special case of the optical observation agent OA(c); the Galilean transformation is the special case of the idealized observation agent OA_∞.

The transformation of IOR spacetime, or the general Lorentz transformation, extends the Lorentz transformation from the optical observation agent OA(c) to the general observation agent OA(η) in which the observation medium can be any form of matter motion, and the information-wave speed η can be any speed value. According to the theory of OR [26-28], different observation agents OA(η) have different spacetime-transformation factors $\Gamma(\eta)$, and hence present different degrees of relativistic effects or relativistic phenomena. However, the spacetime transformations of different observation agents, and even the inertial relativity theories of different observation agents, have the exactly same form and structure, or in other words, have the corresponding relationship of isomorphic consistency.

So, according to the logic of relativity principle: **All observation agents are equal or have equal rights**.

Therefore, the spacetime models of different observation agents could be transformed into each other in the way of isomorphically-consistent correspondence; more generally, the inertial relativity theories of different observation agents could also be transformed into each other in the way of isomorphically-consistent correspondence.

Let OA(η_1) and OA(η_2) are two different observation agents, then:

$$\lim_{\eta 1\to\eta 2}\Gamma(\eta_1)=\lim_{\eta 1\to\eta 2}\frac{1}{\sqrt{1-v^2/\eta_1^2}}=\frac{1}{\sqrt{1-v^2/\eta_2^2}}=\Gamma(\eta_2)$$

$$\lim_{\eta 1\to\eta 2}\begin{Bmatrix} x=\Gamma(\eta_1)(x'+vt') \\ y=y' \\ z=z' \\ t=\Gamma(\eta_1)\left(t'+\dfrac{vx'}{\eta_1^2}\right)\end{Bmatrix}=\begin{Bmatrix} x=\Gamma(\eta_2)(x'+vt') \\ y=y' \\ z=z' \\ t=\Gamma(\eta_2)\left(t'+\dfrac{vx'}{\eta_2^2}\right)\end{Bmatrix} \tag{11.18}$$

which means that, by substituting the information-wave speed η_2 for η_1, the spacetime transformation of OA(η_1) as well as the inertial relativity theory of OA(η_1) could be transformed into that of OA(η_2) in the way of isomorphically-consistent correspondence, and vice versa.

As stated in Sec. 4.3 **The General Lorentz Transformation** of Chapter 4, the general Lorentz transformation has generalized and unified the Lorentz transformation and the Galilean transformation: if $\eta\to c$, then the general Lorentz transformation in Eq. (4.18) strictly converges to the Lorentz transformation in Eq. (4.12); if $\eta\to\infty$, then the general Lorentz transformation in Eq. (4.18) strictly converges to the Galilean transformation in Eq. (4.4).

So, in the sense of isomorphic consistency, the general Lorentz transformation is strictly corresponding not only to the Lorentz transformation but also to the Galilean transformation.

From the perspective of Bohr's correspondence principle and quantum theory, the GPC identity $h_\eta\eta=hc$ (Eq. (6.31)) of OR matter-waves theory has clarified that: **All observation agents are equal**. Then, from the perspective of Galileo's relativity principle and the theory of IOR, the general Lorentz transformation (Eq. (6.18)) has further clarified that: **All observation agents are equal**.

It should be pointed out that the general Lorentz transformation in Eq. (6.18) provides another annotation or interpretation for the principle of correspondence, generalizing Galileo's relativity principle.

11.4 The Unity of the Principle of Correspondence and the Principle of Relativity

According to the statement in Sec. 11.1, Bohr's correspondence principle is one principle on the corresponding relationship between quantum physics and classical physics, or one principle on the corresponding relationship between the optical observation agent and the idealized observation agent, which implies the profound thought of **All Observers of Different Observation Agents are Equal**. According to the statement in Sec. 11.2, Galileo's relativity principle is one principle on the corresponding relationship between the spacetime transformations of different inertial reference frames, or one principle on the corresponding relationship between the spacetime transformations of different inertial observers, which implies the profound thought of **All Observers of Different Reference Frames are Equal**.

According to the statement in Sec. 11.3, the theory of IOR and the general Lorentz transformation further clarify that all observers, regardless of their reference frames and regardless of their observation agents, are equal or have equal rights, and their physical laws or physical models have the corresponding relationship of isomorphic consistency, and could be transformed into each other in the way of isomorphically consistent correspondence.

It is thus clear that the principle of correspondence and the principle of relativity could be unified under the idea and concept of **All Observers are Equal**.

Summing up the conclusions in Secs. 11.1-3, according to the established theory of IOR and the established theory of OR matter waves, we have reason to link or unify Bohr's correspondence principle and Galileo's relativity principle, and to integrate the two principles into the principle of general correspondence with more universal significance.

The Principle of General Correspondence (GC): The universe or spacetime is symmetrical, so all observers in the universe or spacetime are equal or have equal rights; regardless of observers' observation systems (including reference frames and observation agents), physical laws or physical models must have the same form and structure in all reference frames and under all observation agents. In other words, the physical laws or physical models of all reference frames or all observation agents must be isomorphically consistent or have the corresponding relationship of isomorphic consistency.

It is worth noting that, in the principle of GC, the observers might be not only ones of different reference frames but also ones of different observation agents: **the observers of different reference frames are equal**; **the observers of different observation agents are equal**. The principle of GC is one principle on the corresponding relationship between the physical laws or physical models of different observation systems (including reference frames and observation agents). The principle of GC has generalized and unified Bohr's correspondence principle and Galileo's relativity principle, and has transcended Bohr's correspondence principle and Galileo's relativity principle.

By means of the principle of GC, one could make the corresponding transformation of isomorphic consistency between the physical laws or physical models of different reference frames or different observers O and O' who employ the same observation agent OA(η):

$$O' \to O \rightleftarrows O \to O': \; U' \rightleftarrows U \quad \left\{ \begin{array}{l} U = \{t, x, y, z, v, \cdots\} \\ U' = \{t', x', y', z', v', \cdots\} \end{array} \right\} \tag{11.19}$$

which only needs to make the corresponding transformation between the corresponding physical quantities U and U' of O and O', just like the corresponding relationship of the $O \to O'$ and the $O' \to O$ in the Galilean transformation or the Lorentz transformation.

By means of the principle of GC, one could make the corresponding transformation of isomorphic consistency between the physical laws or physical models of different observation agents OA(η_1) and OA(η_2) employed by the identical observer O:

$$\begin{aligned}&\mathrm{OA}(\eta_1)\to\mathrm{OA}(\eta_2):\\&\eta_1\to\eta_2,\ U(\eta_1)\to U(\eta_2)\quad\left\{\begin{aligned}&\mathrm{OA}(\eta):\ \infty>\eta>0\\&U(\eta)=\{h_\eta,t_\eta,x_\eta,y_\eta,z_\eta,v_\eta,\cdots\}\end{aligned}\right\}\end{aligned}\tag{11.20}$$

which only needs to make the corresponding transformation between the corresponding physical quantities $U(\eta_1)$ and $U(\eta_2)$ of OA(η_1) and OA(η_2), just like the corresponding relationship between the general Lorentz transformation of the general observation OA(η) and the Lorentz transformation of the optical observation agent OA(c), or the corresponding relationship between the general Lorentz transformation of the general observation agent OA(η) and the Galilean transformation of the idealized observation agent OA_∞.

In particular, according to the principle of GC, in the idealized observation agent OA_∞ ($\eta\to\infty$ or $h_\eta\to 0$), all the theoretical systems of physics converge to classical physics: $h_\eta\to 0$ implies that quantum physics converges to classical physics, just like the limit correspondence of Planck's constant: $h\to 0$ for Bohr's correspondence principle; and at the same time, $\eta\to\infty$ implies that the theory of observational relativity (OR) converges to classical physics, that is, the case of the idealized observation agent OA_∞.

In order to make the corresponding transformation of isomorphic consistency between the spacetime models or theoretical systems of different observation agents OA(η_1) and OA(η_2) under the principle of GC, one could follow the following two different logic routes.

PGC Logic Route 1

Under the principle of GC, directly replace the information-wave speed η_1 of OA(η_1) with the η_2 of OA(η_2), then the observed quantities $U(\eta_1)$ of OA(η_1) would correspondingly be transformed into the observed quantities $U(\eta_2)$ of OA(η_2), and the physical models of OA(η_1) would in the way of isomorphically-consistent correspondence be transformed into the physical models of OA(η_2).

PGC Logic Route 2

That has two steps:

(i) Under the principle of GC, correspondingly transform the logical premises of the theoretical system of OA(η_1) into that of the theoretical system of OA(η_2);

(ii) from the logical premises of the theoretical system of OA(η_2) and by analogizing or following the logic of the theoretical system of OA(η_1), deduce the theoretical system of OA(η_2), which must be isomorphically consistent with the theoretical system of OA(η_1).

Under the principle of GC, by following both PGC logic route 1 and PGC logic route 2, the theory of OR attempt to extend the theory of IOR from inertial spacetime to gravitational spacetime, to extend Einstein's theory of general relativity from the optical observation agent OA(c) to the general observation agent OA(η), and finally, to establish the theory of gravitationally (or general) observational relativity (GOR), or the theory of GOR for short.

The theory of GOR will clarify the root and essence of the relativistic phenomena in Einstein's theory of general relativity or the relativistic phenomena in gravitational spacetime, and finally, generalized and unify Newton's theory of universal gravitation and Einstein's theory of general relativity.

Perhaps, the principle of general correspondence (GC) might become a sharp weapon of physics to provide the important ideological basis or guiding principle for the development of new theories and the unification of old theories, and moreover, for the test of logical consistency and logical self-consistency of the theoretical systems in physics.

11.5 IOR Three Principles: from Einstein's Special Relativity to IOR

The theory of IOR is originally the theoretical system that has been deduced from the definition of time and the invariance of time-frequency ratio [26,27]. However, the theory of IOR also obeys the principle of general correspondence (GC): by substituting the information-wave speed η of the general observation agent OA(η) for the speed c of light, i.e., the information-wave speed c of the optical observation agent OA(c), Einstein's theory of special relativity could be generalized from OA(c) to OA(η), and then could isomorphically and uniformly be transformed into the theory of IOR.

Here, it would contribute to our understanding of the principle of GC, or to our understanding of the role and effect of the principle of GC to deduce the theory of IOR under the principle of GC.

As stated in Sec. 11.4, under the principle of GC, no matter through PGC logic route 1 or through PGC logic route 2, one could isomorphically and uniformly transform Einstein's theory of special relativity into the theory of IOR.

11.5.1 The Deduction of IOR Theory through PGC Logic Route 1

The Fundamental Formulae in Einstein's Special Relativity

(are simply be summarized as follows):

(i) Minkowski 4d inertial spacetime $X^{4\text{d}}(c)$ (the optical agent OA(c)):

$$\mathrm{OA}(c) \equiv \left\{ \begin{array}{l} X^{4\text{d}}(c): \left\{ \begin{array}{l} x^0 = ct; \\ x^1 = x,\ x^2 = y,\ x^3 = z \end{array} \right\} \\ \mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \end{array} \right\}$$

(ii) The Lorentz factor in inertial spacetime: $\gamma(v) = \dfrac{\mathrm{d}t}{\mathrm{d}\tau} = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

(iii) The Lorentz transformation $O'\to O$: $\begin{cases} x=\gamma\left(x'+vt'\right) \\ y=y' \\ z=z' \\ t=\gamma\left(t'+vx'/c^2\right) \end{cases}$

(iv) The mass-speed relation: $m=\dfrac{m_o}{\sqrt{1-v^2/c^2}}$

(v) The law of Einstein's speed-addition: $u=\dfrac{u'+v}{1+u'v/c^2}$

(vi) Einstein mass-energy relation: $E=mc^2$.

The Fundamental Formulae in the Theory of IOR

Under the principle of GC, through PGC logic route 1 and by directly substituting the information-wave speed η of the general observation agent OA(η) for the information-wave speed c of the optical observation agent OA(c), the fundamental formulae of Einstein's special relativity could isomorphically and uniformly be transformed into that of IOR theory as follows:

(i) The IOR inertial spacetime $X^{4\text{d}}(\eta)$ (the general agent OA(η)):

$$\text{OA}\left(\eta\right)\equiv\begin{Bmatrix} X^{4\text{d}}\left(\eta\right):\begin{Bmatrix} x^0=\eta t; \\ x^1=x,\ x^2=y,\ x^3=z \end{Bmatrix} \\ \text{d}s^2=\eta_{\mu\nu}\text{d}x^{\mu}\text{d}x^{\nu}=\eta^2\text{d}t^2-\text{d}x^2-\text{d}y^2-\text{d}z^2 \end{Bmatrix}$$

(ii) The IOR factor of in inertial spacetime: $\varGamma\left(\eta\right)=\dfrac{\text{d}t}{\text{d}\tau}=\dfrac{1}{\sqrt{1-v^2/\eta^2}}$

(iii) The general Lorentz transformation $O'\to O$: $\begin{cases} x=\varGamma\left(x'+vt'\right) \\ y=y' \\ z=z' \\ t=\varGamma\left(t'+vx'/\eta^2\right) \end{cases}$

(iv) The IOR mass-speed relation: $m=\dfrac{m_o}{\sqrt{1-v^2/\eta^2}}$

(v) The law of IOR speed-addition: $u=\dfrac{u'+v}{1+u'v/\eta^2}$

(vi) The IOR mass-energy relation: $E=m\eta^2$.

The fundamental formulae of IOR theory transformed correspondingly under the principle of GC is exactly the same as that of IOR theory derived from the definition of time and the invariance of time-frequency ratio [26,27]. It is thus clear that, under the principle of GC, the whole theoretical system of IOR could directly be obtained from Einstein's theory of special relativity by the corresponding transformation of isomorphic consistency: OA(c)→OA(η), and vice versa.

In this way, under the principle of GC and through PGC logic route 1, the whole theoretical system of IOR could be established.

11.5.2 The Deduction of IOR Theory through PGC Logic Route 2

The principle of the invariance of light speed is the most important logical premise of Einstein's theory of special relativity.

However, the invariance of light speed has no the feature of self-evidence that a principle should have. People could understand why energy is conserved, so **the principle of conservation of energy** has the rationality as a principle. However, people could not understand why the speed of light is invariant.

Through PGC logic route 1 and by the simply corresponding transformation of $c \rightarrow \eta$, one could directly obtain the theory of IOR that is isomorphically consistent with Einstein's theory of special relativity. However, PGC logic Route 1 could not clarify the root and essence of the invariance of light speed and even that of all other relativistic phenomena, and also could not clarify the causality between of the general observation agent OA(η) and the optical observation agent OA(c).

Originally, the theory of IOR employs the definition of time and the invariance of time-frequency ratio as **the most basic logical premises**. In the theoretical system of IOR, there is an important logical consequence: **the invariance of information-wave speeds**.

The invariance of information-wave speeds has revealed the essence of the invariance of light speed: the speed of light is not really invariant; the so-called invariance of light speed is actually a sort of observational effect, that is, an apparent phenomenon while light is acting as the observation medium to transmit the information of observed objects for observers, rather than the objectively physical reality. The hypothesis of the invariance of light speed is the indispensable logical premise of Einstein's special relativity. Therefore, Einstein's special relativity could not explain by itself why the speed of light is invariant.

Perhaps, only when physical theories originated from the most basic logical prerequisites, could we really know both what the relativistic phenomena were and why the relativistic phenomena did. However, in any case, PGC logic route 2 contributes more to our understanding of the root and essence of relativistic phenomena including the invariance of light speed than PGC logic route 1 do.

As stated in Chapter 4 of **the 1st volume of OR: Inertially Observational Relativity**, the axiom system of Einstein's theory of special relativity consists of three principles, so-called **Einstein's three principles of special relativity**:

(i) The principle of simplicity;

(ii) The principle of relativity;

(iii) The principle of the invariance of light speed

Under the principle of GC, by the corresponding transformation between the optical observation agent OA(c) and the general observation agent OA(η): OA(c)$\rightarrow$OA(η), Einstein's three principles of special relativity could isomorphically and uniformly be transformed into the following three principles of OR theory and become the axiom system of the theory of IOR, so-called **the IOR three principles**:

(i) The principle of simplicity;

(ii) The principle of relativity;

(iii) The principle of the invariance of information-wave speeds.

In the logical premises of IOR theory, the principle of simplicity and the principle of relativity remain valid; however, under the principle of GC, Einstein's principle of the invariance of light speed is correspondingly transformed into the principle of the invariance of information-wave speeds, in which the information-wave speed η of OA(η) replaces the light speed c of OA(c).

Thus, through PGC logic route 2, on the basis of **the IOR three principles**, and by analogizing or following the logic of Einstein's special relativity, one could deduce the theory of IOR. In this way, one could establish the whole theoretical system of IOR that must be isomorphically consistent with Einstein's theory of special relativity [28] (or see Secs. 4.2-3 of Chapter 4 in **the 1st volume of OR: Inertially Observational Relativity**).

Compared with PGC logic route 1, PGC logic route 2 is more helpful in clarifying the logical ideas and the causality of IOR theory, and at the same time, is more helpful in our understanding of the root and essence of the invariance of light speed and even all other inertial relativistic phenomena.

However, both PGC logical route 1 and PGC logical route 2 are logical shortcuts. Taking shortcuts comes at a cost. Whether through PGC logic route 1 or through PGC logic route 2, we would miss the invariance of time-frequency ratio and the transformation of IOR spacetime in differential form, and in particular, we would fail to establish the theory of OR matter waves.

11.6 GOR Three Principles: from Einstein's General Relativity to GOR

As stated in Sec. 11.5, the principle of general correspondence (GC) could isomorphically and uniformly transform Einstein's special relativity of the optical observation agent OA(c) into the IOR theory of the general observation agent OA(η). Likewise, the principle of GC could also isomorphically and uniformly transform Einstein's general relativity of the optical observation agent OA(c) into the GOR theory of the general observation agent OA(η).

The 2nd volume of OR: Gravitationally Observational Relativity (GOR), under the principle of GC, attempts to extend Einstein's general relativity from the optical agent OA(c) to the general observation agent OA(η), and ultimately, to establish the whole theoretical system of GOR.

As stated in Sec. 11.4, According to the principle of GC, we have two logical routes to follow for deducing the theory of GOR.

11.6.1 The Deduction of GOR Theory through PGC Logic Route 1

The Fundamental Formulae in Einstein's General Relativity

(are simply be summarized as follows):

(i) Minkowski 4d gravitational spacetime $X^{4\text{d}}(c)$ (the optical agent OA(c)):

$$\mathrm{OA}(c) \triangleq \left\{ \begin{array}{l} X^{4\text{d}}(c): \left\{ \begin{array}{l} x^0 = ct; \\ x^1 = x,\ x^2 = y,\ x^3 = z \end{array} \right\} \\ \mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \quad \left(g_{\mu\nu} = g_{\mu\nu}\left(c, x^\alpha\right)\right) \end{array} \right\}$$

(ii) The Lorentz factor in gravitational spacetime:

$$\gamma(v,\chi) = \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{1}{\sqrt{\left(\sqrt{1+2\chi/c^2} - \gamma_i v^i/c\right)^2 - v^2/c^2}}$$

$$\left(\chi = c^2\left(g_{00}-1\right)/2;\ \gamma_i = -g_{0i}\big/\sqrt{g_{00}}\right)$$

(iii) The geodesic equation: for matter motion

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}s^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s} = 0 \quad (\mu = 0,1,2,3)$$

$$\left(\Gamma^\mu_{\alpha\beta}(c) = \frac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta} + g_{\nu\beta,\alpha} - g_{\beta\alpha,\nu}\right)\right)$$

(iv) Einstein's field equation: for spacetime curvature

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa T_{\mu\nu} \quad \left(\kappa = \frac{8\pi G}{c^4}\right)$$

$$\left(R_{\mu\nu}(c) = \Gamma^\sigma_{\mu\sigma,\nu} - \Gamma^\sigma_{\mu\nu,\sigma} + \Gamma^\alpha_{\mu\sigma}\Gamma^\sigma_{\alpha\nu} + \Gamma^\alpha_{\mu\nu}\Gamma^\sigma_{\alpha\sigma}\right)$$

The Fundamental Formulae in the Theory of GOR

Under the principle of GC, through PGC logic route 1 and by directly substituting the information-wave speed η of the general observation agent OA(η) for the information-wave speed c of the optical observation agent OA(c), the fundamental formulae of Einstein's general relativity could isomorphically and uniformly be transformed into that of GOR theory as follows:

(i) The GOR gravitational spacetime $X^{4\text{d}}(\eta)$ (the general agent OA(η)):

$$\mathrm{OA}(\eta) \triangleq \left\{ \begin{array}{l} X^{4\text{d}}(\eta): \left\{ \begin{array}{l} x^0 = \eta t; \\ x^1 = x,\ x^2 = y,\ x^3 = z \end{array} \right\} \\ \mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \quad \left(g_{\mu\nu} = g_{\mu\nu}\left(\eta, x^\alpha\right)\right) \end{array} \right\}$$

(ii) The GOR factor in gravitational spacetime:

$$\Gamma(\eta)=\frac{\mathrm{d}t}{\mathrm{d}\tau}=\frac{1}{\sqrt{\left(\sqrt{1+2\chi/\eta^2}-\gamma_i v^i/\eta\right)^2-v^2/\eta^2}}$$

$$\left(\chi=\eta^2\left(g_{00}-1\right)/2;\gamma_i=-g_{0i}\Big/\sqrt{g_{00}}\right)$$

(iii) The GOR geodesic equation: for matter motion

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}s^2}+\Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s}=0\quad\left(\mu=0,1,2,3\right)$$

$$\left(\Gamma^\mu_{\alpha\beta}\left(\eta\right)=\frac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta}+g_{\nu\beta,\alpha}-g_{\beta\alpha,\nu}\right)\right)$$

(iv) The GOR field equation: for spacetime curvature

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\kappa T_{\mu\nu}\quad\left(\kappa=\frac{8\pi G}{\eta^4}\right)$$

$$\left(R_{\mu\nu}\left(\eta\right)=\Gamma^\sigma_{\mu\sigma,\nu}-\Gamma^\sigma_{\mu\nu,\sigma}+\Gamma^\alpha_{\mu\sigma}\Gamma^\sigma_{\alpha\nu}+\Gamma^\alpha_{\mu\nu}\Gamma^\sigma_{\alpha\sigma}\right)$$

PGC logic route 1 is the most convenient logical channel for the theory of GOR.

It could be predicted that, under the principle of GC, even through such a simple and direct logical channel, one could establish the whole theoretical system of **Gravitationally Observational Relativity** (GOR).

11.6.2 The Deduction of GOR Theory through PGC Logic Route 2

The axiom system of Einstein's theory of general relativity also consists of three principles, so-called **Einstein's three principles of general relativity**:

(i) The principle of equivalence;

(ii) The principle of general covariance;

(iii) The principle of the invariance of light speed

It is just because the principle of the invariance of light speed is employed as the logical premise of Einstein's special relativity that the speed of light c appears in Einstein's theory of special relativity, for instance, appears in the transformation factor $\gamma(v)$ of inertial spacetime: $\gamma(v)=1/\sqrt{(1-v^2/c^2)}$. Likewise, it is also because the principle of the invariance of light speed is employed as the logical premise of Einstein's general relativity that the speed of light c appears in Einstein's theory of general relativity, for instance, appears in the transformation factor $\gamma(\chi)$ of gravitational spacetime: $\gamma(\chi)=1/\sqrt{(1+2\chi/c^2)}$. As stated repeatedly by the theory of OR, the principle of the invariance of light speed is the logical premise of both Einstein's special relativity and Einstein's general relativity. With the help of the principle of equivalence, the invariance of light speed could play its role as a principle or a logical premise in Einstein's theory of general relativity. In other words, it was in order to play the role of the invariance of light speed as the principle or logical premise in Einstein's theory of general relativity that Einstein constructed the principle of equivalence [52,53].

The optical observation agent OA(c) in Eq. (10.1) and the general observation agent OA(η) in Eq. (10.2) have the corresponding relationship of isomorphically consistency. Under the principle of GC, OA(c) can isomorphically and uniformly be transformed into OA(η). Therefore, under the principle of GC, by the corresponding transformation of isomorphic consistency: OA(c)$\rightarrow$OA(η), Einstein's three principles as the axiom system of general relativity could isomorphically and uniformly be transformed into **the GOR three principles** as the axiom system of the theory of gravitationally observational relativity (GOR):

(i) The principle of equivalence;

(ii) The principle of general covariance;

(iii) The principle of the invariance of information-wave speeds.

In the logical premises of GOR theory, the principle of equivalence and the principle of general covariance proposed by Einstein remain valid, and moreover, under the principle of GC, have gained more universal significance: the observers in which could not only serve different reference frames but also have different observation agents. As in the case of IOR theory in Sec. 11.5.2, under the principle of GC, Einstein's principle of the invariance of light speed is correspondingly transformed into the principle of the invariance of information-wave speeds, in which the information-wave speed η of the general observation agent OA(η) replaces the light speed c of OA(c).

In this way, on the basis of the axiom system or logical premises of GOR theory, i.e., **the GOR three principles**, and by analogizing or following the logic of Einstein's general relativity, one could deduce the theory of GOR. So, we could establish the whole theoretical system of GOR that must be isomorphically consistent with Einstein's theory of general relativity.

Compared with PGC logic route 1, PGC logic route 2 is more helpful in our understanding of Einstein's general relativity, more helpful in our understanding of the phenomena of spacetime curvature and even all other relativistic effects in gravitational spacetime, and at the same time, more helpful in clarifying the logical idea and the causality of GOR theory.

12 The Spacetime Theory of GOR

The GOR spacetime, like the IOR spacetime, is the observational spacetime of the general observation agent OA(η). However, the IOR spacetime is inertial Spacetime, while the GOR spacetime is gravitational spacetime.

The spacetime that human beings could perceive depends on observation, which is the spacetime $X^{4d}(\eta)$ that the physical world presents to observer through certain observation agents OA(η) ($\eta \in (0,+\infty)$), rather than the objectively real spacetime. The realistic observational agents, including the human sensors and the optical agent OA(c), all have the observational locality ($\eta<\infty$), which makes the gravitational spacetime we perceive or observe exhibit a certain degree of gravitational relativistic effects, appearing to be somewhat curved.

It is the primary problem of the theory of gravitationally observational relativity (GOR) how to define and describe the gravitational spacetime in observation.

Alternatively, how should physicist observe and measure the space and time of gravitational spacetime in observation?

Are the gravitational relativistic effects, including the phenomenon of spacetime curvature, the essential characteristics of the physical world, or the observational effects caused by the observational locality?

12.1 The Problem of the Locality of Gravitational Spacetime

With regard to gravity or gravitational interaction, there are two theoretical systems in physics: one is Newton's theory of universal gravitation [81]; the other is Einstein's theory of general relativity [8].

Both Newton's and Einstein's involve two classes of locality problems:

(i) The gravitational locality;

(ii) The observational locality.

12.1.1 The Gravitational Locality: the Gravitation-Wave Speed is Finite

As formalized models of the physical world, all theories in physics must have certain idealized characteristics.

Actually, both Newton's theory of universal gravitation and Einstein's theory of general relativity imply an important idealized hypothesis: gravity is **an action at a distance**, or the speed of gravitational radiation is infinite.

Newton's law of universal gravitation does not take into account the locality of gravitational interaction: when the distribution of matter in a physical system changes, the gravitational field described by Poisson's equation also change instantaneously [125], or in other words, it takes no time for gravity to cross space.

Newton's law of universal gravitation is the gravitational law of the idealized agent OA_∞, while Einstein's field equation is the gravitational law of optical agent OA(c). According to Einstein's theory of general relativity, under the condition of weak-field approximation, Einstein's field equation reduces to Newton's law of universal gravitation in the form of Poisson's equation.

You could not imagine that the speed of gravitational radiation would be higher in a weaker gravitational field.

So, this suggests that, just like Newton's gravity or gravitational interaction, Einstein's gravity or gravitational interaction in his general relativity is also idealized as **an action at a distance**: the speed of gravitational radiation is infinite.

According to the principle of physical observability (see Chapter 2), there is no action at a distance in the physical world [26-28]: all physical quantities, including the speed of gravitational radiation, must be finite or limited.

In 1859, French astronomer Verrier found that the elliptical orbit of Mercury was precessing at a rate different from that predicted by Newton's law [126], which aroused people's attention to the speed of gravity. In 1805, Laplace reached a conclusion through calculation [43]: the speed of gravitational radiation was greater than 7×10^6c. American physicist Flandern believed that [127]: the speed of gravitational radiation is much greater than the speed c of light, otherwise, the galaxies in the universe would lose their existing stable structures; in 1998, he calculated that the speed of gravitational radiation was $2\times10^{10}c$.

Based on his theory of general relativity [8], Einstein derived a so-called gravitational wave equation, in which the speed of gravitational waves is exactly the speed of light. However, logically or as far as the logic goes, both Newton's theory of universal gravitation and Einstein's theory of general relativity have no prior information about gravitational waves or the speed of gravitational radiation, and therefore, it is simply impossible for Einstein to calculate the speed of gravitational waves or derive the equation for predicting gravitational waves.

So, what does Einstein's equation on gravitational waves mean? Why is the speed of gravitational waves predicted by Einstein exactly the speed of light?

The theory of GOR will reveal the mystery for us.

It should be pointed out that, like Newton's theory of universal gravitation and Einstein's theory of general relativity, the theory of GOR also have no prior information about gravitational waves or the speed of gravitational radiation. The theory of GOR also contains such an important idealized hypothesis: gravity is an action at a distance, or the speed of gravitational radiation is infinite.

12.1.2 The Observational Locality: the Information-Wave Speeds are finite

Newton's theory of universal gravitation is the product of the idealized observation agent OA_∞, in which the speed of information wave is infinite: there is no observational locality ($\eta\to\infty$) in Newton's gravitational spacetime, and it takes no time for observed information to cross space. However, Einstein's theory of general relativity is the product of the optical observation agent OA(c), in which the speed of information wave is the speed of light c and is finite or limited: there is the observational locality ($c<\infty$) in Einstein's gravitational spacetime, and so it takes time for observed information to cross space.

Like Einstein's special relativistic phenomena (including the invariance of light speed) in inertial spacetime, the root and essence of Einstein's general relativistic phenomena (including spacetime curvature) in gravitational spacetime also lie in the observational locality ($c<\infty$) of the optical observation agent OA(c). By examining Schwarzschild metric [80], the theory of OR suggest that [26,27], actually, the so-called spacetime curvature is only a sort of observational effect. The optical agent OA(c) with observational locality ($c<\infty$) is just like a wide-angle lens, making the gravitational spacetime look or observe somewhat curved or deformed.

The theory of GOR will further clarify that, if we could employ the idealized agent OA_∞ ($\eta\to\infty$) to observe the physical world, then the gravitational spacetime would present its objectively real face: flat and not curved.

In Newton's theory of universal gravitation [83], gravity, like electromagnetic force, weak force and strong force, is a kind of **force**, i.e., one of the four fundamental interactions between matter and matter, rather than the geometric effect of spacetime curvature. However, in Einstein's theory of general relativity [8], the gravity or gravitational force is removed from the universe, and only left the geometrized effects of gravity: curved spacetime. Einstein's theory of general relativity geometrizes gravity and equates the gravitational effect with spacetime curvature: matter makes spacetime curved; curved spacetime makes matter moved. Thus, the earth's motion around the sun is no longer the effect of gravity or gravitational force, but that of curved spacetime.

The geometrization of gravitational effects can be regarded as a formalized method for matter interactions. Actually, under the principle of general correspondence (GC), such method will penetrate into the theory of GOR and the process of GOR logical deduction. However, such a method for geometrizing gravitational effects is only a formalized means after all, which does not represent the real physical characters of gravity or gravitational force as a fundamental interaction, and moreover, does not mean there is no gravitational force in the universe. Otherwise, we should also have geometrized the other three fundamental interactions, including electromagnetic force, weak force and strong force.

The theory of GOR will tell us that the so-called spacetime curvature is not due to the distribution or accumulation of matter or energy, but just a sort of observational effect or apparent phenomenon.

12.2 The Spacetime Theory of Einstein's General Relativity

Before relating and analyzing the spacetime theory of GOR under the principle of general correspondence (GC), we need to first recognize the spacetime theory of Einstein's general relativity, and to understand the observational spacetime $X^{4d}(c)$ of gravitational field under the optical agent OA(c).

Einstein's theory of general relativity is the product of the optical observation agent OA(c), in which, the basic task of OA(c) is to employ light or electromagnetic interaction as the observation medium to quantify and measure the gravitational spacetime $X^{4d}(c)$ of OA(c), including the determinations of space and time.

It should be pointed out that the determination of the gravitational spacetime $X^{4d}(c)$ of OA(c) in Einstein's general relativity needs to make use of the principle of equivalence and the principle of the invariance of light speed. In his theory of general relativity, Einstein introduced the local inertial spacetime based on the principle of equivalence, the aim or effect of which is to make gravitational spacetime locally equivalent to inertial spacetime or to the free spacetime S_F where the principle of the invariance of light speed holds true, so that one or OA(c) could:

(i) determine the time of $X^{4d}(c)$: to transform **the observational or observed time** dt into **the standard time** dτ;

(ii) determine the space of $X^{4d}(c)$: to employ the standard time dτ and the invariant light speed c for calculating the physical space dl.

12.2.1 Einstein's Concept of Time

Actually, the coordinate framework of Minkowski 4d spacetime is the observational spacetime $X^{4d}(c)$ of the optical agent OA(c), implying Einstein's theory of spacetime, in which $X^{4d}(c)$ is the observational 4d spacetime of OA(c) with the 1d time and the 3d space:

(i) the coordinate of the 1d time: $x^0=ct$;

(ii) the coordinate of the 3d space: $x^1=x$, $x^2=y$, $x^3=z$.

where $t=x^0/c$ implies the invariance of light speed.

Suppose that a moving object, i.e., the observed object P, moves in $X^{4d}(c)$, then its spacetime trajectory (including the time-element dt and line-element ds) can be described based on the definition of the coordinate framework $X^{4d}(c)$ of Minkowski 4d spacetime in Eq. (10.1) of Chapter 1:

$$\begin{cases} \mathrm{d}t = \mathrm{d}x^0/c \\ \mathrm{d}s^2 = g_{\mu\nu}(c)\mathrm{d}x^\mu \mathrm{d}x^\nu = c^2 g_{00}\mathrm{d}t^2 + 2cg_{0i}\mathrm{d}x^i\mathrm{d}t + g_{ik}\mathrm{d}x^i\mathrm{d}x^k \end{cases} \tag{12.1}$$

where dt is the observational time observed by the observer with the optical observation agent OA(c), x^μ (μ=1,2,3,4) are the coordinates of the observational spacetime $X^{4d}(c)$ of OA(c), $g_{\mu\nu}(c)$ is the spacetime metric of $X^{4d}(c)$ of OA(c), ds is the line-element of P's spacetime trajectory, and $x^i(t)$ (i=1,2,3) are the space coordinates at the specific time t.

The theory of OR (both IOR and GOR) refers to the time t of the observational spacetime $X^{4d}(c)$ of OA(c) as the observational or observed time of the optical observation agent OA(c). However, in his theory of relativity, Einstein did not make clear about the status and effect of observation. In the observational spacetime $X^{4d}(c)$ of OA(c) in Eq. (10.1) or (12.1), we are not quite clear about where the clock for indicating the time t and where the observer for observing the time t.

So, what does the time dt in the observational spacetime $X^{4d}(c)$ mean?

Suppose that the observed object P itself is a clock, or T_P is **the intrinsic clock** of P. According to Def. 1.2 in Chapter 1, if the observer O and the observed object P

are relatively stationary in inertial spacetime S_I or the free spacetime S_F, then the time obtained by O observing P or T_P is exactly the objective and real time (**proper time**): $\mathrm{d}\tau=\mathrm{d}s/c$; otherwise, the time obtained by O observing P or T_P can only be the observational or observed time of O of OA(c): $\mathrm{d}t=\mathrm{d}x^0/c$.

The Time of Inertial Spacetime in Einstein's Special Relativity

The observational spacetime $X^{4\mathrm{d}}(c)$ in Einstein's theory of special relativity is inertial spacetime: $g_{\mu\nu}=\eta_{\mu\nu}=\mathrm{diag}(+1,-1,-1,-1)$ is Minkowski metric.

Therefore, from Eq. (12.1), it follows that:

$$\mathrm{d}t=\frac{\mathrm{d}x^0}{c}=\frac{\mathrm{d}\tau}{\sqrt{1-v^2/c^2}}\quad\left(\mathrm{d}\tau=\frac{\mathrm{d}s}{c},\ v=\frac{\mathrm{d}l}{\mathrm{d}t},\ \mathrm{d}l=\sqrt{\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2}\right)\tag{12.2}$$

where Einstein called τ **the standard time** and t **the coordinate time**; v is the relative speed between the observer O and the observed object P or T_P in the inertial spacetime $X^{4\mathrm{d}}(c)$ of the optical observation OA(c).

According to the relation (Eq. (12.2)) of Einstein's theory of special relativity:

(i) If $v=0$ then $\mathrm{d}t=\mathrm{d}\tau$, which suggests that Einstein's concept of the standard time ($\mathrm{d}\tau$) is consistent with the concept of the intrinsic time defined in Def. 1.2 of the theory of OR;

(ii) If $v\neq 0$ then $\mathrm{d}t>\mathrm{d}\tau$, which is known as the time dilation in inertial spacetime, and according to Eq. (12.2), different inertial speeds v of P would lead to different degrees of time dilation.

In inertial spacetime, motion is relative and so is rest.

With regard to the time-element $\mathrm{d}t$ in the relation (Eq. (12.2)) of Einstein's special relativity, you could imagine that the observer O is at rest, while the observed object P moves relative to O at the inertial speed v. In this case, $\mathrm{d}t$ should be the time the moving clock T_P indicates to the static observer O. You could also imagine that P is at rest, while O moves relative to P at the inertial speed v. In this case, $\mathrm{d}t$ should be the time the static clock T_P indicates to the moving observer O.

In Einstein's special relativity, the clock static in inertial spacetime S_I is regarded as **the standard clock**, and the time it indicates to the observer static in inertial spacetime S_I is **the standard time**, i.e., proper time: $\mathrm{d}\tau$.

It is generally thought that the proper time $\mathrm{d}\tau$ represents the time rate of the clock static in inertial spacetime, and the observed time $\mathrm{d}t$ represents the time rate of the clock moving in inertial spacetime. However, if $\mathrm{d}t$ is interpreted as the time rate of moving clock, then $\mathrm{d}t>\mathrm{d}\tau$ means that moving clock runs faster, which is contrary to the cognition of Einstein's special relativity that moving clock runs slower. So, $\mathrm{d}t$ is actually the time the static standard clock indicates to the moving observer.

The theory of IOR has clarified that [26-28], in the inertial spacetime $X^{4\mathrm{d}}(c)$ of the optical agent OA(c), the intrinsic clock T_P of the observed object P should be regarded the standard clock static in the free spacetime S_F, and $\mathrm{d}t$ is the time the static standard clock T_P indicates to the moving observer O by OA(c).

The Time of Gravitational Spacetime in Einstein's General Relativity

The observational spacetime $X^{4d}(c)$ in Einstein's theory of general relativity is gravitational spacetime, its spacetime metric $g_{\mu\nu}=g_{\mu\nu}(c,x^{\alpha})$ depends on the spacetime coordinates x^{α} (α=0,1,2,3) of $X^{4d}(c)$ and is related to the speed c of light: different coordinates might have different time rates observationally.

In terms of time, like in his theory of special relativity, there are also two concepts in Einstein's theory of general relativity:

(i) The standard time: $d\tau=ds/c$;

(ii) The coordinate time: $dt=ds/(c\sqrt{(g_{00})})$

It is generally thought that: the standard time is namely the time of the standard clock; the coordinate time is namely the time of the coordinate clock. Thus, according to Eq. (12.1), **the coordinate clock** should be static in the space coordinate x^i ($dx^i=0$) of the observational spacetime $X^{4d}(c)$, and **the standard clock** should be static in inertial spacetime S_I or the free spacetime S_F ($g_{\mu\nu}=\eta_{\mu\nu}$, $dx^i=0$).

According to the coordinate framework of Minkowski 4d spacetime or the definition of the optical observation agent OA(c) in Eq. (10.1), the time rate dt in Eq. (12.1) of the observational spacetime $X^{4d}(c)$ should be $dt=dx^0/c$. Actually, the time $dt=dx^0/c$ in Einstein's general relativity is one of the observed quantities defined in Def. 1.2, and can be called **the observed time**; the standard time $d\tau=ds/c$ in Einstein's general relativity is one of the intrinsic physical quantities defined in Def. 1.2, and can be called **the intrinsic time** (**proper time**): the objectively real time.

It is thus clear that, in general, the observational time $dt=dx^0/c$ of OA(c) is not quite the same as the coordinate time $dt=ds/(c\sqrt{(g_{00})})$ of the observational spacetime $X^{4d}(c)$, unless P is stationary in the observational spacetime $X^{4d}(c)$.

It is worth noting that, because of this, Einstein made the neighborhood of the point P in the gravitational spacetime $X^{4d}(c)$ locally become an equivalent inertial spacetime under the principle of equivalence, in which P is instantaneously static.

If P is static in $X^{4d}(c)$, then $dx^i=0$ (i=1,2,3); according to Eq. (12.1), it follows

$$dt=\frac{dx^0}{c}=\frac{ds}{c\sqrt{g_{00}}}=\frac{d\tau}{\sqrt{1+2\chi/c^2}}\quad\left(d\tau=\frac{ds}{c},\ g_{00}=1+2\chi/c^2\right)\qquad(12.3)$$

where χ ($=-GM/r\leq 0$) is the gravitational potential at the specific space coordinate x^i (i=1,2,3) in $X^{4d}(c)$.

According to Eq. (12.3) in Einstein's general relativity:

(i) If $\chi=0$ then $dt=d\tau$, which suggests that the concept of **the standard time** in Einstein's general relativity is equivalent to the concept of **the intrinsic time** in the theory of OR (both IOR and GOR);

(ii) If $\chi\neq 0$ then $dt>d\tau$, which is known as the time dilation of gravitational spacetime, and according to Eq. (12.3), different gravitational potential χ of P would lead to different degrees of time dilation.

In gravitational spacetime, there might be a certain difference of gravitational potential between the observer O and the observed object P. With regard to the time-element dt in the relation (Eq. (12.3)) of Einstein's general relativity, you could imagine that O is static in the free spacetime S_F (χ=0), while P is located at χ ($\neq$0) in the gravitational spacetime $X^{4d}(c)$. In this case, dt should be the time the clock T_P at χ in the potential field indicates to the observer O at χ=0. You could also imagine that P is static in the free spacetime S_F (χ=0), while O is located at χ ($\neq$0) in the gravitational spacetime $X^{4d}(c)$. In this case, dt should be the time the standard clock T_P at χ=0 indicates to the observer O at χ ($\neq$0) in the potential field.

In Einstein's general relativity, the clock static at the null potential (χ=0) is regarded as the standard clock, and the time it indicates to the observer static at the null potential (χ=0) is the standard time, i.e., proper time: dτ.

It is generally thought that the proper time dτ represents the time rate of the clock static at the null potential (χ=0), and the observed time dt represents the time rate of the clock in the potential field. However, if dt is interpreted as the rate of potential clock, then dt>dτ means that potential clock runs faster, which is contrary to the cognition of Einstein's general relativity that potential clock runs slower. So, dt is actually the time the null-potential standard clock indicates to the observer in the potential field.

The theory of GOR will clarify that, in the gravitational spacetime $X^{4d}(c)$ of the optical agent OA(c), the intrinsic clock T_P of the observed object P should be regarded the standard clock static in the free spacetime S_F (at the null potential χ=0), and dt is the time the null-potential standard clock T_P indicates to the observer O in the potential field through OA(c).

The concepts of **the standard time** and **the coordinate time** are important concepts of Einstein's general relativity, which are employed to measure or determine both the time and the space in the gravitational spacetime $X^{4d}(c)$ of the optical observation agent OA(c).

12.2.2 The Determination of the Standard Time in Einstein's General Relativity [8,128,129]

As stated in Sec. 12.2.1, observers moving at different inertial speeds (v) have different rates of observed time dt in inertial spacetime; observers located at different gravitational potentials (χ) also have different rates of observed time dt in gravitational spacetime.

The spacetime in Einstein's special relativity is inertial spacetime, belonging to the observational spacetime $X^{4d}(c)$ of the optical agent OA(c). As shown in Eq. (12.2), dτ is the time when v=0, and Einstein interpreted dτ as the time rate of the clock static in inertial spacetime; dt is the time when $v\neq$0, and Einstein interpreted dt as the time rate of the clock moving in inertial spacetime. The observed dt depends on the inertial speed v of the observed object P in $X^{4d}(c)$, but does not depend on the space coordinates of P in $X^{4d}(c)$. Therefore, in his theory of special relativity, Einstein seemed to prefer to use dt rather than dτ to describe the movement of P. And moreover, based on the observed time-element dt that is not the objectively real time, Einstein interpreted his views of **the relativity of simultaneity**, **time dilation**, and **length contraction**, and exaggerated his thought that relativistic phenomena are the essential characteristics of spacetime and matter motion.

The spacetime in Einstein's general relativity is gravitational spacetime, also belonging to the observational spacetime $X^{4d}(c)$ of the optical agent OA(c). As shown in Eq. (12.3), $d\tau$ is the time when $\chi\neq 0$, and Einstein interpreted $d\tau$ as the time rate of the clock located at the null potential; dt is the time when $\chi\neq 0$, and Einstein interpreted dt as the time rate of the clock located at χ in gravitational spacetime. With regard to gravitational spacetime, different space coordinates have different gravitational potential χ, and therefore, have different time rates dt. In this case, Einstein's theory of general relativity could not directly employ the time rate dt of the coordinate framework of Minkowski 4d spacetime to describe the movement of the observed object P.

Perhaps, for this reason, Einstein realized that the time rate dt in the coordinate framework of Minkowski 4d spacetime did not represent the objectively real time.

Einstein's theory of general relativity needed a uniform time.

So, the concept of **the standard time** ($d\tau$) appeared in Einstein's theory of general relativity to distinguish from the concept of **the coordinate time** (dt).

Naturally, the standard time should be independent of spacetime and matter motion, and not depend on v and χ.

Perhaps, this reminded Einstein of the clock static in inertial spacetime and the proper time $d\tau$ in Eq. (12.2). The standard clock in Einstein's general relativity is located at the null potential (χ=0), equivalent to the standard clock static in inertial spacetime (v=0) of Einstein's special relativity. This implies that, with regarded to the standard clock T_P as the observed object P, $g_{\mu\nu}=\eta_{\mu\nu}$ and $dx^i=0$ (i=1,2,3). Thus, one could calculate the standard time $d\tau$ independent of v and χ from Eq. (12.1):

$$\begin{cases} ds^2 = g_{\mu\nu}(c)dx^\mu dx^\nu = c^2 d\tau^2 \\ d\tau = \dfrac{ds}{c} \end{cases} \tag{12.4}$$

Obviously, the standard time $d\tau$ in Eq. (12.4) is the intrinsic physical quantity defined in Def. 1.2 of Chapter 1, i.e., the intrinsic time (proper time): the objectively real time.

Einstein's special relativity needs to transform the observed time dt that depends on the inertial speed v into the standard time $d\tau$; likewise, Einstein's general relativity needs to transform the observed time dt that depends on the gravitational potential χ into the standard time $d\tau$.

If the intrinsic clock T_P of the observed object P is static in the free spacetime S_F, then T_P is namely the standard clock, and its time is namely the standard time. However, in Einstein's view, P is in the gravitational potential field ($\chi\neq 0$), T_P is not the standard clock, and its time is not the standard time; if P is located at the specific space coordinate x^i (i=1,2,3) of $X^{4d}(c)$, then T_P is the coordinate clock at x^i, and the time dt is the coordinate time at x^i. So, Einstein needed to standardize the time in his general relativity: to transform the gravitational spacetime $X^{4d}(c)$ into inertial spacetime, to transform the clock T_P into the standard clock, and to transform the coordinate time dt into the standard time $d\tau$.

Let D_0 ($x^i(t)$ (i=1,2,3)) be the space coordinate of the observed object P at the specific time $t=x^0/c$. The observer O located at D_0 originally belongs to the gravitational spacetime $S_G=X^{4d}(c)$; S_G is the observational spacetime S_O of O with OA(c). As depicted in Fig. 12.1(a1), under the principle of equivalence, one could introduce a locally inertial spacetime S_I at D_0, in which P and O are instantaneously static at D_0, so that the principle of the invariance of light speed holds true in the neighborhood of D_0.

In this way, the principle of equivalence embodies its special value and significance: through the principle of equivalence, the gravitational spacetime S_G is instantaneously and locally transformed into the equivalent inertial spacetime S_I.

According to Eq. (12.4), the standard time $\mathrm{d}\tau$ is proportional to the line-element ds of the observed object P in the observational spacetime $X^{4d}(c)$ of the optical agent OA(c). As depicted in Fig. 12.1(a1), let L_G be the world line of P in the gravitational spacetime S_G of OA(c), and L_I be the world line of P in the equivalent inertial spacetime S_I. Reasonably, the standard time in S_G can be defined as $\mathrm{d}\tau_G=\mathrm{d}s_G/c$; the standard time in S_I can be defined as $\mathrm{d}\tau_I=\mathrm{d}s_I/c$.

Obviously, according to the definition of the standard time $\mathrm{d}\tau$ in Eq. (12.4), the $\mathrm{d}\tau_I$ in inertial spacetime S_I is the objectively real time, i.e., the intrinsic time (proper time) $\mathrm{d}\tau$: $\mathrm{d}\tau_I=\mathrm{d}\tau$.

According to differential geometry, the line-element $\mathrm{d}s_G$ of the curve L_G is equal to the line-element $\mathrm{d}s_I$ of the tangent L_I to L_G: $\mathrm{d}s_G=\mathrm{d}s_I$. So, it follows that

$$\mathrm{d}\tau_G = \frac{\mathrm{d}s_G}{c} = \frac{\mathrm{d}s_I}{c} = \mathrm{d}\tau_I = \mathrm{d}\tau \tag{12.5}$$

Equation (12.5) is of the important and profound implication: different observational spacetimes $X^{4d}(c)$ of the optical agent OA(c), including S_G and S_I, has the identical standard time, which is exactly the objective and real time, i.e., the intrinsic time (proper time) $\mathrm{d}\tau$.

Under the principle of equivalence, the gravitational spacetime $S_G=X^\mu(c)$ becomes the inertial spacetime S_I locally equivalent S_G, in which the observed object P and the observer O located at D_0 (x^i) is instantaneously static in S_I: $\mathrm{d}x^i=0$ (i=1,2,3). Therefore, according to Eq. (12.1) and Eqs. (12.4-5), the standard time $\mathrm{d}\tau$ in Einstein's theory of general relativity can be determined with the coordinate time dt:

$$\begin{aligned}\mathrm{d}\tau &= \frac{\mathrm{d}s}{c} = \frac{1}{c}\sqrt{g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu} \\ &= \frac{1}{c}\sqrt{g_{00}}\mathrm{d}x^0 = \sqrt{g_{00}}\mathrm{d}t = \sqrt{1+\frac{2\chi}{c^2}}\mathrm{d}t \quad \left(g_{00} = 1+\frac{2\chi}{c^2}\right)\end{aligned} \tag{12.6}$$

where χ is the gravitational potential at the space coordinate x^i (i=1,2,3) in $X^{4d}(c)$.

Equation (12.6) suggests that: in Einstein's theory of general relativity, the standard time could be determined; more importantly, the objective real time, i.e., the proper time, could be determined by observing and measuring.

In particular, it should be pointed out that, as one of the logical premises in

Einstein's theory of general relativity, the hypothesis of **the invariance of light speed** plays an important role in the determination of time.

As stated in Sec. 1.4 of Chapter 1, Minkowski spacetime, or the coordinate framework of 4d spacetime $X^{4d}(c)$ of the optical agent OA(c), is a sort of formalized expression of the principle of the invariance of light speed, in which the time axis $x^0=ct$ (see Eq. (10.1) or Eq. (12.1)) represents the invariance of light speed. It was based on the time-element $\mathrm{d}t=\mathrm{d}x^0/c$ in Eq. (12.1) and the line-element $\mathrm{d}s=g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$ in Eq. (12.1) of the observed object P that Einstein derived Eq. (12.6), i.e., the formula of the standard time $\mathrm{d}\tau$.

12.2.3 The Determination of the Physical Space in Einstein's General Relativity [8,128,129]

In Einstein's theory of general relativity, the determination of physical space depends not only on the standard time $\mathrm{d}\tau$ but also on the hypothesis of the invariance of light speed. So, Einstein had to make use of the principle of equivalence to transform the gravitational spacetime $S_G=X^{4d}(c)$ into the equivalent inertial spacetime S_I at D_0 instantaneously and locally.

As depicted in Fig. 12.1(a2), let D be a space point close enough to D_0 in the gravitational spacetime $S_G=X^{4d}(c)$; send an optical signal from D_0 to D and then be reflected from D back to D_0. The time it takes obeys:

$$\mathrm{d}x^0=\mathrm{d}x^0_{\text{out}}+\mathrm{d}x^0_{\text{back}} \quad \left(\mathrm{d}x^0=c\mathrm{d}t\right) \tag{12.7}$$

where the gravitational spacetime S_G is not necessarily isotropic, and therefore, the time the $\mathrm{d}x^0_{\text{out}}$ takes is not necessarily equals to the time the $\mathrm{d}x^0_{\text{back}}$ takes.

If the spatial displacement from D_0 to D is $\mathrm{d}x^i$ (i=1,2,3), then the spatial displacement from D to D_0 is $-\mathrm{d}x^i$ (i=1,2,3). Under the optical observation agent OA(c), the spacetime line-element $\mathrm{d}s^2=0$ of light, and therefore, we have:

$$\begin{cases} 0=g_{00}\left(\mathrm{d}x^0_{\text{out}}\right)^2+2g_{0i}\mathrm{d}x^i\mathrm{d}x^0_{\text{out}}+g_{ik}\mathrm{d}x^i\mathrm{d}x^k \\ 0=g_{00}\left(\mathrm{d}x^0_{\text{back}}\right)^2-2g_{0i}\mathrm{d}x^i\mathrm{d}x^0_{\text{back}}+g_{ik}\mathrm{d}x^i\mathrm{d}x^k \end{cases}$$
$$\mathrm{d}x^0=\mathrm{d}x^0_{\text{out}}+\mathrm{d}x^0_{\text{back}}=2\frac{\sqrt{\left(g_{0i}g_{0k}-g_{00}g_{ik}\right)\mathrm{d}x^i\mathrm{d}x^k}}{g_{00}} \tag{12.8}$$

Particularly, according to the principle of equivalence and the principle of general covariance, the locally inertial spacetime S_I at D_0 must be isotropic, in which the invariance of light speed holds true. Therefore, the physical space distance $\mathrm{d}l$ between D_0 and D can be defined as $\mathrm{d}l=c\mathrm{d}\tau/2$ based on the standard time $\mathrm{d}\tau$ and the speed c of light in the vacuum.

According to Eq. (12.6), the standard time (proper time) $\mathrm{d}\tau=\sqrt{(g_{00})}\mathrm{d}x^0/c$.

Then, the physical space distance $\mathrm{d}l$ can be measured or determined as follows:

$$\mathrm{d}l = c\frac{\mathrm{d}\tau}{2} = \frac{\sqrt{g_{00}}}{2}\mathrm{d}x^0 = \sqrt{\left(\frac{g_{0i}g_{0k}}{g_{00}} - g_{ik}\right)\mathrm{d}x^i\mathrm{d}x^k}$$

$$\text{or}\quad \mathrm{d}l^2 = \gamma_{ik}(c)\mathrm{d}x^i\mathrm{d}x^k \quad \left(\gamma_{ik}(c) = \frac{g_{0i}g_{0k}}{g_{00}} - g_{ik}\right) \tag{12.9}$$

where γ_{ik} (i,k=1,2,3) is the physical space metric.

12.2.4 The Factor of GOR Spacetime Transformation in Einstein's General Relativity [8,129,130]

In Einstein's theory of relativity, the factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ of spacetime transformation is the ratio of the observed time-element dt to the intrinsic time-element dτ, which is an important physical quantity: the larger the factor γ, the more significant the relativistic phenomena of the observed object.

Therefore, γ may also be referred to as **the relativistic factor** for characterizing the relativistic effects of spacetime and matter motion

The Factor of Inertial Spacetime Transformation: $\gamma=\gamma(v)$

In Einstein's theory of special relativity, the factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ of spacetime transformation is the relativistic factor of inertial spacetime transformation, i.e., the Lorentz factor:

$$\gamma = \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{1}{\sqrt{1-v^2/c^2}} \tag{12.10}$$

where $\gamma=\gamma(v)$ depends on the inertial speed v of the observed object P: the larger the $|v|$, the more significant the relativistic effects of inertial motion.

The Factor of Gravitational Spacetime Transformation: $\gamma=\gamma(\chi)$

In Einstein's theory of general relativity, the factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ of spacetime transformation is the relativistic factor of gravitational spacetime transformation. In the case of simple gravity (v=0):

$$\gamma = \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{1}{\sqrt{1+2\chi/c^2}} \tag{12.11}$$

where $\gamma=\gamma(\chi)$ depends on the gravitational potential χ of the observed object P: the larger the $|\chi|$, the more significant the relativistic effects of gravitational interaction.

The Factor of Gravitational Spacetime Transformation: $\gamma=\gamma(v,\chi)$

Suppose that the observed object P moves in the gravitational field, then in Einstein's general relativity, the factor of spacetime transformation should be:

$$\gamma = \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{1}{\sqrt{1+2\chi/c^2 - v^2/c^2}} \tag{12.12}$$

where $\gamma=\gamma(v,\chi)$ depends on both the gravitational potential χ of the observed object

P and the speed v of the observed object P relative the observer O.

The Factor of Gravitational Spacetime Transformation: $\gamma=\gamma(v,\chi,\gamma_i)$

It is worth noting that, in Eq. (12.12), the χ is a scalar, representing the **scalar potential** of gravity. Einstein imagined that, like the electromagnetic field, gravitational field might have both **scalar potential** and **vector potential**: Following the definition of the strength of electromagnetic field, the field strength **g** at a space point in gravitational field could be defined as follows:

$$\begin{gathered}\mathbf{g}=\lim_{\Delta m\to 0}\frac{\Delta\mathbf{F}}{\Delta m}\quad\left(\Delta\mathbf{F}=\Delta m\mathbf{a}\right)\\ a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2 l}{\mathrm{d}t^2}=\sqrt{\gamma_{ik}a^i a^k}\quad\left(a^i=\frac{\mathrm{d}^2x^i}{\mathrm{d}t^2},\ a_i=\gamma_{ik}a^k\right)\end{gathered}\tag{12.13}$$

where Δm is the mass. According to the principle of equivalence, there is no distinction between gravitational mass and inertial mass; the strength **g** of gravitational field is equivalent to the gravitational acceleration **a** (a^1,a^2,a^3) at the space point x^i (i=1,2,3).

According to Einstein's theory of general relativity:

$$a_i=-\frac{\partial\chi}{\partial x^i}-c\sqrt{1+\frac{2\chi}{c^2}}\frac{\partial\gamma_i(c)}{\partial t}\quad\left(\gamma_i(c)\equiv-\frac{g_{0i}}{\sqrt{g_{00}}}\right)\tag{12.14}$$

where γ_i (i=1,2,3) is the **vector potential** and χ the **scalar potential** of gravity.

If the vector potential of gravity really exists in gravitational field, then $\exists i\in(1,2,3)\ g_{0i}\neq 0$; according to Eq. (12.9), in the observational spacetime $X^{4\mathrm{d}}(c)$ of the optical agent OA(c), the space metric γ_{ik} (i,k=1,2,3) is related to the time axis x^0 of $X^{4\mathrm{d}}(c)$. This would mean that the space axis $x^i\ i\in(1,2,3)$ of $X^{4\mathrm{d}}(c)$ might not be orthogonal to the time axis x^0 of $X^{4\mathrm{d}}(c)$: space and time are interdependent.

Conversely, if the vector potential of gravity does not exist in gravitational field, then $g_{0i}=0$ (i=1,2,3); according to Eq. (12.9), in the observational spacetime $X^{4\mathrm{d}}(c)$ of the optical agent OA(c), the space metric $\gamma_{ik}=-g_{ik}$ (i,k=1,2,3) is not related to the time axis x^0 of $X^{4\mathrm{d}}(c)$. This would mean that the space axis $x^i\ i\in(1,2,3)$ of $X^{4\mathrm{d}}(c)$ is orthogonal to the time axis x^0 of $X^{4\mathrm{d}}(c)$: space and time are orthogonal and independent of each other; the corresponding coordinate system of $X^{4\mathrm{d}}(c)$ could be referred to as **the orthogonal spacetime**.

So, is there really the vector potential of gravity in gravitational field?

This involves the problem about whether the time axis and the space axes are orthogonal in 4d spacetime. More importantly, this has led to the problem about whether space and time are interdependent.

Under the optical agent OA(c) in Eq. (10.1), the observed time $\mathrm{d}t=\mathrm{d}x^0/c$ and the intrinsic time $\mathrm{d}\tau=\mathrm{d}s/c$. Considering both the scalar potential χ and the vector potential γ_i (i=1,2,3) in gravitational field, then

$$\gamma=\frac{\mathrm{d}t}{\mathrm{d}\tau}=\frac{\mathrm{d}x^0}{\mathrm{d}s}=\frac{\mathrm{d}x^0}{\sqrt{g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu}}=\frac{\mathrm{d}x^0}{\sqrt{g_{00}\mathrm{d}x^0\mathrm{d}x^0+2g_{0i}\mathrm{d}x^0\mathrm{d}x^i+g_{ik}\mathrm{d}x^i\mathrm{d}x^k}}$$

$$=\frac{1}{\sqrt{\left(\sqrt{g_{00}}+\frac{g_{0i}}{\sqrt{g_{00}}}\frac{\mathrm{d}x^i}{\mathrm{d}x^0}\right)^2+\left(g_{ik}-\frac{g_{0i}g_{0k}}{g_{00}}\right)\frac{\mathrm{d}x^i}{\mathrm{d}x^0}\frac{\mathrm{d}x^k}{\mathrm{d}x^0}}} \tag{12.15}$$

where $\gamma=\gamma(v,\chi,\gamma_i)$ depends on the speed v of the observed object P relative the observer O, and meanwhile, depends on the scalar potential χ and the vector potential γ_i (i=1,2,3) of P.

Equation (12.15) is the most general factor γ of spacetime transformation in Einstein's theory of general relativity. By contrasting Eq. (12.15) with Eqs. (12.6), (12.9) and (12.14), the factor γ in Eq. (12.15) could be rewritten as:

$$\gamma=\frac{\mathrm{d}t}{\mathrm{d}\tau}=\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{c^2}}-\gamma_i\frac{v^i}{c}\right)^2-\frac{v^2}{c^2}}} \tag{12.16}$$

where $v=\mathrm{d}l/\mathrm{d}t$ and $v^i=\mathrm{d}x^i/\mathrm{d}t$ (i=1,2,3).

The factor of spacetime transformation in Einstein's general relativity (Eq. (12.16)) has generalized the relativistic effects of both inertial spacetime and gravitational spacetime: without gravitational field (χ=0, γ_i=0), Eq. (12.16) reduces to the factor $\gamma=1/\sqrt{(1-v^2/c^2)}$ (Eq. (12.10)) of inertial spacetime transformation; without inertial motion (v=0), Eq. (12.16) reduces to the factor $\gamma=1/\sqrt{(1+2\chi/c^2)}$ (Eq. (12.11)) of gravitational spacetime transformation.

12.2.5 The Problem of the Root and Essence of Relativistic phenomena

Perhaps, the earlies observation of relativistic phenomena should be traced back to the Michelson-Morley experiment [2], which led to the birth of the Fitzgerald-Lorentz transformation [3-6]. Based on the Michelson-Morley experiment, Einstein set up the principle of the invariance of light speed, theoretically derived the Lorentz transformation, established the theory of special relativity in 1905 [7], and then, established the theory of general relativity in 1915 [8]. However, neither Fitzgerald, Lorentz, nor Einstein understand what role light played in the Fitzgerald transformation [3] and the Lorentz transformation [4-6].

Up to today, the mainstream school of physics still does not fully understand what role light plays in Einstein's theory of relativity, including the special relativity and the general relativity.

Why does the speed of light c appear in Einstein's theory of relativity? In particular, why does the speed of light c occupy the specific position in the factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ of spacetime transformation?

Naturally, it is due to the hypothesis of the invariance of light speed that the speed c of light appears in Einstein's theory of relativity, including in the special and in the general, and particularly, appears in the factor $\gamma=1/\surd(1+2\chi/c^2-v^2/c^2)$ of spacetime transformation. However, as the logical premise of Einstein's theory of relativity, the hypothesis of the invariance of light speed itself cannot explain what role light plays in Einstein's theory of relativity.

Einstein believed and nowadays the mainstream school of physics also believe that: the speed of light was the ultimate speed of the universe and could not be exceeded, and therefore, the speed c of light was invariant or the same relative to all inertial observers. So, the factor $\gamma=\gamma(v,\chi,\gamma_i)$ of spacetime transformation in Eq. (12.16) depends on the speed v of matter motion as well as the scalar potential χ and vector potential (i=1,2,3) of gravitational field: if $v=0$ as well as $\chi=0$ and $\gamma_i=0$ (i=1,2,3), then $\gamma=\Gamma_\infty\equiv1$, and there is no relativistic phenomenon; if $v\neq0$ or $\chi\neq0$ or $\exists i\ \gamma_i\neq0$, then $\gamma>\Gamma_\infty\equiv1$, and spacetime and matter motion would present relativistic effects or relativistic phenomena.

Accordingly, Einstein believed and nowadays the mainstream school of physics also believes that: relativistic phenomena, including **the invariance of light speed** and the effect of **spacetime curvature**, were the essential characteristics of spacetime and matter motion, and rooted from matter motion (v) and matter interactions (χ and γ_i).

However, the theory of IOR has already clarified that, in Einstein's theory of special relativity, the root of inertial relativistic effects does not lie in matter motion, and the essence lies in the observational locality ($c<\infty$) of the optical agent OA(c). The theory of GOR will further clarify that, in Einstein's theory of general relativity, the root of gravitational relativistic effects does not lie in matter interactions, and the essence also lies in the observational locality ($c<\infty$) of the optical agent OA(c).

12.3 The Measurement of GOR Spacetime

In his theory of general relativity, based on the principle of equivalence, Einstein transformed gravitational spacetime into inertial spacetime, so that the hypothesis of the invariance of light speed could holds true in the observational spacetime $X^{4d}(c)$ of optical agent OA(c). Thus, **the coordinate time** could be transformed into **the standard time**, and the time of gravitational spacetime could be measured or determined; the gravitational space could be transformed into the inertial space, and so, the space of gravitational spacetime could be measured or determined.

Now, based on the principle of general correspondence (GC), we intend to extend the concepts of the standard time and the coordinate time in Einstein's theory of general relativity, as well as Einstein's logical way of determining the time and space of gravitational spacetime, from the optical observation agent OA(c) to the general observation agent OA(η) in the theory of GOR.

Einstein employed the optical observation agent OA(c) to measure spacetime, including the inertial and the gravitational; while the theory of OR, including IOR and GOR, employ the general observation agent OA(η) to measure spacetime, including the inertial and the gravitational.

12.3.1 GOR Logic Route: OA(c)→OA(η)

In theory, the observation medium of an observation agent OA(η) could be any form of matter motion, not necessarily light; the transmitting speed η of observed information could be any value, not necessarily the speed c of light.

In the objective world, the speed of matter motion must be finite or limited.

Therefore, like the optical agent OA(c), all realistic observation agents (OA(η): $\eta\in(0,+\infty)$) have the observational locality of their own. Restricted by the observation locality ($\eta<\infty$), the observational spacetime $X^{4d}(\eta)$ of OA(η), like the observational spacetime $X^{4d}(c)$ of OA(c), also appears to be somewhat **curved** in observation. However, as depicted in Fig. 12.1(b1), different observation agents have different information-wave speeds and different degrees of observational locality, and therefore, their observational spacetimes would present different degrees of curvature.

Based on the principle of GC, you could make the corresponding transformation of isomorphic consistency between different observation agents, including between the optical agent OA(c) and the general observation agents OA(η). Actually, it is based on the principle of GC that the optical observation agent OA(c) in Eq. (10.1) is isomorphically and uniformly transformed into the general observation agent OA(η) (Eq. (10.2)) in Def. 10.1 of Chapter 10.

As stated in Sec. 11.4 of Chapter 11, based on the principle of GC, there are two logic routes for you to deduce the formulae of measuring the space and time in gravitational spacetime of GOR theory.

Following PGC Logical Route 1

Based on the principle of GC, transforming the optical agent OA(c) into the general observation agent OA(η) in the way of isomorphically-consistent correspondence, the observational spacetime $X^{4d}(c)$ of OA(c) in Einstein's general relativity would be transformed into the GOR observational spacetime $X^{4d}(\eta)$ of OA(η). In this way, the formulae for measuring the space and time of gravitational spacetime in Einstein's general relativity could isomorphically and uniformly be transformed into that in the theory of GOR.

Firstly, substituting η for the light speed c, Eq. (12.6) for measuring the standard time in Einstein's general relativity could directly be transformed into the formula for measuring the standard time in the theory of GOR:

$$\mathrm{d}\tau=\sqrt{g_{00}(\eta)}\mathrm{d}t=\sqrt{1+\frac{2\chi}{\eta^2}}\mathrm{d}t\quad\left(g_{00}(\eta)=1+\frac{2\chi}{\eta^2}\right)\tag{12.17}$$

where $g_{00}=g_{00}(\eta)$ is the 00-element of the metric tensor $g_{\mu\nu}(\eta)$ in the observational spacetime $X^{4d}(\eta)$ of OA(η), distinguished from the $g_{00}(c)$ in the optical observation spacetime $X^{4d}(c)$ of OA(c)

Thus, **the standard time** in the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) (not just OA(c)) can be measured or determined.

Secondly, substituting η for the light speed c, Eq. (12.9) for measuring the physical space in Einstein's general relativity could directly be transformed into the formula for measuring the physical space in the theory of GOR:

$$\begin{aligned} \mathrm{d}l &= \sqrt{\gamma_{ik}(\eta)\mathrm{d}x^i\mathrm{d}x^k} \\ &= \sqrt{\left(\frac{g_{0i}g_{0k}}{g_{00}} - g_{ik}\right)\mathrm{d}x^i\mathrm{d}x^k} \quad \left(\gamma_{ik}(\eta) = \frac{g_{0i}g_{0k}}{g_{00}} - g_{ik}\right) \end{aligned} \tag{12.18}$$

where $\gamma_{ik}=\gamma_{ik}(\eta)$ is the space metric in the observational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η), distinguished from the $\gamma_{ik}(c)$ in the optical observation spacetime $X^{4\mathrm{d}}(c)$ of OA(c).

Thus, **the physical space** of the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agents OA(η) (not just OA(c)) can be measured or determined.

Following PGC Logical Route 2

Perhaps, based on the principle of GC, following PGC logic Route 2, it is more helpful for us to understand the spacetime theory of Einstein's general relativity, and then to deduce the spacetime theory of GOR more logically.

Actually, what followed by Def. 10.1 is PGC logic route 2: in the optical agent OA(c), $x^0=ct$ implies the invariance of light speed; while in the general observation agent OA(η), $x^0=\eta t$ implies the invariance of information-wave speeds. Therefore, the corresponding transformation of isomorphic consistency between OA(η) and OA(c) is actually that between the invariance of information-wave speeds and the invariance of light speed, rather than simply that between the information-wave speed η and the light speed c.

As stated in Sec. 12.2, in Einstein's theory of general relativity, the measurement of gravitational spacetime, including the determination of time and the determination of space, depends on **Einstein's three principles of general relativity**: (i) the principle of equivalence; (ii) the principle of general covariance; (iii) the principle of the invariance of light speed.

As stated in Sec. 11.6 of Chapter 11, based on the principle of GC, following PGC logic route 2, by replacing the invariance of light speed with the invariance of information-wave speeds, the theory of GOR have had **the GOR three principles**: (i) the principle of equivalence; (ii) the principle of general covariance; (iii) the principle of the invariance of information-wave speeds. Naturally, taking such three principles as the logical premised or axiom system, by analogizing and following the logic of Einstein's general relativity, the theory of GOR could derive the metric relations of gravitational spacetime, including Eqs. (12.17-18), which must be isomorphically consistent with that of Einstein's general relativity.

Under the principle of equivalence, the **curved gravitational spacetime** $S_G=X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η) is instantaneously and locally transformed into the equivalent **flat inertial spacetime** S_I, i.e., the free spacetime S_F, in which the principle of general covariance and the principle of the invariance of information-wave speeds hold true. Thus, firstly, as what will be stated in Sec.12.3.3, the standard time of GOR gravitational spacetime $X^{4\mathrm{d}}(\eta)$ could be determined: following Einstein's logic, Eq. (12.17) could be derived; secondly, as what will be stated in Sec. 12.3.4, the physical space of GOR gravitational spacetime $X^{4\mathrm{d}}(\eta)$

could be determined: following Einstein's logic, Eq. (12.18) could be derived.

12.3.2 The Time Concept of GOR

As stated in Sec. 10.1 of Chapter 10, the definition of time (Def. 2.2 in Chapter 2) and its inference: **the invariance of time-frequency ratio** (Eq. (2.3)), remain valid in the theory of GOR.

According to Def. 10.1, the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) is isomorphically consistent with the Minkowski spacetime $X^{4d}(c)$ of the optical agent OA(c), and has the same spacetime structure of 1d time and 3d space:

(i) The 1d time coordinate: $x^0=\eta t$;

(ii) The 3d space coordinates: $x^1=x$, $x^2=y$, $x^3=z$,

where the time coordinate $t=x^0/\eta$ implies the invariance of information-wave speeds.

Let P the observed object and O the observer: P exists in the intrinsic spacetime O_o (the free spacetime S_F); O employs the observation agent OA(η) to observe P. In the view of O, P moves in the observational spacetime $X^{4d}(\eta)$ of OA(η). According to Def 10.1, the spacetime trajectory of P could be described by the time-element dt and the line-element ds in the observational spacetime $X^{4d}(\eta)$ of OA(η):

$$\begin{cases} \mathrm{d}t = \mathrm{d}x^0/\eta \\ \mathrm{d}s^2 = g_{\mu\nu}(\eta)\mathrm{d}x^\mu \mathrm{d}x^\nu = \eta^2 g_{00}\mathrm{d}t^2 + 2\eta g_{0i}\mathrm{d}x^i\mathrm{d}t + g_{ik}\mathrm{d}x^i\mathrm{d}x^k \end{cases} \tag{12.19}$$

where dt is the observational time observed by O with OA(η), x^μ (μ=1,2,3,4) the spacetime coordinates of $X^{4d}(\eta)$, $x^i(t)$ (i=1,2,3) the space coordinates of P at a specific time $t=x^0/\eta$, ds the line-element of spacetime trajectory of P, and $g_{\mu\nu}(\eta)$ the spacetime metric of $X^{4d}(\eta)$ of OA(η).

The time-element dt in the definition of time in Def. 2.2 of Chapter 2 or in Eq. (12.19) can be referred to as **the GOR time**.

Then, what relationships are there among **the GOR time** and **the IOR time** in Def. 1.1 of Chapter 1 as well as Einstein's **coordinate time** and **standard time**?

The GOR time is the time of general observation agent OA(η) in Def 10.1, which is consistent with the time concept in Def. 2.2 of Chapter 2. According to Sec. 12.2.1 and Sec. 12.3.1, the GOR time dt=dx^0/η is not only the observed time of the general observation agent OA(η), but also the observed time in Def. 2.2. However, generally, dt=dx^0/η is neither the standard time nor the coordinate time.

The concept of time in Def. 2.2 generalizes Einstein's concepts of both the coordinate time and the standard time: no matter the coordinate time or the standard time is only a special case of the observed time in Def. 2.2. In particular, the intrinsic time of Def. 2.2 is consistent with or equivalent to Einstein's standard time, which is the time of the standard clock static in the free spacetime S_F.

However, Einstein did not specify the status and role of observers in the observation or measurement of time.

Based on the definition of time in Def. 2.2, the concept of time in the theory of

OR (including IOR and GOR) needs to be linked to observation and observers: in the observation spacetime $X^{4d}(\eta)$ of an observation agent OA(η), the time dt is the observed time of the observer O in Def. 2.2; particularly, according to Def. 1.2 of Chapter 1, the intrinsic time in Def. 2.2 is the objectively real time, which could be measured or determined only if the standard clock is static in the free spacetime S_F, or O could employ the idealized observation agent OA_∞.

It is conceivable that the time dt in Def. 10.1 (see Eq. (10.2) or Eq. (12.19)) is the observed time indicated by the standard clock to the observer O of OA(η).

Then, in terms of the spacetime trajectory of the observed object P (Eq. (12.19)), the problems are that:

Firstly, what and where the standard clock is;

And secondly, who and where the observer O is.

According to the definition of clock in Def. 2.3, the standard clock is static in the free spacetime S_F, or vice versa, all periodic signal sources static in the free spacetime S_F, including P as a matter wave or its intrinsic clock T_P, could be employed as the standard clock.

The basic task of the observer O is to measure or determine the spacetime coordinates x^μ (μ=1,2,3,4) of the observed object P, that is,

(i) to determine the specific time $t=x^0/\eta$ of P;

(ii) to determine the space coordinates $x^i(t)$ (i=1,2,3) of P at the specific time t.

As the observational or observed time of the observer O, the measurement of the GOR time $\mathrm{d}t=\mathrm{d}x^0/\eta$ needs to specify the relative relationship between the observer O of OA(η) and the observed object P.

Observation implies the spacetime transformation: by means of a certain observation agent OA(η), the observer O transforms the observed object P from the intrinsic spacetime O_o of its own to the observational spacetime $X^{4d}(\eta)$ of OA(η). Originally, the observed object P belongs to the intrinsic spacetime O_o static in the free spacetime S_F; P itself or its intrinsic clock T_P is namely the standard clock. Therefore, the time dt in the definition of the general observation agent OA(η) (Def. 10.1: Eq. (10.2) or Eq.(12.19)) is the observational or observed time of the observer O of OA(η) and indicated by T_P as the standard clock.

In particular, in gravitational field, observers located at different space coordinates might have different observed times dt.

So, where is the observer O of the time dt in Def 10.1 (Eq. (10.2) or Eq. (12.19))? As an observer, O observes the spacetime trajectory of P as shown in Eq. (12.19): at the specific time $t=x^0/\eta$, P arrives at or is located at the specific space coordinates $x^i(t)$ (i=1,2,3); the corresponding dt must be the observed time rate of P or T_P at the space coordinates $x^i(t)$ (i=1,2,3). Therefore, the observer O should be located at the space coordinates $x^i(t)$ (i=1,2,3): at the specific time $t=x^0/\eta$, the corresponding time rate dt of spacetime trajectory of P in Eq. (12.19) is the observed time rate dt of the observer O who is instantaneously located at the same space coordinate $x^i(t)$ (i=1,2,3) as that of the observed object P or T_P.

In terms of spatial location, the standard clock T_P is always static relative to P.

Naturally, if O and P are relatively static in the free spacetime S_F at one moment $t=x^0/\eta$, then the observed time $\mathrm{d}t=\mathrm{d}x^0/\eta$ ($g_{\mu\nu}=\eta_{\mu\nu}$, $\mathrm{d}x^i=0$; $i=1,2,3$) of O is exactly the standard time; if O and P are relatively static at the specific space coordinates $x^i(t)$ ($i=1,2,3$) of $X^{4\mathrm{d}}(\eta)$ at one moment $t=x^0/\eta$, then the observed time $\mathrm{d}t=\mathrm{d}x^0/\eta$ ($\mathrm{d}x^i=0$; $i=1,2,3$) of O is exactly the coordinate time

To sum up, the GOR times, including the standard time and the observed time are subject to the definition of time in Def. 2.2 of Chapter 2: the standard time of GOR is consistent with Einstein's standard time; the coordinate time of GOR is consistent with Einstein's coordinate time. Both can be derived from the definition of the general observation agent OA(η) in Def. 10.1 or from Eq. (12.19), i.e., the spacetime trajectory of the observed object P.

The Standard Time of GOR: if O and P are relatively static in the free spacetime S_F, then $g_{\mu\nu}=\eta_{\mu\nu}$ and $\mathrm{d}x^i=0$ ($i=1,2,3$), we have:

$$\begin{cases} \mathrm{d}s^2 = g_{\mu\nu}(\eta)\mathrm{d}x^\mu \mathrm{d}x^\nu = \left(\mathrm{d}x^0(\eta)\right)^2 \\ \mathrm{d}t(\eta) = \dfrac{\mathrm{d}x^0}{\eta} = \dfrac{\mathrm{d}s}{\eta} = \mathrm{d}\tau \end{cases} \tag{12.20}$$

where the observed time $\mathrm{d}t$ of O is exactly the intrinsic time or the standard time $\mathrm{d}\tau$, consistent with the concept of the standard time in Einstein' general relativity.

The Coordinate Time of GOR: if O and P are static at the specific coordinates $\mathrm{d}x^i$ ($i=1,2,3$) of the observational spacetime $X^{4\mathrm{d}}(\eta)$, then $\mathrm{d}x^i=0$ ($i=1,2,3$), we have:

$$\begin{cases} \mathrm{d}s^2 = g_{\mu\nu}(\eta)\mathrm{d}x^\mu \mathrm{d}x^\nu = g_{00}(\eta)\left(\mathrm{d}x^0\right)^2 \\ \mathrm{d}t(\eta) = \dfrac{\mathrm{d}x^0}{\eta} = \dfrac{\mathrm{d}s}{\eta\sqrt{g_{00}}} = \dfrac{\mathrm{d}\tau}{\sqrt{1+2\chi/\eta^2}} \quad \left(g_{00} = 1+\dfrac{2\chi}{\eta^2}\right) \end{cases} \tag{12.21}$$

where the observed time $\mathrm{d}t$ of O is the GOR coordinate time $\mathrm{d}t(\eta)$ of the general observation agent OA(η), consistent with the concept of the coordinate time in Einstein' general relativity: $\mathrm{d}t(c)=\mathrm{d}\tau/\sqrt{(1+2\chi/c^2)}$.

The Observed Time of GOR: Even if O and P are instantaneously located at the identical coordinates $\mathrm{d}x^i$ ($i=1,2,3$) of $X^{4\mathrm{d}}(\eta)$, it does not necessarily imply that they are relatively static; it is the most general situation of the observed time $\mathrm{d}t$ of O that $\mathrm{d}t=\mathrm{d}x^0/\eta$, obeying Def. 2.2 of Chapter 2. If only considering the case of orthogonal spacetime, we have:

$$\begin{cases} \mathrm{d}s^2 = g_{\mu\nu}(\eta)\mathrm{d}x^\mu \mathrm{d}x^\nu = g_{00}\mathrm{d}x^0\mathrm{d}x^0 + g_{ik}\mathrm{d}x^i\mathrm{d}x^k \\ \mathrm{d}t(\eta) = \dfrac{\mathrm{d}x^0}{\eta} = \dfrac{\mathrm{d}x^0}{\mathrm{d}s}\mathrm{d}\tau = \dfrac{\mathrm{d}\tau}{\sqrt{g_{00} + g_{ik}\dfrac{\mathrm{d}x^i}{\mathrm{d}x^0}\dfrac{\mathrm{d}x^k}{\mathrm{d}x^0}}} = \dfrac{\mathrm{d}\tau}{\sqrt{1+2\chi/\eta^2 - v^2/\eta^2}} \end{cases} \tag{12.22}$$

which involves the case of O in moving relative to P (v is the instantaneous speed of O relative to P).

It is worth noting that Eq. (12.22) of the general observation agent OA(η) in the theory of GOR is isomorphically consistent with Eq. (12.11) of the optical agent OA(c) in Einstein's theory of general relativity: $\mathrm{d}t(c)=\mathrm{d}\tau/\surd(1+2\chi/c^2-v^2/c^2)$.

As shown in Eqs. (12.20-22), the GOR time, i.e., the observed time of GOR in Eq. (12.22), generalizes the standard time of GOR and the coordinate time of GOR: if $v=0$, then Eq. (12.22) reduces to Eq. (12.21), the observed time $\mathrm{d}t=\mathrm{d}x^0/\eta$ of GOR is namely the coordinate time of GOR; if $v=0$ and $\chi=0$, then Eq. (12.22) reduces to Eq. (12.20), the observed time $\mathrm{d}t=\mathrm{d}x^0/\eta$ of GOR is namely the standard time $\mathrm{d}\tau=\mathrm{d}s/\eta$ of GOR.

The concept of observed time is the important concept of time in the theory of OR (including IOR and GOR). The concept of observed time of GOR in Def. 2.2 is the time of the general observation agent OA(η), suitable for all observation agents, including the optical observation agent OA(c) and the idealized observation agent OA_∞. The observed time dt of GOR in Eq. (12.22) is the time observed by the observer O in the observational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η). Like in Einstein's theory of relativity, the observed time dt in the theory of OR (including IOR and GOR) also tends to dilate: if $v\neq0$ or $\chi\neq0$, then $\mathrm{d}t>\mathrm{d}\tau$; in the view of observers, the standard clock goes faster.

In particular, Eq. (12.22) shows that:

(i) In inertial spacetime ($\chi=0$): $\mathrm{d}t=\mathrm{d}\tau/\surd(1-v^2/\eta^2)\geq\mathrm{d}\tau$.

This means that, if an observer in moving ($v\neq0$) observes the standard clock static in the free spacetime S_F, then the observed time dt is greater than the standard time dτ. In other words, in the view of inertial observers, the clock static in inertial spacetime goes faster. This is consistent with the cognition or judgment of Einstein's theory of special relativity: the clock in moving goes slower.

(ii) In gravitational spacetime ($v=0$): $\mathrm{d}t=\mathrm{d}\tau/\surd(1+2\chi/\eta^2)\geq\mathrm{d}\tau$.

This means that, if an observer in potential field ($\chi\neq0$) observes the standard clock static in the free spacetime S_F, then the observed time dt is greater than the standard time dτ. In other words, in the view of observers in potential field or gravitational field, the clock static at the null potential goes faster. This is consistent with the cognition or judgment of Einstein's theory of general relativity: the clock in potential field or gravitational field goes slower.

However, it is worth noting that, Eqs. (12.20-22) shows that such phenomena of time dilation depend on observation: the observed times of different observation agents dilate in different degrees; in particular, as $\eta\to\infty$, the observational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η) would revert to the Galilean spacetime $X^{4\mathrm{d}}_\infty$, in which the observed time of GOR is exactly the intrinsic time or the standard time, that is, the objective real time: $\mathrm{d}t=\mathrm{d}\tau$.

There is the corresponding relationship of isomorphic consistency between the GOR time and Einstein's time of general relativity. Therefore, Einstein's standard time and coordinate time of general relativity could correspondingly be transformed from the optical observation agent OA(c) to the general observation agent OA(η). In particular, Einstein's logic way of general relativity for measuring or determining the observational spacetime $X^{4d}(c)$ the optical agent OA(c) could be extended to the theory of GOR for measuring or determining the observational spacetime $X^{4d}(\eta)$ of the general observational agent OA(η).

12.3.3 The Determination of the Standard Time in GOR

Actually, as stated in Sec. 12.3.2: the GOR time, i.e., $dt=dx^0/\eta$ in Eq. (12.19), should be the observed time of GOR, $d\tau=ds/\eta$ in Eq. (12.20) the standard time of GOR, and $dt=dx^0/(\eta\sqrt{(g_{00})})$ in Eq. (12.21) the coordinate time of GOR.

Like the observational spacetime $X^{4d}(c)$ of Einstein's general relativity, the GOR observational spacetime $X^{4d}(\eta)$ is also gravitational spacetime: different space coordinates have different gravitational potentials; hence, different space coordinates have different time rates. Thus, the theory of GOR could not directly employ the coordinate time dt or the observed time dt of OA(η) to describe or determine the motion of the observed object P.

So, like Einstein's theory of general relativity, the theory of GOR also needs the uniform standard time.

Under the principle of GC, through PGC logic route 1, by directly substituting η for c, Einstein's standard time $d\tau=ds/c$ could be transformed into the standard time of GOR: $d\tau=ds/\eta$. Alternatively, under the principle of GC, through PGC logic route 2, analogizing or following the logic of Einstein's general relativity: the standard clock is static in S_F, $g_{\mu\nu}=\eta_{\mu\nu}$ and $dx^i=0$ (i=1,2,3), one could derive the objectively real time $d\tau$, i.e., the standard time of GOR, from Eq. (12.19):

$$\begin{cases} ds^2 = g_{\mu\nu}(\eta)dx^\mu dx^\nu = \eta^2 d\tau^2 \\ d\tau = \dfrac{ds}{\eta} \end{cases} \tag{12.23}$$

According to Def. 2.3 in Chapter 2, the clock at rest in the free spacetime S_F is namely the standard clock.

Suppose that the intrinsic clock T_P of the observed object P is originally static in S_F, then T_P is the standard clock, and the observed time of the observer O in S_F (at the null potential (χ=0)) static relative to T_P is the standard time. However, in the theory of GOR, the observational spacetime $X^{4d}(\eta)$ of OA(η) is gravitational spacetime, and the observer O at any specific space coordinate is located in the gravitational field ($\chi\neq0$); therefore, the observed time dt of O is not the standard time $d\tau$. If P is static at the specific space coordinate x^i (i=1,2,3) and O is the observer static at x^i (i=1,2,3), then the observed time dt of O is namely the coordinate time of O. So, in the theory of GOR, the observed time $dt=dx^0/\eta$ of the observation agent OA(η) has to be standardized.

Under the principle of GC, through PGC logic route 2, by analogizing or following the logic of Einstein's general relativity, based on the principle of equivalence and the invariance of information-wave speeds, the theory of GOR could transform the observed time $dt=dx^0/\eta$ (Eq. (12.19)) of observers in the observational spacetime $X^{4d}(\eta)$ of OA(η) into the standard time $d\tau$.

Let D_0 ($x^i(t)$ (i=1,2,3)) be the space coordinate of the observed object P at the specific time $t=x^0/\eta$. The observer O at D_0 is located at potential χ of the gravitational spacetime $S_G=X^{4d}(\eta)$, and measures or determines the time in the GOR gravitational spacetime by means of the observation agent OA(η); as stated above, the observed time dt of O is not the standard time dτ. As depicted in Fig. 12.1(b1), under the principle of equivalence, one could introduce a locally inertial spacetime S_I at D_0, in which P and O are instantaneously static at D_0, so that the invariance of information-wave speeds holds true.

Like in Einstein's theory of general relativity, the principle of equivalence embodies its special value and significance: under the principle of equivalence, the GOR gravitational spacetime S_G is instantaneously and locally transformed into the equivalent inertial spacetime S_I.

According to Eq. (12.23), the standard time dτ is proportional to the line-element ds of the observed object P in the observational spacetime $X^{4d}(\eta)$ of the observation agent OA(η). As depicted in Fig. 12.1(b1), let L_G be the world line of P in the gravitational spacetime S_G of OA(η), and L_I be the world line of P in the equivalent inertial spacetime S_I. Reasonably, the standard time in S_G could be defined as $\mathrm{d}\tau_G=\mathrm{d}s_G/\eta$; the standard time in S_I can be defined as $\mathrm{d}\tau_I=\mathrm{d}s_I/\eta$.

Obviously, according to the definition of the standard time dτ in Eq. (12.23), the dτ_I in inertial spacetime S_I is exactly the intrinsic time (proper time), i.e. the objectively real time, dτ: $\mathrm{d}\tau_I=\mathrm{d}\tau$.

According to differential geometry, the line-element ds_G of the curve L_G is equal to the line-element ds_I of the tangent L_I to L_G: $\mathrm{d}s_G=\mathrm{d}s_I$. So, it follows that

$$\mathrm{d}\tau_G=\frac{\mathrm{d}s_G}{\eta}=\frac{\mathrm{d}s_I}{\eta}=\mathrm{d}\tau_I=\mathrm{d}\tau \tag{12.24}$$

Equation (12.24) is of the important and profound implication: different observational spacetimes $X^{4d}(\eta)$ of the general observation agent OA(η), including S_G and S_I, has the identical standard time, which is exactly the objectively real time, i.e., the intrinsic time (proper time) dτ.

Under the principle of equivalence, the gravitational spacetime $S_G=X^\mu(\eta)$ becomes the inertial spacetime S_I locally equivalent S_G, in which the observed object P and the observer O located at D_0 (x^i) is instantaneously static in S_I: $\mathrm{d}x^i=0$ (i=1,2,3). Therefore, according to Eq. (12.19) and Eqs. (12.23-24), the standard time dτ of GOR could be determined with the coordinate time dt of GOR:

$$\begin{aligned}\mathrm{d}\tau&=\frac{\mathrm{d}s}{\eta}=\frac{1}{\eta}\sqrt{g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu}\\&=\frac{1}{\eta}\sqrt{g_{00}}\mathrm{d}x^0=\sqrt{g_{00}}\mathrm{d}t=\sqrt{1+\frac{2\chi}{\eta^2}}\mathrm{d}t\quad\left(g_{00}=1+\frac{2\chi}{\eta^2}\right)\end{aligned} \tag{12.25}$$

where χ is the gravitational potential at the space coordinate $D_0(x^i)$ of $X^{4d}(\eta)$.

Obviously, the standard time of GOR in Eq. (12.25) is isomorphically consistent with Einstein's standard time in Eq. (12.6) of general relativity.

Equation (12.25) suggests that the standard time is the objectively real time, could be measured or determined not only by the optical observation agent OA(c), but also, in theory, by any observation agent OA(η).

It should be pointed out that, in the theory of GOR, the invariance of information-wave speeds plays an important role in the determination of time.

As stated in Sec. 12.3.2, the coordinate framework of 4d spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) is a sort of formalized expression of the invariance of information-wave speeds, in which the time axis $x^0=\eta t$ implies the invariance of information-wave speeds. The standard time $d\tau$ of GOR in Eq. (12.25) is derived based on the time-element $dt=dx^0/\eta$ (Eq. (12.19)) and the line-element $ds=g_{\mu\nu}dx^\mu dx^\nu$ (Eq. (12.19)) of the observed object P in the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η).

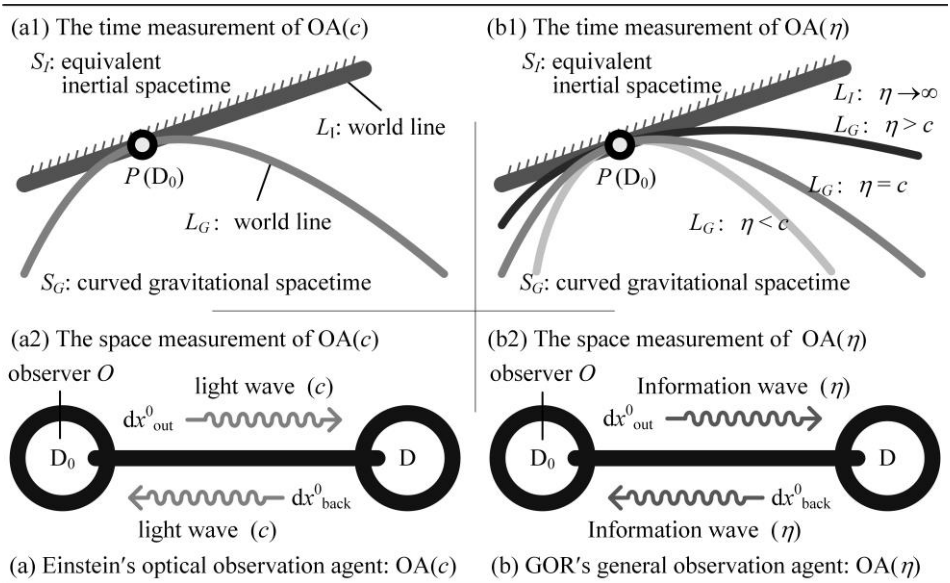

Figure. 12.1 The Measurement of Observational Spacetimes. (a) The observational spacetime of Einstein's general relativity: it employs the optical observation agent OA(c) to measure time and space; the speed c of light is limited ($c<\infty$), and therefore, OA(c) has the observational locality and the gravitational spacetime looks a litter curved. (b) The observational spacetime of GOR: it employs the general observation agent OA(η) to measure time and space, the slower the information-wave speed η, the more significant the observational locality of OA(η), the more curved the gravitational spacetime appears to be; under the idealized observation agent OA_∞, as $\eta\to\infty$, without observational locality, the gravitational spacetime would revert to flat.

12.3.4 The Determination of the Physical Space in GOR

Under the principle of GC, through PGC logic route 2, by analogizing or following Einstein's logic of general relativity, and by means of the principle of equivalence, the theory of GOR could transform the gravitational spacetime $S_G=X^{4d}(\eta)$ of OA(η) into the equivalent inertial spacetime S_I at D_0 instantaneously and locally. Thus, based on the invariance of information-wave speeds, one could define and determine the physical space distance dl of the gravitational spacetime $S_G=X^\mu(\eta)$ of OA(η) with the information-wave speed η and the standard time dτ in Eq. (12.25) of GOR theory.

Following the logic of the determination of physical space in Einstein's general relativity, as stated in Sec. 12.2.3 and as depicted in Fig. 12.1(b2), let D be a space point close enough to D_0 in the observational spacetime $S_G=X^{4\mathrm{d}}(\eta)$; send an information wave from D_0 to D and then be reflected from D back to D_0. The time the information wave takes obeys:

$$\mathrm{d}x^0(\eta)=\mathrm{d}x^0_{\text{out}}(\eta)+\mathrm{d}x^0_{\text{back}}(\eta)\quad\left(\mathrm{d}x^0=\eta\mathrm{d}t\right)\tag{12.26}$$

where the gravitational spacetime S_G is not necessarily isotropic, and therefore, the time the dx^0_{out} takes is not necessarily equals to the time the dx^0_{back} takes.

If the spatial displacement from D_0 to D is dx^i (i=1,2,3), then the spatial displacement from D to D_0 is –dx^i (i=1,2,3). Under the general observation agent OA(η), the spacetime line-element ds^2=0 of informons, and therefore, we have:

$$\begin{gathered}\begin{cases}0=g_{00}\left(\mathrm{d}x^0_{\text{out}}\right)^2+2g_{0i}\mathrm{d}x^i\mathrm{d}x^0_{\text{out}}+g_{ik}\mathrm{d}x^i\mathrm{d}x^k\\0=g_{00}\left(\mathrm{d}x^0_{\text{back}}\right)^2-2g_{0i}\mathrm{d}x^i\mathrm{d}x^0_{\text{back}}+g_{ik}\mathrm{d}x^i\mathrm{d}x^k\end{cases}\\\mathrm{d}x^0(\eta)=\mathrm{d}x^0_{\text{out}}(\eta)+\mathrm{d}x^0_{\text{back}}(\eta)=2\frac{\sqrt{\left(g_{0i}g_{0k}-g_{00}g_{ik}\right)\mathrm{d}x^i\mathrm{d}x^k}}{g_{00}}\end{gathered}\tag{12.27}$$

Particularly, according to the principle of equivalence and the principle of general covariance, the locally inertial spacetime S_I at D_0 must be isotropic, in which the invariance of information-wave speeds holds true. Therefore, the physical space distance dl between D_0 and D can be defined as d$l=\eta$d$\tau/2$ based on the proper time dτ and the information-wave speed η.

According to Eq. (12.25), the standard time (proper time) d$\tau=\sqrt{(g_{00})}\mathrm{d}x^0/\eta$. Then, the GOR physical space distance dl could be measured or determined as follows:

$$\begin{gathered}\mathrm{d}l=\eta\frac{\mathrm{d}\tau}{2}=\frac{\sqrt{g_{00}}}{2}\mathrm{d}x^0=\sqrt{\left(\frac{g_{0i}g_{0k}}{g_{00}}-g_{ik}\right)\mathrm{d}x^i\mathrm{d}x^k}\\\text{or }\ \mathrm{d}l^2=\gamma_{ik}(\eta)\mathrm{d}x^i\mathrm{d}x^k\quad\left(\gamma_{ik}(\eta)=\frac{g_{0i}g_{0k}}{g_{00}}-g_{ik}\right)\end{gathered}\tag{12.28}$$

where $\gamma_{ik}=\gamma_{ik}(\eta)$ (i,k=1,2,3) is the physical space metric in the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η).

Obviously, the physical space distance of GOR in Eq. (12.28) is isomorphically consistent with Einstein's physical space distance in Eq. (19.9) of general relativity.

It should be pointed out that: the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(c)$ and the space metric $\gamma_{ik}=\gamma_{ik}(c)$ in Eq. (12.9) are that of the observational spacetime $X^{4\mathrm{d}}(c)$ of the optical observation agent OA(c), depending on the light speed c; while the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$ and the space metric $\gamma_{ik}=\gamma_{ik}(\eta)$ in Eq. (12.28) are that of the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η), depending on OA(η) and its information-wave speed η.

12.4 The GOR Factor of Spacetime Transformation

Observation means spacetime transformation: from the objective spacetime to the subjective spacetime or the observational spacetime.

All matter objects originally belong to the objective spacetime $X^{4d}{}_{\infty}$, more purely, belong to the intrinsic spacetime O_o or the free spacetime S_F. An observer O perceives or observes a specific matter object P by means of a specific observation agent OA(η), which actually means that the observer O is transforming the observed object P from the objective spacetime $X^{4d}{}_{\infty}$ to the subjective or observational spacetime $X^{4d}(\eta)$ of OA(η), or more purely, from the intrinsic spacetime O_o or the free spacetime S_F to observational spacetime $X^{4d}(\eta)$ of OA(η).

The factors of spacetime transformation in the theory of OR include the IOR factor and the GOR factor.

As clarified in **the 1st volume of OR: Inertially Observational Relativity** (IOR), the IOR factor is that of inertial spacetime transformation, an extremely important physical quantity in the theory of IOR, characterizing the relativistic effects of inertial spacetime and inertial motion. Now, in **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR), the GOR factor is that of gravitational spacetime transformation, an extremely important physical quantity in the theory of GOR, characterizing the relativistic effects of gravitational spacetime and gravitational interaction.

The factors of spacetime transformation in the theory of OR is defined as the ratio of the observed time dt in the observational spacetime $X^{4d}(\eta)$ to the standard time dτ in the intrinsic spacetime O_o of the objective spactime $X^{4d}{}_{\infty}$: $\Gamma(\eta)=\mathrm{d}t/\mathrm{d}\tau$, i.e., the ratio of the observed time dt of the observer O to the objectively real time dτ.

In the theory of GOR, the GOR spacetime is the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η), belongs to gravitational spacetime, and could be set as different gravitational scenes. Suppose P is the observed objects: (i) P is at rest in gravitational field; (ii) P moves in gravitational scalar-field; or (iii) P moves in gravitational vector-field. Thus, like Einstein's general relativity, the theory of GOR also has different forms of spacetime-transformation factor.

Actually, the GOR factor may be referred to as the OR factor of spacetime transformation, which not only generalizes the factor of spacetime transformation of Einstein’s general relativity, but also generalizes the IOR factor (including the Lorentz factor and the Galilean factor), i.e., the OR factor of inertial spacetime transformation.

12.4.1 The IOR Factor: Inertial Spacetime

As stated in **the 1st volume of OR: Inertially Observational Relativity** [26-28], the IOR factor $\Gamma(\eta,v)$ is that of inertial-spacetime transformation of the general observation agent OA(η), generalizing the Lorentz factor $\gamma=1/\sqrt{(1-v^2/c^2)}$ of the optical agent OA(c) and the Galilean factor $\Gamma_\infty\equiv 1$ of the idealized agent OA_∞.

Suppose that the observer O observes the object P by means of a specific observation agent OA(η): the intrinsic clock T_P of P is the standard clock; O moves at the inertial speed v relative to P. This implies that the observer O transforms the observed object P by means of OA(η) from the free spacetime S_F where P is at rest to the inertial spacetime $S_I=X^{4d}(\eta)$ where O moves.

In Einstein's theory of special relativity, the factor of spacetime transformation is namely the Lorentz factor: $\gamma=\mathrm{d}t/\mathrm{d}\tau=1/\sqrt{(1-v^2/c^2)}$, i.e., the factor $\gamma=\Gamma(c,v)$ of inertial spacetime transformation of the optical agent OA(c), characterizing the inertial relativistic effects of P in the inertial spacetime $X^{4d}(c)$ of OA(c).

Actually, the Lorentz factor $\gamma=\Gamma(c,v)$ is only a special case of the IOR factor.

According to **the 1st volume of OR: Inertially Observational Relativity** [26-28], the IOR factor of spacetime transformation is

$$\Gamma(\eta,v)=\frac{\mathrm{d}t(\eta)}{\mathrm{d}\tau}=\frac{1}{\sqrt{1-v^2/\eta^2}} \tag{12.29}$$

characterizing the inertial relativistic effects of P presenting to the observer O in the inertial spacetime $X^{4d}(\eta)$ of the general observation agent OA(η).

Originally, the factor $\Gamma(\eta)$ of spacetime transformation of OA(η) in Eq. (12.29) is a logical consequence derived from the most basic logical premise of OR theory [26-28], but it also follows the principle of GC.

Equations (12.29) and (12.10) show that the IOR factor $\Gamma(\eta,v)$ in theory of IOR is isomorphically consistent with the Lorentz factor $\gamma=\Gamma(c,v)$ in Einstein's theory of special relativity. This not only reflects the logical validity of the principle of GC, but also reflects the logical consistency between the theory of IOR and Einstein's theory of special relativity.

Actually, under the principle of GC, through PGC logic route 1, by substituting the information-wave speed η for the light speed c, one could directly transform the Lorentz factor $\gamma=1/\sqrt{(1-v^2/c^2)}$, from the optical agent OA(c) to the general observation agent OA(η), from Einstein's theory of special relativity to the theory of IOR: $\Gamma(\eta)=1/\sqrt{(1-v^2/\eta^2)}$ in Eq. (12.29); vice versa.

Obviously, under the principle of GC, through PGC logic route 2, by transforming the invariance of light speed into the invariance of information-wave speeds and following the logic of Einstein's special relativity, one could also derive the IOR factor of inertial spacetime transformation in Eq. (12.29). In particular, following the PGC logic Route 2 is more helpful for us to understand the factor of inertial spacetime transformation of the general observation agent OA(η) in the theory of IOR.

12.4.2 The GOR Factor: Static Field

Suppose that the observer O observes the object P by means of a specific observation agent OA(η): the intrinsic clock T_P of P is the standard clock; O is located at the potential χ in the gravitational spacetime $S_G=X^{4d}(\eta)$, and static relative to P; S_G is a static field. This implies that the observer O transforms the observed object P by means of OA(η) from the free spacetime S_F where P is at rest to the gravitational spacetime $S_G=X^{4d}(\eta)$ where O is located at the potential χ.

In Einstein's theory of general relativity, the factor $\gamma=\mathrm{d}t/\mathrm{d}\tau=1/\sqrt{(1+2\chi/c^2)}$ of spacetime transformation is the factor of gravitational spacetime transformation of the optical agent OA(c), characterizing the gravitational relativistic effects of P presenting to O in the static gravitational spacetime $X^{4d}(c)$ of the optical agent OA(c) when P and O are relatively static.

Under the principle of GC, through PGC logic route 1, by substituting the information-wave speed η for the light speed c in $\gamma=\Gamma(c)$, one could directly obtain the GOR factor $\Gamma(\eta)=\mathrm{d}t/\mathrm{d}\tau=1/\sqrt{(1+2\chi/\eta^2)}$ of gravitational spacetime transformation from Eq. (12.11) in Einstein's theory of general relativity, characterizing the gravitational relativistic effects of P presenting to O in the gravitational static spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) when P and O are relatively static or stationary.

Under the principle of GC, through PGC logic route 2, by transforming the invariance of light speed into the invariance of information-wave speeds and following the logic of Einstein's general relativity, one could also derive the GOR factor of gravitational spacetime transformation. In particular, following PGC logic Route 2 is more helpful for us to understand the factor of gravitational spacetime transformation of the general observation agent OA(η) in the theory of GOR.

Naturally, by following PGC logic route 2, one could also transform the optical agent OA(c) into the general observation agent OA(η) in Def. 10.1; then, based on Eq. (12.19), one could derive the GOR factor of gravitational spacetime transformation when O and P are relatively static ($\mathrm{d}x^i=0$ (i=1,2,3)):

$$\begin{aligned}\Gamma(\eta,\chi)&=\frac{\mathrm{d}t(\eta)}{\mathrm{d}\tau}=\frac{\mathrm{d}x^0}{\mathrm{d}s}=\frac{\mathrm{d}x^0}{\sqrt{g_{\mu\nu}(\eta)\mathrm{d}x^\mu\mathrm{d}x^\nu}}=\frac{1}{\sqrt{g_{00}(\eta)}}\\&=\frac{1}{\sqrt{1+2\chi/\eta^2}}\quad\left(\mathrm{d}x^i=0\ \ (i=1,2,3);\ g_{00}(\eta)=1+2\chi/\eta^2\right)\end{aligned}\tag{12.30}$$

Obviously, Eq. (12.30) in the theory of GOR is isomorphically consistent with Eq. (12.11) in Einstein's general relativity. So, the result from PGC logic route 2 is exactly the same as that from PGC logic route 1.

12.4.3 The GOR Factor: Scalar Field

Suppose that the observer O observes the object P by means of a specific observation agent OA(η): the intrinsic clock T_P of P is the standard clock; O is located at the potential χ in the gravitational spacetime $S_G=X^{4d}(\eta)$, and moves at the speed v relative to P; S_G is a scalar field. This implies that the observer O transforms the observed object P by means of OA(η) from the free spacetime S_F where P is static to the gravitational spacetime $S_G=X^{4d}(\eta)$ where O is located at the potential χ.

In Einstein's theory of general relativity, if P and O relatively moves in the gravitational scalar spacetime $S_G=X^{4d}(c)$ of the optical agent OA(c), then the factor of spacetime transformation is $\gamma=1/\sqrt{(1+2\chi/c^2-v^2/c^2)}$, characterizing the gravitational relativistic effects of P presenting to O in the gravitational scalar spacetime $S_G=X^{4d}(c)$ of OA(c) when P and O relatively move.

Under the principle of GC, through PGC logic route 1, by substituting the information-wave speed η for the light speed c in $\gamma=\Gamma(c)$, one could directly obtain the GOR factor $\Gamma(\eta)=1/\sqrt{(1+2\chi/\eta^2-v^2/\eta^2)}$ of gravitational spacetime transformation from Eq. (12.12) in Einstein's theory of general relativity, characterizing the gravitational relativistic effects of P presenting to O in the gravitational scalar spacetime $S_G=X^{4d}(\eta)$ of the general observation agent OA(η) when P and O relatively move.

Under the principle of GC, through PGC logic route 2, by transforming the invariance of light speed into the invariance of information-wave speeds and following the logic of Einstein's general relativity, one could also derive the GOR factor of gravitational spacetime transformation when P and O relatively move in the scalar field $S_G= X^{4d}(\eta)$ of the general observation agent OA(η).

If the observational spacetime $S_G=X^{4d}(\eta)$ of OA(η) is a scalar gravitational field, then the gravitational vector potential $\gamma_i=0$ ($g_{0i}=g_{i0}=0$ (i=1,2,3)). According to Eq. (12.19) of definition 1.1, we have:

$$\Gamma(\eta)=\frac{\mathrm{d}t(\eta)}{\mathrm{d}\tau}=\frac{\mathrm{d}x^0}{\mathrm{d}s}=\frac{\mathrm{d}x^0}{\sqrt{g_{\mu\nu}(\eta)\mathrm{d}x^\mu\mathrm{d}x^\nu}}=\frac{1}{\sqrt{g_{00}+g_{ik}\dfrac{\mathrm{d}x^i}{\mathrm{d}x^0}\dfrac{\mathrm{d}x^k}{\mathrm{d}x^0}}} \tag{12.31}$$

$$\left(g_{0i}=g_{i0}=0\quad(i=1,2,3)\right)$$

By contrasting Eq. (12.31) and Eqs. (12.27-28), we have:

$$\Gamma(\eta,\chi,v)=\frac{\mathrm{d}t(\eta)}{\mathrm{d}\tau}=\frac{1}{\sqrt{1+2\chi/\eta^2-v^2/\eta^2}}$$
$$\left(v=\frac{\mathrm{d}l}{\mathrm{d}t},\ \mathrm{d}l=\sqrt{\left(\frac{g_{0i}g_{0k}}{g_{00}}-g_{ik}\right)\mathrm{d}x^i\mathrm{d}x^k}=\sqrt{-g_{ik}\mathrm{d}x^i\mathrm{d}x^k}\right) \tag{12.32}$$

where v=dl/dt is the speed of O relative to P, dl the physical space distance that P moves in the observation spacetime $X^{4d}(\eta)$ within the time dt.

Obviously, Eq. (12.32) in the theory of GOR is isomorphically consistent with Eq. (12.12) in Einstein's general relativity. So, the result from PGC logic route 2 is exactly the same as that from PGC logic route 1.

12.4.4 The GOR Factor: Vector Field

Perhaps, as Einstein imagined, gravitational field is similar to electromagnetic field, not only has the scalar potential but also has the vector potential.

Suppose that the observer O observes the object P by means of a specific observation agent OA(η): the intrinsic clock T_P of P is the standard clock; O is located at the potential χ in the gravitational spacetime $S_G=X^{4d}(\eta)$, and moves at the speed v relative to P; S_G is a vector field. This implies that the observer O transforms the observed object P by means of OA(η) from the free spacetime S_F where P is static to the gravitational spacetime $S_G=X^{4d}(\eta)$ where O is located at the potential χ.

In Einstein's theory of general relativity, if P and O relatively moves in the gravitational vector spacetime $S_G=X^{4d}(c)$ of the optical agent OA(c), then the factor of spacetime transformation is $\gamma=1/\surd(((1+2\chi/c^2)^{1/2}-\gamma_i v^i/c)^2 -v^2/c^2)$, characterizing the gravitational relativistic effects of P presenting to O in the gravitational vector spacetime $S_G=X^{4d}(c)$ of OA(c) when P and O relatively move.

Under the principle of GC, through GC logic route 1, by substituting the information-wave speed η for the light speed c in $\gamma=\Gamma(c)$, one could directly obtain the GOR factor $\Gamma(\eta)=1/\surd(((1+2\chi/\eta^2)^{1/2}-\gamma_i v^i/\eta)^2-v^2/\eta^2)$ of gravitational spacetime transformation from Eq. (12.16) in Einstein's theory of general relativity, characterizing the gravitational relativistic effects of P presenting to the observer O in the gravitational vector spacetime $S_G=X^{4d}(\eta)$ of the general observation agent OA(η) when P and O relatively move.

Under the principle of GC, through PGC logic route 2, by transforming the invariance of light speed into the invariance of information-wave speeds and following the logic of Einstein's general relativity, one could also derive the GOR factor of gravitational-spacetime transformation when P and O relatively move in the vector field $S_G=X^{4d}(\eta)$ of the general observation agent OA(η).

Following the definition of the strength of electrostatic field, the field strength **g** of any spatial coordinates x^i (i=1,2,3) in gravitational field can be defined as:

$$\begin{aligned} &\mathbf{g}=\lim_{\Delta m\to 0}\frac{\Delta\mathbf{F}}{\Delta m}\quad(\Delta\mathbf{F}=\Delta m\mathbf{a}) \\ &a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2 l}{\mathrm{d}t^2}=\sqrt{\gamma_{ik}(\eta)a^i a^k}\quad\left(a^i(\eta)=\frac{\mathrm{d}^2x^i}{\mathrm{d}t^2},\, a_i(\eta)=\gamma_{ik}a^k(\eta)\right) \end{aligned}\tag{12.33}$$

where Δm is mass, there is no distinction between gravitational mass and inertial mass according to the principle of equivalence; the gravitational field strength **g** is equivalent to the gravitational acceleration **a** of the point.

Under the principle of GC, Eq. (12.14) in Einstein's general relativity can isomorphically and uniformly be transformed into:

$$a_i=-\frac{\partial\chi}{\partial x^i}-\eta\sqrt{1+\frac{2\chi}{\eta^2}}\frac{\partial\gamma_i(\eta)}{\partial t}\quad\left(\gamma_i(\eta)\equiv-\frac{g_{0i}}{\sqrt{g_{00}}}\right)\tag{12.34}$$

where $\gamma_i=\gamma_i(\eta)$ (i=1,2,3) is the vector potential of gravitational field under the general observation agent OA(η), which depends on the information-wave speed η of OA(η).

According to Def. 10.1 for the general observation agent OA(η): the observed time is $\mathrm{d}t=\mathrm{d}x^0/\eta$, the intrinsic time is $\mathrm{d}\tau=\mathrm{d}s/\eta$; the observed object P moves in the observational spacetime $X^{4d}(\eta)$ of OA(η), and its spacetime trajectory, including the time-element dt of P and the line-element ds of P, is described by Eq. (12.19).

Considering the scalar potential χ and the vector potential γ_i (i=1,2,3) of the gravitational field in Eq. (12.34), according to Eq. (12.19), we have:

$$\Gamma(\eta)=\frac{\mathrm{d}t(\eta)}{\mathrm{d}\tau}=\frac{\mathrm{d}x^0}{\mathrm{d}s}=\frac{\mathrm{d}x^0}{\sqrt{g_{\mu\nu}(\eta)\mathrm{d}x^\mu\mathrm{d}x^\nu}}=\frac{\mathrm{d}x^0}{\sqrt{g_{00}\mathrm{d}x^0\mathrm{d}x^0+2g_{0i}\mathrm{d}x^0\mathrm{d}x^i+g_{ik}\mathrm{d}x^i\mathrm{d}x^k}}$$

$$=\frac{1}{\sqrt{\left(\sqrt{g_{00}}+\frac{g_{0i}}{\sqrt{g_{00}}}\frac{\mathrm{d}x^i}{\mathrm{d}x^0}\right)^2+\left(g_{ik}-\frac{g_{0i}g_{0k}}{g_{00}}\right)\frac{\mathrm{d}x^i}{\mathrm{d}x^0}\frac{\mathrm{d}x^k}{\mathrm{d}x^0}}} \qquad (12.35)$$

By contrasting Eq. (12.25), Eq. (12.28), and Eqs. (12.34-35), we have the general factor of spacetime transformation in the theory $\Gamma(\eta)$ of OR (generalizing the IOR factor and the GOR factor):

$$\Gamma(\eta,\chi,\gamma_i,v)=\frac{\mathrm{d}t(\eta)}{\mathrm{d}\tau}=\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{\eta^2}}-\gamma_i\frac{v^i}{\eta}\right)^2-\frac{v^2}{\eta^2}}} \qquad (12.36)$$

where η is the information-wave speed of OA(η); dt=dx^0/η is the observed time of the observer O with OA(η), i.e., the time the intrinsic clock T_P of the observed object P as the standard clock presents to O; dτ=ds/η is the intrinsic time (proper time), i.e., the objectively real time; v=dl/dt is the speed of P relative to O, v^i=dx^i/dt (i=1,2,3); χ is the Newtonian gravitational potential, i.e., the scalar potential gravitational field; and $\gamma_i=\gamma_i(\eta)=-g_{0i}/\sqrt{(g_{00})}$ (i=1,2,3) is the vector potential of gravitational field, depending on the observation agent OA(η).

Obviously, Eq. (12.36) in the theory of GOR is isomorphically consistent with Eq. (12.16) in Einstein's general relativity. So, the result from PGC logic route 2 is exactly the same as that from PGC logic route 1. It should be pointed out that the GOR factor $\Gamma(\eta)$ of spacetime transformation in Eq. (12.36) is that of the general observation agent OA(η), that is, the most general form of the factor of spacetime transformation.

Firstly, the OR (both IOR and GOR) factor (Eq. (12.36)) generalizes the factors of spacetime transformation under different observation agents OA(η), including the optical agent OA(c) and the idealized agent OA_∞: as $\eta\to c$, Eq. (12.36) strictly reduces to Einstein's factor of spacetime transformation in Eq. (12.16); as $\eta\to\infty$, Eq. (12.36) strictly reduces to the Galilean factor Γ_∞ (≡1).

Secondly, the OR (both IOR and GOR) factor (Eq. (12.36)) generalizes the factors of inertial and gravitational spacetime transformations: without gravitational interaction (χ=0 and γ_i=0), Eq. (12.36) strictly reduces to the IOR factor $\Gamma(\eta)=1/\sqrt{(1-v^2/\eta^2)}$ (Eq. (12.29)) of inertial spacetime transformation; without relative movement, Eq. (12.36) strictly reduces to the GOR factor $\Gamma(\eta)=1/\sqrt{(1+2\chi/\eta^2)}$ (Eq. (12.30)) of gravitational spacetime transformation.

In addition, the OR (both IOR and GOR) factor (Eq. (12.36)) generalizes the factors of gravitational scalar spacetime (χ) vector spacetime (γ_i (i=1,2,3)). If γ_i=0

(i=1,2,3), then Eq. (12.36) strictly reduces to the GOR factor of scalar spacetime transformation in Eq. (12.32): $\Gamma(\eta)=1/\sqrt{(1+2\chi/\eta^2-v^2/\eta^2)}$.

12.5 All Relativistic Effects are Observational Effects and Apparent Phenomena

The GOR factor of spacetime transformation has important implications, giving us new insights different from Einstein's theory of general relativity.

In Einstein's theory of relativity, the speed c of light in vacuum is a cosmic constant and the ultimate speed of the universe, which is invariant and could not be exceeded. Therefore, according to Eq. (12.12): $\gamma=1/\sqrt{(1+ 2\chi/c^2-v^2/c^2)}$, in Einstein's theory of general relativistic, the factor $\gamma=\gamma(v,\chi)$ of spacetime transformation does not seem to depend on the light speed c, but depends on the speed v of matter motion and the Newtonian gravitational potential χ: $\gamma>\Gamma_\infty\equiv1$ only if $v\neq0$ or $\chi\neq0$; at this moment, the observed object P presents relativistic phenomena. On these grounds, Einstein believed that, the mainstream school of physics also believes that, relativistic effects are the essential characteristics of spacetime and matter motion, and the root lies in matter motion ($|v|>0$) and the interactions between matter and matter ($|\chi|>0$).

According to the IOR factor, i.e., the OR factor of inertial spacetime transformation, **the 1st volume of OR: Inertially Observational Relativity** has clarified that all inertial relativistic effects, including the invariance of light speed, are observational effects or apparent phenomena [26-28]. The GOR factor, i.e., the OR factor of gravitational spacetime transformation will further clarify that all relativistic phenomena, including the inertial and the gravitational, are observational effects and apparent phenomena.

12.5.1 The Root and Essence of Relativistic Phenomena

Relativistic effects or phenomena, no matter inertial or gravitational, are not the essential characteristics of the physical world.

According to the spacetime theory of GOR, relativistic effects are not as claimed by Einstein and the mainstream school of physics: the root lies in matter motion ($|v|>0$) or matter interactions ($\chi|>0$).

Actually, the spacetime-transformation factor $\gamma=\gamma(c)$ in Einstein's theory of general relativity only characterizes the relativistic effects of the optical agent OA(c). It is the spacetime-transformation factor of the optical observation agent OA(c), and only a special case of the spacetime-transformation factor $\Gamma=\Gamma(\eta)$ of the general observation agent OA(η), i.e., the special case of the OR factor of spacetime transformation, does not suggests that the relativistic effects or phenomena depend on matter motion ($|v|>0$) or matter interactions ($|\chi|>0$).

The theory of OR has clarified that the root and essence of inertial relativistic phenomena lie in the observational locality of observation agents [26-28]. Now, the theory of GOR will further clarify that the root and essence of gravitational relativistic phenomena also lie in the observational locality of observation agents.

Different Observation Agents
Present Different Degrees of Relativistic Effects.

The theory of GOR suggests that different observation agents present different degrees of relativistic effects.

According to Eq. (12.32): $\Gamma=1/\sqrt{(1+2\chi/\eta^2-v^2/\eta^2)}$, the GOR factor $\Gamma=\Gamma(\eta)$ of spacetime transformation depends on the observation agent OA(η). A specific observation agent OA(η) has the specific observation medium and the specific information-wave speed η; correspondingly, the GOR factor $\Gamma=\Gamma(\eta)$ has the specific value, representing specific degree of relativistic effects.

Suppose there are observation agents OA(η_1) and OA(η_2) ($\eta_2>\eta_1$). Given v and χ, the spacetime-transformation factors of OA(η_1) and OA(η_2) have different values: $\Gamma(\eta_1)>\Gamma(\eta_2)\geq\Gamma_\infty\equiv1$. So, in the observational spacetimes $X^{4d}(\eta)$ of different observation agents OA(η), the identical matter motion (v) or the identical matter interaction (χ) presents different degrees of relativistic effects: the lower the information-wave speed η, the more significant the relativistic effects.

This suggests that relativistic effects depend on observation, depends on observations agents, depends on observation media, and depends on the speed at which the observed information is transmitted by observation media. Therefore, the so-called relativistic phenomena are only the observationally relativistic effect presented by observation agents, i.e., observational effects.

There is no Relativistic Phenomena in the Objectively Real Spacetime.

In particularly, as $\eta\to\infty$, $X^{4d}(\eta)\to X^{4d}{}_\infty$: the observational spacetime $X^{4d}(\eta)$ of observation agent OA(η) would revert to the Galilean spacetime, i.e., the objective real spacetime; $\Gamma(\eta)=\Gamma_\infty\equiv1$: the GOR factor $\Gamma(\eta)$ of spacetime transformation would revert to the Galilean factor Γ_∞.

So, there is no relativistic phenomena in the objectively real spacetime.

The GOR factor of spacetime transformation in Eq. (12.32) shows that

(i) $v^2/\eta^2\to0$ as $\eta\to\infty$: in the objectively real spacetime $X^{4d}{}_\infty$, matter motions (v) do not present relativistic phenomena;

(ii) $\chi/\eta^2\to0$ as $\eta\to\infty$: in the objectively real spacetime $X^{4d}{}_\infty$, matter interactions (χ) also do not present relativistic phenomena;

So, as far as the objective and real physical world is concerned, spacetime and matter motion have no relativistic effects or relativistic phenomena.

This suggests that the so-called relativistic effects or relativistic phenomena are not objectively physical reality, but observational effects and apparent phenomena, the root and essence of which lie in the observational locality: the information-wave speed η ($<\infty$) of a realistic observation agent must be finite or limited.

We should realize that although light is extremely fast, the speed of light is still finite: $c<\infty$, and so the optical agent OA(c) still has the observation locality of its own. It is the observational locality ($c<\infty$) of the optical agent OA(c) to be the making of Einstein's theory of relativity, including the special and the general.

Observed Information is Both Objective and Apparent.

As stated in Sec. 7.2.3 of Chapter 7, in a sense, the observational or observed physical quantities are both objective and apparent.

Our observations, characterized by the OR factor $\Gamma(\eta)$ of spacetime transformation, not only contain the objectively real information about spacetime and matter motion, which is characterized by the Galileo factor Γ_∞, but also include the observational effects or apparent phenomena rooted from the observational locality of observation agent OA(η), which are characterized by the observational-effect $\Delta\Gamma(\eta)=\Gamma(\eta)-\Gamma_\infty$.

So, our observations are not completely objective or real.

In the theory of OR, according to Eq. (7.2), the IOR factor $\Gamma(\eta)=1/\sqrt{(1-v^2/\eta^2)}$ of spacetime transformation can be decomposed into the Galilean factor $\Gamma_\infty\equiv1$ and the observational-effect factor $\Delta\Gamma(\eta)$ by Taylor series expansion: $\Gamma(\eta)=\Gamma_\infty+\Delta\Gamma(\eta)$. This suggests that the observation of inertial spacetime by a realistic observation agent OA(η) not only contains objectively real information (Γ_∞) about inertial motion, but also contains the observational effects or apparent phenomena ($\Delta\Gamma(\eta)$) rooted from the observational locality ($\eta<\infty$) of the observation agent OA(η).

Likewise, in the theory of GOR, the GOR factor $\Gamma(\eta)=1/\sqrt{(1+2\chi/\eta^2)}$ of spacetime transformation can be decomposed into the Galilean factor Γ_∞ and the observational-effect factor $\Delta\Gamma(\eta)$ by Taylor series expansion:

$$\Gamma(\eta)=\Gamma_\infty+\Delta\Gamma(\eta)$$
$$\begin{cases} \Gamma_\infty=\lim\limits_{\eta\to\infty}\Gamma(\eta)=1 \\ \Delta\Gamma(\eta)=\dfrac{1}{2}\dfrac{\alpha}{\eta^2}+\dfrac{1\cdot3}{2\cdot4}\dfrac{\alpha^2}{\eta^4}+\dfrac{1\cdot3\cdot5}{2\cdot4\cdot6}\dfrac{\alpha^3}{\eta^6}+\cdots \quad (\alpha=-2\chi) \end{cases} \tag{12.37}$$

where, $\alpha=-2\chi\geq0$ and $\Delta\Gamma(\eta)\geq0$.

Like the IOR observational-effect factor $\Delta\Gamma(\eta)$ in Eq. (7.2), the GOR observational-effect factor $\Delta\Gamma(\eta)$ in Eq. (12.37) represents relativistic effects, purely belonging to observational effects or apparent phenomena, rather than the objectively physical reality; only the Galilean factor $\Gamma_\infty\equiv1$, as stated in the theory of IOR, represents the objectively physical reality.

Like the IOR factor $\Gamma(\eta)=\Gamma_\infty+\Delta\Gamma(\eta)$ of spacetime transformation in Eq. (7.2), the GOR factor $\Gamma(\eta)=\Gamma_\infty+\Delta\Gamma(\eta)$ of spacetime transformation in Eq. (12.37) suggests that the observation of gravitational spacetime by a realistic observation agent OA(η) not only contains objectively real information (Γ_∞) about gravitational interaction, but also contains the observational effects or apparent phenomena ($\Delta\Gamma(\eta)$) rooted from the observational locality ($\eta<\infty$) of the observation agent OA(η).

12.5.2 Could Spacetime Really be Curved?

Spacetime could never be curved.

In fact, **spacetime curvature** is a relativistic phenomenon, a gravitational relativistic effect. Like all relativistic effects, spacetime curvature is also an observation effect or an apparent phenomenon, and the root and essence also lie in the observational locality ($\eta<\infty$) of observation agent OA(η).

As stated in Sec. 12.1.2, gravity, like electromagnetic force, weak force and strong force, is one of the most basic interactions between matter and matter, and is a kind of force, rather than the geometric effect of spacetime curvature. It could be regarded as a formalization method for describing gravitational interaction to geometrize gravitational effects or to make gravitational spacetime equivalent to curved spacetime.

However, spacetime is not really curved.

We are not sure whether Einstein really thought that gravitational spacetime was curved, or Einstein's theory of general relativity needed curved spacetime. According to Einstein's theory of general relativity [8], the accumulation of matter and energy makes spacetime curved, so that the earth moves around the sun in the curved spacetime of the sun. However, we wonder how the sun moves in the curved spacetime of the earth? Perhaps, we could imagine how space is curved; however, in any case, we could not imagine how time is curved.

Now, the theory of GOR tell us that the so-called spacetime curvature in Einstein's general relativity is not due to the accumulation of matter and energy, but a kind of observational effects, rooted from the observational locality ($c<\infty$) of the optical agent OA(c), just as what we observe or photograph through wide-angle lenses, that is, the effect of wide-angle lenses.

The IOR factor $\Gamma(\eta)=1/\sqrt{(1-v^2/\eta^2)}$ of spacetime transformation represents the inertial observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η), in which the spacetime metric $g_{\mu\nu}$ is Minkowski metric: $\eta_{\mu\nu}=\mathrm{diag}(+1,-1,-1,-1)$. Obviously, $\eta_{\mu\nu}$ does not depend on the coordinates x^α ($\alpha=1,2,3,4$) of inertial spacetime and the information-wave speeds η, and therefore, the inertial spacetime $X^{4d}(\eta)$ of IOR is flat, not curved.

The GOR factor $\Gamma=1/\sqrt{(1+2\chi/\eta^2-v^2/\eta^2)}$ of spacetime transformation represents the gravitational observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η), in which the spacetime metric $g_{\mu\nu}$ depend on the coordinates x^α ($\alpha=1,2,3,4$) of gravitational spacetime and the information-wave speeds η, and therefore, the gravitational spacetime $X^{4d}(\eta)$ of GOR appears to be a little curved. It is worth noting that whether spacetime is curved and to what extent spacetime is curved do not really depend on the gravitational potential χ, but depends on the observation agent OA(η), depends on the information-wave speeds η.

According to the GOR factor $\Gamma(\eta)=1/\sqrt{(1+2\chi/\eta^2-v^2/\eta^2)}$ of spacetime transformation, the observational spacetimes of different observation agents presents different degrees of curvature: the slower the information-wave speed η of OA(η), the more curved the observational spacetime $X^{4d}(\eta)$ of OA(η) appears to be; conversely, the more flat. In particularly, if $\eta\to\infty$, then $|\chi|/\eta^2\to 0$, $\Delta\Gamma(\eta)\to 0$, $\Gamma(\eta)\to\Gamma_\infty\equiv 1$, and $X^{4d}(\eta)\to X^{4d}{}_\infty$: the observational spacetime $X^{4d}(\eta)$ of the observation agent OA(η) would revert to the Galilean spacetime $X^{4d}{}_{\infty}$ which is objectively real, and flat, rather than curved.

It is thus clear that the objectively real spacetime could never be curved.

The so-called spacetime curvature is essentially an observation effect. Like all relativistic effects, the root of spacetime curvature lies in the observational locality ($\eta<\infty$) of observation agent OA(η). Under the idealized observation agent OA_∞, the observational spacetime would present the really natural face of its own.

12.5.3 Is There Really the Vector Potential in Gravitational Field?

The gravitational field is not a vector field, and there is no gravitational vector potential in gravitational spacetime.

As stated in Sec. 12.4.4, by following Einstein's logic of general relativity, the theory of GOR could also deduce the factor of spacetime transformation containing the vector potential $\gamma_i=-g_{0i}/\sqrt{(g_{00})}$ (i=1,2,3), which is the spacetime-transformation factor of the general observation agent: $\Gamma(\eta)=1/\sqrt{(((1+2\chi/\eta^2)^{1/2}-\gamma_i v^i/\eta)^2-v^2/\eta^2)}$.

If there is really the vector potential $\gamma_i=-g_{0i}/\sqrt{(g_{00})}$ (i=1,2,3) in gravitational field, then it must be that $\exists i\in(1,2,3)$ $g_{0i}\neq 0$. According to Eq. (12.28), the space metric $\gamma_{ik}=g_{0i}g_{0k}/g_{00}-g_{ik}$ (i,k=1,2,3) must depend on the time axis x^0 of $X^{4d}(\eta)$. This implies that the time axis x^0 and the space axes x^i ($i\in(1,2,3)$) of the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) might be non-orthogonal: time and space might be interdependent.

Conversely, if there is no vector potential in gravitational field, then it must be that $g_{0i}=0$ ($i=(1,2,3)$). According to Eq. (12.28), the space metric $\gamma_{ik}=-g_{ik}$ (i,k=1,2,3) must be independent of the time axis x^0 of $X^{4d}(\eta)$. This implies that the time axis x^0 and the space axes x^i ($i\in(1,2,3)$) of the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) are orthogonal: time and space must be independent of each other.

Therefore, in the spacetime theory of GOR, there also exists the problem of whether spacetime is orthogonal or whether space and time is orthogonal.

Originally, whether space and time are orthogonal is a philosophical problem.

As stated in Sec. 1.5 of **Chapter 1 in the 1st volume of OR: Inertially Observational Relativity**, Galileo and Newton held the absolutist view of spacetime [54-57]: space and time are independent of each other; time flows silently, space exists immutably. This implies that spacetime is originally orthogonal, or space and time are originally orthogonal.

However, if as Einstein imagined, there is not only **the scalar potential** χ but also **the vector potential** $\gamma_i=-g_{0i}/\sqrt{(g_{00})}$ (i=1,2,3) in gravitational spacetime, then the space and time of gravitational spacetime may be non-orthogonal. This would be consistent with Mach and Einstein's relativist view of spacetime [58-60]: space is time, time is space; space and time are interdependent, and under certain conditions could be transformed into each other.

It is said that the vector potential of gravitational field has indeed been observed experimentally. However, according to the theory of GOR, the so-called gravitational vector potential does not exist objectively. If the vector potential is present in observation, it could only be an observational effect or an apparent phenomenon under a specific observation agent.

The scalar potential χ of gravitational field in the GOR factor of spacetime transformation (Eq. (12.36)(is the Newtonian gravitational potential, which is objectively real and the intrinsic physical quantity. There is no doubt that Newton's gravitational potential is objective existence: no matter whether we observe or not, all matter on the earth's surface must be affected by it.

However, the gravitational vector potential $\gamma_i=\gamma_i(\eta)$ in the GOR factor of spacetime transformation (Eq. (12.36)) depends on the observation agent OA(η): different observation agents has different gravitational vector potentials. According to Eq. (12.36): if $\eta\to\infty$, then $\Gamma(\eta)\to\Gamma_\infty\equiv1$ and $X^{4d}(\eta)\to X^{4d}_\infty$, that is, the observational spacetime $X^{4d}(\eta)$ of OA(η) would revert to the Galilean spacetime X^{4d}_∞. From this, we could presume that, and the next chapter will prove that, $g_{\mu\nu}(\eta)\to\eta_{\mu\nu}$ as $\eta\to\infty$. In other words, the metric tensor $g_{\mu\nu}$ of the Galilean spacetime is exactly the metric tensor $\eta_{\mu\nu}$ of Minkowski spacetime. This suggests that if $\eta\to\infty$, then $g_{0i}=g_{i0}=0$ and $\gamma_i=0$ (i=1,2,3), in which there are two important implications:

(i) The objectively gravitational field has no gravitational vector potential;

(ii) Space and time are originally orthogonal and independent of each other.

So, according to the spacetime theory of GOR, the objective and real gravitational field is the gravitational scalar field that Newton described for us; the gravitational vector field that Einstein imagined is not the objectively real existence. Space and time are absolute, orthogonal, and independent of each other, which conforms to the absolutist spacetime view of Galileo and Newton, rather than to the relativist spacetime view of Mach and Einstein.

12.5.4 The Idealized Observation Agent and Superluminal Observational Agents

The theory of GOR suggests that Newton's theory of gravitation and Einstein's theory of gravitation belong to different observation systems: Einstein's theory of general relativity is the product of the optical observation system, serving the optical observation agent OA(η), what it presents to us is only the optical image of gravitational spacetime, rather than the objective and real gravitational world; Newton's theory of universal gravitation is the product of the idealized observation system, serving the idealized observation agent OA_∞, what it presents to us is the objective and real gravitational world.

According to the GOR factor $\Gamma(\eta)$ of spacetime transformation, if we had the idealized observation agent OA_∞ to observe spacetime and matter motion, then all relativistic phenomena or relativistic effects of spacetime and matter motion would disappear: $\eta\to\infty$, $\Delta\Gamma(\eta)\to0$, $\Gamma(\eta)\to\Gamma_\infty\equiv1$, and $X^{4d}(\eta)\to X^{4d}_\infty$. Then, the spacetime in our observations, no matter inertial or gravitational, would be the objective and real face of the physical world.

However, there is no the idealized observation agent OA_∞ in the natural world.

According to the theory of OR, according to the principle of physical observability, all realistic observation agents OA(η) have the observational locality ($\eta<\infty$) of its own, and the natural landscapes the OA(η) present to us is always only an image of the objective world, and could never be equal to the objective and real physical world.

Limited by the current level of science and technology, most of our observations and experiments rely on the optical observation agent OA(c). This is the reason why Einstein's theory of relativity, including the special and the general, is supported by most observations and experiments. In fact, those observations or experiments are not so much support for Einstein's theory of relativity as support for the theory of observational relativity (OR). Naturally, those observations and experiments have tested and verified the theory of OR (including IOR and GOR) under the case of the optical observation agent OA(c).

Of course, the optical observation agent OA(c) or the observational locality ($c<\infty$) of OA(η) is not necessarily be the observational barrier that human could not surpass [52,53]. According to the theory of OR [26-28], the invariance of light speed is only an observational effect when light acts as the observation medium; the speed of light is not really invariant or really cannot be exceeded. With the progress of science and technology, human beings will discover and even invent superluminal matter motion. Actually, more and more experiments on quantum entanglement have shown superluminal phenomena [131-136]. Such **spooky action at a distance** *s*eems to be more and more strongly challenging Einstein's concept of locality that takes the speed of light as the limit of matter motion.

It can be imagined that mankind will master the superluminal observation agents in the future [53], so that we can observe a more real natural world. Then, we will observe the natural landscapes different from that presented to us by the optical observation agent, and observe the **gravitational deflection**, the **gravitational redshift**, and the **Mercury precession** different from that predicted by Einstein's theory of general relativity. At that time, Superluminal astronomy, such as gravitational wave astronomy [42], will replace traditional optical astronomy, including radio astronomy.

13 Cartesian Spacetime and Idealized Convergence

Logically, as the theoretical systems of physics describing gravitational interaction in the identical physical world, Einstein's theory of general relativity and Newton's theory of universal gravitation should have the intrinsic corresponding relationship and the logical consistency.

According to the factor of spacetime transformation in his general relativity, Einstein conceived that: jus as the Lorentz transformation of special relativity is approximately corresponding to the Galilean transformation **at slower speed**, the field equation of general relativity should be approximately corresponding to Newton's law of universal gravitation in the form of Poisson's equation **in weaker field**. In Einstein's theory of general relativity, the **weak-field approximation** is not only an important concept but also a kind of logical skill, and plays an important role in the establishment of Einstein's theory of general relativity.

Originally, under the principle of general correspondence (GC), by analogizing or following Einstein's the logic of weak-field approximation, no matter through PGC logic Route 1 or PGC logic Route 2, the theory of GOR could extend Einstein's theory of general relativity from the optical agent OA(c) to the general observation agent OA(η), and could derive the GOR field equation, that is, the gravitational-field equation of GOR theory, which must be isomorphically consistent with the Einstein field equation.

However, under the principle of GC, both PGC logic route 1 and PGC logic route 2 are logical shortcuts; there must be a price to be paid for taking shortcuts. If the deduction of GOR theory completely relies on the principle of GC, then we might fail to understand the root and essence of gravitational relativistic phenomena.

Newton's law of universal gravitation belongs to the idealized agent OA_∞; while Einstein's field equation belongs to the optical agent OA(c). There is no the direct corresponding relationship between the Einstein's and the Newton's. So, Einstein could only correspond his field equation approximately to Newton's law of universal gravitation by the weak-field approximation. The theory of GOR belongs to the general observation agent OA(η). So, we could correspond the GOR field equation strictly to Newton's law of universal gravitation by the idealized convergence, that is, by the idealized agent OA_∞: $\eta\rightarrow\infty$. Therefore, the derivation and calibration of the gravitational-field equation of GOR theory need the idealized convergence under the idealized agent, rather than the weak-field approximation.

This chapter attempts to clarify the logical thought of the idealized convergence of GOR theory. At the same time, an important theorem will be proved in this chapter: so-called **the theorem of Galilean spacetime**.

13.1 Einstein's Logic of Weak-Field Approximation

Before discussing the logical way of idealized convergence in the theory of

GOR, we first need to discuss and analyze the logical thought of Einstein's weak-field approximation.

By analogizing Einstein's weak-field approximation, we will clarify the logical thought of the idealized convergence of GOR theory.

13.1.1 The Logical Thought of Weak-Field Approximation

In Einstein's theory of general relativity, the so-called weak-field approximation means that, if the strength **g** of gravitational field (Eq. (12.13)) is weak enough, then Einstein's field equation approximates Newton's law of universal gravitation.

Newton's gravitational spacetime is flat; while the gravitational spacetime of Einstein's general relativity is curved. Einstein conceived that: if gravitational field is weak enough, that is, has a much weaker potential, then the curved gravitational spacetime must be approximately flat. In this way, Einstein's theory of general relativity could be linked or corresponded to Newton's law of universal gravitation.

The basic idea of Einstein's weak-field approximation originated from Einstein's understanding on the root and essence of relativistic effects. In Einstein's view, relativistic phenomena, no matter the inertial effects or the gravitational effects, are the essential characteristics of matter motion and matter interactions. Einstein's weak-field approximation in general relativity could be analogized with Einstein's slow-speed approximation in special relativity. Or, more precisely, the logic of Einstein's weak-field approximation is stemmed from the idea of Einstein's slow-speed approximation.

The speed c of light is a cosmic constant, and naturally, is invariant.

Therefore, in Einstein's theory of special relativity, the factor $\gamma=1/\sqrt{(1-v^2/c^2)}$ of inertial spacetime transformation only depends on the movement speed v of matter: the greater the $|v|$, the greater the inertial factor $\gamma=\gamma(v)$, and the more significant the inertial relativistic effects. Accordingly, Einstein believed that, and the mainstream school of physic also believe that, the root and essence of inertial relativistic effects lie in the motions of matter. In the case of slower speed ($|v|<<c$), the Lorentz factor γ approximates the Galilean factor Γ_∞: $\gamma=1/\sqrt{(1-v^2/c^2)}\approx 1\equiv\Gamma_\infty$; the Lorentz transformation approximates the Galilean transformation. Thus, Einstein believed that, the mainstream school of physic also believe that, the Lorentz transformation and the Galilean transformation, as well as, Einstein's special relativity and Newton's classical mechanics, have the corresponding relationship of slow-speed approximation. Moreover, the mainstream school of physics believe that the Lorentz transformation is better, while the Galilean transformation is only an approximation, that is, approximately true only in the case of slower speeds.

Similarly, in Einstein's theory of general relativity, due to the speed c of light is invariant; the factor $\gamma=1/\sqrt{(1+2\chi/c^2)}$ of gravitational spacetime transformation depends on the gravitational potential χ: the greater the $|\chi|$, the greater the gravitational factor $\gamma=\gamma(\chi)$, and the more significant the gravitational relativistic effects. Accordingly, Einstein believed that, the mainstream school of physic also believe that, the root and essence of gravitational relativistic effects lie in the interactions of matter. In the case of weaker field ($|\chi|<<c^2$), the gravitational factor γ

also approximates the Galilean factor Γ_∞: $\gamma=1/\sqrt{(1+2\chi/c^2)}\approx1\equiv\Gamma_\infty$; gravitational spacetime is approximately flat. Thus, Einstein believed that, Einstein's field equation and Newton's law of universal gravitation are approximate in the form of Poisson's equation; Einstein's theory of general relativity and Newton's theory of universal gravitation have the logical consistency in weaker gravitational fields, that is, have the corresponding relationship of weak-field approximation. Moreover, the mainstream school of physics believe that Einstein's theory of general relativity is a better theory of gravity; while Newton's theory of universal gravitation is only an approximation, that is, approximately true only in the case of weaker fields.

It is worth noting that, no matter the slow-speed approximation or the weak-field approximation, the factor γ of spacetime transformation is required to approximate the Galilean factor Γ_∞. The Galilean factor $\Gamma_\infty\equiv1$ represents the Cartesian spacetime that is flat, in which the observed time dt is namely the intrinsic time dτ: dt=dτ; and the spacetime metric $g_{\mu\nu}$ is the Minkowski metric: $\eta_{\mu\nu}$=diag(+1,–1,–1,–1). (we will specially discuss the problem of Cartesian spacetime in Sec. 13.2.)

Under the conditions of the weak-field approximation: $\gamma\approx\Gamma_\infty$. This implies that a weak gravitational field is approximately flat, in which Newton's theory of universal gravitation is approximately true. Thus, by making use of the weak-field approximation, Einstein's theory of general relativity could approximately be reduced to Newton's theory of universal gravitation, and Einstein's field equation could approximately be reduced to Newton's law of universal gravitational equation. In this way, the coefficient of Einstein field equation could be calibrated.

13.1.2 The Spacetime Metric of Weak Gravitational Fields

Generally, as stated in Sec. 12.4 of chapter 12, the factor γ of spacetime transformation in Einstein's general relativity is related the movement speed v of matter as well as the scalar potential χ and vector potential γ_i (i=1,2,3) of gravitational spacetime: $\gamma=\gamma(v,\chi,\gamma_i)$.

According to Eq. (12.16) of Einstein's theory of general relativity,

the factor of spacetime transformation is $\gamma(v,\chi,\gamma_i)=\dfrac{1}{\sqrt{\left(\sqrt{1+\dfrac{2\chi}{c^2}}-\gamma_i\dfrac{v^i}{c}\right)^2-\dfrac{v^2}{c^2}}}$.

To make the **curved** gravitational spacetime approximate the **flat** Cartesian spacetime: $\gamma\approx\Gamma_\infty\equiv1$, in addition to the assumption: (i) the weak field ($|\chi|<<c^2$), it also needs: (ii) the slow speed ($|v|<<c$), and moreover, it often needs: (iii) the spacetime orthogonality (g_{0i}=0 (i=1,2,3)). Actually, the spacetime orthogonality g_{0i}=0 means the gravitational vector potential γ_i=0 (i=1,2,3).

As a matter of fact, the weak-field approximation is a linearization method designed by Einstein specifically for his theory of general relativity.

The gravitational spacetime in Einstein's theory of general relativity is **curved** or nonlinear: $\gamma>\Gamma_\infty$; In the case of the slow-speed and weak-field, $\gamma\approx\Gamma_\infty$, the gravitational spacetime tends to be **flat** or linear, and the gravitational spacetime metric $g_{\mu\nu}$ approximates the Minkowski metric: $g_{\mu\nu}(x^{\alpha},c)\approx\eta_{\mu\nu}=\text{diag}(+1,-1,-1,-1)$.

According to the factor γ of spacetime transformation in Einstein's theory of general relativity, in order to achieve $g_{\mu\nu}(x^{\alpha},c)\approx\eta_{\mu\nu}$, it is necessary to create the scenarios of slow-speed and weak-field. If $|\chi|<<c^2$ and $|\gamma_i v^i|<<c$, then the gravitational spacetime metric $g_{\mu\nu}(x^{\alpha},c)$ could be linearized as:

$$\begin{aligned}&|\chi|\ll c^2 \text{ and } \left|\gamma_i v^i\right|\ll c: \\ &g_{\mu\nu}\left(x^{\alpha},c\right)=\eta_{\mu\nu}+h_{\mu\nu}\left(x^{\alpha},c\right) \quad \left(\left|h_{\mu\nu}\right|\ll\left|\eta_{\mu\nu}\right|\right)\end{aligned} \tag{13.1}$$

where $h_{\mu\nu}$ and its derivative of each order are infinitesimal; $\eta_{\mu\nu}$ can be called **the flat gauge**, while $h_{\mu\nu}$ can be called **the curved gauge**.

Equation (13.1) is **the weak-field metric** of Einstein's general relativity, that is, the metric of a weak gravitational field, and is the core relationship of Einstein's logical way or weak-field approximation. The weak-field metric in Eq. (13.1) is a linearized equation of the gravitational spacetime metric $g_{\mu\nu}$ in Einstein's general relativity, which is the result of the linearization of the weak-field approximation or the formalized expression of the weak-field approximation.

It is worth noting that, according to Einstein's logic of weak-field approximation: $|h_{\mu\nu}|<<|\eta_{\mu\nu}|$ in a weak gravitational field; $g_{\mu\nu}=\eta_{\mu\nu}$ and $h_{\mu\nu}=\mathbf{0}$ if without gravitational field. So, Einstein believed that the curved gauge $h_{\mu\nu}$ in Eq. (13.1) represented the weak gravitational potential, was the gravitational radiation or the **gravitational wave**, which had laid the groundwork for Einstein to deduce the **gravitational wave** equation, and later to make the prediction of **gravitational wave**.

The theory of gravitationally observational relativity (GOR), or the theory of GOR, will clarify that the **curved gauge** $h_{\mu\nu}$ in Eq. (13.1) does not represent gravitational radiation, let alone the so-called **gravitational wave**.

13.1.3 The Conditions of Weak-Field Approximation

Einstein tried to make use of the logical way of weak-field approximation to approximately correspond Einstein's gravitational-field equation in general relativity to Newton's law of universal gravitation in the form of Poisson equation, so that he could calibrated the coefficient of Einstein field equation.

Actually, the conditions involved in the weak-field approximation are not only the weak field, the slow speed, and the spacetime orthogonality. The calibration of the Einstein field-equation coefficient involves several linearized assumptions related to the logic of the weak-field approximation.

Einstein's approach of weak-field approximation has five assumptions.

(i) The weak field

The spacetime is flat: $g_{\mu\nu}=\eta_{\mu\nu}$ if Newtonian potential $\chi=0$. Thus, the spacetime of a weak gravitational field ($|\chi|<<c^2$) is approximately flat, and it follows that:

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu} \quad \left(\left|h_{\mu\nu}\right|\ll\left|\eta_{\mu\nu}\right|\right)$$

(ii) The slow speed

The speed v of the observed object P relative to the observer O is much slower than the speed c of light: $|v|<<c$, or

$$\left|v^i\right|=\left|\frac{\mathrm{d}x^i}{\mathrm{d}t}\right|\ll\frac{\mathrm{d}x^0}{\mathrm{d}t}=c\quad\left(i=1,2,3;\ x^0=ct\right)$$

(iii) The static field

The metric $g_{\mu\nu}$ or the curved metric $h_{\mu\nu}$ does not change over time, that is,

$$g_{\alpha\beta,0}=\frac{\partial g_{\alpha\beta}}{\partial x^0}=\frac{\partial g_{\alpha\beta}}{c\partial t}=0\ \text{ or }\ h_{\alpha\beta,0}=\frac{\partial h_{\alpha\beta}}{\partial x^0}=\frac{\partial g_{\alpha\beta}}{c\partial t}=0$$
$$\left(\alpha,\beta=0,1,2,3\right)$$

In Einstein's theory of general relativity, the condition of static field is **approximate**, only requiring that the metric $g_{\mu\nu}$ or $h_{\mu\nu}$ does not change significantly with time t. Moreover, it is worth noting that the condition of static field does not mean $\partial h_{\alpha\beta}/\partial t=0$, but $h_{\mu\nu,0}=\partial h_{\alpha\beta}/\partial x^0\approx 0$.

(iv) The spacetime orthogonality

The time axis x^0 is orthogonal to the space axes x^i (i=1,2,3), that is,

$$g_{i0}=g_{0i}=0\quad\left(i=1,2,3\right)$$

It is noteworthy that the condition of spacetime orthogonality seems redundant.

On the one hand, as stated in Sec. 12.5.3 of Chapter 12, the objectively real spacetime is the Cartesian spacetime $X^{4\mathrm{d}}{}_{\infty}$, and its time axis x^0 and space axes x^i (i=1,2,3) are originally orthogonal. Later, the theorem of Cartesian spacetime will prove that. On the other hand, under the condition of weak field, γ_i is naturally a weak potential: $\gamma_i\approx 0$, and the spacetime tends to be orthogonal: $g_{0i}\approx 0$. Combining the condition of slow speed, the gravitational vector potential in the factor γ of spacetime transformation is naturally subject to $|\gamma_i v^i|<<c$.

(v) The harmonic coordinates

$$\Box x^{\mu}=\frac{1}{\sqrt{(-g)}}\frac{\partial}{\partial x^{\nu}}\left(\sqrt{(-g)}g^{\mu\nu}\right)=0\quad\left(\mu=0,1,2,3\right)$$

where $g=\det(g_{\mu\nu})$ is the determinant of the spacetime metric $g_{\mu\nu}$; in particular, the extreme case of harmonic coordinates is $\sqrt{(-g)}=1$ or $g_{\mu\nu}=\eta_{\mu\nu}$.

The theory of OR will clarify that, no matter the theory of GOR or Einstein's theory of general relativity, the logic deduction of gravitational-field equation, including linearization, does not need the weak-field approximation or the logical way of weak-field approximation.

Actually, the corresponding relationship between the theory of GOR and Newton's theory of universal gravitation is not that of the weak-field approximation, but the corresponding relationship between the general observation agent OA(η) and the idealized observation agent OA_{∞}.

Therefore, the theory of GOR does not need to be approximately corresponded to Newton's theory of universal gravitation through the weak-field approximation. Instead, the theory of GOR will be strictly corresponded to Newton's theory of universal gravitation through the idealized convergence ($\eta \to \infty$), that is, the GOR logical wave of the idealized observation agent OA_∞.

The theory of GOR will clarify that the conditions of the weak-field approximation in Einstein's theory of general relativity, including the weak field, slow speed, static field, spacetime orthogonality, and harmonic coordinates, could be satisfied by the idealized convergence or the logical way of the idealized agent OA_∞: as the information-wave speed $\eta \to \infty$ of the general observation agent $OA(\eta)$, all the conditions of Einstein's weak-field approximation would hold true.

13.1.4 Newton's Gravitational Scene

Einstein's weak-field approximation is the approximation between Einstein's field equation and Newton's law of universal gravitation in the form of Poisson equation [125] under the conditions of weak-field approximation.

Naturally, in order for Einstein's field equation to be correspond to Newton's law of universal gravitation in the form of Poisson equation, the gravitational-field equation should set up the same scene of gravitational interaction as Newton's law of universal gravitation.

Here, the so-called Newton's gravitational scene is that set up by Newton in his law of universal gravitation: in the flat gravitational spacetime, there are quietly two particles M and m at a distance of r, M is the gravitational source and center of gravity (forming a spherically symmetric gravitational-field), and m (with the matter density ρ) is the object affected by gravity. Let m be located at gravitational potential χ, according to Newton's law of universal gravitation: $\chi = -GM/r$. Without loss of generality, let M be located at the origin of the space coordinate of gravitational spacetime, then the gravitational potential is the same or equal everywhere on the surface of the sphere with the same radius r.

It should be pointed out that Newton's gravitational scene does not mean that Newton's gravitational field must be a **weak field**: the masses of both M and m could be arbitrarily large, and could even be conceived as **black holes** with huge mass or gravity. In this regard, Einstein's theory of general relativity and Newton's theory of universal gravitation do not necessarily have the corresponding relationship of so-called weak-field approximation.

Actually, whether Newton's gravitational scene is a weak gravitational field or not, the Cartesian spacetime is flat. In order to correspond his theory of general relativity to Newton's theory of universal gravitation, Einstein must flatten the gravitational spacetime in his theory of general relativity. However, as far as Einstein's gravitational spacetime is corresponded to Newton's gravitational spacetime, restricted by the optical observation, the logical way Einstein could make use of was only the weak-field approximation: as $\chi \approx 0$ or $r \approx \infty$, the metric $g_{\mu\nu}(x^\alpha, c)$ of the observational spacetime $X^{4d}(c)$ in Einstein's theory of general relativity could approximate the Minkowski metric $\eta_{\mu\nu}$: $g_{\mu\nu}(x^\alpha, c) \approx \eta_{\mu\nu}$. In this way, $X^{4d}(c)$ could be approximately flat.

However, the theory of GOR does not need the logical way of weak-field approximation. The logical way of GOR theory is the idealized convergence, i.e., the logic of the idealized agent OA_∞: as $\eta\rightarrow\infty$, the metric $g_{\mu\nu}(x^\alpha,c)$ of the GOR observational spacetime $X^{4d}(\eta)$ in the theory of GOR strictly converges to the Minkowski metric $\eta_{\mu\nu}$: $g_{\mu\nu}(x^\alpha,c)\rightarrow\eta_{\mu\nu}$.

In this way, $X^{4d}(\eta)$ would naturally be flat.

According to the conditions of weak-field approximation in Sec. 13.1.3, the five assumptions, including the weak field, slow speed, static field, spacetime orthogonality, and harmonic coordinates, naturally hold true in Newton's gravitational scene:

(i) The weak field: spacetime is flat in Newton's gravitational scene, naturally, $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ ($|\eta_{\mu\nu}|>>|h_{\mu\nu}|=0$);

(ii) The slow speed: M and m are relatively stationary in Newton's gravitational scene;

(iii) The static field: the gravitational potential χ does not change over time in Newton's gravitational scene;

(iv) The spacetime orthogonality: $g_{\mu\nu}=\eta_{\mu\nu}$, naturally, $g_{0i}=g_{i0}=0$;

(v) The harmonic coordinates: $g_{\mu\nu}=\eta_{\mu\nu}$, naturally, $\Box x^\mu=0$.

Perhaps, it was by reference to Newton's gravitational scene that Einstein set up the conditions of weak-field approximation for his theory of general theory. On the contrary, when we set the gravitational scene for Einstein field equation by reference to Newton's gravitational scene, the Einstein's conditions of weak-field approximation naturally hold true.

The question left to us by Newton's gravitational scene is that:

Is Newton's gravitational scene really a flat spacetime? Or, is the spacetime metric $g_{\mu\nu}$ of Newton's gravitational scene really the Minkowski metric $\eta_{\mu\nu}$?

According to the theory of OR, Newton's gravitational scene belongs to the Cartesian spacetime X^{4d}_∞. So, the issue evolves into that: Is the metric $g_{\mu\nu}$ of the Cartesian spacetime X^{4d}_∞ really the Minkowski metric $\eta_{\mu\nu}$?

The issue involves **the theorem of Cartesian spacetime**.

13.2 The Theorem of Cartesian Spacetime

The theorem of Cartesian spacetime, based on the definition (Def. 10.1) of the general observation agent $OA(\eta)$ in Chapter 10, proves that the Cartesian spacetime X^{4d}_∞ is a flat spacetime, and the metric $g_{\mu\nu}$ of Cartesian spacetime is exactly the Minkowski metric: $\eta_{\mu\nu}=\mathrm{diag}(+1,-1,-1,-1)$.

Einstein believed that: Newton's gravitational scene was a flat spacetime; or, in a flat spacetime, Newton's theory of universal gravitation held true. It was based on such an understanding that Einstein conceived the logical way of weak-field approximation, and then corresponded his theory of general relativity to Newton's theory of universal gravitation to calibrate Einstein's field equation.

According to the theory of GOR, Newton's theory of universal gravitation is the product of the idealized observation agent OA_∞ with the idealized observational spacetime $X^{4d}{}_\infty$, called the **Cartesian spacetime** in the theory of OR, representing the objective and real physical world.

It should be pointed out that Newton's theory of universal gravitation is not necessarily the theory of weak gravitational fields; the flat spacetime of Cartesian spacetime is not due to weak gravitational fields or no gravitational fields, but the objectively physical reality presented to observers by the idealized agent OA_∞.

13.2.1 The Idealized Agent and Cartesian Spacetime

The theory of OR has clarified that, in order to perceive or observes the natural world, observers must make use of certain observation agents.

In theory, any form of matter motion could be employed as the observation medium to transmit observed information to observers. According to Def. 10.1 in Sec. 10.3 of Chapter 10, the general observation agent OA(η) can be formalized as:

$$\mathrm{OA}(\eta) \triangleq \left\{ \begin{array}{l} X^{4\mathrm{d}}(\eta): \left\{ \begin{array}{l} x^0 = \eta t; \\ x^1 = x,\ x^2 = y,\ x^3 = z \end{array} \right\} \\ \mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \quad \left(g_{\mu\nu} = g_{\mu\nu}\left(x^\alpha, \eta\right)\right) \end{array} \right\}$$

where η is the information-wave speed of OA(η); $X^{4d}(\eta)$ is the observational spacetime of OA(η): x^0 is the 1d time, (x^1,x^2,x^3) is the 3d space; $g_{\mu\nu}=g_{\mu\nu}(x^\alpha,\eta)$ is the metric of the observational spacetime $X^{4d}(\eta)$.

As stated in Sec. 1.4 **Observation Agents** of Chapter 1 and as shown in Tab. 1.1, the so-called **Cartesian spacetime** refers to the idealized observational spacetime $X^{4d}{}_\infty$ of the idealized observation agent OA_∞: $X^{4d}(\eta)\to X^{4d}{}_\infty$ as $\eta\to\infty$, where there is no observational locality: it takes no time for information to cross space.

Based on Def. 10.1, the spacetime theory of GOR derives the GOR factor $\Gamma(\eta)$ (see Sec. 12.4 in Chapter 12), that is, the factor of spacetime transformation in the observational spacetime $X^{4d}(\eta)$ of the general agent OA(η) (Eq. (12.36)):

$$\Gamma(\eta) = \frac{\mathrm{d}t(\eta)}{\mathrm{d}\tau} = \frac{1}{\sqrt{\left(\sqrt{1+\dfrac{2\chi}{\eta^2}} - \gamma_i \dfrac{v^i}{\eta}\right)^2 - \dfrac{v^2}{\eta^2}}}$$

The GOR factor $\Gamma(\eta)$ of spacetime transformation generalizes the factors of the optical agent OA(c) and the idealized agent OA_∞: as $\eta\to c$, $\Gamma(\eta)$ is strictly reduced to the spacetime-transformation factor $\Gamma(c)=\gamma$ of Einstein's relativity theory; as $\eta\to\infty$, $\Gamma(\eta)$ is strictly reduced to the Galilean factor Γ_∞ (≡1).

Here, the Galilean factor Γ_∞ (≡1) represents the Cartesian spacetime, that is, the idealized observational spacetime $X^{4d}{}_\infty$: $\Gamma(\eta)\to\Gamma_\infty\equiv 1$ and $X^{4d}(\eta)\to X^{4d}{}_\infty$ as $\eta\to\infty$.

It could be imagined (as stated in Sec. 13.1, Einstein also thought the same way) that the Cartesian spacetime $X^{4d}{}_\infty$ is **flat**, and its metric $g_{\mu\nu}$ is exactly the Minkowski metric $\eta_{\mu\nu}$: if $\eta\to\infty$ then $g_{\mu\nu}\to\eta_{\mu\nu}$. Such an idea could be formalized as the following Cartesian-spacetime theorem.

The Theorem of Cartesian Spacetime: Let OA(η) be the general observation agent, $g_{\mu\nu}=g_{\mu\nu}(x^{\alpha},\eta)$ the metric of the observational spacetime $X^{4d}(\eta)$ of OA(η), and η the information-wave speed, then $g_{\mu\nu}\to\eta_{\mu\nu}$ as $\eta\to\infty$.

Naturally, proof is required for $g_{\mu\nu}\to\eta_{\mu\nu}$ as $\eta\to\infty$.

Alternatively, it needs to be proven that the Cartesian spacetime X^{4d}_{∞} is flat, and the metric $g_{\mu\nu}$ of Cartesian spacetime X^{4d}_{∞} is exacely the Minkowski metric $\eta_{\mu\nu}$, that is, $g_{\mu\nu}=\eta_{\mu\nu}=\text{diag}(+1,-1,-1,-1)$.

Newton's gravitational scene belongs to the Cartesian spacetime X^{4d}_{∞}. If the theorem of Cartesian spacetime holds true, then without the need of the weak-field approximation, the theory of GOR could strictly converge to Newton's theory of universal gravitation as $\eta\to\infty$ under the idealized agent OA_{∞}. In this way, the GOR gravitational-field equation could be corresponded to Newton's law of universal gravitation in the form of Poisson equation.

13.2.2 The Lemmas of Cartesian-Spacetime Theorem

The proof of Cartesian-spacetime theorem consists of several lemmas.

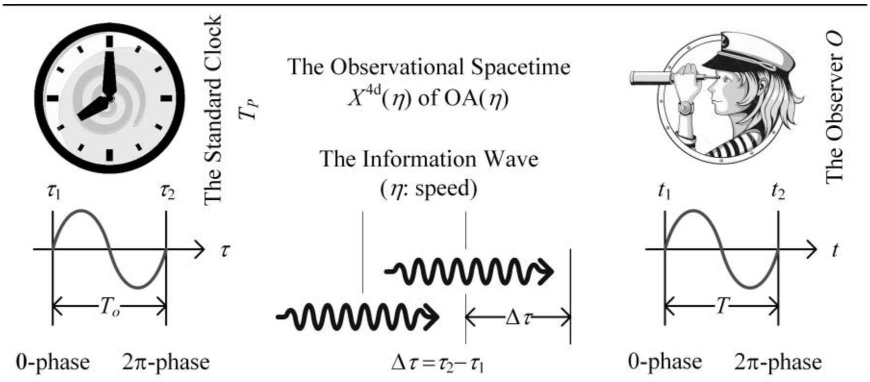

Figure 13.1 The Observer *O* and the Observed Time *t*: The observer O observes the standard clock T_P; the time dt observed by O depends on the observation agent OA(η) that O is armed with. The general observation agent OA(η) has the observational locality ($\eta<\infty$), and therefore, the observed time dt is not the objectively real time dτ. However, in the Cartesian spacetime X^{4d}_{∞}, the observation agent is the idealized agent OA_{∞} ($\eta\to\infty$), and has is no observation locality, thus dt=dτ.

Lemma 13.1 and Corollary 13.1

Lemma 13.1 is intended to prove that, in the Cartesian spacetime X^{4d}_{∞}, the observational (observed) time dt of observers is exactly the objectively real time, i.e., the intrinsic time (proper time) dτ.

Lemma 13.1: Let OA(η) be the general observation agent, and η the information-wave speed of OA(η), then in the observational spacetime $X^{4d}(\eta)$ of

OA(η), as $\eta\to\infty$, the observed time dt of any observer is exactly the standard time dτ: dt=dτ (dτ=ds/η).

Proof:

As depicted in Fig. 13.1, the observer O observes the standard T_P, the intrinsic period of T_P is T_o. According to the definition of standard time, if O and T_P are relatively stationary in the free spacetime S_F ($g_{\mu\nu}=\eta_{\mu\nu}$ and dx^i=0 (i=1,2,3)), then the observed time dt of O is the standard time dτ.

According to the definition of observation agents (Def. 10.1) and the symmetry of the spacetime metric $g_{\mu\nu}$, the line-element ds of the observational spacetime $X^{4d}(\eta)$ of OA(η) obeys

$$\begin{aligned} \mathrm{d}s^2 &= g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\ &= \eta^2 g_{00}\mathrm{d}t^2 + 2\eta g_{0i}\mathrm{d}x^i\mathrm{d}t + g_{ik}\mathrm{d}x^i\mathrm{d}x^k \\ &= \eta^2\eta_{00}\mathrm{d}t^2 \quad \left(g_{\mu\nu}=\eta_{\mu\nu};\, \mathrm{d}x^i=0\,(i=1,2,3)\right) \\ &= \eta^2\mathrm{d}\tau^2 \quad (\eta_{00}=1) \end{aligned} \tag{13.2}$$

Therefore, the standard dτ=ds/η.

Suppose that the standard clock T_P emits the signals of the 0-phase and 2π-phase of the clock period outward at times τ_1 and τ_2, respectively, then naturally, the intrinsic period T_o of T_P is $T_o=\Delta\tau=\tau_2-\tau_1$.

Suppose that the observer O receives the signals of the 0-phase and 2π-phase of the clock period at times t_1 and t_2, respectively, then the period T of the standard clock T_P observed by O is $T=\Delta t=t_2-t_1$.

If $\eta\to\infty$, then the observation agent OA(η) has no observation locality, the observation medium $M(\eta)$ takes no time to transmit observed information. Therefore, $t_1=\tau_1$ and $t_2=\tau_2$, the period T of the standard clock T_P observed by O is exactly the intrinsic period T_o of the standard clock T_P: $T=t_2-t_1=\tau_2-\tau_1=T_o$.

Thus, as $\eta\to\infty$, in the observational spacetime $X^{4d}(\eta)$ of OA(η), the observed time dt of an observer is exactly the standard time: dt=dτ (dτ=ds/η).

Q.E.D.

According to Lemma 13.1, if $\eta\to\infty$, or in the Cartesian spacetime X^{4d}_∞ of the idealized observation agent OA$_\infty$, the observational time dt observed by an observer O (regardless of the space coordinates of X^{4d}_∞) observing the standard clock T_P is the standard time, i.e., the intrinsic time (proper time) dτ. This means that, in the Cartesian spacetime X^{4d}_∞ of the idealized observation agent OA$_\infty$, time does not depend on space: space and time are independent of each other.

Thus, Lemma 13.1 has the following corollary.

Corollary 13.1: Let OA(η) be the general observation agent, and η the information-wave speed of OA(η), then as $\eta\to\infty$, in the metric $g_{\mu\nu}$ of the observational spacetime $X^{4d}(\eta)$ of OA(η), the metric elements of 0i and i0 are zero: $g_{0i}=g_{i0}=0$ (i=1,2,3).

Corollary 13.1 is of important significance.

(i) Corollary 13.1 suggests that space and time in the objectively physical world are originally orthogonal: time is just time, flowing quietly; space is just space, existing quietly.

(ii) Corollary 13.1 indicates that the objectively physical world has no the so-called gravitational vector potential imagined by Einstein.

However, it is worth noting that, in Lemma 13.1, $\gamma_i=0$ (i=1,2,3) requires $\eta\to\infty$. This means that when the observation agent OA(η) has the observation locality ($\eta<\infty$), the gravitational vector potential γ_i (i=1,2,3) might appear in the observational spacetime as a pure observational effect.

Lemma 13.2 and its Proof

Lemma 13.2 is intended to prove that, in the Cartesian spacetime X^{4d}_{∞}, the 00 element g_{00} of the metric $g_{\mu\nu}$ is the 00 element η_{00} of the Minkowski metric $\eta_{\mu\nu}$.

Lemma 13.2: Let OA(η) be the general observation agent, and η the information-wave speed of OA(η), then as $\eta\to\infty$, the metric element g_{00} in the metric $g_{\mu\nu}$ of the observational spacetime $X^{4d}(\eta)$ of OA(η) equals to one, i.e., the 00 element η_{00} of the Minkowski metric $\eta_{\mu\nu}$: $g_{00}=\eta_{00}=1$.

Proof:

According to Def. 10.1, the line-element ds in the observational spacetime $X^{4d}(\eta)$ of the observation agent OA(η) could be written as follows:

$$\frac{\mathrm{d}s^2}{\eta^2}=g_{00}\mathrm{d}t^2+2\frac{g_{0i}}{\eta}\mathrm{d}x^i\mathrm{d}t+\frac{g_{ik}}{\eta^2}\mathrm{d}x^i\mathrm{d}x^k \tag{13.3}$$

According to Lemma 13.1, $\mathrm{d}t=\mathrm{d}\tau$ as $\eta\to\infty$, and $\mathrm{d}\tau=\mathrm{d}s/\eta$.

So, it follows as $\eta\to\infty$ that

$$\mathrm{d}\tau^2=g_{00}\mathrm{d}t^2 \tag{13.4}$$

By contrasting Eq. (13.4) and Eq. (13.2), we have: $g_{00}=\eta_{00}=1$.

Q.E.D.

Lemma 13.3 and its Proof

Lemma 13.3 is intended to prove that, in the Cartesian spacetime X^{4d}_{∞}, the ii element g_{ii} of the metric $g_{\mu\nu}$ is the ii element η_{ii} of the Minkowski metric $\eta_{\mu\nu}$.

Lemma 13.3: Let OA(η) be the general observation agent, and η the information-wave speed of OA(η), then as $\eta\to\infty$, the metric element g_{ii} in the metric $g_{\mu\nu}$ of the observational spacetime $X^{4d}(\eta)$ of OA(η) equals to -1, i.e., the ii element η_{ii} of the Minkowski metric $\eta_{\mu\nu}$: $g_{ii}=\eta_{ii}=-1$ (i=1,2,3).

Proof:

The metric $g_{\mu\nu}$ in the observational spacetime $X^{4d}(\eta)$ of the observation agent OA(η) is independent of the observer O and the observed object T_P. Therefore, we could set the motion scene of T_P moving uniformly along the axis x^1 relative to O:

$dx^1 \neq 0$ and $dx^2 = dx^3 = 0$.

According to Def. 10.1, as well as Lemma 13.1 and Corol. 13.1, based on the given conditions of Lemma 13.3 and the motion scene, it follows that:

$$\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= \eta^2 g_{00} dt^2 + 2\eta g_{0i} dx^i dt + g_{ik} dx^i dx^k \\
&= \eta^2 g_{00} dt^2 + g_{ik} dx^i dx^k \quad \left(g_{0i} = g_{i0} = 0\right) \\
&= \eta^2 g_{00} dt^2 + g_{11} dx^1 dx^1 \quad \left(dx^2 = dx^3 = 0\right) \\
&= \eta^2 g_{00} dt^2 - dl^2
\end{aligned}$$

$$dl = \sqrt{-g_{11}}\, dx^1 \tag{13.5}$$

where dl is **the physical space distance** or **the pure space distance** that the observed object T_P moves during the period dt in the observational spacetime $X^{4d}(\eta)$ of the observation agent OA(η).

If $\eta \to \infty$, then OA(η)$\to$OA$_\infty$ and $X^{4d}(\eta) \to X^{4d}_\infty$. According to the motion scene, in the Cartesian spacetime X^{4d}_∞ of the idealized observation agent OA$_\infty$, the physical space distance (pure space distance) that the observed object T_P moves during the period $dt = d\tau$ must be $dl = dx^1$. Therefore, by contrasting with the $dl = \sqrt{(-g_{11})}dx^1$ in Eq. (13.5), it follows that: $g_{11} = -1$ as $\eta \to \infty$.

Similarly, $g_{22} = g_{33} = -1$ as $\eta \to \infty$.

Or, $g_{ii} = \eta_{ii} = -1$ (i=1,2,3) as $\eta \to \infty$.

Q.E.D.

Lemma 13.4 and its Proof

Lemma 13.4 is intended to prove that, in the Cartesian spacetime X^{4d}_∞, the ik element g_{ik} of the metric $g_{\mu\nu}$ is the ik element η_{ik} of the Minkowski metric $\eta_{\mu\nu}$.

Lemma 13.4: Let OA(η) be the general observation agent, and η the information-wave speed of OA(η), then as $\eta \to \infty$, the metric element g_{ik} in the metric $g_{\mu\nu}$ of the observational spacetime $X^{4d}(\eta)$ of OA(η) equals to zero, i.e., the ik element η_{ik} of the Minkowski metric $\eta_{\mu\nu}$: $g_{ik} = \eta_{ik} = 0$ (i,k=1,2,3; $i \neq k$).

Proof:

The metric $g_{\mu\nu}$ in the observational spacetime $X^{4d}(\eta)$ of the observation agent OA(η) is independent of the observer O and the observed object T_P. Therefore, we could set the motion scene of T_P moving uniformly in the axis x^1-x^2 plane: $dx^1 \neq 0$ and $dx^2 \neq 0$; $dx^3 = 0$.

According to Def. 10.1, as well as Lemma 13.1 and Corol. 13.1, based on the given conditions of Lemma 13.4 and the motion scene, it follows that:

$$
\begin{aligned}
\mathrm{d}s^2 &= g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\
&= \eta^2 g_{00}\mathrm{d}t^2 + 2\eta g_{0i}\mathrm{d}x^i\mathrm{d}t + g_{ik}\mathrm{d}x^i\mathrm{d}x^k \\
&= \eta^2 g_{00}\mathrm{d}t^2 + g_{ik}\mathrm{d}x^i\mathrm{d}x^k \quad \left(g_{0i} = g_{i0} = 0\right) \\
&= \eta^2 g_{00}\mathrm{d}t^2 - \mathrm{d}l^2 \quad \left(\mathrm{d}x^3 = 0\right)
\end{aligned}
$$

$$
\mathrm{d}l = \sqrt{-\left(2g_{12}\mathrm{d}x^1\mathrm{d}x^2 + g_{11}\left(\mathrm{d}x^1\right)^2 + g_{22}\left(\mathrm{d}x^2\right)^2\right)} \tag{13.6}
$$

where dl is **the physical space distance** or **the pure space distance** that the observed object T_P moves during the period dt in the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the observation agent OA(η).

According to Lemma 13.3, $g_{ii}=-1$ (i=1,2,3) as $\eta\to\infty$, and then

$$
\mathrm{d}l = \sqrt{\left(\left(\mathrm{d}x^1\right)^2 + \left(\mathrm{d}x^2\right)^2\right) - 2g_{12}\mathrm{d}x^1\mathrm{d}x^2} \tag{13.7}
$$

If $\eta\to\infty$, then OA(η)$\to$OA$_\infty$ and $X^{4\mathrm{d}}(\eta)\to X^{4\mathrm{d}}_\infty$. According to the motion scene, in the Cartesian spacetime $X^{4\mathrm{d}}_\infty$ of the idealized observation agent OA$_\infty$, the physical space distance (pure space distance) that the observed object T_P moves during the period dt=dτ must be dl=$\sqrt{((\mathrm{d}x^1)^2+(\mathrm{d}x^2)^2)}$. Therefore, by contrasting with the dl in Eq. (13.7), it follows that: $g_{12}=g_{21}=0$ as $\eta\to\infty$.

Similarly, $g_{23}=g_{32}=0$ as $\eta\to\infty$.

Or, $g_{ik}=\eta_{ik}=0$ (i,k=1,2,3; $i\neq k$) as $\eta\to\infty$.

Q.E.D.

13.2.3 The Proof of Cartesian-Spacetime Theorem

According to Lemmas 13.1-4, we can prove the theorem of Cartesian spacetime.

Proof:

According to Lemma 13.1, the observed time dt is the standard time dτ: dt=dτ.

According to Corol. 13.1, spacetime is orthogonal: $g_{0i}=g_{i0}=0$ (i=1,2,3).

According to Lemma 13.2, $g_{00}=\eta_{00}=1$.

According to Lemma 13.3, $g_{ii}=\eta_{ii}=-1$ (i=1,2,3).

According to Lemma 13.4, $g_{ik}=\eta_{ik}=0$ (i,k=1,2,3; $i\neq k$).

In summary, the theorem of Cartesian spacetime holds true: $g_{\mu\nu}\to\eta_{\mu\nu}$ as $\eta\to\infty$, where, $\eta_{\mu\nu}$ is the Minkowski metric: $\eta_{\mu\nu}=\mathrm{diag}(+1,-1,-1,-1)$.

Q.E.D.

The theorem of Cartesian spacetime means that the objective and real spacetime, or, the Cartesian spacetime, is originally **flat** rather than **curved**.

13.2.4 Verifying the Cartesian-Spacetime Theorem

In his general relativity, Einstein made use the weak-field approximation to solve his gravitational-field equation, and obtained the first solution of Einstein field equation [8], i.e., Einstein's approximate solution, in which the metric $g_{\mu\nu}(x^{\alpha},c)$ in the observational spacetime $X^{4\mathrm{d}}(c)$ of the optical observation agent OA(c) was:

$$\begin{cases} g_{00}(c) = 1 + \dfrac{2\chi}{c^2} \quad \left(\chi = -\dfrac{GM}{r}\right) \\ g_{0i}(c) = 0 \\ g_{ik}(c) = -\delta_{ik} + \dfrac{2\chi}{c^2}\dfrac{x^i x^k}{r^2} \quad (i,k = 1,2,3) \end{cases} \tag{13.8}$$

In Eq. (13.8), $\chi = -GM/r$ is the Newtonian gravitational potential; as $r \to \infty$, $\chi \to 0$ and $g_{\mu\nu}(x^{\alpha},c) \to \eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$. This is in line with the expectation of Einstein's logic of weak-field approximation.

However, as clarified by the theory of GOR in Sec. 12.5 of Chapter 12, the root and essence of gravitational relativistic effects do not lie in gravitational interaction, but in the observational locality of the observation agent OA(η) ($\eta < \infty$). Based on the principle of GC, through PGC logic route 1, substituting the information-wave speed η for the light speed c, one could directly obtain the metric $g_{\mu\nu}(x^{\alpha}, \eta)$ in the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η), that is, the approximate solution of the GOR gravitational-field equation:

$$\begin{cases} g_{00}(\eta) = 1 + \dfrac{2\chi}{\eta^2} \quad \left(\chi = -\dfrac{GM}{r}\right) \\ g_{0i}(\eta) = 0 \\ g_{ik}(\eta) = -\delta_{ik} + \dfrac{2\chi}{\eta^2}\dfrac{x^i x^k}{r^2} \quad (i,k = 1,2,3) \end{cases} \tag{13.9}$$

Obviously, regardless of χ and r, if $\eta \to \infty$ then

$$\lim_{\eta\to\infty} g_{\mu\nu}(\eta) = \eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1) \tag{13.10}$$

It is thus clear that, for both the approximate solution of Einstein field equation (Eq. (13.8)) or the approximate solution of the GOR field equation (Eq. (13.9)), the theorem of Cartesian spacetime holds true: $g_{\mu\nu}(x^{\alpha}, \eta) \to \eta_{\mu\nu}$ as $\eta \to \infty$.

Likewise, the Schwarzschild metric [80], the first exact solution of Einstein field equation, can also be applied to verify the theorem of Cartesian spacetime and to draw the conclusion similar to that Einstein's approximate solution.

13.2.5 The Significance of Cartesian-Spacetime Theorem

The theorem of Cartesian spacetime makes the fuzzy image of Cartesian spacetime in Chapter 10 and Chapter 12 much clearer. Actually, the theorem of Cartesian spacetime is consistent with the statement about the intrinsic spacetime and the observational spacetime in Chapter 1 and Chapter 10, as well as, with the statement about the GOR factor of spacetime transformation in Chapter 12.

According to the GOR factor of spacetime transformation in Eq. (12.36), $\Gamma(\eta) \to \Gamma_{\infty}$ as $\eta \to \infty$, which has already clarified that the Cartesian spacetime $X^{4\mathrm{d}}_{\infty}$

of the idealized agent OA_∞ is flat. Now, the theorem of Cartesian spacetime proves that: $g_{\mu\nu}(\eta)\to\eta_{\mu\nu}$ as $\eta\to\infty$, which more clearly indicates that the Cartesian spacetime $X^{4d}{}_\infty$ (including Newton's gravitational field) is flat rather than curved. Actually, in Einstein's subconscious, Newton's gravitational field was originally flat, which was reflected in Einstein's logical way of weak-field approximation.

According to the theorem of Cartesian spacetime, in the theory of OR (including IOR and GOR), the general observation agent OA(η) generalizes all observation agents, including the optical agent OA(c) and the idealized agent OA_∞, unifies the Minkowski 4d spacetime of OA(c) and the Cartesian 4d spacetime (with the independent 1d time and the independent 3d space) of OA_∞.

Naturally, as $\eta\to c$, the general observation agent OA(η) becomes the optical agent OA(η). In particular, as $\eta\to\infty$, the general observation agent OA(η) becomes the idealized agent OA_∞, and the observational spacetime $X^{4d}(\eta)$ becomes the Cartesian spacetime $X^{4d}{}_\infty$; according to the theorem of Cartesian spacetime, the metric $g_{\mu\nu}(\infty)$ of the Cartesian spacetime $X^{4d}{}_\infty$ is exactly the Minkowski metric $\eta_{\mu\nu}$: $\eta_{\mu\nu}$=diag(+1, –1,–1,–1).

The metric $g_{\mu\nu}$ of the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) depends on the information-wave speed η of OA(η): $g_{\mu\nu}=g_{\mu\nu}(\eta)$, which could be decomposed into:

$$g_{\mu\nu}(\eta)=\eta_{\mu\nu}+h_{\mu\nu}(\eta)\quad\left(\lim_{\eta\to\infty}h_{\mu\nu}(\eta)=\mathbf{0}\right) \tag{13.11}$$

where the Minkowski metric $\eta_{\mu\nu}$ (i.e., **the flat metric**) does not rely on OA(η), while **the curved metric** $h_{\mu\nu}$ relies on OA(η): $h_{\mu\nu}=h_{\mu\nu}(\eta)$; in particular, according to the theorem of Cartesian spacetime, $h_{\mu\nu}\to\mathbf{0}$ as $\eta\to\infty$.

According to the definition (Def. 10.1) of **Observation Agent** in Chapter 10 and Eq. (10.2), the line-elements ds of the observation spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) is described in:

$$\begin{aligned}\mathrm{d}s^2&=g_{\mu\nu}(\eta)\mathrm{d}x^\mu\mathrm{d}x^\nu=\left(\eta_{\mu\nu}+h_{\mu\nu}(\eta)\right)\mathrm{d}x^\mu\mathrm{d}x^\nu\\&=\eta^2\mathrm{d}t^2-\mathrm{d}l^2+h_{\mu\nu}(\eta)\mathrm{d}x^\mu\mathrm{d}x^\nu\quad\left(\mathrm{d}l=\sqrt{\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2}\right)\end{aligned} \tag{13.12}$$

where dt is the observed time, dl is the physical space distance.

Equation (13.12) can be rewritten as:

$$\mathrm{d}\tau^2=\mathrm{d}t^2-\frac{\mathrm{d}l^2}{\eta^2}+\frac{h_{\mu\nu}(\eta)}{\eta^2}\mathrm{d}x^\mu\mathrm{d}x^\nu\quad\left(\mathrm{d}\tau=\frac{\mathrm{d}\tau}{\eta}\right) \tag{13.13}$$

According to the theory of Cartesian spacetime: $h_{\mu\nu}\to\mathbf{0}$ as $\eta\to\infty$.

Thus, as $\eta\to\infty$, the spacetime line-element ds in Eq. (13.13) splits into two independent relations in the Cartesian spacetime $X^{4d}{}_\infty$ (i.e., the objectively physical world), the independent time-line dt and the independent space-line dl:

$$X^{4d}{}_{\infty}:\quad \begin{cases} dt = d\tau \\ dl = \sqrt{dx^2 + dy^2 + dz^2} \end{cases} \tag{13.14}$$

where the observed time dt is the objectively real time dτ; the physical space distance (pure space distance) dl is the Cartesian 3d space distance.

The theorem of Cartesian spacetime has clarified the spacetime characteristics of Cartesian spacetime as the intrinsic spacetime of the objective world:

(i) time is uniform everywhere: different observers have the same time, and simultaneity is absolute;

(ii) time and space are independent of each other: time flows quietly and evenly, while space exists quietly and eternally.

What Eq. (13.14) presents is exactly the scene of the Galilean transformation, that is, the scene in observational space time $X^{4d}{}_{\infty}$ of the idealized agent OA_{∞}, which is the Cartesian spacetime $X^{4d}{}_{\infty}$: the true portrayal of the objectively physical world. As $\eta\rightarrow\infty$, the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) would logically revert to Cartesian spacetime, which from one aspect confirms the formal and logical consistency between the general observation agent OA(η) and the idealized agent OA_{∞}.

Based on the theorem of Cartesian spacetime, the theory of GOR could construct the logical way of idealized convergence under the idealized observation agent OA_{∞} and the conditions of idealized convergence: when the information-wave speed η of the general observation agent OA(η) is large enough, it follows that $g_{\mu\nu}(\eta)=\eta_{\mu\nu}+h_{\mu\nu}(\eta)$ ($|h_{\mu\nu}|<<|\eta_{\mu\nu}|$).

The logical way of idealized convergence under the idealized observation agent OA_{∞} will lay the theoretical foundation for the deduction and calibration of GOR gravitational-field equation.

13.3 The GOR Logical Way of Idealized Convergence

It is the fundamental thought of the principle of general correspondence (GC) that: **One physical world, one logical system**.

Like Einstein's theory of general relativity, the theory of GOR also needs to maintain the logical consistency with Newton's theory of universal gravitation; like the Einstein field equation, the GOR gravitational-field equation also needs to maintain logical consistency with Newton's law of universal gravitation in the form of Poisson equation. However, unlike Einstein's theory of general relativity, the theory of GOR follows the GOR logic of idealized convergence or the GOR logical way of idealized convergence under the idealized observation agent OA_{∞} rather than Einstein's logical way of weak-field approximation.

The logical consistency between the general Lorentz transformation of OR theory and the Galilean transformation does not lie in the slow-speed approximation, but in the idealized convergence under the idealized observation agent OA_{∞}, which is the corresponding relationship of isomorphic consistency between the general observation agent OA(η) and the idealized agent OA_{∞}, that is, the strictly logical consistency: as $\eta\to\infty$, the general Lorentz transformation strictly reduces to the Galilean transformation.

Likewise, the logical consistency between the theory of GOR and Newton's theory of universal gravitation does not lie in the weak-field approximation, but in the idealized convergence under the idealized observation agent OA_∞, which is the corresponding relationship of isomorphic consistency between the general observation agent OA(η) and the idealized agent OA_∞, that is, the strictly logical consistency: as $\eta\to\infty$, the GOR gravitational-field equation strictly reduces to Newton's law of universal gravitation in the form of Poisson equation.

13.3.1 The Logic of Idealized Convergence

Restricted by the optical observation agent OA(c), Einstein believed that the essence of relativistic effects lies in matter motion and matter interactions: the Galilean transformation could only approximately be corresponded to the Lorentz transformation in special theory by the slow-speed approximation; Newton's law of universal gravitation could only approximately be corresponded to Einstein field equation in general relativity by the weak-field approximation.

However, the theory of observational relativity (OR) has discovered that all relativistic effects are observational effects, and the essence lies in the observational locality ($\eta<\infty$) of the observational agent OA(η): different observational agents have different degrees of observational locality, and therefore, present different degrees of relativistic effects. Newton's theory of universal gravitation is the theory of the idealized observation agent OA_∞; therefore, in logic, the theory of GOR could be corresponded to Newton's theory of universal gravitation by the idealized convergence under the idealized observation agent OA_∞, rather than by Einstein's weak-field approximation.

This is so-called the GOR logical way of idealized convergence.

As $\eta\to\infty$, the observation agent OA(η) converges to the idealized agent OA_∞; the observational spacetime $X^{4d}(\eta)$ converges to the Cartesian spacetime $X^{4d}{}_\infty$. According to the theorem of Cartesian spacetime, $g_{\mu\nu}(\eta)\to\eta_{\mu\nu}$ as $\eta\to\infty$, and therefore, the Cartesian spacetime $X^{4d}{}_\infty$ (including Newton's gravitational field) is a **flat spacetime**. At such a case, logically or in logic, the gravitational theory of GOR should strictly be corresponded to Newton's theory of universal gravitation.

The theory of GOR repeatedly emphasizes that: Newton's theory of universal gravitation is not the theory of the weak-field approximation of Einstein's general relativity; Einstein's theory of general relativity and Newton's theory of universal gravitation belong to different observational agents and do not have the direct corresponding relationship. Einstein's theory of general relativity is the theory of optical agent OA(c), which holds true only if light acts as the observation medium; Newton's theory of universal gravitation is the theory of the idealized agent OA_∞, representing the objectively gravitational interaction, which could only be converged or corresponded by the idealized convergence under the idealized agent OA_∞.

The GOR factor $\Gamma=\Gamma(\eta)$ of gravitational spacetime transformation depends on the observation agent OA(η): $\Gamma=1/\sqrt{(1+2\chi/\eta^2)}$. For a given gravitational potential χ, the larger the χ, the flatter the gravitational spacetime, the closer the GOR factor $\Gamma=\Gamma(\eta)$ is to the Galilean factor $\Gamma_\infty\equiv1$, and the closer the gravitational spacetime is to the Cartesian spacetime $X^{4d}{}_\infty$. In particular, as $\eta\to\infty$, $\Gamma(\eta)\to\Gamma_\infty\equiv1$, the GOR gravitational spacetime $X^{4d}(\eta)$ becomes the flat Cartesian spacetime $X^{4d}{}_\infty$. At such a case, the gravitational theory of GOR and Newton's theory of universal gravitation have strict logical consistency: so-called **the idealized convergence**.

In this way, the gravitational theory of GOR could strictly converge to Newton's theory of universal gravitation; the GOR gravitational-field equation could strictly converge to Newton's law of universal gravitation in the form of Poisson equation.

13.3.2 Quasi Cartesian Spacetime and its Metric

According to the theorem of Cartesian spacetime, the metric $g_{\mu\nu}$ of the Cartesian spacetime $X^{4d}{}_\infty$ of the idealized observation agent OA$_\infty$ is exactly the Minkowski metric: $\eta_{\mu\nu}=\text{diag}(+1, -1,-1,-1)$.

Based on the definition (Def. 10.1) of the general observation agent OA(η), the theory of GOR has derived the GOR factor (Eq. (12.36)) that is isomorphically consistent with Einstein's general factor γ of gravitational spacetime transformation (Eq. (12.16)). So, we have the GOR factor of the general observational agent OA(η):

Equation (12.3): $$\Gamma=\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{\eta^2}}-\gamma_i\frac{v^i}{\eta}\right)^2-\frac{v^2}{\eta^2}}}\,.$$

The GOR factor $\Gamma=\Gamma(\eta)$ of spacetime transformation depends on the information-wave speed η of OA(η), which could be any speed value; while the factor γ of spacetime transformation in Einstein's theory of general relativity is only a special case of the GOR factor $\Gamma=\Gamma(\eta)$ of spacetime transformation, where η could only be the speed c of light: $\gamma=\Gamma(c)$.

In his theory of general relativity, restricted by the perspective of optical observation, Einstein could only use the weak-field approximation to make the curved gravitational spacetime $X^{4d}(c)$ of the optical agent OA(c) approximate to the Cartesian spacetime $X^{4d}{}_\infty$: $\gamma\approx\Gamma_\infty$, and to make Einstein's field equation approximate to the Newton's law of universal gravitation in the form of Poisson equation, by assuming: (i) weak field ($|\chi|<<c^2$); (ii) slow speed ($|v|<<c$); (iii) the static field ($g_{\alpha\beta,0}=h_{\alpha\beta,0}=0$ ($\alpha,\beta=0,1,2,3$)); (iv) Spacetime orthogonality ($g_{0i}=g_{i0}=0$, ($i=1,2,3$)); (v) The harmonic coordinates ($\sqrt{(-g)}=1$).

As clarified by the theory of OR, however, Einstein's theory of general relativity does not have the direct corresponding relationship with Newton's theory of universal gravitation, and particularly, Newton's gravitational field is not necessarily a weak gravitational field.

In the theory of GOR, Newton's gravitational spacetime is the gravitational spacetime $X^{4d}{}_\infty$ of the idealized agent OA$_\infty$, i.e., Cartesian spacetime: OA(η)$\to$OA$_\infty$ as $\eta\to\infty$. At such a case, the gravitational theory of GOR converges to Newton's theory of universal gravitation.

Therefore, in order to make the curved gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) strictly converge to the Cartesian spacetime X^{4d}_{∞}, and to make the GOR field equation strictly converge to Newton's law of universal gravitation in the form of Poisson equation, the theory of GOR should replace Einstein's logic of weak-field approximation with the logic of idealized convergence. (It should be pointed out that this is also a logical way of isomorphically consistent correspondence under the principle of GC.)

In the theory of GOR, the idealized convergence under the idealized agent OA_{∞} does not require to set the conditions of weak-field approximation, including the weak field, slow speed, static field, spacetime orthogonality, and harmonic coordinates; simply requires the information-wave speed η of OA(η) to be large enough or $\eta\to\infty$, then the curved gravitational spacetime of OA(η) could be flattened or linearized, and become the Cartesian spacetime X^{4d}_{∞}:

$$\lim_{\eta\to\infty}\Gamma(\eta)=\lim_{\eta\to\infty}\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{\eta^2}}-\gamma_i\frac{v^i}{\eta}\right)^2-\frac{v^2}{\eta^2}}}=1=\Gamma_{\infty} \tag{13.15}$$

Actually, the GOR idealized convergence is also a linearization method.

The general observation agent OA(η) has the observation locality ($\eta<\infty$). Therefore, according to the logic of Einstein's theory of general relativity, the GOR gravitational spacetime of OA(η) should also be described as **curved** or **nonlinear**: $\Gamma(\eta)>\Gamma_{\infty}\equiv1$. Under the idealized agent OA_{∞}, $\Gamma(\eta)\to\Gamma_{\infty}\equiv1$ as $\eta\to\infty$. At such a case, the gravitational theory of GOR would strictly converge to Newton's theory of universal gravitation.

According to the theorem of Cartesian spacetime: as $\eta\to\infty$, the GOR gravitational spacetime $X^{4d}(\eta)$ converges to the Cartesian spacetime X^{4d}_{∞}, and the GOR gravitational spacetime metric $g_{\mu\nu}(x^{\alpha},\eta)$ converges to the Minkowski metric $\eta_{\mu\nu}=\text{diag}(+1,-1,-1,-1)$. Therefore, based on the GOR idealized convergence, let the information-wave speed η be large enough, then the GOR gravitational spacetime could be regarded as a quasi Cartesian-spacetime, and the GOR gravitational spacetime metric $g_{\mu\nu}(x^{\alpha},\eta)$ could be regarded as a quasi Cartesian-spacetime metric that could be decomposed and linearized as:

$$\begin{aligned}&\eta\gg\sqrt{|\chi|}\quad\text{and}\quad\eta\gg\left|\gamma_i v^i\right|:\\ &g_{\mu\nu}\left(x^{\alpha},\eta\right)=\eta_{\mu\nu}+h_{\mu\nu}\left(x^{\alpha},\eta\right)\\ &\left(\left|h_{\mu\nu}\right|\ll\left|\eta_{\mu\nu}\right|\ \text{and}\ \lim_{\eta\to\infty}h_{\mu\nu}\left(x^{\alpha},\eta\right)=\mathbf{0}\right)\end{aligned} \tag{13.16}$$

where $h_{\mu\nu}$ and its derivative of each order are infinitesimal; According to the theorem of Cartesian spacetime: $g_{\mu\nu}(x^{\alpha},\eta)\to\eta_{\mu\nu}$ as $\eta\to\infty$, and hence, $h_{\mu\nu}\to\mathbf{0}$.

Obviously, Eq. (13.16) is isomorphically consistent with Einstein's condition of the weak field in Eq. (13.1). Equation (13.16) is the condition of GOR idealized convergence; the metric $g_{\mu\nu}(x^{\alpha},\eta)$ in Eq. (13.16) is the quasi Cartesian-spacetime metric, $\eta_{\mu\nu}$ is the flat metric, and $h_{\mu\nu}$ is the curved metric.

Equation (13.1) is the core relationship of Einstein's weak-field approximation; while Eq. (13.16) is the core relationship of the GOR idealized convergence under the idealized agent OA_∞ in the theory of GOR, and is a linearized equation of the gravitational spacetime metric $g_{\mu\nu}(x^\alpha, \eta)$ in the theory of GOR. It is the result of idealized converging and the formalized expression of idealized convergence.

It is worth noting that the curved metric $h_{\mu\nu}=h_{\mu\nu}(\eta)$ depends on the observation agent OA(η): under different observation agents, the same gravitational scene has different curved metrics, and would exhibit different curvatures.

This fact suggests that $h_{\mu\nu}$ is not so-called **the weak gravitational potential**, does not represent gravitational radiation, nor is **the gravitational wave**, but the carrier wave, that is, the information wave loaded with observed information about matter motion (v) and gravitational interaction (χ).

The theory of Gravitationally Observational Relativity (GOR) will reveal the mystery of Einstein's prediction for gravitational waves in Chapter 18.

13.3.3 The Conditions of Idealized Convergence

The GOR logical way of idealized convergence theory can be analogized with Einstein's logical way of weak-field approximation; the conditions of idealized convergence for the theory of GOR can be corresponded to the conditions of weak-field approximation for Einstein's theory of general relativity.

As stated in Sec. 13.1.3, Einstein's logical way of weak-field approximation has five assumptions; while the COR logical way of idealized convergence under the idealized observation agent OA_∞ only means one assumption or condition, that is, the information-wave speed η of the observation agent OA(η) is large enough or $\eta\to\infty$. According to the theorem of Cartesian spacetime, if η is large enough or $\eta\to\infty$, then the linearized equation (13.16) of the idealized convergence holds, and naturally, Einstein's five conditions of weak-field approximation hold true.

(i) The flat spacetime (corresponding Einstein's assumption of the weak field)

In theory of GOR, the gravitational spacetime is the observational spacetime $X^{4\text{d}}(\eta)$ of the general observation agent OA(η), the spacetime metric $g_{\mu\nu}(\eta)$ of $X^{4\text{d}}(\eta)$ could be decomposed into **the flat metric** $\eta_{\mu\nu}$ and **the curved metric** $h_{\mu\nu}(\eta)$: $g_{\mu\nu}(\eta)=\eta_{\mu\nu}+h_{\mu\nu}(\eta)$.

According to the theorem of Cartesian spacetime: $g_{\mu\nu}=\eta_{\mu\nu}$ as $\eta\to\infty$. Therefore, for any gravitational scene with Newton's gravitational potential χ, if η is large enough ($\eta>>\sqrt{|\chi|}$), the gravitational spacetime $X^{4\text{d}}(\eta)$ tends to be **flat**, which could be regarded as the **weak-field** scene in Einstein's logical way of weak-field approximation. The condition of the weak field condition holds (see Sec. 13.1.3):

$$g_{\mu\nu}\left(x^{\alpha},\eta\right)=\eta_{\mu\nu}+h_{\mu\nu}\left(x^{\alpha},\eta\right)\quad\left(\left|h_{\mu\nu}\right|\ll\left|\eta_{\mu\nu}\right|\right)$$

where $h_{\mu\nu}$ is the curved metric, does not represent the weak gravitational potential, but represents the curved state of the observational spacetime $X^{4\text{d}}(\eta)$: $h_{\mu\nu}\to\mathbf{0}$ as

$\eta\to\infty$, or in other words, $|h_{\mu\nu}|<<|\eta_{\mu\nu}|$ if η is large enough. At such a case, the GOR observational spacetime $X^{4d}(\eta)$ of OA(η) converges to the Cartesian spacetime X^{4d}_∞ and tends to be **flat**.

(ii) The slow speed

As $\eta\to\infty$, the speed v of any observed object P relative to the observer O could be regarded as a **slow speed**. Therefore, if η is large enough ($\eta>>|v|$), then the condition of the slow speed holds (see Sec. 13.1.3):

$$\left|v^i\right|=\left|\frac{\mathrm{d}x^i}{\mathrm{d}t}\right|\ll\frac{\mathrm{d}x^0}{\mathrm{d}t}=\eta\quad\left(i=1,2,3;\ x^0=\eta t\right)$$

(iii) The static field

As $\eta\to\infty$ or η is large enough, according to the logic of Einstein's logical way of weak-field approximation, the gravitational spacetime at such a case could be regarded as a **static gravitational field** that does not change with time, or in other words, the metric $g_{\mu\nu}(\eta)$ or $h_{\mu\nu}(\eta)$ does not change **significantly** with time t:

$$g_{\alpha\beta,0}=\frac{\partial g_{\alpha\beta}}{\partial x^0}=\frac{\partial g_{\alpha\beta}}{\eta\partial t}=0\quad\text{or}\quad h_{\alpha\beta,0}=\frac{\partial h_{\alpha\beta}}{\partial x^0}=\frac{\partial g_{\alpha\beta}}{\eta\partial t}=0$$
$$\left(\alpha,\beta=0,1,2,3\right)$$

However, it is worth noting that, in the idealized convergence of GOR theory, the condition of the static field does not require $g_{\mu\nu}(\eta)$ or $h_{\mu\nu}(\eta)$ not to significantly change with time t, but requires $g_{\mu\nu}(\eta)$ or $h_{\mu\nu}(\eta)$ not to significantly change with time-line $x^0=\eta t$, which only needs η to be large enough or $\eta\to\infty$.

(iv) The spacetime orthogonality

In the theorem of Cartesian spacetime, Corol. 13.1 of Lemma 13.1 has proven that, as $\eta\to\infty$, the metric elements of $0i$ and $i0$ are zero: $g_{0i}=g_{i0}=0$ (i=1,2,3), the time axis x^0 and space axes x^i (i=1,2,3) of the observational spacetime $X^{4d}(\eta)$ of OA(η) tend to be orthogonal. It is thus clear that the spacetime of the objectively physical world is originally orthogonal. (This is consistent with the relevant statement in Sec. 12.5.3 of Chapter 12.)

(v) The harmonic coordinates

According to the theorem of Cartesian spacetime, $g_{\mu\nu}=\eta_{\mu\nu}$ as $\eta\to\infty$, and therefore, if η is large enough, then $|h_{\mu\nu}|<<|\eta_{\mu\nu}|$ or $g_{\mu\nu}\approx\eta_{\mu\nu}$ or $\sqrt{(-g)}\approx1$. Thus, the following relation of harmonic coordinates holds true:

$$\Box x^\mu=\frac{1}{\sqrt{\left(-g\left(\eta\right)\right)}}\frac{\partial}{\partial x^\nu}\left(\sqrt{\left(-g\left(\eta\right)\right)}g^{\mu\nu}\left(\eta\right)\right)=0\quad\left(\mu=0,1,2,3\right)$$

where $g(\eta)=\det(g_{\mu\nu}(\eta))$ is the determinant of the metric $g_{\mu\nu}(\eta)$ of the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η).

To sum up, in the theory of GOR, the condition of GOR idealized convergence under the idealized agent OA_∞ only has one item: the information-wave speed η of the general observation agent OA(η) is large enough or $\eta \to \infty$.

Thus, as $\eta \to \infty$ or η is large enough, the conditions of Einstein's weak-field approximation, including the weak field, slow speed, static field, spacetime orthogonality, and harmonic coordinates, hold true.

So, it could imagine that, based on the condition of GOR idealized convergence: **$\eta \to \infty$ or η is large enough**, by analogizing or following the logic of Einstein's weak-field approximation in Einstein's theory of general relativity, the theory of GOR could correspond the GOR gravitational-field equation to Newton's law of universal gravitation in the form of Poisson equation, and finally build and calibrate the GOR field equation in the theory of GOR.

This is namely the GOR logical way of idealized convergence.

No matter Einstein's weak-field approximation or the GOR idealized convergence is a linearization method, intended to linearize the nonlinear gravitational-field equation, so that it could calibrate the coefficient of the gravitational-field equation or solve the gravitational-field equation. However, Einstein's weak-field approximation is only an approximate linearization method that could only approximately linearize the Einstein field equation; the GOR idealized convergence under the idealized agent OA_∞ is a strict linearization method that could precisely linearize the GOR field equation.

More importantly, Einstein's weak-field approximation, restrictedly by the perspective of optical observation, is the product of the misconception of **spacetime curvature**; while, the GOR idealized convergence is the logical necessity, which is on the basis of universal observations and from the broadest perspective of the general observation agent.

Based on the theorem Cartesian spacetime, following the logic of GOR idealized convergence, the theory of GOR will under the idealized agent OA_∞ calibrate the coefficient of GOR field equation, and finally, build up the GOR field equation.

The GOR logical way of idealized convergence is the embodiment of the logical thought of isomorphic-consistency correspondence in the principle of general correspondence (GC), which is the corresponding relationship of isomorphic consistency between the GOR general observation agent OA(η) and the idealized observation agent OA_∞; the corresponding relationship of isomorphic consistency between the GOR observational spacetime $X^{4d}(\eta)$ and Cartesian spacetime $X^{4d}{}_\infty$; the corresponding relationship of isomorphic consistency between the GOR gravitational-field equation and Newton's law of universal gravitation in the form of Poisson equation; the corresponding relationship of isomorphic consistency between the gravitational theory of GOR and Newton's theory of universal gravitation.

Ultimately, the gravitational theory of GOR will generalize unify Einstein's theory of general relativity and Newton's theory of universal gravitation.

14 The GOR Gravitational-Field Equation

Einstein's field equation is the most fundamental formula in Einstein's theory of general relativity. In a sense, the Einstein field equation represents Einstein's theory of general relativity.

Likewise, the GOR gravitational-field equation, or the GOR field equation, is the most fundamental formula in the theory of Gravitationally Observational Relative (GOR), which represents the theory of GOR.

This chapter is intended to build up the GOR gravitational-field equation.

Based on the principle of general correspondence (GC), through PGC logic route 1, substituting the information-wave speed η of the general observation agent OA(η) for the light speed c, and directly transforming the Einstein field equation from the optical agent OA(c) to the general observation agent OA(η) isomorphically and uniformly, we could easily obtain the GOR field equation. However, relying simply on PGC logic route 1, we might fail to understand the essence of gravitational interaction and gravitational relativistic phenomena.

Based on the principle of GC, through PGC logic route 2, by analogizing or following the logic of Einstein's of weak-field approximation in the theory of general relativity, we could also deduce and calibrate the GOR field equation. However, relying simply on PGC logic route 2, we might fail to understand the logical flaws of Einstein's weak-field approximation, and like Einstein, might mistakenly understand the essence of gravitational phenomena and mistakenly support the Einstein's prediction of gravitational waves.

So, on the one hand, we strive to deduce the GOR field equation based on the principle of GC, combining PGC logic route 1 and PGC logic route 2, by analogizing or following the logic of Einstein's general relativity; on the other hand, we strive to calibrate the GOR field equation based on the theorem of Cartesian spacetime, taking advantage of the GOR logical way of idealized convergence under the idealized agent OA_∞ that is stated in Chapter 13.

14.1 The Establishment of Einstein's Field Equation

Before deducing the GOR gravitational-field equation under the principle of GC, before calibrating the GOR gravitational-field equation by the GOR logical way of idealized convergence based on the theorem of Cartesian spacetime, we need to analyze and examine Einstein's logical way of weak-field approximation in the theory of general relativity, which will contribute us to understand Einstein' logic of weak-field approximation and the logic of the GOR idealized convergence, and at the same time, to reexamine Newton's theory of universal gravitation and Einstein's theory of general relativity.

With regard to Einstein's theory of general relativity, after the form of Einstein's gravitational-field equation had been determined, the remaining problem for Einstein to have to solve was how to calibrate the coefficient κ_E of Einstein field equation. Einstein expected that his field equation could be related or corresponded to Newton's law of universal gravitation through a certain logical way, so that the coefficient κ_E of Einstein field equation could be calibrated with Newton's gravitational potential χ or Newton's gravitational constant G.

14.1.1 Newton's Law of Universal Gravitation and Poisson Equation

Beyond doubt, gravity or universal gravitation is Newton's great discovery; Newton's law of universal gravitation is a great monument of physics, and is Newton's great contribution to human beings and physics.

Newton believed that [81]: all things on earth attract or gravitate toward each other, and there is the gravitational interaction between any two objects M and m. The gravitational force F is directly proportional to the product of the masses M and m, and inversely proportional to the square of the distance r between M and m:

$$F = \frac{GMm}{r^2} = -\frac{m}{r}\chi \quad \left(\chi = -\frac{GM}{r}\right) \tag{14.1a}$$

$$\Delta\chi = \nabla^2\chi = 4\pi G\rho \quad \left(\Delta = \nabla^2 = \sum_{i=1}^{3}\frac{\partial}{\partial x^i}\frac{\partial}{\partial x^i}\right) \tag{14.1b}$$

where G=6.754×10 Nm^2/kg^2 is the Universal Gravitational Constant, χ is the Newtonian gravitational potential, ρ is the material density of m, $\Delta=\nabla^2$ is the Laplace operator; Eq. (14.1a) is Newton's law of universal gravitation, and Eq. (14.1b) is the Poisson equation, a second-order partial differential equation, i.e., the partial-differential-equation form of Newton' law of universal gravitation [125].

Naturally, all doctrines or theories in physics are the idealized models of the objectively physical world, which inevitably contain idealized assumptions or hypotheses. Newton's law of universal gravitation is no exception.

As stated in Sec. 12.1 **The Problem of the Locality of Gravitational Spacetime** of Chapter 12, Newton's gravitational theory, including Newton's law of universal gravitation, has two idealized hypotheses:

(i) Gravitational interaction acts at a distance, with infinite radiation speed: it takes no time for gravity to cross space;

(ii) Observational information acts at a distance, with infinite transmit speed: it takes no time for information to cross space.

The first hypothesis is the condition of **Action at a distance** about gravity.

Newton realized from the very beginning that his law of universal gravitation had the problem of gravitational action at a distance. But Newton's reason told Newton that **action at a distance** is not the objectively physical reality, and gravitational radiation must have a limited speed. Although Newton as well as later physicists failed to solve the problem of gravitational action at a distance, intuition told Newton and later people like Laplace [43] and Flandern [127] that the speed of gravitational radiation must be extremely fast: far faster than the speed of light. Otherwise, photons would be difficult to be affected by gravity or universal gravitation, and the celestial bodies in the universe would be difficult to maintain such a stable operating structure (see Flandern's literature [127]).

The second hypothesis may be said the condition of **Medium at a distance**, which is actually a class of **Action at a distance**, too.

Newton did not realize that the theories or models of physics needed to be linked with observation. He did not realize that his theory, including all his physical laws or models, depended on the idealized observation system, was the theory of idealized observation; while there was no the idealized observation system (agent) in the objectively physical world. However, the hypothesis of **medium at a distance** is implicitly embedded in Newton's law of universal gravitation. Of course, the hypothesis of medium at a distance not only implicitly exists in Newton's law of universal gravitation, but also in all laws or theoretical models of classical physics, including Kepler's three laws of planetary motion, the Galilean transformation, and Newton's three laws of motion.

The hypothesis of **Medium at a distance** endows classical physics with an idealized observation system: **the idealized observation agent** OA_∞. As the theory of OR has elucidated, Galileo's doctrine and Newton's theory are that of idealized observation, belonging to the idealized observation agent OA_∞. Newton's law of universal gravitation (Eq. (14.1)) is no exception: strictly speaking, the law is true or valid if and only if under the idealized observation agent OA_∞.

According to the theorem of Cartesian spacetime (see Sec. 13.2 of Chapter 13): $g_{\mu\nu}=\eta_{\mu\nu}$ as $\eta\rightarrow\infty$. Newton's gravitational field belongs to the Cartesian spacetime $X^{4d}{}_\infty$ of OA_∞, and therefore, Newton's law of universal gravitation must be true if in the Cartesian spacetime $X^{4d}{}_\infty$ or under the idealized observation agent OA_∞.

14.1.2 Einstein's Field Equation and Motion Equation

Einstein believed that: the earth's motion around the sun was due to spacetime curvature; spacetime curvature was due to the presence of matter and energy. Therefore, Einstein conceived that his theory of general relativity should contain two basic equations: one was field equation; the other was motion equation.

(i) Field Equation: to describe how spacetime is curved, it has the form of {**Spacetime Curvature = Matter or Energy**}

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa_E T_{\mu\nu} \tag{14.2}$$

where $G_{\mu\nu}$ is known as **Einstein tensor**, $R_{\mu\nu}$ (i.e. the curvature of spacetime) is known as **Ricci tensor**, R is the Gaussian curvature, $g_{\mu\nu}$ is the spacetime metric, $T_{\mu\nu}$ is the energy-momentum tensor, and κ_E is the coefficient of Einstein field equation.

Equation (14.2) does not include the cosmological term: $\Lambda g^{\mu\nu}$, or the cosmological constant $\Lambda=0$. (We are not discussing the cosmological term $\Lambda g^{\mu\nu}$ of Einstein field equation for the moment).

(ii) Motion Equation: to describe how an object moves in curved spacetime, it is determined as the geodesic equation

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}s^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}s} \frac{\mathrm{d}x^\beta}{\mathrm{d}s} = 0 \quad (\mu = 0,1,2,3) \tag{14.3}$$

where $\Gamma^\alpha{}_{\mu\nu}$ is referred to as **Connection**.

Later, Einstein et al [137] and Fock [138] successively proved that Einstein's field equation (Eq. (14.2)) could be derived from Einstein's motion equation (Eq. (14.3)). This means that Einstein's theory of general relativity actually has only one basic equation: the Einstein field equation. However, this does not mean the denial of the value and significance of Einstein motion equation, but only means that Einstein's field equation and Einstein's motion equation are equivalent, and have the intrinsic relevance.

It is worth noting that the calibration of the coefficient κ_E of Einstein field equation depends not only on Einstein's field equation (Eq. (14.2)) but also on Einstein's motion equation (Eq. (14.3)).

As a theory or a model of the objectively physical world, Einstein's theory of general relativity, including Einstein's field equation and Einstein's motion, must have the idealized characteristics and hypothetical preconditions of its own.

As stated in Sec. 12.1 **The Problem of the Locality of Gravitational Spacetime** of Chapter 12, Einstein's theory of general relativity also has two idealized hypotheses:

(i) Gravitational interaction acts at a distance, with infinite radiation speed: it takes no time for gravity to cross space;

(ii) Observational information acts at the speed c of light, employing light wave or electromagnetic interaction as the observation medium: it takes time for information to cross space.

The first hypothesis is also the condition of **Action at a distance** about gravity.

With regard to the first hypothesis of action at a distance, Einstein did not realize that, like Newton's theory of universal gravitation, his theory of general relativity also contains the same idealized hypothesis: gravity or gravitational interaction is **action at a distance**. Actually, as stated in Sec. 12.1 of Chapter 12, like Newton's theory of universal gravitation, Einstein's theory of general relativity, including his field equation and motion equation, also is implicitly embedded the hypothesis of action at a distance about gravity or gravitational interaction, and therefore, there is no prior knowledge or information about gravitational waves and the speed of gravitational radiation. Logically or theoretically, it is impossible for Einstein or Einstein's theory of general relativity to predict gravitational waves, let alone the speed of gravitational waves.

The second hypothesis may be said the condition of **optical observation**.

Like Newton, Einstein did not realize that the theories or models of physics needed to be linked with observation. Einstein also did not realize that his theory of relativity, including the special and the general depended on the optical observation system, was the theory of optical observation, and restricted by the observational locality ($c<\infty$) of the optical observation agent OA(c). However, the hypothesis of **optical observation** is implicitly embedded in Einstein's theory of relativity, including the special and the general, and in particular, embedded in Einstein's principle of the invariance of light speed. (Perhaps, Einstein never considered that his theory of relativity was the theory of optical observation and needed to employ light or electromagnetic interaction as the observation medium.)

Einstein's hypothesis of the invariance of light speed endows Einstein's theory of relativity with the optical observation system or the optical observation agent OA(c), the most common means by which human beings perceive or observe the objectively world. Minkowski formalized the optical agent OA(c) as the 4d spacetime coordinate framework of optical observation, known as the Minkowski spacetime [50,51]. Although Minkowski spacetime was created specifically for Einstein's theory of special relativity, it was later extended to the theory of general relativity by Einstein. It is thus clear that, as the theory of OR has clarified, Einstein's theory of relativity, including the special and the general, is the theory of optical observation and belong to optical observation agent OA(c). So, Einstein's theory of relativity is true or valid if and only if under the optical agent OA(c).

The gravitational spacetime in Einstein's theory of general relativity is the observational spacetime $X^{4\text{d}}(c)$ of the optical agent OA(c), which is a curved spacetime. So, how is the Einstein field equation in curved spacetime related to or corresponded to Newton's law of universal gravitation in flat spacetime?

14.1.3 Einstein's Weak-Field Approximation

Logically or in logic, as the theoretical systems describing the gravitational interaction in the same physical world, Einstein's theory of general relativity and Newton's theory of universal gravitation should have the intrinsic linkage or corresponding relationship.

Einstein supposed that, as the Lorentz transformation in the theory of special relativity approximately reduced to the Galilean transformation in the case of slow-speed approximation, his gravitational-field equation in the theory of general relativity should approximately reduce to Newton' law of universal gravitation in the form of Poisson equation in the case of weak-field approximation.

However, as clarified in Chapter 13, there is no the direct corresponding relationship between Einstein's field equation (as an optical observation model) and Newton's law of universal gravitation (as an idealized observation model).

As far as the calibration of the coefficient κ_E of Einstein field equation is concerned, the logical way of weak-field approximation in Einstein's theory of general relativity seemed to be valid or effective: through the weak-field approximation, Einstein field equation was linked and corresponded to Newton's law of universal gravitation in the form of Poisson equation, and then, Einstein successfully calibrated and built up the gravitational field equation in his theory of general relativity.

Einstein's field equation is actually a complex system of Nonlinear partial differential equations, which is very difficult to calibrate and solve. Einstein had to use certain linearization method to process it: the most basic task was to determine the coefficient κ_E of Einstein field equation; the most basic method was Einstein's logical way of weak-field approximation.

In order to correspond Einstein's field equation in **curved** spacetime with Newton's law of universal gravitation or the form of Poisson equation in **flat** spacetime, Einstein had to flatten the curved spacetime, so that the factor γ of gravitational spacetime transformation was approximate to the Galilean factor: $\gamma\approx\Gamma_\infty\equiv1$. According to the factor $\gamma=1/\sqrt{(1+2\chi/c^2)}$ of gravitational spacetime transformation in Einstein's theory of general relativity, the speed c of light in vacuum is a constant. So, intended to make $\gamma\approx\Gamma_\infty\equiv1$, one has to make the Newtonian gravitational potential $\chi\approx0$ or $|\chi|<<c^2$, so that Newton's gravitational field becomes the so-called weak Gravitational field.

The Galilean factor $\Gamma_\infty\equiv1$ represents the **flat** Cartesian spacetime, where $\mathrm{d}t=\mathrm{d}\tau$. (Einstein's logical way of weak-field approximation implies such a claim; while strict theoretical proof comes from the theorem of Cartesian spacetime.)

According to Eqs. (12.15-16) in Chapter 12, the most general form of the Einstein factor γ of gravitational spacetime transformation can be written as

$$\gamma=\frac{1}{\sqrt{\left(\sqrt{g_{00}}+\dfrac{g_{0i}}{\sqrt{g_{00}}}\dfrac{\mathrm{d}x^i}{\mathrm{d}x^0}\right)^2+\left(g_{ik}-\dfrac{g_{0i}g_{0k}}{g_{00}}\right)\dfrac{\mathrm{d}x^i}{\mathrm{d}x^0}\dfrac{\mathrm{d}x^k}{\mathrm{d}x^0}}}$$
$$=\frac{1}{\sqrt{\left(\sqrt{1+2\chi/c^2}-\gamma_i v^i/c\right)^2-v^2/c^2}} \tag{14.4}$$

which suggests that, to make $\gamma\approx\Gamma_\infty\equiv1$, in addition to the gravitational scalar potential χ, it is also necessary to consider the gravitational vector potential γ_i (i=1,2,3) and the speed v of the observed object P. So, as stated in Sec. 13.1 of Chapter 13, Einstein's conditions of weak-field approximation have five hypotheses: (i) weak field ($|\chi|<<c^2$); (ii) slow speed ($|v|<<c$); (iii) static field ($g_{\alpha\beta,0}=h_{\alpha\beta,0}=0$ (α,β=0,1,2,3)); (iv) spacetime orthogonality ($g_{0i}=g_{i0}=0$, (i=1,2,3)); (v) harmonic coordinates ($\sqrt{(-g)}=1$).

Under the case of weak-field approximation, $\gamma\approx\Gamma_\infty\equiv1$ means that the spacetime of weak gravitational field is approximately flat. Einstein imagined that, since the spacetime of weak field is approximately flat, Newton's law of universal gravitation would approximately be true under the weak-field conditions, and therefore, his field equation and motion equation in the case of weak field could approximately be corresponded to Newton's law of universal gravitation in the form of Poisson equation. In this way, the coefficient κ_E of Einstein field equation could be calibrated or determined with Newton's gravitational potential χ or Newton's universal gravitational constant G.

As stated in Chapter 13, the metric $g_{\mu\nu}$ of curved gravitational spacetime could be decomposed into the flat metric (Minkowski metric) $\eta_{\mu\nu}$ and the curved metric $h_{\mu\nu}$; in the case of weak field, it follows that

The condition of weak field: $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}\quad\left(|h_{\mu\nu}|\le|\eta_{\mu\nu}|\right)$

Naturally, in the absence of gravity or gravitational field, $g_{\mu\nu}=\eta_{\mu\nu}$ and $h_{\mu\nu}=\mathbf{0}$. Therefore, in Einstein's view, the curved metric $h_{\mu\nu}$ ($\approx\mathbf{0}$) under the conditions weak-field approximation represented the weak gravitational potential.

It is worth noting that, under the condition ($g_{0i}=g_{i0}=0$ (i=1,2,3)) of spacetime orthogonality, the factor γ (Eq. (14.4)) of spacetime transformation in Einstein's theory of general relativity would reduce to:

$$\gamma=\frac{\mathrm{d}t}{\mathrm{d}\tau}=\frac{1}{\sqrt{g_{00}-v^2/c^2}}=\frac{1}{\sqrt{1-v^2/c^2+h_{00}}} \tag{14.5}$$
$$\left(g_{00}=\eta_{00}+h_{00};\,\eta_{00}\equiv 1\right)$$

Thus, the curved state of gravitational spacetime in Einstein's theory of general relativity only depends on the 00-element h_{00} of the curved metric $h_{\mu\nu}$. Equation (14.5) shows that, according to Einstein's logic of weak-field approximation, if $|h_{00}|<<1$ (weak field) and $|v|<<c$ (slow speed), then $\gamma\approx\Gamma_\infty\equiv 1$, the curved gravitational spacetime would approximately be flat, and Newton's law of universal gravitation would approximately be true. In this way, Einstein's field equation and motion equation in curved spacetime could approximately be corresponded to the Newton's law of universal gravitation in flat spacetime.

The weak-field approximation is actually Einstein's linearization theory made specifically for his theory of general relativity, in particular, for the linearization and calibration of his field equation.

According to Eq. (14.5), Einstein's logical way of weak-field approximation could be designed as follows.

The Logical Program of Weak-Field Approximation

With the goal of calibrating the coefficient κ_E of Einstein field equation, according to the Einstein factor γ (Eq. (14.5)) of spacetime transformation in the weak-field situation, based on Einstein's logic of weak-field approximation, by linking the curved-metric 00-element h_{00} with Newton's gravitational potential χ and the field-equation coefficient κ_E, then the logical deduction program could be divided into the following two steps.

(i) The first step: the weak-field approximation of Einstein motion equation, that is, linking the curved-metric 00-element h_{00} in Einstein's motion equation (Eq. (14.3)) with the Newtonian gravitational potential χ ($h_{00}\sim\chi$).

(ii) The second step: the weak-field approximation of Einstein field equation: that is, linking the curved-metric 00-element h_{00} in Einstein's field equation (Eq. (14.2)) with the field-equation coefficient κ_E ($h_{00}\sim\kappa_E$).

Finally, under the scene of Newton's law of universal gravitation, by contrasting the relations $h_{00}\sim\chi$ and $h_{00}\sim\kappa_E$ of weak-field approximation, the coefficient κ_E of Einstein field equation could be calibrated with Newton's gravitational potential χ or Newton's universal gravitational constant G.

The Conditions of Weak-Field Approximation

As the linearization theory of Einstein's general relativity, Einstein's logical way of weak-field approximation has a set of linearization conditions. According to the conditions of weak-field approximation stated in Sec. 13.1.3 of Chapter 13, in addition to the assumption of

(i) The weak field: $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ ($|h_{\mu\nu}|<<|\eta_{\mu\nu}|$);

Einstein also had to assume

(ii) The slow speed: $|v|<<c$;

(iii) The static field: $g_{\mu\nu,0}=\partial g_{\alpha\beta}/\partial x^0=0$ or $h_{\mu\nu,0}=\partial h_{\alpha\beta}/\partial x^0=0$;

(iv) The spacetime orthogonality: $g_{0i}=g_{i0}=0$ (i=1,2,3); and

(v) The harmonic coordinates: $\Box x^{\mu}=g^{\alpha\beta}\Gamma^{\mu}{}_{\alpha\beta}=0$ (μ=0,1,2,3).

Newton's Gravitational Scene

As described in Sec. 13.1.4 of Chapter 13, in Newton's gravitational scene, that is, the scene set by Newton's law of universal gravitation, there are only two matter particles M and m at a distance of r in the gravitational spacetime: M is the gravitational source and the gravitational center, forming the spherically symmetric gravitational-field, and therefore, the gravitational potential is the same or equal everywhere on the sphere with the same radius r; m (with the matter density ρ) is the object affected by gravity. Let m be located at gravitational potential χ, then according to Newton's law of universal gravitation: $\chi=-GM/r$.

Naturally, in order to be approximately corresponded to Newton's law of universal gravitation, Einstein's field equation or Einstein's motion equation should be set up the same gravitational scene as that Newton's law of universal gravitation.

It should be pointed out again, Newton's gravitational scene does not mean that the gravitational field in Newton's theory of universal gravitation is the so-called **weak field**. Actually, in the gravitational scene of Newton's law of universal gravitation, the masses of M and m could also be arbitrarily large.

However, as stated in Sec. 13.1.4 of Chapter 13, Einstein's conditions of weak-field approximation naturally hold true in Newton's gravitational scene; in other words, it was based on Newton's gravitational scene that Einstein set up his conditions of weak-field approximation. It should be pointed out that, for the first condition of the weak field, $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ ($h_{\mu\nu}|<<|\eta_{\mu\nu}|$) is not because that Newton's gravitational field is the so-called weak field, but that Newton's gravitational field is flat spacetime. In Einstein's view, flat spacetime means weak field.

14.1.4 The Weak-Field Approximation of Motion Equation

Einstein's motion equation (Eq. (14.3)) in general relativity can be written as:

$$\begin{aligned}&\frac{\mathrm{d}^2x^{\mu}}{\mathrm{d}\tau^2}+\Gamma^{\mu}_{\alpha\beta}\frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{\beta}}{\mathrm{d}\tau}=0 \quad (\mu=0,1,2,3)\\ &\Gamma^{\mu}_{\alpha\beta}=\frac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta}+g_{\nu\beta,\alpha}-g_{\beta\alpha,\nu}\right) \quad \left(,\sigma=\frac{\partial}{\partial x^{\sigma}}\right)\end{aligned} \tag{14.6}$$

What it describes is the geodesic line of the observed object P moving in the curved spacetime of gravitational field, which is related with the connection $\Gamma^{\mu}{}_{\alpha\beta}$, and the connection $\Gamma^{\mu}{}_{\alpha\beta}$ is related with the spacetime metric $g_{\mu\nu}$.

The Imagination on
the Weak-Field Approximation of Motion Equation

In Einstein's view, under the conditions of weak-field approximation, the gravitational spacetime was approximately flat, his motion equation (Eq. (14.6)), the so-called geodesic line in curved spacetime, would approximately reduce to a straight line in Euclidean space of Newton's Gravitational field:

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = -\nabla\chi \quad \left(\chi = -\frac{GM}{r}\right) \tag{14.7}$$

where χ is the Newtonian gravitational potential.

In particular, Multiplying the left and right ends of Eq. (14.7) by m, one could get Newton's law of universal gravitation in the form of Newton's second law.

In this way, under the conditions of weak-field approximation, the curved metric $h_{\mu\nu}$ could be related with the Newtonian gravitational potential χ.

The Operation on
the Weak-Field Approximation of Motion Equation

The procedure of weak-field approximation for Einstein's motion equation (Eq. (14.3) is the linearization procedure of Einstein's motion equation: the nonlinear geodesic equation is approximate to a linear equation, or in other words, the curved geodesic line is approximate to a straight line.

In Einstein's condition of the weak field: $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ ($|h_{\mu\nu}|<<|\eta_{\mu\nu}|$), $h_{\mu\nu}$ represents the weak gravitational potential, $h_{\mu\nu}$ and its derivative of each order are infinitesimal. Therefore, the connection $\Gamma^{\mu}{}_{\alpha\beta}$ in the geodesic (Eq. (14.6)) only needs to reserve the linear term of $h_{\mu\nu}$ and ignore the high-order terms.

In this way, the connection $\Gamma^{\mu}{}_{\alpha\beta}$ in Einstein's motion equation (Eq. (14.6)) could form an approximate linear relationship with the weak potential $h_{\mu\nu}$:

$$\begin{aligned} \Gamma^{\mu}_{\alpha\beta} &= \frac{1}{2} g^{\mu\nu}\left(g_{\alpha\nu,\beta} + g_{\nu\beta,\alpha} - g_{\beta\alpha,\nu}\right) \\ &\approx \frac{1}{2}\eta^{\mu\nu}\left(h_{\alpha\nu,\beta} + h_{\nu\beta,\alpha} - h_{\beta\alpha,\nu}\right) \\ &\left(g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu};\, \eta^{\mu\nu} = \eta_{\mu\nu},\, h^{\mu\nu} = \eta^{\mu\alpha}\eta^{\beta\nu}h_{\alpha\beta}\right) \end{aligned} \tag{14.8}$$

It is known that $\mathrm{d}t\approx\mathrm{d}\tau$ under the conditions of weak-field approximation. According to the condition of slow speed $|v^i|=|\mathrm{d}x^i/\mathrm{d}t|<<c$ and $x^0=ct$, it follows that $|\mathrm{d}x^i/\mathrm{d}\tau|<<|\mathrm{d}x^0/\mathrm{d}\tau|$. Therefore, it holds that:

$$
\begin{aligned}
&\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2}+\Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}\\
&\quad=\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2}+\Gamma^\mu_{00}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}+\Gamma^\mu_{i0}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}+\Gamma^\mu_{0k}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^k}{\mathrm{d}\tau}+\Gamma^\mu_{ik}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^k}{\mathrm{d}\tau} \qquad (14.9)\\
&\quad=\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2}+\Gamma^\mu_{00}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\quad(\mu=0,1,2,3)
\end{aligned}
$$

Equation (14.6) approximately reduces to:

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2}+\Gamma^\mu_{00}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}=0\quad(\mu=0,1,2,3) \qquad (14.10)$$

Based on the condition of static field, it follows from Eq. (14.8) that:

$$
\begin{aligned}
\Gamma^\mu_{00}&\approx\frac{1}{2}\eta^{\mu\nu}\left(h_{0\nu,0}+h_{\nu0,0}-h_{00,\nu}\right)=-\frac{1}{2}\eta^{\mu\nu}h_{00,\nu}\\
&=-\frac{1}{2}\left(\eta^{\mu0}h_{00,0}+\eta^{\mu i}h_{00,i}\right)=-\frac{1}{2}\eta^{\mu i}h_{00,i}
\end{aligned} \qquad (14.11)
$$

For the Minkowski metric $\eta_{\mu\nu}$, $\eta^{0i}=0$ and $\eta^{ii}=-1$ (i=1,2,3). It follows that:

$$\Gamma^0_{00}=0\quad\text{and}\quad\Gamma^i_{00}=\frac{1}{2}h_{00,i} \qquad (14.12)$$

Thus, the geodesic equation (Eq. (14.6) in curved spacetime splits into two sets of equations in flat spacetime: independent time and independent space, that is:

$$\frac{\mathrm{d}^2 t}{\mathrm{d}\tau^2}=0 \qquad (14.13a)$$

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}\tau^2}=-\frac{1}{2}h_{00,i}\left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\right)^2\quad(i=1,2,3) \qquad (14.13b)$$

This split between time and space reminds us of the Galilean transformation. In the Galilean transformation, time and space are independent of each other: spacetime splits into independent time and independent space.

In Eq. (14.13), the time equation (14.13a) suggests that, in the weak gravitational field, Einstein's coordinate time-element dt is approximate to Einstein's standard time-element dτ. So, one could get the solution of Eq. (14.13a):

$$t=a\tau+b \qquad (14.14)$$

Appropriately selecting the time unit (let a=1) and calibrating the time (let b=0), then one would have the conclusion of $t=\tau$.

Actually, according to Eq. (14.4) and Eq. (14.5), it is known that, under the conditions of the weak field, slow speed, and spacetime orthogonality, the Einstein factor of spacetime transformation: $\gamma=\Gamma(c)\approx\Gamma_\infty\equiv1$, i.e., d$t\approx$d$\tau$.

Therefore, the space equations (Eq. (14.13b)) reduce to:

$$\frac{d^2x^i}{dt^2} = -\frac{c^2}{2}\nabla_i h_{00} \quad (i = 1,2,3) \tag{14.15}$$

The geodesic (Eq. (14.6)) in the curved spacetime $X^{4d}(c)$ reduces to the straight line of the Cartesian spacetime $X^{4d}{}_{\infty}$ (Eq. (14.15)). By contrasting the geodesic (Eq. (14.15)) in the weak-field approximation and the straight line (Eq. (14.7)) in Newton's gravitational field, one would have that:

$$h_{00} = \frac{2\chi}{c^2} \quad \text{and} \quad g_{00} = 1 + \frac{2\chi}{c^2} \tag{14.16}$$

In this way, the curved metric 00-element h_{00} of weak gravitational spacetime is related with the Newtonian gravitational potential: $h_{00} \sim \chi$.

14.1.5 The Weak-Field Approximation of Field Equation

As shown in Eq. (14.2), Einstein's field equation can be expressed as: $G_{\mu\nu}=\kappa_E T_{\mu\nu}$, where the left end $G_{\mu\nu}$ represents the curvature of gravitational spacetime, and the right end $T_{\mu\nu}$ represents the distribution of matter and energy. The curvature $G_{\mu\nu}$ of spacetime and the distribution $T_{\mu\nu}$ of matter and energy are related by the coefficient κ_E of Einstein field equation.

The Imagination on
the Weak-Field Approximation of Field Equation

On the one hand, the Einstein tensor $G_{\mu\nu}$ at the left end of Einstein field equation is related with the spacetime curvature $R_{\mu\nu}$, $R_{\mu\nu}$ is related with the connection $\Gamma^{\mu}{}_{\alpha\beta}$, and $\Gamma^{\mu}{}_{\alpha\beta}$ is related with the spacetime metric $g_{\mu\nu}$, forming the complicated nonlinear relationship between $G_{\mu\nu}$ and $R_{\mu\nu}$ or $g_{\mu\nu}$. On the other hand, the Einstein tensor $G_{\mu\nu}$ at the left end of Einstein field equation and the energy-momentum tensor $T_{\mu\nu}$ at the right end of Einstein field equation are related by the coefficient κ_E of Einstein field equation, forming the simple linear relationship between $G_{\mu\nu}$ and $T_{\mu\nu}$.

In Einstein's view, under the conditions of weak-field approximation, with the weakening of the gravitational potential, the nonlinear relationship between the Einstein tensor $G_{\mu\nu}$ and the spacetime curvature $R_{\mu\nu}$ or the spacetime metric $g_{\mu\nu}$ would reduce to the approximate linear relationship between $G_{\mu\nu}$ and the weak potential $h_{\mu\nu}$, and the nonlinear Einstein field equation in curved spacetime would approximately be reduced to the linear field equation in flat spacetime.

Thus, the curved metric $h_{\mu\nu}$ under the conditions of weak-field approximation could be related with the coefficient κ_E of Einstein field equation.

The Operation on
the Weak-Field Approximation of Field Equation

Multiplying the left and right ends of Eq. (14.2) by $g^{\mu\nu}$, one could get

$$g^{\mu\nu}R_{\mu\nu}-\frac{1}{2}g^{\mu\nu}g_{\mu\nu}R=-\kappa_E g^{\mu\nu}T_{\mu\nu}$$

$$\begin{cases} g^{\mu\nu}g_{\mu\nu}=\delta^{\mu}_{\mu}=4 \\ g^{\mu\nu}R_{\mu\nu}=R^{\mu}_{\mu}=R \\ g^{\mu\nu}T_{\mu\nu}=T^{\mu}_{\mu}=T \end{cases} \tag{14.17}$$

where R is the Gaussian curvature, $g_{\mu\nu}$ is the spacetime metric, and T is the trace of the energy-momentum tensor $T_{\mu\nu}$.

Thus, $R=\kappa_E T$, and Einstein's field equation (14.2) could be rewritten as

$$R_{\mu\nu}=-\kappa_E\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right) \tag{14.18}$$

According to the definition of the spacetime curvature, the Ricci tensor $R_{\mu\nu}$ is

$$\begin{aligned} R_{\mu\nu} &\triangleq R^{\alpha}_{\mu\nu\alpha} \\ &=\frac{\partial}{\partial x^{\nu}}\Gamma^{\alpha}_{\mu\alpha}-\frac{\partial}{\partial x^{\alpha}}\Gamma^{\alpha}_{\mu\nu}+\Gamma^{\sigma}_{\mu\alpha}\Gamma^{\alpha}_{\sigma\nu}-\Gamma^{\sigma}_{\mu\nu}\Gamma^{\alpha}_{\sigma\alpha} \end{aligned} \tag{14.19}$$

It is thus clear that Einstein's field equation is a complex system of Nonlinear partial differential equations and very difficult to calibrate and solve.

Like the weak-field approximation of Einstein motion equation, by means of the logical way of weak field approximation, removing the high-order small quantities of the curved metric $h_{\mu\nu}$ and reserving the linear term of $h_{\mu\nu}$, Einstein's field equation and the spacetime curvature $R_{\mu\nu}$ would approximately be reduced to linear equations of the curved metric $h_{\mu\nu}$.

From Eq. (14.8), ignoring the high-order terms of the curved metric $h_{\mu\nu}$, the Ricci tensor $R_{\mu\nu}$ is approximate to:

$$R_{\mu\nu}\approx\frac{\partial}{\partial x^{\nu}}\Gamma^{\alpha}_{\mu\alpha}-\frac{\partial}{\partial x^{\alpha}}\Gamma^{\alpha}_{\mu\nu} \tag{14.20}$$

From Eq. (14.8), ignoring the high-order terms of the curved metric $h_{\mu\nu}$, the connection $\Gamma^{\mu}{}_{\alpha\beta}$ is approximate to:

$$\begin{aligned} \Gamma^{\alpha}_{\mu\nu} &\approx\frac{1}{2}\eta^{\alpha\beta}\left(h_{\mu\beta,\nu}+h_{\beta\nu,\mu}-h_{\nu\mu,\beta}\right) \\ &=\frac{1}{2}\left(\frac{\partial}{\partial x^{\nu}}h^{\alpha}_{\mu}+\frac{\partial}{\partial x^{\mu}}h^{\alpha}_{\nu}-\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\nu\mu}\right) \end{aligned} \tag{14.21}$$

By substituting α for ν in Eq. (14.21), one could get $\Gamma^{\alpha}{}_{\mu\alpha}$ from $\Gamma^{\alpha}{}_{\mu\nu}$:

$$\Gamma^{\alpha}_{\mu\alpha} \approx \frac{1}{2}\left(\frac{\partial}{\partial x^{\alpha}}h^{\alpha}_{\mu} + \frac{\partial}{\partial x^{\mu}}h^{\alpha}_{\alpha} - \frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\alpha\mu}\right)$$
$$= \frac{1}{2}\left(\frac{\partial}{\partial x^{\alpha}}h^{\alpha}_{\mu} + \frac{\partial}{\partial x^{\mu}}h - \frac{\partial}{\partial x^{\beta}}h^{\beta}_{\mu}\right) = \frac{1}{2}\frac{\partial}{\partial x^{\mu}}h \tag{14.22}$$

Thus, Eq. (14.20) could be rewritten as

$$R_{\mu\nu} = \frac{1}{2}\frac{\partial}{\partial x^{\nu}}\frac{\partial}{\partial x^{\mu}}h - \frac{1}{2}\frac{\partial}{\partial x^{\alpha}}\left(\frac{\partial}{\partial x^{\nu}}h^{\alpha}_{\mu} + \frac{\partial}{\partial x^{\mu}}h^{\alpha}_{\nu} - \frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\nu\mu}\right)$$
$$= \frac{1}{2}\frac{\partial}{\partial x^{\alpha}}\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\mu\nu}$$
$$-\frac{1}{2}\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial}{\partial x^{\alpha}}h^{\alpha}_{\mu} - \frac{1}{2}\frac{\partial}{\partial x^{\mu}}h\right) - \frac{1}{2}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial}{\partial x^{\alpha}}h^{\alpha}_{\nu} - \frac{1}{2}\frac{\partial}{\partial x^{\nu}}h\right) \tag{14.23}$$

According to the condition of harmonic coordinates (see Sec. 13.1.3 of Chapter 13), the formula of harmonic coordinates could be written as $\square x^{\mu}=g^{\alpha\beta}\Gamma^{\mu}{}_{\alpha\beta}=0$. Based on the theory of linearization for weak-field approximation, such a normalized condition is equivalent to [139]:

$$\frac{\partial}{\partial x^{\sigma}}h^{\sigma}_{\lambda} - \frac{1}{2}\frac{\partial}{\partial x^{\lambda}}h = 0 \tag{14.24}$$

Therefore, under the conditions of weak field approximation, the curvature $R_{\mu\nu}$ could further be reduced to:

$$R_{\mu\nu} = \frac{1}{2}\frac{\partial}{\partial x^{\alpha}}\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\mu\nu}$$
$$= \frac{1}{2}\square h_{\mu\nu} \quad \left(\square = \frac{\partial}{\partial x^{\alpha}}\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta} = \frac{\partial^2}{c^2\partial t^2} - \nabla^2\right) \tag{14.25}$$

where the symbol "□" is d' Alembert operator of the optical observation agent OA(c), $\Delta=\nabla^2$ is Laplace operator.

It should be pointed out that, originally, Laplace operator $\Delta=\nabla^2$ is a second-order partial differential operator in the 3d Cartesian space; while the 4d Minkowski spacetime extends it from 3d space to 4d spacetime, that is, the so-called d' Alembert operator. Actually, the d' Alembert operator "□" is only a second-order partial differential operator in the 4d observational spacetime $X^{4\text{d}}(c)$ of the optical agent OA(c). The theory of OR further extends it to that in the 4d observational spacetime $X^{4\text{d}}(\eta)$ of the general observation agent OA(η) (see Sec. 5.5 **D' Alembert Operator in OR Theory** in Chapter 5).

According to Eq. (14.25), Einstein's nonlinear gravitational-field equation (Eq. (14.2)) is ultimately linearized by the weak-field approximation as:

$$\frac{1}{2}\square h_{\mu\nu} = -\kappa_E\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) \tag{14.26}$$

In this way, the curved metric 00-element h_{00} of weak gravitational spacetime is related with the coefficient κ_E of Einstein field equation: $h_{00} \sim \kappa_E$.

14.1.6 Calibrating the Coefficient κ_E of Field Equation

Einstein expected that the linearized field equation (14.26) of weak-field approximation could approximately be corresponded to Newton's law of universal gravitation in the form of Poisson equation.

In Einstein's view, in Newton's gravitational scene (see Sec. 10.2 in Chapter 10), the larger matter particle M makes the surrounding spacetime curved, and the smaller matter particle m moves in the curved spacetime. Let the 3d speed (or 3d **Proper Speed**) of m be $v=(v^1,v^2,v^3)$ ($v^i=\mathrm{d}x^i/\mathrm{d}t$ ($i=1,2,3$)), and the 4d speed (or **Four-Speed**) be $u=(u^0,u^1,u^2,u^3)$ ($u^\mu=\mathrm{d}x^\mu/\mathrm{d}\tau$ ($\mu=0,1,2,3$)). Then, under the conditions of weak-field and slow-speed, $\mathrm{d}t\approx\mathrm{d}\tau$, $g_{\mu\nu}\approx\eta_{\mu\nu}$, and $|v|<<c$, it follows that

$$\begin{cases} u^\mu = \dfrac{\mathrm{d}x^\mu}{\mathrm{d}\tau} \approx \dfrac{\mathrm{d}x^\mu}{\mathrm{d}t} = \begin{cases} v^0 = c & \mu = 0 \\ v^i = 0 & \mu = i = 1,2,3 \end{cases} \\ T^{\mu\nu} = \rho u^\mu u^\nu \\ T_{\mu\nu} = \rho g_{\mu\alpha} g_{\nu\beta} u^\alpha u^\beta \approx \rho c^2 \eta_{\mu 0}\eta_{\nu 0} \quad \left(T_{00} = \rho c^2\right) \\ T = T^\mu_\mu = \rho u^\mu u_\mu = \rho c^2 \end{cases} \tag{14.27}$$

where the gravitational spacetime belongs to the observational spacetime $X^{4\mathrm{d}}(c)$ of the optical observation agent OA(c) with the time axis $x^0=ct$; ρ is the material density of the matter particle m, $T_{\mu\nu}$ is the energy-momentum tensor of the matter particle m in the observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c), and T is the trace of $T_{\mu\nu}$.

It should be pointed out that the concept of **Four-Speed** comes from Einstein's theory of special relativity and is the extension of the concept of 3d speed or 3d proper speed from the 3d Cartesian space to the 4d Minkowski spacetime. Actually, the concept of Einstein's four-speed is only the speed concept of the 4d observational spacetime $X^{4\mathrm{d}}(c)$ of the optical agent OA(c). The theory of OR has further extended the concept of four-speed to that of the 4d observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η) (see Sec. 5.4 **The Four-Speed in OR Spacetime** in Chapter 5).

Under the conditions of weak-field and slow-speed, the four-speed of m (the observed object P) is $u\approx(c,0,0,0)$, and therefore, as shown in Eq. (14.27), the energy-momentum tensor $T_{\mu\nu}$ of m reduces to a scalar: T_{00}.

Thus, the field equation (Eq. (14.18) and Eq. (14.26)) could be rewritten as:

$$\begin{aligned} R_{\mu\nu} &\approx -\kappa_E\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right) = -\frac{1}{2}\kappa_E T_{00}\delta_{\mu\nu} \\ \Box h_{\mu\nu} &\approx -\kappa_E T_{00}\delta_{\mu\nu} = -\kappa_E \rho c^2 \delta_{\mu\nu} \end{aligned} \tag{14.28}$$

where $\delta_{\mu\nu}$ is Kronecker tensor.

Under the conditions of weak-field and slow-speed, d' Alembert operator "□" of the optical agent OA(c) is approximate to $-\Delta$ (see Sec. 5.5 in Chapter 5):

$$\square = \frac{\partial^2}{c^2 \partial t^2} - \nabla^2 \approx -\nabla^2 = -\Delta \tag{14.29}$$

According to Eq. (14.16), Eq. (16.28), and Eq. (14.29), one could get that

$$\begin{aligned} &\square h_{00} = -\kappa_E \rho c^2 \quad \text{or} \quad \Delta h_{00} = \kappa_E \rho c^2 \ \left(h_{00} = 2\chi/c^2\right) \\ &\text{so that } \nabla^2 \chi = \frac{1}{2} \kappa_E \rho c^4 \end{aligned} \tag{14.30}$$

So, by contrasting Eq. (14.30) with the Poisson-equation form of Newton's law of universal gravitation: $\nabla^2\chi = 4\pi G\rho$, one could get the coefficient κ_E of Einstein's field equation:

$$\kappa_E = \frac{8\pi G}{c^4} \tag{14.31}$$

Einstein linearized his field equation and his motion equation by the logical way of weak-field approximation, so that he could calibrated the coefficient κ_E of his field equation by the approximate correspondence between Einstein's equation of gravitational field and Newton's law of universal gravitation in the form of Poisson equation. In this way, Einstein's field equation had been built up finally.

Obviously, Einstein's logical way of weak-field approximation played an important role in the calibration of Einstein field equation and even in the establishment of Einstein's theory of general relativity. Afterwards, Einstein also applied the logical way of weak-field approximation to solve his field equation.

14.2 GOR Field Equation and GOR Motion Equation

Beyond doubt, Einstein's gravitational-field equation is the most important formula in Einstein's theory of general relativity and the core of Einstein's theory of general relativity. Likewise, the GOR gravitational-field equation will be the most important formula in the theory of Gravitationally Observational Relativity (GOR) and the core of the theory of GOR.

Like Einstein's field equation, the establishment of GOR gravitational-field equation in the theory of GOR, involves two tasks:

(i) The deduction of the logical form of GOR field equation;

(ii) The calibration of the coefficient of GOR field equation.

It is said that Einstein ever sighed that, it only took him five weeks to establish the theory of special relativity, while the theory of general relativity took him ten years. It can be speculated that Einstein's field equation took up most of the time.

However, taking advantage of the principle of general correspondence (GC), it will not take us ten years to establish the GOR gravitational-field equation.

14.2.1 The GOR Field Equation

Under the principle of GC, through PGC logic route 1, directly substituting the information-wave speed η of the general observation agent OA(η) for the light speed c in Einstein's field equation (Eq. (14.2)) including that in the coefficient κ_E (Eq. (14.31), one could extend Einstein's field equation from the optical agent OA(c) to the general observation agent OA(η), and transform Einstein's field equation into the GOR gravitational-field equation:

$$R_{\mu\nu}(\eta)-\frac{1}{2}g_{\mu\nu}R=-\kappa_{\text{GOR}}T_{\mu\nu}\quad\left(\kappa_{\text{GOR}}=\frac{8\pi G}{\eta^4}\right)\tag{14.32}$$

where κ_{GOR} is the coefficient of GOR field equation.

It is worth noting that, unlike Einstein's field equation, in the GOR field equation, the Ricci tensor $R_{\mu\nu}=R_{\mu\nu}(\eta)$, the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$, the Gaussian curvature $R=R(\eta)$, the energy-momentum tensor $T_{\mu\nu}=T_{\mu\nu}(\eta)$, and the field-equation coefficient $\kappa_{\text{GOR}}=\kappa_{\text{GOR}}(\eta)$, all depend on the information-wave speed η of the general observation agent OA(η), rather than the light speed c.

Under the principle of GC, through PGC logical route 1, we do not even have to spend time calibrating the coefficient κ_{GOR} of GOR field equation (Eq. (14.32)).

Naturally, under the principle of GC, through PGC logical route 2, we could transform Einstein's three principles in his general relativity: (i) the principle of equivalence, (ii) the principle of general covariance, and (iii) the principle of the invariance of light speed, into the GOR three principles: (i) the principle of equivalence; (ii) the principle of general covariance; (iii) the principle of the invariance of information-wave speeds. Then, by analogizing or following the logic of Einstein's general relativity, and starting from the GOR three principles, we also could logically and theoretically deduce the gravitational-field equation of GOR theory, which must be the same as Eq. (14.32).

Obviously, the GOR field equation of Eq. (14.32) is isomorphically consistent with the Einstein field equation of Eq. (14.2). Moreover, it is worth noting that, if $\eta=c$, the GOR field equation is exactly the Einstein field equation, where $\kappa_{\text{GOR}}=\kappa_E$ is exactly the coefficient κ_E of Einstein field equation.

So, the GOR field equation will generalize Einstein's field equation.

14.2.2 The GOR Motion Equation

As we know, Einstein ever imagined that his theory of general relativity should contain two basic equations: one is the field equation; the other is the motion equation. It was later found that the field equation and the motion equation are equivalent [137,138]. Now that they are equivalent, Einstein's field equation and motion equation have the same value and significance. In particular, as stated in Sec. 14.1, the calibration of the coefficient κ_E of Einstein field equation needs the support of Einstein's motion equation.

In the same logic, the theory of GOR should also have two fundamental formulae that are mutually equivalent: one is the GOR field equation; the other is the GOR motion equation.

Naturally, like the GOR field equation, under the principle of GC, through the

PGC logic route 1, directly substituting the information-wave speed η of the general observation agent OA(η) for the light speed c in Einstein's motion equation (Eq. (14.3)), one could extend Einstein's motion equation from the optical agent OA(c) to the general observation agent OA(η), and transform Einstein's motion equation into the GOR motion equation:

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}s^2}+\Gamma^\mu_{\alpha\beta}(\eta)\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s}=0 \quad (\mu=0,1,2,3) \tag{14.33}$$

where, in particular, unlike Einstein's motion equation, in the GOR motion equation, the connection $\Gamma^\mu{}_{\alpha\beta}=\Gamma^\mu{}_{\alpha\beta}(\eta)$ and the time axis $x^0=\eta t$ depend on the information-wave speed η of the general observation agent OA(η), rather than the speed c of light in vacuum.

Naturally, under the principle of GC, through PGC logical route 2, we could transform Einstein's three principles of general relativity: (i) the principle of equivalence, (ii) the principle of general covariance, and (iii) the principle of the invariance of light speed, into the GOR three principles: (i) the principle of equivalence; (ii) the principle of general covariance; (iii) the principle of the invariance of information-wave speeds. Then, by analogizing or following the logic of Einstein's general relativity, and starting from the GOR three principles, we also could logically and theoretically deduce the motion (or geodesic) equation of GOR theory, which must be the same as Eq. (14.33).

Obviously, the GOR motion equation of Eq. (14.33) is isomorphically consistent with the Einstein motion equation of Eq. (14.3). Moreover, it is worth noting that, if $\eta=c$, the GOR motion equation is exactly the Einstein motion equation.

So, the GOR motion equation will generalize Einstein's motion equation.

14.2.3 The Problem about the Calibration of GOR Field Equation

Now, under the principle of GC, through PGC logical routes 1 and 2, we have established the gravitational-field equation of GOR theory (Eq. (14.32)) and the motion equation of GOR theory (Eq. (14.33)). The remaining problem is how to calibrate the coefficient $\kappa_{\mathrm{GOR}}=\kappa_{\mathrm{GOR}}(\eta)$ of GOR field equation and firmly establish the GOR field equation.

The simplest or most convenient way to calibrate the GOR field equation is that, under the principle of GC, through PGC logic route 1, directly substituting the information-wave speed η of the general observation agent OA(η) for the light speed c in the coefficient $\kappa_E=8\pi G/c^4$ of Einstein field equation, one could get the coefficient $\kappa_{\mathrm{GOR}}=8\pi G/\eta^4$ of GOR gravitational-field equation.

However, it should be pointed out that, in that way, we might pay a price for the logical simplicity or convenience, fail to correctly understand Einstein's logic of weak-field approximation, fail to correctly understand Einstein's gravitational-field equation and Newton's law of universal gravitation, fail to correctly understand of Einstein's theory of general relativity and Newton's theory of universal gravitation, and fail to correctly recognized the essence of gravitational relativistic effects.

To avoid **knowing the result but not knowing the reason**, the theory of GOR does not intend to simply copy the coefficient $\kappa_E=8\pi G/c^4$ of Einstein field equation and to directly get the coefficient $\kappa_{\text{GOR}}=8\pi G/\eta^4$ of GOR field equation by substituting η for c through PGC logic route 1.

Moreover, the theory of GOR does not intend to follow the Einstein's logical way of weak-field approximation to deduce the GOR field equation and calibrate the GOR field-equation coefficient $\kappa_{\text{GOR}}=\kappa_{\text{GOR}}(\eta)$. Under the theorem of Cartesian spacetime, the theory of GOR strives to deduce the GOR field equation and calibrate the GOR field-equation coefficient $\kappa_{\text{GOR}}=\kappa_{\text{GOR}}(\eta)$ through the GOR logical way of idealized convergence that is stated and conceived in Chapter 13.

14.3 The GOR Idealized Convergence vs Einstein's Weak-Field Approximation

No matter Einstein's theory of general relativity or the theory of GOR, the calibration of its gravitational-field equation has to employ certain logical way to correspond the field equation in curved spacetime to Newton's law of universal gravitation in flat spacetime. The difference is that: Einstein's theory of general relativity employed the logical way of weak-field approximation; while the theory of GOR will employ the logical way of idealized convergence.

Based on the theorem of Cartesian space-time, Chapter 13 specifically examines Einstein's logic of weak-field approximation and designs the GOR logical way of idealized convergence. Here, we will further sort it out. In particular, by further analyzing the logical flaws of the weak-field approximation, we could correctly understand Einstein's weak-field approximation; by further analyzing the condition of idealized convergence, we could properly apply the GOR logical way of idealized convergence to deduce the GOR gravitational-field equation and calibrate the GOR field-equation coefficient κ_{GOR}.

14.3.1 The Logic Flaws of Weak-Field Approximation

In Einstein's theory of general relativity, in order to correspond the Einstein field equation in curved spacetime to Newton's law of universal gravitation in flat spacetime, Einstein conceived and designed the logical way of weak-field approximation, intended to flatten the curved gravitational spacetime to let the spacetime metric $g_{\mu\nu}\approx\eta_{\mu\nu}$ or the spacetime-transformation factor $\gamma\approx\Gamma_\infty\equiv1$. In this way, Einstein's field equation could be corresponded to Newton's law of universal gravitation in the form of Poisson equation.

According Einstein's factor of gravitational spacetime transformation:

$$\gamma=\gamma(v,\chi,\gamma_i)=\frac{1}{\sqrt{\left(\sqrt{1+2\chi/c^2}-\gamma_i v^i/c\right)^2-v^2/c^2}} \tag{14.34}$$

$$\lim_{v\to0,\,\chi\to0,\,\gamma_i\to0}\gamma(v,\chi,\gamma_i)=1=\Gamma_\infty$$

we know that, the speed c of light is a cosmic constant, the Galilean factor $\Gamma_\infty\equiv1$

represents the flat Cartesian spacetime, including the flat Newtonian gravitational field, such as the gravitational scene in Newton's law of universal gravitation.

Equation (14.34) shows that, to make $\gamma=\Gamma_\infty$, it is required that $v=0$ and $\chi=0$. However, $v=0$ and $\chi=0$ are trivial: no matter motion and no matter interaction. So, for the gravitational spacetime in his theory of general relativity, Einstein had to compile the logical way of weak-field approximation and assume: (i) the weak field ($|\chi|<<c^2$); (ii) the slow speed ($|v|<<c$); (iii) the static field ($g_{\alpha\beta,0}=h_{\alpha\beta,0}=0$ ($\alpha,\beta=0,1,2,3$)); (iv) the spacetime orthogonality ($g_{0i}=g_{i0}=0$, ($i=1,2,3$)); (v) the harmonic coordinates ($\sqrt{(-g)}=1$). In this way, $\gamma\approx\Gamma_\infty\equiv1$, the curved gravitational spacetime could approximately be flattened.

This is Einstein's logic of weak-field approximation.

Einstein's logical way of weak-field approximation seemed to be valid or effective. By means of the weak-field approximation, Einstein had corresponded his field equation to Newton's law of universal gravitation in the form of Poisson equation and had calibrated the coefficient of the field equation: $\kappa_E=8\pi G/c^4$. In this way, Einstein had finally established his gravitational-field equation.

However, the logical flaws of Einstein's weak-field approximation mislead physics and Einstein himself.

The theory of GOR repeatedly emphasizes that Einstein's theory of general relativity is the gravitational theory of the optical agent OA(c), while Newton's theory of universal gravitation is the gravitational theory of the idealized agent OA(η). Therefore, there is no direct corresponding relationship between them. By means of the weak-field approximation, the theoretical model of the optical agent OA(c) is weakly linked to the theoretical model of the idealized agent OA_∞, which is the root of the logical flaws of Einstein's weak-field approximation.

As stated in Sec. 13.1 **Einstein's Logic of Weak-Field Approximation** of Chapter 13, actually, both the slow-speed approximation and the weak-field approximation reflect Einstein's mistaken understanding of the essence of relativistic effects or relativistic phenomena. Limited by the perspective of the optical agent OA(c), Einstein mistakenly believed that: the essence of inertial relativistic effects lies in matter motion; the essence of gravitational relativistic effects lies in gravitational interaction between matter and matter.

It is such mistaken understanding of the essence of relativistic effects or relativistic phenomena that gave birth to Einstein's logical way of weak-field approximation. Conversely, Einstein's logical way of the weak-field approximation further strengthens the mistaken understanding of the essence of gravitational relativistic effects or gravitational phenomena in the physics community. Up to now, the mainstream school of physics still insist that Newton's law of universal gravitation is only the weak-field approximation of Einstein's gravitational-field equation, and Newton's theory of universal gravitation is only the weak-field approximation of Einstein's theory of general relativity, holding true only in the case of macroscopic, slow-speed, and weak-field.

The logical flaws of Einstein's logical way of weak-field approximation are further amplified in Einstein's theory of general relativity: the information wave that loads the information of gravitational spacetime is mistaken for **gravitational wave**. With no prior knowledge about gravitational radiation, solely relying on the weak-field equation, Einstein had predicted the specious gravitational wave according to his theory of general relativity. And surprisingly, the speed of gravitational waves predicted by Einstein was exactly the speed c of light.

The theory of GOR strives to make the gravitational theory in physics bakd to the correct logical way: following the correct logical way, correctly deduce the GOR gravitational-field equation, correctly understand gravitational relativistic effects, including gravitational deflection, gravitational redshift, Mercury's anomalous perihelion shift, and Einstein's gravitational wave.

14.3.2 The GOR Logic of Idealized Convergence

Einstein's theory of general relativity is the gravitational theory of the optical agent OA(c); Newton's theory of universal gravitation is the gravitational theory of the idealized agent OA_∞. OA(c) and OA_∞ has no direct corresponding relationship.

The theory of GOR is the gravitational theory of the general agent OA(η).

Under the principle of GC, the theory of GOR could strictly correspond to Einstein's the theory of general relativity: if $\eta=c$, then OA(η) is the optical agent OA(c), and the theory of GOR strictly reduces to Einstein's theory of general relativity; the theory of GOR could strictly correspond to Newton's the theory of universal gravitation: if $\eta\to\infty$, then OA(η) is the idealized agent OA_∞, and the theory of GOR strictly reduces to Newton's theory of universal gravitation.

According to the theorem of Cartesian spacetime, as $\eta\to\infty$, the curved gravitational spacetime $X^{4d}(\eta)$ of OA(η) strictly converges to the flat Cartesian spacetime $X^{4d}{}_\infty$ of OA_∞. Therefore, under the idealized agent OA_∞, the GOR gravitational-field equation could strictly correspond to Newton's law of universal gravitation in the flat Cartesian spacetime $X^{4d}{}_\infty$. In other words, as $\eta\to\infty$, the GOR field equation could converge to Newton's law of universal gravitation in the form of Poisson equation. This is the logic or ideological foundation of the GOR logical way of idealized convergence.

The theory of GOR has conceptualized and designed the logical way of idealized convergence based on the theorem of Cartesian spacetime in Chapter 13. The GOR logical way of idealized convergence has set up the condition of idealized convergence: the information-wave speed η of the general observation agent OA(η) is large enough or $\eta\to\infty$. Here, we will briefly sum up the idealized convergence and further compare the GOR idealized convergence with Einstein's weak-field approximation, so that we could properly apply the idealized convergence to deduce and calibrate the GOR gravitational-field equation.

Similar to Einstein's theory of general relativity, the theory of GOR needs to deduce and calibrate the gravitational-field equation by corresponding the GOR field equation to Newton's law of universal gravitation in the flat Cartesian spacetime $X^{4d}{}_\infty$. Therefore, the theory of GOR needs to flatten the curved spacetime $X^{4d}(\eta)$ of the general observation agent OA(η): to make the spacetime metric $g_{\mu\nu}(\eta)=\eta_{\mu\nu}$ or the spacetime-transformation factor $\Gamma(\eta)=\Gamma_\infty$.

By observing the OR factor of spacetime transformation:

$$\Gamma = \Gamma(\eta) == \frac{1}{\sqrt{\left(\sqrt{1+2\chi/\eta^2} - \gamma_i v^i/\eta\right)^2 - v^2/\eta^2}} \tag{14.35}$$

$$\forall\ v,\ \chi,\ \gamma_i\ (i=1,2,3) \quad \lim_{\eta\to\infty}\Gamma(\eta) = 1 = \Gamma_\infty$$

we know that, in essence, the OR factor $\Gamma=\Gamma(\eta)$ depends on the information-wave speed η of the general observation agent OA(η), rather than v, χ, γ_i (i=1,2,3).

Equation (14.35) suggests that, if $\eta\to\infty$, then the observation agent OA(η) converges to the idealized agent OA_∞, and the curved gravitational spacetime $X^{4d}(\eta)$ converges to the flat Cartesian spacetime X^{4d}_∞.

Actually, according to the theorem of Cartesian spacetime: $g_{\mu\nu}(\eta)\to\eta_{\mu\nu}$ as $\eta\to\infty$, which means that, under the idealized observation agent OA_∞, the gravitational spacetime metric $g_{\mu\nu}(\eta)$ of the general observation agent OA(η) converges to the Minkowski metric $\eta_{\mu\nu}$. The theorem Cartesian spacetime has more clearly clarified that the gravitational spacetime X^{4d}_∞ of the idealized agent OA_∞ has the characteristic of flat spacetime and lays the theoretical foundation for the GOR logical way of idealized convergence under the idealized agent OA_∞.

Naturally, **the $g_{\mu\nu}(\eta)\to\eta_{\mu\nu}$ as $\eta\to\infty$** of the Cartesian-spacetime theorem is consistent or equivalent with **the $\Gamma(\eta)\to\Gamma_\infty$ as $\eta\to\infty$** of the OR factor of spacetime transformation in Eq. (14.35).

There is an important corollary from Lemma 13.1 of the theorem of Cartesian Spacetime, that is, Corol. 13.1 (see Sec. 13.2 in Chapter 13): $g_{0i}=g_{i0}=0$ (i=1,2,3) as $\eta\to\infty$, which suggests that, under the idealized agent OA_∞, or under the condition of GOR idealized convergence, spacetime is orthogonal. So, in the Cartesian spacetime, or in the objectively real spacetime (including Newton's gravitational field), space and time are originally orthogonal or independent of each other.

It is worth noting that, in the case of orthogonal spacetime ($g_{0i}=g_{i0}=0$ (i=1,2,3)), the OR factor Γ of spacetime transformation (Eq. (12.35)) reduces to:

$$\begin{aligned}\Gamma &= \Gamma(\eta) = \mathrm{d}t(\eta)/\mathrm{d}\tau \\ &= \frac{1}{\sqrt{\left(\sqrt{g_{00}} + \frac{g_{0i}}{\sqrt{g_{00}}}\frac{\mathrm{d}x^i}{\mathrm{d}x^0}\right)^2 + \left(g_{ik} - \frac{g_{0i}g_{0k}}{g_{00}}\right)\frac{\mathrm{d}x^i}{\mathrm{d}x^0}\frac{\mathrm{d}x^k}{\mathrm{d}x^0}}} \\ &= \frac{1}{\sqrt{g_{00} - v^2/\eta^2}} = \frac{1}{\sqrt{1 - v^2/\eta^2 + h_{00}}} \quad \left(g_{00}(\eta) = \eta_{00} + h_{00}(\eta);\ \eta_{00} \equiv 1\right)\end{aligned} \tag{14.36}$$

So, the curved state of GOR gravitational spacetime only depends on the 00-element h_{00} of the curved metric $h_{\mu\nu}$.

It is worth noting that, in Einstein's theory of general relativity, the spacetime orthogonality is a hypothetical condition of Einstein's weak-field approximation as a linearization method. However, in the theory of GOR, the spacetime orthogonality is a logical consequence of the theorem of Cartesian spacetime and the condition of idealized convergence, and in particular, is the essential characteristic of the objectively real spacetime.

According to the theorem of Cartesian spacetime, under the condition of idealized convergence, it must be true that: $|h_{00}|<<1$ and $|v|<<\eta$. According to the spacetime-transformation factor (Eq. (14.35)) in orthogonal spacetime, $\Gamma(\eta)\to\Gamma_\infty$ as $\eta\to\infty$, the curved gravitational spacetime $X^{4d}(\eta)$ of the observation agent OA(η) tends to be flat, and therefore, Newton's law of universal gravitation holds true. In this way, the GOR field equation and the GOR motion equation could correspond to Newton's law of universal gravitation in the form of Poisson equation.

The GOR logical way of idealized convergence is the linearization theory of GOR theory for processing the GOR field equation and the GOR motion equation. According to Eq (14.36), by analogizing Einstein's logical way of weak-field approximation, the GOR logical way of idealized convergence under the idealized observation agent OA_∞ could be conceived or designed as follows.

The Logical Program of Idealized Convergence

With the goal of calibrating the coefficient κ_{GOR} of GOR field equation, according to the GOR factor Γ (Eq. (14.36)) of spacetime transformation in the idealized observation situation, based on the GOR logic of idealized convergence, by linking the curved-metric 00-element h_{00} with Newton's gravitational potential χ and the field-equation coefficient κ_{GOR}, then the logical deduction program could be divided into the following two steps.

(i) The first step: the idealized convergence of GOR motion equation, that is, linking the GOR curved-metric 00-element h_{00} in the GOR motion equation (Eq. (14.33)) with the Newtonian gravitational potential χ ($h_{00}\sim\chi$).

(ii) The second step: the idealized convergence of GOR field equation, that is, linking the GOR curved-metric 00-element h_{00} in the GOR field equation (Eq. (14.32)) with the field-equation coefficient κ_{GOR} ($h_{00}\sim\kappa_{GOR}$).

Finally, under the scene of Newton's law of universal gravitation, by contrasting the relations $h_{00}\sim\chi$ and $h_{00}\sim\kappa_{GOR}$ of idealized convergence, the coefficient κ_{GOR} of GOR field equation could be calibrated with Newton's gravitational potential χ or Newton's universal gravitational constant G.

The Condition of Idealized Convergence

The theorem of Cartesian spacetime and Eq. (14.35) mean that the theory of GOR only needs to set $\eta\to\infty$ to satisfy $\Gamma(\eta)=\Gamma_\infty$ and $g_{\mu\nu}(\eta)=\eta_{\mu\nu}$. Therefore, the theory of GOR does not need to make the assumption of weak-field approximation. As stated in Sec. 13.3 **The GOR Logical Way of Idealized Convergence** of Chapter 13, the GOR logical way of idealized convergence is based on the theorem of Cartesian spacetime and only has one hypothetical condition, i.e., the so-called condition of idealized convergence.

The Condition of Idealized Convergence: The information-wave speed η of the observation agent OA(η) is large enough or $\eta\to\infty$.

As clarified in Sec. 13.3 of Chapter 13, based on the theorem of Cartesian spacetime, under the condition of idealized convergence: **η is large enough or $\eta\to\infty$**, the conditions of Einstein's weak-field approximation, including

(i) The weak field: $|\chi|<<\eta^2$ (or $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ ($|h_{\mu\nu}|<<|\eta_{\mu\nu}|$));

(ii) The slow speed: $|v|<<\eta$;

(iii) The static field: $g_{\mu\nu,0}=\partial g_{\alpha\beta}/\partial x^0=0$ or $h_{\mu\nu,0}=\partial h_{\alpha\beta}/\partial x^0=0$ ($x^0=\eta t$);

(iv) The spacetime orthogonality: $g_{0i}=g_{i0}=0$ (i=1,2,3); and

(v) The harmonic coordinates: $\Box x^\mu=g^{\alpha\beta}\Gamma^\mu{}_{\alpha\beta}=0$ (μ=0,1,2,3),

all hold true. Therefore, based on the condition of idealized convergence: η is large enough or $\eta\to\infty$, by analogizing Einstein's logic of weak-field approximation, the theory of GOR could calibrate the coefficient κ_{GOR} of GOR field equation.

Newton's Gravitational Scene

Similar to Einstein's theory of general relativity, in order to be corresponded to Newton's law of universal gravitation, the GOR field equation and the GOR motion equation should be set with the same gravitational-interaction scene as that of Newton's law of universal gravitation.

Chapter 10 has defined the GOR gravitational scene in Sec. 10.2, that is, the Newton's gravitational-interaction scene set by Newton in Newton's law of universal gravitation (see Sec. 13.1.4 of Chapter 13).

It should be pointed out that, as repeatedly stressed, Newton's gravitational scene is the gravitational field under the idealized observation agent OA(η), represents the objectively gravitational spacetime, and therefore, does not mean weak gravitational fields.

Now, there is no need to limit the gravitational spacetime to weak gravitational fields. The theory of GOR will follow the GOR logic of idealized convergence based on the theorem of Cartesian spacetime and the condition of idealized convergence: η is large enough or $\eta\to\infty$, corresponding the GOR field equation to Newton's law of universal gravitation in the form of Poisson equation. In this way, the theory of GOR could deduce the GOR gravitational-field equation and calibrate the GOR field-equation coefficient κ_{GOR}.

14.4 The Idealized Convergence of GOR Motion Equation

The GOR motion equation (14.33) can be written as:

$$\begin{aligned}&\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\tau^2}+\Gamma^\mu_{\alpha\beta}(\eta)\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}=0\quad(\mu=0,1,2,3)\\&\Gamma^\mu_{\alpha\beta}(\eta)=\frac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta}+g_{\nu\beta,\alpha}-g_{\beta\alpha,\nu}\right)\end{aligned}\tag{14.37}$$

where, it is worth noting that, unlike Einstein's motion equation, in the GOR motion equation, the connection $\Gamma^\mu{}_{\alpha\beta}=\Gamma^\mu{}_{\alpha\beta}(\eta)$, the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$, and the time axis $x^0=\eta t$, all depend on the information-wave speed η of the general observation agent OA(η), rather than the speed c of light in vacuum.

Like Einstein's motion equation, the GOR motion equation represents the geodesic line of the observed object P moving in curved gravitational spacetime. The difference is that Einstein's motion equation belongs to the optical agent observation OA(c), while the GOR motion equation belongs to the general observation agent OA(η).

The Imagination on
the Idealized Convergence of GOR Motion Equation

Actually, the idealized convergence of GOR motion equation is to drive the idealized-agent form of GOR motion equation under the condition of idealized convergence: the information-wave speed η of OA(η) is large enough or $\eta\to\infty$.

Naturally, under the condition of idealized convergence: η is large enough or $\eta\to\infty$, gravitational spacetime tends to be flat, the geodesic line in curved spacetime (Eq. (14.37)) reduces to the straight line of Newton's law of universal gravitation in the flat Cartesian space:

The straight line (Eq. (14.7)) $\dfrac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = -\nabla\chi \quad \left(\chi = -\dfrac{GM}{r}\right)$

Multiplying the left and right ends of Eq. (14.7) by m, one could get Newton's law of universal gravitation in the form of Newton's second law.

$$\begin{cases} F = -m\nabla\chi = -\dfrac{\partial V}{\partial r} = \dfrac{GMm}{r^2} \\ F = ma = m\dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2} \quad \left(a = \dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}, F = -\dfrac{\partial V}{\partial r}\right) \end{cases} \tag{14.38}$$

where χ is the Newtonian gravitational potential, V is the Newtonian gravitational energy, and F is gravity or universal gravitation.

In this way, under the condition of idealized convergence, the curved metric $h_{\mu\nu}$ could be related with the Newtonian gravitational potential χ.

The Operation on
the Idealized Convergence of GOR Motion Equation

The GOR motion equation of the general observation agent OA(η) has generalized Einstein's motion equation of the optical observation agent OA(c): Einstein's motion equation holds true only if OA(η)$\to$OA(c) as $\eta\to c$.

One could imagine the case of GOR motion equation under the idealized observation agent OA_∞, where the information-wave speed η of OA(η) were large enough or $\eta\to\infty$, OA(η)$\to\mathrm{OA}_\infty$, the curved observational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η) would reduce to the flat Cartesian spacetime $X^{4\mathrm{d}}{}_\infty$ of OA_∞, the GOR geodesic line (Eq. (14.37)) would reduce to the straight line (Eq. (14.7)) in Cartesian spacetime. Here, the idealization procedure of the observation agent OA(η) is the linearization procedure of GOR motion equation: as $\eta\to\infty$, the nonlinear GOR geodesic equation is transformed into the linear equation.

Under the condition of idealized convergence: η is large enough or $\eta\to\infty$, the gravitational-spacetime metric $g_{\mu\nu}(\eta)=\eta_{\mu\nu}+h_{\mu\nu}(\eta)$ ($|h_{\mu\nu}|<<|\eta_{\mu\nu}|$), $h_{\mu\nu}$ and its derivative of each order become infinitesimal. Therefore, the connection $\Gamma^{\mu}{}_{\alpha\beta}$ in the GOR geodesic equation (14.37) only needs to reserve the linear term of the curved metric $h_{\mu\nu}$ and ignore the high-order terms:

$$\begin{aligned}\Gamma^{\mu}_{\alpha\beta}(\eta)&=\frac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta}+g_{\nu\beta,\alpha}-g_{\beta\alpha,\nu}\right)\\&=\frac{1}{2}\eta^{\mu\nu}\left(h_{\alpha\nu,\beta}+h_{\nu\beta,\alpha}-h_{\beta\alpha,\nu}\right)\\&\left(g^{\mu\nu}=\eta^{\mu\nu}-h^{\mu\nu};\eta^{\mu\nu}=\eta_{\mu\nu},h^{\mu\nu}=\eta^{\mu\alpha}\eta^{\beta\nu}h_{\alpha\beta}\right)\end{aligned}\tag{14.39}$$

If the information-wave speed η of OA(η) is large enough or $\eta\to\infty$, then $\mathrm{d}t=\mathrm{d}\tau$ and $|v^i|=|\mathrm{d}x^i/\mathrm{d}t|<<\eta$; since $x^0=\eta t$, $|\mathrm{d}x^i/\mathrm{d}\tau|<<|\mathrm{d}x^0/\mathrm{d}\tau|$. So, it follows that

$$\begin{aligned}&\frac{\mathrm{d}^2x^{\mu}}{\mathrm{d}\tau^2}+\Gamma^{\mu}_{\alpha\beta}(\eta)\frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{\beta}}{\mathrm{d}\tau}\\&=\frac{\mathrm{d}^2x^{\mu}}{\mathrm{d}\tau^2}+\Gamma^{\mu}_{00}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}+\Gamma^{\mu}_{i0}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}+\Gamma^{\mu}_{0k}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^k}{\mathrm{d}\tau}+\Gamma^{\mu}_{ik}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^k}{\mathrm{d}\tau}\\&=\frac{\mathrm{d}^2x^{\mu}}{\mathrm{d}\tau^2}+\Gamma^{\mu}_{00}(\eta)\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\quad(\mu=0,1,2,3)\end{aligned}\tag{14.40}$$

Thus, the GOR geodesic equation (14.37) reduces to:

$$\frac{\mathrm{d}^2x^{\mu}}{\mathrm{d}\tau^2}+\Gamma^{\mu}_{00}(\eta)\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}=0\quad(\mu=0,1,2,3)\tag{14.41}$$

If the information-wave speed η of OA(η) is large enough or $\eta\to\infty$, then $\partial/\eta\partial t=0$ and $g_{\alpha\beta,0}=\partial g_{\alpha\beta}/\partial x^0=\partial h_{\alpha\beta}/\partial x^0=\partial h_{\alpha\beta}/\eta\partial t^0=0$. So, it follows from Eq. (14.39):

$$\begin{aligned}\Gamma^{\mu}_{00}(\eta)&=\frac{1}{2}\eta^{\mu\nu}\left(h_{0\nu,0}+h_{\nu0,0}-h_{00,\nu}\right)=-\frac{1}{2}\eta^{\mu\nu}h_{00,\nu}\\&=-\frac{1}{2}\left(\eta^{\mu0}h_{00,0}+\eta^{\mu i}h_{00,i}\right)=-\frac{1}{2}\eta^{\mu i}h_{00,i}\end{aligned}\tag{14.42}$$

For the Minkowski metric $\eta_{\mu\nu}$, $\eta^{0i}=0$ and $\eta^{ii}=-1$ (i=1,2,3). It follows that:

$$\Gamma^{0}_{00}(\eta)=0\quad\text{and}\quad\Gamma^{i}_{00}(\eta)=\frac{1}{2}h_{00,i}\tag{14.43}$$

Thus, the GOR geodesic equation (14.37) splits into two sets of equations in the Cartesian spacetime $X^{4\mathrm{d}}{}_{\infty}$: independent time and independent space, that is:

$$\frac{\mathrm{d}^2t}{\mathrm{d}\tau^2}=0\tag{14.44a}$$

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}\tau^2}=-\frac{1}{2}h_{00,i}\left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\right)^2\quad(i=1,2,3)\tag{14.44b}$$

This split between time and space is exactly the characteristic of the Galilean transformation: in the Cartesian spacetime $X^{4d}{}_{\infty}$, time and space are independent of each other.

In Eq. (14.44), the time equation (14.44a) suggests that, in the Cartesian spacetime $X^{4d}{}_{\infty}$, the GOR observational (observed) time-element d*t* is equivalent to the intrinsic time-element dτ. So, one could get the solution of Eq. (14.13a):

$$t = a\tau + b \tag{14.45}$$

Appropriately selecting the time unit (let a=1) and calibrating the time (let b=0), then one would have the conclusion of $t=\tau$.

This has the same meaning as that in the Galilean transformation, where the times t and t' of the observers O and O' are the same: $t=t'$.

This is the same conclusion as the theorem of Cartesian spacetime: dt=dτ.

Therefore, in Eq. (14.44), the space equations (Eq. (14.44b)) reduce to:

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}t^2} = -\frac{\eta^2}{2}\nabla_i h_{00} \quad (i = 1,2,3) \tag{14.46}$$

Under the idealized observation agent OA_{∞}, the GOR geodesic line (Eq. (14.37)) in the curved spacetime $X^{4d}(\eta)$ of OA(η) reduces to the straight line (Eq. (14.46)) in the Cartesian spacetime $X^{4d}{}_{\infty}$ of OA_{∞}. By contrasting the straight line (Eq. (14.46)) in the Cartesian spacetime $X^{4d}{}_{\infty}$ and the straight line (Eq. (14.7)) in Newton's gravitational field, one would have that:

$$h_{00}(\eta) = \frac{2\chi}{\eta^2} \quad \text{and} \quad g_{00}(\eta) = 1 + \frac{2\chi}{\eta^2} \tag{14.47}$$

In this way, the curved metric 00-element h_{00} in the GOR gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) is related with the Newtonian gravitational potential: $h_{00}\sim\chi$.

14.5 The Idealized Convergence of GOR Field Equation

The GOR field equation (Eq. (14.32): $G_{\mu\nu}(\eta)=\kappa_{\mathrm{GOR}}T_{\mu\nu}(\eta)$) is isomorphically consistent with Einstein field equations (Eq. (14.2): $G_{\mu\nu}=\kappa_E T_{\mu\nu}$); Similarly, it could be interpreted in accordance with the logic of Einstein field equation: the left end $G_{\mu\nu}(\eta)$ represents the curvature of GOR gravitational spacetime, and the right end $T_{\mu\nu}(\eta)$ represents the distribution of matter and energy in GOR gravitational spacetime. The curvature $G_{\mu\nu}$ of gravitational spacetime and the distribution $T_{\mu\nu}$ of matter and energy are related by the coefficient κ_{GOR} of GOR field equation.

The difference is that Einstein's field equation belongs to the optical observation agent OA(c), while the GOR field equation belongs to the general observation agent OA(η). The GOR field equation has generalized Einstein's field equation, while Einstein's field equation is only a special case of GOR field equation, which holds true only if the observation agent OA(η) is the optical agent OA(c).

The Imagination on
the Idealized Convergence of GOR Field Equation

One could imagine the case of GOR field equation under the idealized observation agent OA_∞, where the information-wave speed η of OA(η) were large enough or $\eta\to\infty$, OA(η)$\to OA_\infty$, the curved observational spacetime $X^{4d}(\eta)$ of OA(η) would reduce to the flat Cartesian spacetime $X^{4d}{}_\infty$ of OA_∞, the GOR field equation (Eq. (14.32)) would reduce to Newton's law of universal gravitation in the form of Poisson equation. Here, the idealization procedure of the observation agent OA(η) is the linearization procedure of GOR field equation: as $\eta\to\infty$, the nonlinear GOR field equation is transformed into the linear Poisson equation.

In this way, under the condition of idealized convergence, the curved metric $h_{\mu\nu}$ could be related with the coefficient κ_{GOR} of GOR field equation.

The Operation on
the Idealized Convergence of GOR Field Equation

Multiplying the left and right ends of Eq. (14.32) by $g^{\mu\nu}$, one could get

$$g^{\mu\nu}R_{\mu\nu}(\eta)-\frac{1}{2}g^{\mu\nu}g_{\mu\nu}R=-\kappa_{\text{GOR}}g^{\mu\nu}T_{\mu\nu}$$

$$\begin{cases} g^{\mu\nu}g_{\mu\nu}=\delta^{\mu}_{\mu}=4 \\ g^{\mu\nu}R_{\mu\nu}=R^{\mu}_{\mu}=R \\ g^{\mu\nu}T_{\mu\nu}=T^{\mu}_{\mu}=T \end{cases} \tag{14.48}$$

where $R_{\mu\nu}=R_{\mu\nu}(\eta)$ is the Ricci tensor under OA(η), $R=R(\eta)$ is the Gaussian curvature of OA(η), $g_{\mu\nu}=g_{\mu\nu}(\eta)$ is the spacetime metric under OA(η), $T_{\mu\nu}=T_{\mu\nu}(\eta)$ is the energy-momentum tensor under OA(η), and $T=T(\eta)$ is the trace of $T_{\mu\nu}(\eta)$.

Thus, $R=\kappa_{\text{GOR}}T$, and the GOR field equation (14.32) could be rewritten as

$$R_{\mu\nu}(\eta)=-\kappa_{\text{GOR}}\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right) \tag{14.49}$$

According to the definition of the spacetime curvature, the Ricci tensor $R_{\mu\nu}$ is

$$\begin{aligned} R_{\mu\nu}(\eta) &\triangleq R^{\alpha}_{\mu\nu\alpha} \\ &=\frac{\partial}{\partial x^{\nu}}\Gamma^{\alpha}_{\mu\alpha}-\frac{\partial}{\partial x^{\alpha}}\Gamma^{\alpha}_{\mu\nu}+\Gamma^{\sigma}_{\mu\alpha}\Gamma^{\alpha}_{\sigma\nu}-\Gamma^{\sigma}_{\mu\nu}\Gamma^{\alpha}_{\sigma\alpha} \end{aligned} \tag{14.50}$$

This means that the GOR field equation of the general observation agent OA(η) is a nonlinear partial differential equation of the metric tensor $g_{\mu\nu}(\eta)$ of the general observation agent OA(η).

Under the condition of idealized convergence: η is large enough or $\eta\to\infty$, the curved metric $h_{\mu\nu}(\eta)$ and its derivative of each order become infinitesimal. Therefore, by reserving the linear term of the curved metric $h_{\mu\nu}$ and ignoring the high-order terms, the spacetime curvature $R_{\mu\nu}(\eta)$ under OA(η) and even the GOR field equation could be reduced to linear relations of $h_{\mu\nu}(\eta)$.

Under the condition of idealized convergence: η is large enough or $\eta\to\infty$, according to Eq. (14.39), ignoring the high-order terms of $h_{\mu\nu}(\eta)$, the Ricci tensor

$R_{\mu\nu}(\eta)$ under OA(η) reduces to:

$$R_{\mu\nu}(\eta)=\frac{\partial}{\partial x^{\nu}}\Gamma^{\alpha}_{\mu\alpha}-\frac{\partial}{\partial x^{\alpha}}\Gamma^{\alpha}_{\mu\nu} \tag{14.51}$$

Under the condition of idealized convergence: η is large enough or $\eta\to\infty$, according to Eq. (14.39), ignoring the high-order terms of $h_{\mu\nu}(\eta)$, the connection $\Gamma^{\alpha}{}_{\mu\nu}(\eta)$ under OA(η) reduces to:

$$\begin{aligned}\Gamma^{\alpha}_{\mu\nu}(\eta)&=\frac{1}{2}\eta^{\alpha\beta}\left(h_{\mu\beta,\nu}+h_{\beta\nu,\mu}-h_{\nu\mu,\beta}\right)\\&=\frac{1}{2}\left(\frac{\partial}{\partial x^{\nu}}h^{\alpha}_{\mu}+\frac{\partial}{\partial x^{\mu}}h^{\alpha}_{\nu}-\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\nu\mu}\right)\end{aligned} \tag{14.52}$$

By substituting α for ν in Eq. (14.52), one could get $\Gamma^{\alpha}{}_{\mu\alpha}$ from $\Gamma^{\alpha}{}_{\mu\nu}$:

$$\begin{aligned}\Gamma^{\alpha}_{\mu\alpha}(\eta)&=\frac{1}{2}\left(\frac{\partial}{\partial x^{\alpha}}h^{\alpha}_{\mu}+\frac{\partial}{\partial x^{\mu}}h^{\alpha}_{\alpha}-\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\alpha\mu}\right)\\&=\frac{1}{2}\left(\frac{\partial}{\partial x^{\alpha}}h^{\alpha}_{\mu}+\frac{\partial}{\partial x^{\mu}}h-\frac{\partial}{\partial x^{\beta}}h^{\beta}_{\mu}\right)=\frac{1}{2}\frac{\partial}{\partial x^{\mu}}h\end{aligned} \tag{14.53}$$

Thus, Eq. (14.51) could be rewritten as

$$\begin{aligned}R_{\mu\nu}(\eta)&=\frac{1}{2}\frac{\partial}{\partial x^{\nu}}\frac{\partial}{\partial x^{\mu}}h-\frac{1}{2}\frac{\partial}{\partial x^{\alpha}}\left(\frac{\partial}{\partial x^{\nu}}h^{\alpha}_{\mu}+\frac{\partial}{\partial x^{\mu}}h^{\alpha}_{\nu}-\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\nu\mu}\right)\\&=\frac{1}{2}\frac{\partial}{\partial x^{\alpha}}\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\mu\nu}\\&\quad-\frac{1}{2}\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial}{\partial x^{\alpha}}h^{\alpha}_{\mu}-\frac{1}{2}\frac{\partial}{\partial x^{\mu}}h\right)-\frac{1}{2}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial}{\partial x^{\alpha}}h^{\alpha}_{\nu}-\frac{1}{2}\frac{\partial}{\partial x^{\nu}}h\right)\end{aligned} \tag{14.54}$$

Then, the spacetime curvature $R_{\mu\nu}(\eta)$ is linearly related to $h_{\mu\nu}(\eta)$.

According to the condition of harmonic coordinates (see Sec. 13.1.3 of Chapter 13), the formula of harmonic coordinates could be written as $\square x^{\mu}=g^{\alpha\beta}\Gamma^{\mu}{}_{\alpha\beta}=0$. Under the condition of idealized convergence: η is large enough or $\eta\to\infty$, such a normalized condition naturally holds true. In the theory of linearization, this normalization condition is equivalent to [139]:

$$\frac{\partial}{\partial x^{\sigma}}h^{\sigma}_{\lambda}-\frac{1}{2}\frac{\partial}{\partial x^{\lambda}}h=0\qquad\begin{cases}h^{\sigma}_{\lambda}=g^{\sigma\mu}h_{\lambda\mu}\\h=h^{\mu}_{\mu}\end{cases} \tag{14.55}$$

Therefore, under the condition of idealized convergence: η is large enough or $\eta\to\infty$, the curvature $R_{\mu\nu}(\eta)$ could further be reduced to:

$$R_{\mu\nu}(\eta)=\frac{1}{2}\frac{\partial}{\partial x^{\alpha}}\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}h_{\mu\nu}=\frac{1}{2}\square h_{\mu\nu}$$
$$\left(\square=\frac{\partial}{\partial x^{\alpha}}\frac{\partial}{\partial x^{\beta}}\eta^{\alpha\beta}=\frac{\partial^2}{\eta^2\partial t^2}-\nabla^2\right) \tag{14.56}$$

where the symbol "□" is d' Alembert operator of the general observation agent OA(η), $\Delta=\nabla^2$ is Laplace operator.

It should be pointed out that the "□" in Eq. (14.56) is the general d' Alembert operator in the theory of OR, that is, the d' Alembert operator of the general observation agent OA(η) (see Sec. 5.5 in Chapter 5).

Thus, under the condition of idealized convergence: η is large enough or $\eta\rightarrow\infty$, the nonlinear GOR field equation (Eq. (14.32) or (14.49)) finally reduces to the linear GOR field equation:

$$\frac{1}{2}\square h_{\mu\nu}(\eta)=-\kappa_{\mathrm{GOR}}\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right) \tag{14.57}$$

In this way, the curved metric 00-element h_{00} of GOR gravitational spacetime is related with the coefficient κ_{GOR} of GOR field equation: $h_{00}\sim\kappa_{\mathrm{GOR}}$.

14.6 The Calibration of GOR Field Equation

The theory of GOR expects that, under the idealized observation agent OA_{∞}, the GOR field equation (Eq. (14.32) or Eq. (14.49)) could strictly be corresponded to Newton's law of universal gravitation in the form of Poisson equation (Eq. (14.1b)).

In Newton's gravitational scene (see Sec. 10.2 in Chapter 10), the matter particle M as a gravitational source forms the spherically-symmetric gravitational field; under the action of the gravitational force of M, the matter particle m moves in the gravitational field of M. Let the 3d speed (or 3d **Proper Speed**) of m be $v=(v^1,v^2,v^3)$ ($v^i=\mathrm{d}x^i/\mathrm{d}t$ ($i=1,2,3$)), and the 4d speed (or **Four-Speed**) be $u=(u^0,u^1,u^2,u^3)$ ($u^\mu=\mathrm{d}x^\mu/\mathrm{d}\tau$ ($\mu=0,1,2,3$)). Then, under the condition of idealized convergence: η is large enough or $\eta\rightarrow\infty$, $\mathrm{d}t=\mathrm{d}\tau$, $g_{\mu\nu}=\eta_{\mu\nu}$, and $|v|<<\eta$, it follows that

$$\begin{cases} u^{\mu}=\dfrac{\mathrm{d}x^{\mu}}{\mathrm{d}\tau}=\dfrac{\mathrm{d}x^{\mu}}{\mathrm{d}t}=\begin{cases} v^0=\eta & \mu=0 \\ v^i=0 & \mu=i=1,2,3 \end{cases} \\ T^{\mu\nu}=\rho u^{\mu}u^{\nu} \\ T_{\mu\nu}=\rho g_{\mu\alpha}g_{\nu\beta}u^{\alpha}u^{\beta}=\rho\eta^2\eta_{\mu0}\eta_{\nu0} \quad \left(T_{00}=\rho\eta^2\right) \\ T=T_{\mu}^{\mu}=\rho u^{\mu}u_{\mu}=\rho\eta^2 \end{cases} \tag{14.58}$$

where the GOR gravitational spacetime belongs to the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η) with the time axis $x^0=\eta t$; ρ is the material density of the observed object m, $T_{\mu\nu}(\eta)$ is the energy-momentum tensor of m in the observational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η), and T is the trace of $T_{\mu\nu}(\eta)$.

It should be pointed out that the concept of **Four-Speed** is the extension of the concept of Einstein's four-speed from the optical observation agent OA(c) to the general observation agent OA(η) (see Sec. 5.4 **The Four-Speed in OR Spacetime** in Chapter 5).

Under the condition of idealized convergence: η is large enough or $\eta \to \infty$, the four-speed of m (the observed object P) is $u = (\eta, 0, 0, 0)$, and therefore, as shown in Eq. (14.58), the energy-momentum tensor $T_{\mu\nu}(\eta)$ of m reduces to a scalar: T_{00}.

Thus, the field equations (14.49) and (14.57)) could be rewritten as:

$$\begin{aligned} R_{\mu\nu}(\eta) &= -\kappa_{\text{GOR}}\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right) = -\frac{1}{2}\kappa_{\text{GOR}}T_{00}\delta_{\mu\nu} \\ \Box h_{\mu\nu}(\eta) &= -\kappa_{\text{GOR}}T_{00}\delta_{\mu\nu} = -\kappa_{\text{GOR}}\rho\eta^2\delta_{\mu\nu} \end{aligned} \tag{14.59}$$

where $\delta_{\mu\nu}$ is Kronecker tensor.

Under the condition of idealized convergence: η is large enough or $\eta \to \infty$, the general d' Alembert operator "□" of the general observation agent OA(η) reduces to $-\Delta$ (see Sec. 5.5 in Chapter 5):

$$\Box = \frac{\partial^2}{\eta^2 \partial t^2} - \nabla^2 = -\nabla^2 = -\Delta \tag{14.60}$$

According to Eq. (14.47), Eq. (16.59), and Eq. (14.60), one could get that

$$\begin{aligned} &\Box h_{00} = -\kappa_{\text{GOR}}\rho\eta^2 \quad \text{or} \quad \Delta h_{00} = \kappa_{\text{GOR}}\rho\eta^2 \quad \left(h_{00} = 2\chi/\eta^2\right) \\ &\text{so that} \quad \nabla^2\chi = \frac{1}{2}\kappa_{\text{GOR}}\rho\eta^4 \end{aligned} \tag{14.61}$$

By contrasting Eq. (14.61) with the Poisson-equation form of Newton's law of universal gravitation: $\nabla^2\chi = 4\pi G\rho$ (Eq. (14.1b)), one could get the coefficient κ_{GOR} of GOR gravitational-field equation:

$$\kappa_{\text{GOR}} = \frac{8\pi G}{\eta^4} \tag{14.62}$$

This result is exactly the GOR field-equation coefficient in Eq. (14.32) obtained from Einstein field equation through PGC logical route 1.

Now, the GOR field equation has been formally established.

The establishment of GOR Gravitational-field equation represents the birth of a new theory of gravity or gravitational interaction: so-called the **Gravitationally Observational Relativity** (GOR) or the **General Observational Relativity** (GOR), the theory of GOR for short.

14.7 The GOR Field Equation: the Unity of Gravitational Theories

The field equations are the cores of gravitational theories.

Newton's field equation, i.e., the Poisson-equation form of Newton's law of universal gravitation, represents Newton's theory of universal gravitation; Einstein's field equation represents Einstein's theory of general relativity. Now, the GOR field equation represents the theory of GOR.

Newton's theory of universal gravitation [81] and Einstein's theory of general relativity [8] are the two great gravitational theories of physics. The theory of GOR repeatedly emphasizes that Newton's theory of universal gravitation is not the approximate theory of Einstein's theory of general relativity, and is not only applicable to macroscopic, slow-speed, or weak-field. Newton's theory of universal gravitation and Einstein's theory of general relativity belong to different observation systems: Newton's gravitational theory is the theory of idealized observation, and the product of the idealized agent OA_∞; Einstein's gravitational theory is the theory of optical observation, and the product of the optical agent OA(c). To a certain extent or relatively speaking, Newton's theory of universal gravitation is the right one, representing the objectively gravitational world; Einstein's theory of general relativity is the approximate one, only the optical image of the objectively gravitational world.

The GOR field equation, i.e., the gravitational-field equation of OR theory, is the gravitational-field equation of the general observation agent OA(η). The most important value and significance lies in that it has generalized Newton's field equation belonging to the idealized agent OA_∞ and Einstein's field equation belonging to the optical agent OA(c).

Actually, this means the unification of Newton's theory of universal gravitation and Einstein's theory of general relativity.

The most general form of the GOR field equation (14.32) can be rewritten as:

$$R_{\mu\nu}(\eta)-\frac{1}{2}g_{\mu\nu}(\eta)R(\eta)=-\kappa(\eta)T_{\mu\nu}(\eta)$$
$$\left(\kappa(\eta)=\kappa_{\text{GOR}}=8\pi G/\eta^4\right) \tag{14.63}$$

where the Ricci tensor $R_{\mu\nu}=R_{\mu\nu}(\eta)$, the Gaussian curvature $R=R(\eta)$, the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$, the energy-momentum tensor $T_{\mu\nu}=T_{\mu\nu}(\eta)$, and the coefficient $\kappa_{\text{GOR}}=\kappa(\eta)$ of GOR field equation, all depend on the general observation agent OA(η), depend on the information-wave speed η of OA(η).

Naturally, if the information-wave speed η of OA(η) is the light speed c ($\eta\to c$), then the GOR field equation is exactly Einstein's field equation:

$$R_{\mu\nu}(c)-\frac{1}{2}g_{\mu\nu}(c)R(c)=-\kappa(c)T_{\mu\nu}(c)$$
$$\left(\kappa(c)=\kappa_E=8\pi G/c^4\right) \tag{14.64}$$

where the Ricci tensor $R_{\mu\nu}=R_{\mu\nu}(c)$, the Gaussian curvature $R=R(c)$, the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(c)$, the energy-momentum tensor $T_{\mu\nu}=T_{\mu\nu}(c)$, and the coefficient $\kappa_\eta=\kappa(c)$, all depend on the optical agent OA(c), depend on the light speed c.

It is thus clear that Einstein's field equation (14.64) is the gravitational-field equation of the optical agent OA(c), which is only a special case of the GOR field equation (14.63), and is valid or effective only if the observation medium of the observation agent OA(η) is light or electromagnetic interaction, or only if the information-wave speed η of OA(η) is the light speed c.

In particular, considering the situation of the idealized observation agent OA_∞, the GOR field equation (14.63) could be rewritten as:

$$\Box\chi_{\mu\nu} = -\frac{\eta^2}{2}\kappa_{\text{GOR}}T_{\mu\nu} \quad \left(\Box\chi_{\mu\nu} \equiv \frac{\eta^2}{2}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\right) \tag{14.65}$$

where $\chi_{\mu\nu}$ is a tensor, the general Newtonian gravitational potential.

One could prove that, as $\eta\to\infty$, $\chi_{\mu\nu}\to 0$ ($\mu\neq 0$ or $\nu\neq 0$), and particularly, $\chi_{00}\to\chi$ that is a scalar, exactly the Newtonian gravitational potential.

According to the idealized-convergence equations (Eqs. (14.48), (14.49), and (14.57-59)): in the GOR field equation (14.63), as $\eta\to\infty$, the energy-momentum tensor $T_{\mu\nu}$ reduces to the scalar T_{00}, and the trace $T=T_{00}$ of $T_{\mu\nu}$.

Accordingly, the Gaussian curvature R is:

$$\begin{aligned} &T_{\mu\nu} = \begin{cases} T_{00} = \rho\eta^2 & \mu = \nu = 0 \\ T_{\mu\nu} = 0 & \text{else} \end{cases} \\ &R = \kappa_{\text{GOR}}T = \frac{8\pi G}{\eta^4}T_{00} = \frac{8\pi G}{\eta^2}\rho = -\frac{2}{\eta^2}\Box\chi \end{aligned} \tag{14.66}$$

where χ is the Newtonian gravitational potential.

According to Eqs. (14.59), (14.65) and (14.66):

$$\begin{aligned} &R_{00} = \frac{1}{2}\Box h_{00} = \frac{1}{\eta^2}\Box\chi \quad \text{and} \quad R = -\frac{2}{\eta^2}\Box\chi \\ &\Box\chi_{00} \equiv \frac{\eta^2}{2}\left(R_{00} - \frac{1}{2}g_{00}R\right) = \Box\chi = -\nabla^2\chi \end{aligned} \tag{14.67}$$

Originally, the GOR field equation consists of 10 independent partial differential equations. However, under the idealized agent OA_∞, as the energy-momentum tensor $T_{\mu\nu}$ reduces, the GOR field equation reduces correspondingly, leaving only the one nontrivial equation:

$$\begin{aligned} &\Box\chi_{00} = -\frac{\eta^2}{2}\kappa_{\text{GOR}}T_{00} = -4\pi G\rho \\ &\Box\chi_{\mu\nu} = 0 \quad (\mu\neq 0 \text{ or } \nu\neq 0) \end{aligned} \tag{14.68}$$

Thus, the Poisson equation follows from Eqs. (14.67) and (14.68): $\nabla^2\chi=4\pi G\rho$.

Actually, Eq. (14.68) is exactly Newton's gravitational-field equation, which is the same as or equivalent to the Poisson-equation form (14.1b) of Newton's law of universal gravitation.

In this way, the GOR field equation (14.65) strictly reduces to Newton's field equation (14.68)) as $\eta\to\infty$. In other words, under the idealized agent OA_∞, the GOR field equation strictly reduces to Newton's field equation, i.e., the Poisson-equation form of Newton's law of universal gravitation.

It is thus clear that Newton' field equation, i.e., Newton's law of universal gravitation in the form of Poisson equation, also is only a special case of the GOR field equation. It is valid or effective only under the idealized agent OA_∞.

The ancient Poisson equation (14.1b), as a partial-differential-equation form of Newton's law of universal gravitation or as Newton's gravitational-field equation, is isomorphically consistent with the GOR field equation including Einstein's field equation. This phenomenon is worth thought-provoking.

To sum up, the GOR Gravitational-field equation has generalized and unified Newton's Gravitational-field equation and Einstein's gravitational-field equation. Under the principle of GC, the GOR field equation has the strict corresponding relationship of isomorphic consistency with both Newton's field equation and Einstein' field equation: as $\eta \to c$, the GOR field equation strictly reduces to Einstein's field equation; as $\eta \to \infty$, the GOR field equation is strictly reduces to Newton's field equation. This strict corresponding relationship of isomorphic consistency reflects the logical consistency not only between the GOR field equation and Einstein's field equation but also between the GOR field equation and Newton's field equation, and moreover, confirms the logical self-consistency of the GOR gravitational-field equation and even the theory of GOR.

15 The Solution of GOR Field Equation

The calibration and establishment of the GOR gravitational-field equation marks the birth of the theory of Gravitationally Observational Relativity (GOR).

The theory of GOR is the theory of the general observation agent OA(η). So, based on the theory of GOR, we could examine or reexamine both Newton's theory of universal gravitation and Einstein's theory of general relativity from a higher or broader perspective.

Physics is both speculative and empirical.

A new theory of physics must have logical rationality and theoretical validity, and moreover, must be subjected to the testing of observation and experiment, must be in accordance with observation and experiment, with the natural laws, and with the objectively physical reality.

This chapter aims to solve the GOR field equation, preparing for the subsequent test and verification of GOR field equation and even the whole theoretical system of GOR, for the reexamination of Einstein's theory of general relativity, and for the rediscovery of the essence of gravitational relativistic effects or gravitational relativistic phenomena.

15.1 GOR and Einstein's Predictions

Under the principle of general correspondence (GC), by analogizing or following Einstein's logic of making the scientific predictions and testing his theory of general relativity, the theory of GOR will also make the scientific predictions of its own, and then, we could make the test for these predictions and make the verification for the theory of GOR.

The test and verification of GOR theory can be conceived as follows:

(i) The test content: Einstein's three famous predictions, including (a) the gravitational redshift of light, (b) the gravitational deflection of light, and (c) the perihelion precession of Mercury orbit;

(ii) The test scene: a static spherically-symmetric gravitational field in which matter distributes in spherical symmetry and the external-vacuum metric is the external-vacuum solution of GOR field equation;

(iii) The test steps: (a) to solve the GOR field equation, (2) to determine the metric $g_{\mu\nu}(\eta)$ of the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η), (c) to calculate the predictive values, and (d) to compare the observed values and the predictive values.

Actually, all the tests and verifications for Einstein's theory of general relativity are also the tests and verifications for the theory of GOR; all observations and experiments that support Einstein's theory of general relativity are also the observations and experiments that support the theory of GOR. However, with regard to gravitational relativistic effects or gravitational relativistic phenomena, the theory of GOR will give us the different interpretation from Einstein's theory of general relativity, rediscover the root and essence of gravitational relativistic effects or gravitational relativistic phenomena, and correct the wrong ideas of Einstein's theory of general relativity.

15.1.1 Einstein's Three Great Predictions

In order to verify his theory of general relativity, Einstein conceived and designed three tests, i.e., Einstein's famous three scientific predictions:

(i) Gravitational redshift: the frequency of light would present redshift as light travels through a gravitational field;

(ii) Gravitational deflection: light would bend in a gravitational field;

(iii) Perihelion precession: the orbit of a planet would have a perihelion shift, and the precession rate of Mercury's perihelion is φ=43.03″/century.

Einstein's famous three scientific predictions provided the ways to test and verify Einstein's theory of general relativity. Actually, Newton's theory of universal gravitation and the theory of GOR can also predict the gravitational redshift of light, the gravitational deflection of light, and the perihelion precession of Mercury orbit.

The tests for Einstein's theory of general relativity imagined by Einstein, no matter the gravitational redshift of light or the gravitational deflection of light, or the orbital precession of a planet, the predicted value calculated according to Einstein's theory of general relativity depends on the metric $g_{\mu\nu}(c)$ of the optical observation spacetime, i.e., the solution of Einstein's field equations, in which, the dynamic problem is that of the idealized celestial **two-body system**, the gravitational scene is the idealized gravitational field, i.e., the static spherically-symmetric gravitational spacetime. Actually, this is exactly the same as the gravitational scene of Newton's law of universal gravitation or Poisson equation.

People are always struggling for whether Newton or Einstein is right.

The tests of gravitational redshift of light and the gravitational deflection of light were proposed by Einstein based on the principle of equivalence before the formal establishment of his theory of general relativity; while, the prediction or calculation of the perihelion precession of a planet was completed by Einstein after the formal establishment of his theory of general relativity.

The gravitational redshift of light predicted and calculated according to Einstein's theory seems to be the same as that according to Newton's theory, which are difficult to be distinguished in observation. It should be pointed out that there are still doubts about the calculation of the gravitational redshift of light according to Newton's theory, which is open to discussion. Before the formal establishment of Einstein's theory of general relativity, the gravitational deflection of light predicted and calculated by Einstein's theory was the same as that by Newton's theory. However, after the formal establishment of Einstein's theory of general relativity, Einstein recalculated the gravitational deflection of light, and got the new predictive value that was twice that of Newton's theory.

In the idealized gravitational scene, based on the idealized two-body system of celestial bodies, the precession rate of Mercury's perihelion predicted by Einstein's theory of general relativity is 43.03″ per 100 years, while the perihelion precession of a planet predicted or calculated by Newton's theory of universal gravitation theory is null. It is not Newton's theory of universal gravitation that could not predict perihelion precession of a planet. Actually, if the non-idealized factors of celestial motion systems are considered, Newton's theory of universal gravitation could also predict the perihelion precession of a planet. The actual precession rate of Mercury's orbit is 5600.73″ per 100 years, in which the precession of about 5025″ is the precession of the equinoxes caused by the non-inertial geocentric coordinate system. Considering the perturbation of Venus, Earth and Jupiter to Mercury, the precession of Mercury's perihelion needs to be predicted or calculated with Newton's theory of universal gravitation, and the predictive value according to Newton's theory reaches about 532″, far greater than the orbital precession of Mercury predicted by Einstein's theory of general relativity. The theory of GOR will clarify that, as a matter of fact, the orbital precession of Mercury: φ=43.03″/century, predicted by Einstein's theory of general relativity, is not the objective and real orbital precession of Mercury.

15.1.2 The Test Scene for the Theory of GOR

Naturally, the three famous scientific predictions of Einstein's theory of general relativity can also be employed to test or verify the theory of GOR, including the GOR field equation and the GOR motion equation.

The theory of GOR can also predict the gravitational redshift of light and the gravitational deflection of light, as well as the orbital precession of a planet. In particular, the predictions according to the theory of GOR for celestial bodies and even light travelling in gravitational field are the observational phenomena presented by the general observation agent OA(η), generalizing the observational phenomena presented by the idealized agent OA_∞ and the optical agent OA(η). In other words, the theory of GOR can make both the prediction of Einstein's theory of general relativity and the prediction of Newton's theory of universal gravitation.

No matter the gravitational redshift of light or the gravitational deflection of light, or the orbital precession of a planet, can be summed up as the idealized two-body problem of celestial bodies: (M,m), where M is the matter system (such as the sun in the solar system) that forms the gravitational field; m is the observed object P moving in the gravitational field (often idealized as a mass point) that can be a photon in the test of gravitational redshift or gravitational deflection, or a planet in the observation of orbital precession.

The Idealized Test Scene of GOR: There is the two-body system (M,m) of celestial bodies, where M is a material sphere with the radius R whose matter is distributed in spherical symmetry, forming a static spherically-symmetric gravitational field. Without regard to m, the outside of M is a vacuum; m is the observed object P ($m<<M$) moving in the gravitational field of M. (More ideally, both M and m could be regarded as two mass points.)

Like Einstein's way that he predicted the gravitational redshift of light and the gravitational deflection of light, as well as, the orbital precession of Mercury's perihelion according to his theory of general relativity, the primary task of GOR theory is to solve the gravitational-field equation of GOR theory and determine the gravitational spacetime metric $g_{\mu\nu}(\eta)$ of the two-body system (M,m). Thus, the GOR motion equation of the celestial body or photon (m) can be constructed based on the spacetime metric $g_{\mu\nu}(\eta)$, and then, the predictive value of the gravitational redshift of light or the gravitational deflection of light or even the orbital precession rate of Mercury's perihelion can under the general observational agent OA(η) be predicted or calculated according to the theory of GOR.

Actually, the approximate solution (by Einstein [8]) and exact solution (by Schwarzschild [80]) of Einstein's field equation, and even the Poisson-equation form of Newton's law of universal gravitation, are all the external vacuum solutions of the static spherically-symmetric gravitational field of the two-body system (M,m).

15.2 The Solution of Einstein's Field Equation

For the sake of analogy, we first briefly review and analyze the solutions of Einstein gravitational-field equation, including Einstein's approximate solution and Schwarzschild's exact solution.

Einstein's field equation is a nonlinear partial-differential equation system with respect to the spacetime metric $g_{\mu\nu}$, consisting of 10 independent relations. Solving Einstein's field equation means to determine the metric $g_{\mu\nu}=g_{\mu\nu}(c)$ of the observational spacetime $X^{4\mathrm{d}}(c)$ of the optical observation agent OA(η), and calculate the 10 independent elements in the spacetime metric tensor $g_{\mu\nu}$.

15.2.1 The Approximate Solution of Einstein's Field Equation

Due to the nonlinearity and complexity of his field equation, Einstein followed his logic of the weak-field approximation to solve his field equation.

Referring to the scale of the sun in the solar system, if $M=M_S$ is the solar mass and $R=R_S$ is the solar radius, then the Newtonian gravitational potential of the solar surface is $|\chi|=GM/R_S\approx 1.9\times 10^{11}\ \mathrm{m}^2/\mathrm{s}^2 \ll c^2$. Therefore, the gravitational potential outside of the sun could be regarded as a weak gravitational field. Naturally, if r is large enough, then the Newtonian gravitational potential $\chi=-GM/r$ could also be regarded as a weak gravitational potential.

Considering a specific case that conforms to Einstein's conditions of weak-field approximation: $g_{\mu\nu}(c)\approx\eta_{\mu\nu}$, accordingly, the determinant $g=\det(g_{\mu\nu})$ of the spacetime metric $g_{\mu\nu}$ satisfies $\sqrt{(-g)}=1$. According to Riemannian geometry and tensor calculus, it follows that:

$$\Gamma^{\alpha}_{\lambda\alpha}(c)=\frac{\partial \ln\sqrt{(-g)}}{\partial x^{\alpha}}=0 \quad \left(x^0=ct\right) \tag{15.1}$$

According to the conceived spherically-symmetric gravitational scene, outside M or the sun M_S, the material density $\rho=0$, the energy-momentum tensor $T_{\mu\nu}=0$, and Einstein's field equation (14.18) reduces to $R_{\mu\nu}=0$. According to the definition of the Ricci tensor $R_{\mu\nu}(c)$ in Eq. (14.19) and Eq. (15.1), Einstein's field equations (14.18) could be reduced to:

$$R_{\mu\nu}(c) = -\frac{\partial}{\partial x^{\alpha}} \Gamma^{\alpha}_{\mu\nu} + \Gamma^{\sigma}_{\mu\alpha}\Gamma^{\alpha}_{\sigma\nu} = 0 \quad \left(x^0 = ct\right) \tag{15.2}$$

The reduced field equation (15.2) is still a nonlinear partial-differential equation system, which is also difficult to solve. So, Einstein had to employ his logical way of weak-field approximation to linearize the gravitational-field equation (15.2) (see Sec. 13.1.3 in Chapter 13).

Based on the conditions of weak-field approximation (see Sec. 13.1.3 in Chapter 13), including: the weak field, slow speed, static field, spacetime orthogonality, and harmonic coordinates, by combining with the weak-field approximation of Einstein motion equation (Eq. (14.16)), Einstein could solve the gravitational-field equation (15.2) and get the metric $g_{\mu\nu}(c)$ of the gravitational spacetime $X^{4\mathrm{d}}(c)$ of the optical observation agent $g_{\mu\nu}(c)$:

$$\begin{cases} g_{00}(c) = 1 + \dfrac{2\chi}{c^2} \quad h_{00}(c) = \dfrac{2\chi}{c^2} \\ g_{0i}(c) = 0 \quad h_{0i}(c) = 0 \\ g_{ik}(c) = -\delta_{ik} + \dfrac{2\chi}{c^2}\dfrac{x^i x^k}{r^2} \quad h_{ik}(c) = \dfrac{2\chi}{c^2}\dfrac{x^i x^k}{r^2} \end{cases} \tag{15.3}$$
$$\left(i,k = 1,2,3;\ \chi = -GM/r\right)$$

where $h_{\mu\nu}$ was regarded by Einstein as the weak-gravitational potential ($|h_{\mu\nu}| << |\eta_{\mu\nu}|$) under the flat spacetime background.

Equation (15.3) shows that, as clarified by the theory of GOR, Einstein's field equation belongs to the optical agent OA(c), the observation medium is light, the information-wave speed is the light speed c, and the metric $g_{\mu\nu}$ of the observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c) depends on the light speed c: $g_{\mu\nu}=g_{\mu\nu}(c)$ and $h_{\mu\nu}=h_{\mu\nu}(c)$.

Correspondingly, the line-element ds of the gravitational spacetime $X^{4\mathrm{d}}(c)$ is:

$$\begin{aligned} \mathrm{d}s^2 &= g_{00}\left(\mathrm{d}x^0\right)^2 + g_{ik}\mathrm{d}x^i\mathrm{d}x^k \\ &= c^2\left(1 + \frac{2\chi}{c^2}\right)\mathrm{d}t^2 - \left(\delta_{ik} - \frac{2\chi}{c^2}\frac{x^i x^k}{r^2}\right)\mathrm{d}x^i\mathrm{d}x^k \end{aligned} \tag{15.4}$$

where the time axis x^0 of the observationally gravitational spacetime $X^{4\mathrm{d}}(c)$ of the optical observation agent OA(c) is $x^0=ct$.

Before the formal establishment of his theory of general relativity, Einstein had made the predictions of the gravitational redshift of light and the gravitational deflection of light based on the principle of equivalence. After the formal establishment of his theory of general relativity, Einstein employed the approximate solution (in Eq (15.3) or Eq. (15.4)) of his field equation to construct the motion equation of the observed object m for theoretically calculating and predicting the gravitational redshift of light and the gravitational deflection of light, for calculating the orbital precession of the planet m or Mercury, and even for predicting the gravitational waves and the speed of gravitational radiation. Interestingly or puzzlingly, the speed of gravitational radiation predicted by Einstein was exactly the speed c of light in vacuum.

15.2.2 The Exact Solution of Einstein's Field Equation

In 1916, shortly after Einstein's theory of general relativity had been officially published, German astronomer and physicist Schwarzschild obtained the first exact solution of Einstein field equations in the trenches on the front line of World War I, that is known as the Schwarzschild solution [80].

Like the approximate solution of Einstein field equation, the gravitational scene of the Schwarzschild solution is also the static spherically-symmetric gravitational field of the celestial-body two-body system (M,m) without matter outside M. According to Newton's law of universal gravitation, the gravitational potential at the distance r (>R) from the center of M is $\chi=-GM/r$. The Schwarzschild solution is the external-vacuum solution of M, where the spatial coordinate system is the spherical coordinate system rather than the Cartesian coordinate system. Correspondingly, the definition of the optical observation agent OA(c) and the observational spacetime $X^{4\mathrm{d}}(c)$ of OA(c) (in Eq. (10.1)) can be rewritten as:

$$\mathrm{OA}(c) \triangleq \left\{ \begin{array}{l} X^{4\mathrm{d}}(c): \left\{ \begin{array}{l} x^0 = ct; \\ x^1 = r,\ x^2 = \theta,\ x^3 = \varphi \end{array} \right\} \\ \mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu \end{array} \right\} \tag{15.5}$$

where the spatial coordinates $(x^1,x^2,x^3)=(r,\theta,\varphi)$ are the spherical coordinates.

It is worth noting that the observation agents defined in Eq. (10.1) and in Eq. (15.5) are both the optical agent OA(c). Regardless of the definition of the Cartesian coordinates (Eq. (10.1)) or the spherical coordinates (Eq. (15.5)), the definition of the time axis x^0 is the same: $x^0=ct$. And, no matter Eq. (10.1) or Eq. (15.5), the formulae of the line-element ds have the same form:

$$\text{Cartesian:}\quad \mathrm{d}s^2 = g_{\mu\nu}(x,y,z,c)\mathrm{d}x^\mu \mathrm{d}x^\nu$$
$$\begin{pmatrix} x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z \\ \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1) \end{pmatrix} \tag{15.6a}$$

$$\text{Spherical:}\quad \mathrm{d}s^2 = g_{\mu\nu}(r,\theta,\varphi,c)\mathrm{d}x^\mu \mathrm{d}x^\nu$$
$$\begin{pmatrix} x^0 = ct,\ x^1 = r,\ x^2 = \theta,\ x^3 = \varphi \\ \eta_{\mu\nu} = \mathrm{diag}(1,-1,-r^2,-r^2\sin^2\varphi) \end{pmatrix} \quad \left\{ \begin{array}{l} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{array} \right. \tag{15.6b}$$

Naturally, the spherical metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta,\varphi,c)$ and the Cartesian metric $g_{\mu\nu}=g_{\mu\nu}(x,y,z,c)$ have different manifestations. In the Cartesian coordinates, the flat spacetime refers to the Minkowski spacetime, and the corresponding metric $g_{\mu\nu}$ is the Minkowski metric $\eta_{\mu\nu}$=diag(+1,–1,–1,–1); while, in the spherical coordinates, the flat spacetime metric is $\eta_{\mu\nu}$=diag(+1,–1,$-r^2$,$-r^2\sin^2\theta$): the spherical-coordinate form of the Minkowski metric $\eta_{\mu\nu}$.

The Schwarzschild metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta,\varphi,c)$ in the spherical coordinates is [80]:

$$\begin{cases} g_{00}(r,c) = 1 - 2GM/c^2 r \\ g_{11}(r,c) = -\left(1 - 2GM/c^2 r\right)^{-1} \\ g_{22}(r,c) = -r^2 \\ g_{33}(r,c) = -r^2 \sin^2\theta \\ g_{\mu\nu}(r,c) = 0 \quad (\mu \neq \nu) \end{cases} \tag{15.7}$$

Correspondingly, in the spherical coordinates, the Schwarzschild line-elements $\mathrm{d}s=\mathrm{d}s(r,\theta,\varphi,c)$ is [80]:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{c^2 r}\right)c^2\mathrm{d}t^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{15.8}$$

With the Schwarzschild solution (Eq. (15.7) or Eq. (15.8)), based on the exact spacetime metric $g_{\mu\nu}(c)$, we are able to build the exact motion equations of celestial bodies, i.e., the geodesics of celestial-body motion, so that we could make more exact calculations and predictions for the gravitational redshift of light and the gravitational deflection of light, as well as, for the perihelion precession of the planet *m* or Mercury.

15.3 The Approximate Solution of GOR Field Equation

Like Einstein's field equation, the GOR field equation is also a nonlinear partial-differential equation system with respect to the spacetime metric $g_{\mu\nu}$, consisting of 10 independent relations. Solving the GOR field equation means to determine the metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$ of the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η), and calculate the 10 independent elements in the spacetime metric tensor $g_{\mu\nu}(\eta)$

Under the principle of GC, through PGC logic route 1, by substituting the information-wave speed η for the light speed c in Eqs. (15.3-4), the approximate solution Eqs. (15.3-4) of Einstein field equations (i.e., the approximate solution of the gravitational-field equation of the optical agent OA(c)) could be isomorphically and uniformly transformed into the approximate solution of GOR field equation (i.e., the gravitational-field equation of the general observation agent OA(η)). However, in order to rediscover the essence of the spacetime metric $g_{\mu\nu}$ or the curved metric $h_{\mu\nu}$, we prefer to solve the GOR field equation through PGC logic route 2, starting from more basic logical premises.

15.3.1 The GOR Field Equation for Approximate Solution

The approximate solution of the GOR field equation may analogizes and follows Einstein's logic to acquire the approximate solution of Einstein field equation. However, unlike Einstein's logic of weak-field approximation, the theory of GOR employs the logic of GOR idealized convergence (see Sec. 13.3 in Chapter 13), based on the condition of idealized convergence: the information-wave speed η of the observation agent OA(η) is large enough or $\eta\to\infty$, to acquire the approximate solution of GOR field equation.

In Chapter 13, the GOR logical way of idealized convergence has clarified that, based on the theorem of Cartesian spacetime, if η is large enough or $\eta\to\infty$, then the linearization equation (13.16) of the idealized convergence is valid, Einstein's five conditions of weak-field approximation, including the weak field, slow speed, static field, the spacetime orthogonality, and the harmonic coordinates, are naturally valid. In other words, the GOR logical way of idealized convergence could replace Einstein's logical way of weak-field approximation. In this way, under the principle of GC, through PGC logic route 2, by analogizing and following the logic of Einstein's approximate solution, based on the condition of GOR idealized convergence, we will get the approximate solution of GOR field equation.

Under the condition of idealized convergence: η is large enough or $\eta\to\infty$, according to the theorem of Cartesian spacetime, $g_{\mu\nu}(\eta)\to\eta_{\mu\nu}$; accordingly, the determinant g=det($g_{\mu\nu}$) of the spacetime metric $g_{\mu\nu}$ meets $\sqrt{(-g)}=1$.

Thus, according to Riemannian geometry and tensor calculus, it follows that:

$$\Gamma^{\alpha}_{\lambda\alpha}(\eta)=\frac{\partial\ln\sqrt{(-g)}}{\partial x^{\alpha}}=0\quad\left(x^{0}=\eta t\right)\tag{15.9}$$

It is worth noting that, in Einstein's approximate solution [68], $\sqrt{(-g)}=1$ is a forced assumption or condition so as to meet the condition of harmonic coordinates.

According to the conceived spherically-symmetric gravitational scene, without regard to m, there is no matter outside M or the sun M_S, the energy-momentum tensor $T_{\mu\nu}=0$, and the GOR field equation (14.49) reduces to $R_{\mu\nu}=0$. According to the definition of the Ricci tensor $R_{\mu\nu}(c)$ in Eq. (14.50), and according to Eq. (15.9), the GOR field equations (14.49) will be reduced to:

$$R_{\mu\nu}(\eta)=-\frac{\partial}{\partial x^{\rho}}\Gamma^{\rho}_{\mu\nu}+\Gamma^{\sigma}_{\mu\alpha}\Gamma^{\alpha}_{\sigma\nu}=0\quad\left(x^{0}=\eta t;\,\mu,\nu,\rho,\sigma,\alpha=0,1,2,3\right)\tag{15.10}$$

15.3.2 The Approximate Solution Based on the Condition of Idealized Convergence

Finding the approximate solution of GOR field equation is to determine the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$ or $h_{\mu\nu}=h_{\mu\nu}(\eta)$ in the GOR field equation (15.10) under the condition of GOR idealized convergence: η is large enough or $\eta\to\infty$.

The reduced GOR field equation (15.10), like the reduced Einstein field equations (15.2), is still a nonlinear partial-differential equation system, which is also difficult to solve. Einstein followed his logic of weak-field approximation to solve the reduced field equation (15.2); while the theory of GOR follows the GOR logic of idealized convergence to solve the reduced GOR field equation (15.10).

(I) Determining the Coupling Metric Elements of time and space: g_{0i} (i=1,2,3)

In Sec. 13.2 **The Theorem of Cartesian Spacetime** of Chapter 13, Corol. 13.1 from Lemma 13.1 has proven that, under the condition of idealized convergence: η

is large enough or $\eta\to\infty$, the condition of spacetime orthogonality naturally holds true. In other words, under the condition of GOR idealized convergence, the 0*i* and *i*0 elements (*i*=1,2,3) of the gravitational-spacetime metric $g_{\mu\nu}(\eta)$ are zero:

$$\begin{aligned} g_{0i}(\eta) &= g_{i0}(\eta) = 0 \\ h_{0i}(\eta) &= h_{i0}(\eta) = 0 \end{aligned} \quad (i=1,2,3) \tag{15.11}$$

It is worth noting that, in Einstein's approximate solution [68], the spacetime orthogonality is a forced assumption or condition, rather than a logical consequence.

The GOR spacetime line-element d*s*=d*s*(η) is the line-element of the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η); under the condition of GOR idealized convergence, $X^{4\mathrm{d}}(\eta)$ presents the objectively spacetime orthogonality, and hence, the line-element formula reduces to:

$$\begin{aligned} \mathrm{d}s^2 &= g_{\mu\nu}(\eta)\mathrm{d}x^\mu \mathrm{d}x^\nu \quad (\mu,\nu=0,1,2,3) \\ &= g_{00}(\eta)\left(\mathrm{d}x^0\right)^2 + g_{ik}(\eta)\mathrm{d}x^i\mathrm{d}x^k \quad \left(x^0=\eta t;\, i,k=1,2,3\right) \end{aligned} \tag{15.12}$$

(II) Determining the Time Metric Element: g_{00}

Under the condition of idealized convergence: η is large enough or $\eta\to\infty$, all the conditions of weal-field approximation hold true, in which, the spacetime orthogonality means that time and space are independent of each other, and the GOR field equation (15.10) splits into two independent parts: **the time equation** and **the space equation**.

Under the condition of GOR idealized convergence, the independent time equation can be derived as follows:

$$R_{00}(\eta) = -\frac{\partial}{\partial x^\rho}\Gamma^\rho_{00} + \Gamma^\sigma_{0\alpha}\Gamma^\alpha_{\sigma 0} = 0 \quad \left(x^0=\eta t;\, \rho,\sigma,\alpha=0,1,2,3\right) \tag{15.13}$$

At the same time, the weak field means that the curved metric $h_{\mu\nu}$ is small enough, the static field means that spacetime metric $g_{\mu\nu}$ does not change with x^0, and the slow speed means that the speed *v* of the moving object *m* is far slower than the information-wave speed η of the observation agent OA(η), that is:

The weak field: $g_{\mu\nu}(\eta) = \eta_{\mu\nu} + h_{\mu\nu}\left(x^\alpha,\eta\right) \quad \left(\left|h_{\mu\nu}\right| \ll \left|\eta_{\mu\nu}\right|\right)$

The slow speed: $\left|v^i\right| = \left|\frac{\mathrm{d}x^i}{\mathrm{d}t}\right| \ll \frac{\mathrm{d}x^0}{\mathrm{d}t} = \eta \quad \left(x^0=\eta t;\, i=1,2,3\right)$

The static field: $\frac{\partial g_{\mu\nu}}{\partial x^0} = \frac{\partial g_{\mu\nu}}{\eta\partial t} \approx 0$

It is worth noting that, in Einstein's approximate solution [68], the weak field, slow speed, and static field, are forced assumptions or conditions, while the condition of GOR idealized convergence does not require that the gravitational field is really a weak field or a static field, nor that the observed object *m* really moves at a slow speed, but only requires that the information-wave speed η of the observation agent OA(η) is large enough or $\eta\to\infty$.

Thus, by combining the Poisson-equation form $\nabla^2\chi=4\pi G\rho$ of Newton's law of universal gravitation and the GOR motion equation (i.e., the geodesic equation (14.33)), we have the solution of the time equation (15.13) of GOR field equation:

$$g_{00}(\eta)=1+\frac{2\chi}{\eta^2}\quad h_{00}(\eta)=\frac{2\chi}{\eta^2}\quad (\chi=-GM/r) \tag{15.14}$$

This result is the same as Eq. (14.47) in Sec. 14.4 **The Idealized Convergence of GOR Motion Equation** of Chapter 14.

(III) Determining the Space Metric Elements: g_{ik} (i,k=1,2,3)

Under the condition of GOR idealized convergence, the independent space equation can be derived as follows:

$$\begin{gathered} R_{ik}(\eta)=-\frac{\partial}{\partial x^\rho}\Gamma^\rho_{ik}+\Gamma^\sigma_{i\alpha}\Gamma^\alpha_{\sigma k}=0 \\ \left(x^0=\eta t;\, i,k=1,2,3;\, \rho,\sigma,\alpha=0,1,2,3\right) \end{gathered} \tag{15.15}$$

Likewise, under the condition of idealized convergence: η is large enough or $\eta\to\infty$, by combining the Poisson-equation form $\nabla^2\chi=4\pi G\rho$ of Newton's law of universal gravitation and the GOR motion equation (i.e., the geodesic equation (14.33)), we have the solution of the space equation (15.15) of GOR field equation:

$$\begin{gathered} g_{ii}(\eta)=-1+\frac{2\chi}{\eta^2}\frac{(x^i)^2}{r^2}\quad h_{ii}(\eta)=\frac{2\chi}{\eta^2}\frac{(x^i)^2}{r^2} \\ g_{ik}(\eta)=h_{ik}(\eta)=\frac{2\chi}{\eta^2}\frac{x^i x^k}{r^2}\quad (i\neq k) \\ \left(i,k=1,2,3;\, \chi=-GM/r\right) \end{gathered} \tag{15.16}$$

By summarizing Eq. (15.11), Eq. (15.14) and Eq. (15.16), we have the approximate solution of GOR field equation:

$$\begin{cases} g_{00}(\eta)=1+\dfrac{2\chi}{\eta^2}\quad h_{00}(\eta)=\dfrac{2\chi}{\eta^2} \\ g_{0i}(\eta)=0\quad h_{0i}(\eta)=0 \\ g_{ik}(\eta)=-\delta_{ik}+\dfrac{2\chi}{\eta^2}\dfrac{x^i x^k}{r^2}\quad h_{ik}(\eta)=\dfrac{2\chi}{\eta^2}\dfrac{x^i x^k}{r^2} \end{cases} \tag{15.17}$$
$$\left(i,k=1,2,3;\, \chi=-GM/r\right)$$

where $h_{\mu\nu}=h_{\mu\nu}(\eta)$ depends on the observation agent OA(η). It is worth noting that any element of $h_{\mu\nu}$ contains the factor χ/η^2.

According to Eq. (15.17), the line-element ds of gravitational spacetime is

$$\begin{aligned} ds^2 &= g_{00}\left(dx^0\right)^2 + g_{ik}dx^i dx^k \\ &= \eta^2\left(1+\frac{2\chi}{\eta^2}\right)dt^2 - \left(\delta_{ik} - \frac{2\chi}{\eta^2}\frac{x^i x^k}{r^2}\right)dx^i dx^k \end{aligned} \tag{15.18}$$

where the time axis of the GOR gravitational spacetime $X^{4d}(\eta)$ is $x^0=\eta t$, and the space axes are the Cartesian coordinate axes: $x^1=x$, $x^2=y$, $x^3=z$.

The solution in Eq. (15.17) or Eq. (15.18) of the GOR field equation is the approximate solution that requires the information-wave speed η of the observation agent OA(η) to be large enough or $\eta\rightarrow\infty$.

Obviously, the approximate solution of GOR field equation is isomorphically consistent with Einstein's approximate solution of Einstein field equation.

Based on the approximate solution of GOR field equation (15.17)) or according to the GOR line-element formula (15.18), the theory of GOR also could construct the motion equation of the observed object m for theoretically calculating and predicting the gravitational redshift of light and the gravitational deflection of light, for calculating the orbital precession of the planet m or Mercury, and in particular, could deduce the information-wave equation, revealing the essence of Einstein's prediction of gravitational waves.

Based on the approximate solution of the GOR field equation (15.17)) or according to the GOR line-element formula (15.18), the theory of GOR will provide us with new ideas or views different from Einstein's theory of general relativity on the gravitational redshift of light and the gravitational deflection of light, as well as on the relativistic celestial phenomena such as the precession of Mercury, and even on Einstein's gravitational waves.

15.4 The Exact Solution of GOR Field Equation

Under the principle of GC, through PGC logic route 1, by substituting the information-wave speed η for the light speed c in Eqs. (15.7-8), the Schwarzschild exact solution of Einstein field equation [80], i.e., the exact solution of the gravitational-field equation of the optical observation agent OA(c) (Eqs. (15.7-8)) could be isomorphically and uniformly transformed into the exact solution of GOR field equation, i.e., the exact solution of the gravitational-field equation of the general observation agent OA(η).

However, in order to understand the logic behind the exact solution of GOR field equation, we attempt to deduce the exact solution of GOR field equation from a more basic logical premise through PGC logic route 2. In particular, based on the GOR logical way of idealized convergence, the theory of GOR will employ the GOR condition of idealized convergence, rather than Einstein's conditions of weak-field approximation, as the boundary condition for the GOR field equation.

15.4.1 The Metric of Spherically-Symmetric Gravitational Field and the GOR Line-Element

Like the Schwarzschild exact solution, the gravitational scene conceived for the exact solution of GOR field equation is also the static spherically-symmetric gravitational field: let (M,m) be the two-body system, M be a sphere with the radius R, and its matter be distributed in spherical symmetry, forming a spherically-symmetric gravitational field; without regard to m, the outside of M is a vacuum. According to Newton's law of universal gravitation, the Newtonian gravitational potential χ at the distance r (>R) from the center of M is $\chi=-GM/r$.

Like the Schwarzschild exact solution, the exact solution of GOR field equation is also the external-vacuum solution of M, in which the spatial coordinates are the spherical coordinates rather than the Cartesian coordinates. Correspondingly, the definition (Def. 10.1) of the general observation agent OA(η) and the observational spacetime $X^{4d}(\eta)$ of OA(η) (in Eq. (10.2)) can be rewritten as:

$$\mathrm{OA}(\eta) \triangleq \left\{ \begin{array}{l} X^{4\mathrm{d}}(\eta): \left\{ \begin{array}{l} x^0 = \eta t; \\ x^1 = r,\ x^2 = \theta,\ x^3 = \varphi \end{array} \right\} \\ \mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \end{array} \right\} \tag{15.19}$$

where the spatial coordinate (x^1,x^2,x^3)=(r,θ,φ) is the spherical coordinate system.

It is worth noting that the observation agents defined in Eq. (10.2) and in Eq. (15.19) are both the general observation agent OA(η). Regardless of the definition of the Cartesian coordinates (Def. 10.1 in Eq. (10.2)) or the spherical coordinates (in Eq. (15.19)), the definition of the time axis x^0 is the same: $x^0=\eta t$. And, no matter Eq. (10.2) or Eq. (15.19), the formulae of the line-element ds have the same form:

$$\text{Cartesian:}\quad \mathrm{d}s^2 = g_{\mu\nu}(x,y,z,\eta)\mathrm{d}x^\mu \mathrm{d}x^\nu$$
$$\left(\begin{array}{l} x^0 = \eta t,\ x^1 = x,\ x^2 = y,\ x^3 = z \\ \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1) \end{array} \right) \tag{15.20a}$$

$$\text{Spherical:}\quad \mathrm{d}s^2 = g_{\mu\nu}(r,\theta,\varphi,\eta)\mathrm{d}x^\mu \mathrm{d}x^\nu$$
$$\left(\begin{array}{l} x^0 = \eta t,\ x^1 = r,\ x^2 = \theta,\ x^3 = \varphi \\ \eta_{\mu\nu} = \mathrm{diag}(1,-1,-r^2,-r^2\sin^2\varphi) \end{array} \right) \quad \left\{ \begin{array}{l} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{array} \right. \tag{15.20b}$$

where the spherical metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta,\varphi,\eta)$ and the Cartesian metric $g_{\mu\nu}=g_{\mu\nu}(x,y,z,\eta)$ is not only the function of the respective spatial coordinates (r,θ,φ) and (x,y,z), but also the function of the information-wave speed η of the general observation agent OA(η).

As shown in Eq. (15.20a) and Eq. (15.20b), the spherical metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta,\varphi,\eta)$ and the Cartesian metric $g_{\mu\nu}=g_{\mu\nu}(x,y,z,\eta)$ have different manifestations. In the Cartesian coordinates, the flat spacetime refers to the Minkowski spacetime, and the corresponding metric $g_{\mu\nu}$ is the Minkowski metric $\eta_{\mu\nu}$=diag(+1,–1,–1,–1); while, in the spherical coordinates, the flat spacetime metric is $\eta_{\mu\nu}$=diag(+1,–1,$-r^2$,$-r^2\sin^2\theta$): the spherical-coordinate form of the Minkowski metric $\eta_{\mu\nu}$.

In spherically-symmetric gravitational field, the spacetime metric is also spherically symmetric. According to Birkhoff's theorem [140], the spacetime metric $g_{\mu\nu}$ of spherically-symmetric gravitational fields of external vacuum must be static and does not change over time: $\partial g_{\mu\nu}/\partial t=0$.

So, for spherically-symmetric gravitational field of external vacuum, the most general form of the spacetime line-element ds could be

$$\mathrm{d}s^2 = W(r)\eta^2\mathrm{d}t^2 - U(r)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{15.21}$$

where $g_{00}=W(r)$ and $g_{11}=-U(r)$ are the metric elements to be solved. Actually, Eq. (15.21) also means that the space and time of the spherically-symmetric gravitational spacetime are orthogonal: $g_{0i}=g_{i0}=0\,(i=1,2,3)$.

In particular, in the theory of GOR, the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$ of the general observation agent OA(η) depends on the information wave speed η of OA(η); for Eq. (15.21), $g_{00}=W(r,\eta)$ and $g_{11}=-U(r,\eta)$ are the function not only of r, but also of η. Therefore, in the theory of GOR, for spherically-symmetric gravitational field of external vacuum, the most general form of the spacetime metric $g_{\mu\nu}$ and line-element ds under the general observation agent OA(η) could be

$$\mathrm{d}s^2 = W(r,\eta)\eta^2\mathrm{d}t^2 - U(r,\eta)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$$
$$\begin{cases} g_{00}(r,\eta) = W(r,\eta) = \mathrm{e}^{\xi(r,\eta)} \\ g_{11}(r,\eta) = -U(r,\eta) = -\mathrm{e}^{\zeta(r,\eta)} \\ g_{22}(r,\eta) = -r^2 \\ g_{33}(r,\eta) = -r^2\sin^2\theta \\ g_{\mu\nu}(r,\eta) = 0 \quad (\mu \neq \nu) \end{cases} \tag{15.22}$$

where $g_{00}=W(r,\eta)$ and $g_{11}=-U(r,\eta)$ are the elements of spacetime metric $g_{\mu\nu}$ under the general observation agent OA(η) to be solved; in other words, $\xi(r,\eta)$ and $\zeta(r,\eta)$ are the parameters of Eq. (15.22) to be solved.

15.4.2 The GOR Field Equation of Spherically-Symmetric Gravitational Field

According to the spherically-symmetric gravitational scene conceived for external-vacuum solution, the energy-momentum tensor $T_{\mu\nu}=0$.

Thus, by means of the weak-field approximation, Einstein's field equations (14.2) could be reduced from the form of Eq. (14.19) to Eq. (15.2): $R_{\mu\nu}(c)=0$. In this way, Einstein had obtained the approximate solution of Einstein field equation. By means of the GOR idealized convergence, the GOR field equation (14.32) can be reduced from the form of Eq. (14.49) to Eq. (15.9): $R_{\mu\nu}(\eta)=0$. In this way, we have obtained the approximate solution (Eqs. (15.17-18)) of GOR field equation.

However, just as Schwarzschild's exact solution seemingly does not rely on the weak-field approximation, the exact solution of GOR field equation seemingly can also independent of the GOR idealized convergence. Without the GOR idealized convergence, the GOR field equation (14.32) could not be reduced from the form of

Eq. (14.49) to Eq. (15.10): $R_{\mu\nu}(\eta)=0$.

According to the logic of Schwarzschild's exact solution, in view of the spherically-symmetric gravitational scene conceived for external-vacuum solution: $T_{\mu\nu}=0$, from the GOR field equation (14.32), the field equation of the spherically-symmetric gravitational spacetime $X^{4\mathrm{d}}(\eta)$) of the general observation agent OA(η) could be written as:

$$R_{\mu\nu}(\eta)-\frac{1}{2}g_{\mu\nu}R(\eta)=0$$
$$\begin{cases} R_{\mu\nu}(\eta)\triangleq\dfrac{\partial}{\partial x^{\nu}}\Gamma^{\alpha}_{\mu\alpha}-\dfrac{\partial}{\partial x^{\alpha}}\Gamma^{\alpha}_{\mu\nu}+\Gamma^{\sigma}_{\mu\alpha}\Gamma^{\alpha}_{\sigma\nu}-\Gamma^{\sigma}_{\mu\nu}\Gamma^{\alpha}_{\sigma\alpha} \\ \Gamma^{\alpha}_{\mu\nu}(\eta)\triangleq\dfrac{1}{2}g^{\alpha\lambda}\left(g_{\mu\lambda,\nu}+g_{\lambda\nu,\mu}-g_{\nu\mu,\lambda}\right) \end{cases} \tag{15.23}$$

where $R_{\mu\nu}$ can be called the general Ricci tensor, i.e., the curvature of the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η), R the general Gaussian curvature, and $\Gamma^{\alpha}{}_{\mu\nu}$ the connection of the general observation agent OA(η). It is worth noting that, $R_{\mu\nu}=R_{\mu\nu}(\eta)$, $R=R(\eta)$, and $\Gamma^{\alpha}{}_{\mu\nu}=\Gamma^{\alpha}{}_{\mu\nu}(\eta)$ all depend on the information-wave speed η of OA(η).

According to the definition of connection (Eq. (15.23)), we have:

$$\begin{aligned} &\Gamma^{\mu}_{\mu\mu}(\eta)=\frac{1}{2}g^{\mu\mu}\frac{\partial g_{\mu\mu}}{\partial x^{\mu}}=\frac{1}{2}\frac{\partial\ln|g_{\mu\mu}|}{\partial x^{\mu}}\quad\left(g^{\mu\mu}=\frac{1}{g_{\mu\mu}}\right)\\ &\Gamma^{\alpha}_{\mu\mu}(\eta)=-\frac{1}{2}g^{\alpha\alpha}\frac{\partial g_{\mu\mu}}{\partial x^{\alpha}}\quad(\alpha\neq\mu)\\ &\Gamma^{\alpha}_{\mu\alpha}(\eta)=\frac{1}{2}g^{\alpha\alpha}\frac{\partial g_{\alpha\alpha}}{\partial x^{\mu}}=\frac{1}{2}\frac{\partial\ln|g_{\alpha\alpha}|}{\partial x^{\mu}}\quad\left(g^{\alpha\alpha}=\frac{1}{g_{\alpha\alpha}}\right)\\ &\Gamma^{\alpha}_{\mu\nu}(\eta)=0\quad(\mu,\nu,\alpha=0,1,2,3;\ \alpha\neq\mu\neq\nu\neq\alpha) \end{aligned} \tag{15.24}$$

By further calculations, it follows that:

$$\begin{cases} \Gamma^{0}_{10}(\eta)=\Gamma^{0}_{01}(\eta)=\dfrac{1}{2}\dfrac{\mathrm{d}\xi}{\mathrm{d}r}\quad \Gamma^{1}_{00}=\dfrac{1}{2}\mathrm{e}^{\xi-\zeta}\dfrac{\mathrm{d}\xi}{\mathrm{d}r} \\ \Gamma^{1}_{11}(\eta)=\dfrac{1}{2}\dfrac{\mathrm{d}\zeta}{\mathrm{d}r}\quad \Gamma^{1}_{22}(\eta)=-r\mathrm{e}^{-\zeta}\quad \Gamma^{1}_{33}(\eta)=-r\mathrm{e}^{-\zeta}\sin^{2}\theta \\ \Gamma^{2}_{12}(\eta)=\Gamma^{2}_{21}(\eta)=\dfrac{1}{r}\quad \Gamma^{2}_{33}(\eta)=-\sin\theta\cos\theta \\ \Gamma^{3}_{13}(\eta)=\Gamma^{3}_{31}(\eta)=\dfrac{1}{r}\quad \Gamma^{3}_{23}(\eta)=\Gamma^{3}_{32}(\eta)=\cot\theta \end{cases} \tag{15.25}$$

the others: $\Gamma^{\alpha}_{\mu\nu}(\eta)=0$

Making use of the equation

$$\frac{\partial \ln \sqrt{-g}}{\partial x^{\mu}} = \Gamma^{\nu}_{\mu\nu} \quad \left(g = \det\left(g_{\mu\nu}\right) = -\mathrm{e}^{\mu+\lambda} r^4 \sin^2 \theta\right) \tag{15.26}$$

and based on the definition (Eq. (15.23)) of the spacetime metric $R_{\mu\nu}$, we have that:

$$\begin{cases} R_{00}(\eta) = -\dfrac{\partial}{\partial r}\Gamma^1_{00} + 2\Gamma^1_{00}\Gamma^0_{01} - \Gamma^1_{00}\dfrac{\partial}{\partial r}\ln\sqrt{-g} \\ R_{11}(\eta) = \dfrac{\partial^2}{\partial r^2}\ln\sqrt{-g} - \dfrac{\partial}{\partial r}\Gamma^1_{11} - \Gamma^1_{11}\dfrac{\partial}{\partial r}\ln\sqrt{-g} \\ \qquad + \left(\Gamma^0_{10}\right)^2 + \left(\Gamma^1_{11}\right)^2 + \left(\Gamma^2_{12}\right)^2 + \left(\Gamma^3_{13}\right)^2 \\ R_{22}(\eta) = \dfrac{\partial^2}{\partial \theta^2}\ln\sqrt{-g} - \dfrac{\partial}{\partial r}\Gamma^1_{22} - \Gamma^1_{22}\dfrac{\partial}{\partial r}\ln\sqrt{-g} + 2\Gamma^2_{12}\Gamma^1_{22} + \left(\Gamma^3_{23}\right)^2 \\ R_{33}(\eta) = -\left(\dfrac{\partial}{\partial r}\Gamma^1_{33} + \dfrac{\partial}{\partial \theta}\Gamma^2_{33}\right) + 2\left(\Gamma^3_{13}\Gamma^1_{33} + \Gamma^3_{23}\Gamma^2_{33}\right) \\ \qquad -\left(\Gamma^1_{33}\dfrac{\partial}{\partial r}\ln\sqrt{-g} + \Gamma^2_{33}\dfrac{\partial}{\partial \theta}\ln\sqrt{-g}\right) \end{cases} \tag{15.27}$$

the others: $R_{\mu\nu}(\eta) = 0$

Making use of Eq. (15.27), we have the curvature scalar R:

$$\begin{aligned} R(\eta) &= g^{\mu\nu}R_{\mu\nu} \\ &= \mathrm{e}^{-\xi}R_{00} - \mathrm{e}^{-\zeta}R_{11} - \frac{R_{22}}{r^2} - \frac{R_{33}}{r^2\sin^2\theta} \\ &= -\mathrm{e}^{-\zeta}\left(\frac{\mathrm{d}^2\xi}{\mathrm{d}r^2} - \frac{1}{2}\frac{\mathrm{d}\zeta}{\mathrm{d}r}\frac{\mathrm{d}\xi}{\mathrm{d}r} + \frac{1}{2}\left(\frac{\mathrm{d}\xi}{\mathrm{d}r}\right)^2\right) \\ &\quad + \frac{2}{r}\mathrm{e}^{-\zeta}\left(\frac{\mathrm{d}\zeta}{\mathrm{d}r} - \frac{\mathrm{d}\xi}{\mathrm{d}r}\right) + \frac{2}{r^2}\left(1 - \mathrm{e}^{-\zeta}\right) \end{aligned} \tag{15.28}$$

Then, the GOR field equation (15.23) of the spherically-symmetric gravitational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η) leaves four nontrivial equations:

$$R_{\mu\mu}(\eta) - \frac{1}{2}g_{\mu\mu}R(\eta) = 0 \quad \left(x^0 = \eta t;\ \mu = 0,1,2,3\right) \tag{15.29}$$

15.4.3 The Exact Solution of GOR Field Equation and GOR Idealized Convergence

Actually, the Schwarzschild exact solution is a special case of the solution of Eq. (15.29), i.e., the solution as OA(η) is the optical observation agent OA(c) (η=c). The boundary conditions set by Schwarzschild for solving Einstein's field equation are: as $r\to\infty$, g_{00}=1 and g_{11}=−1, in which Schwarzschild followed Einstein's logical thought of weak-field approximation: if $r\to\infty$, the Newtonian gravitational potential

$\chi \to 0$, the spacetime tends to be flat, and therefore, $g_{\mu\nu} \to \eta_{\mu\nu}$.

According to the theorem of Cartesian spacetime in Chapter 13: $g_{\mu\nu} \to \eta_{\mu\nu}$ as $\eta \to \infty$. Therefore, the boundary conditions of the GOR field equation (15.29) could be set based on the logical thought of GOR idealized convergence. Under the condition of idealized convergence: η is large enough or $\eta \to \infty$, we have

$$\begin{aligned} &\lim_{\eta\to\infty} W(r,\eta) = 1 \quad \text{and} \quad \lim_{\eta\to\infty} U(r,\eta) = 1 \\ &\text{or:} \quad \lim_{\eta\to\infty} \xi(r,\eta) = 0 \quad \text{and} \quad \lim_{\eta\to\infty} \zeta(r,\eta) = 0 \end{aligned} \tag{15.30}$$

No matter $r \to \infty$, or $\chi \to 0$, or $\eta \to \infty$, the spacetime metric $g_{\mu\nu}$ would reduce to the flat-spacetime metric: $g_{\mu\nu} \to \eta_{\mu\nu}$. It could be expected that the solution of GOR field equation (15.29) $g_{\mu\nu}=g_{\mu\nu}(r,\chi,\eta)$. In other words, $W=W(r,\chi,\eta)$ and $U=W(r,\chi,\eta)$ are the functions of r, χ, and η.

Under the principle of GC, by analogizing or following the logic of Schwarzschild solution, with the boundary conditions of the GOR idealized convergence (Eq. (15.30)), combining Eqs. (15.24-28), we have

$$W(r,\eta) = 1 + \frac{2\chi}{\eta^2} \quad \text{and} \quad U(r,\eta) = \left(1 + \frac{2\chi}{\eta^2}\right)^{-1} \tag{15.31}$$

where $\chi = -GM/r$ is the Newtonian gravitational potential.

Correspondingly, the line-element ds of the spherically-symmetric gravitational spacetime $X^{4\text{d}}(\eta)$ of the general observation agent OA(η) and the GOR exact solution (the metric $g_{\mu\nu}(\eta)$ of the observational spacetime $X^{4\text{d}}(\eta)$ of OA(η)) could be expressed as follows:

$$\mathrm{d}s^2 = \left(1 + \frac{2\chi}{\eta^2}\right)\eta^2 \mathrm{d}t^2 - \left(1 + \frac{2\chi}{\eta^2}\right)^{-1} \mathrm{d}r^2 - r^2 \mathrm{d}\theta^2 - r^2 \sin^2\theta \mathrm{d}\varphi^2$$

$$\begin{cases} g_{00}(r,\eta) = 1 + 2\chi/\eta^2 \quad (\chi = -GM/r) \\ g_{11}(r,\eta) = -\left(1 + 2\chi/\eta^2\right)^{-1} \\ g_{22}(r,\eta) = -r^2 \\ g_{33}(r,\eta) = -r^2 \sin^2\theta \\ g_{\mu\nu}(r,\eta) = 0 \quad (\mu \neq \nu) \end{cases} \tag{15.32}$$

where both r=0 (representing the mass center of M) and $r=2GM/\eta^2$ are the singularities of Eq. (15.32), i.e., the singularities of the GOR field equation; the time axis of the observational spacetime $X^{4\text{d}}(\eta)$ is $x^0=\eta t$, and the space axes of the observational spacetime $X^{4\text{d}}(\eta)$ are $x^1=r$, $x^2=\theta$, and $x^3=\varphi$.

Based on the exact solution of the GOR field equation (15.31) or according to the GOR line-element formula (15.32), the theory of GOR could exactly construct the motion equation of the observed object m for theoretically predicting the gravitational redshift of light and the gravitational deflection of light, for calculating the orbital precession of the planet *m* or Mercury, and in particular, for deducing the information-wave equation, revealing the essence of Einstein's gravitational wave.

Based on the exact solution of the GOR field equation (15.31) or according to the GOR line-element formula (15.32), the theory of GOR will interpret and analyze the relativistic celestial phenomena, provide us with new ideas or new views different from Einstein's theory of general relativity.

15.5 The Significance of GOR Field-Equation Solution

As stated in Sec. 13.2.5 **The Significance of Cartesian-Spacetime Theorem** of Chapter 13, the metric $g_{\mu\nu}(\eta)$ of the GOR gravitational spacetime $X^{4d}(\eta)$ can be decomposed into the flat metric $\eta_{\mu\nu}$ and the curved metric $h_{\mu\nu}(\eta)$:

The decomposition of spacetime metric: $g_{\mu\nu}(\eta) = \eta_{\mu\nu} + h_{\mu\nu}(\eta)$

where $\eta_{\mu\nu}$ is the Minkowski metric, represents the flat spacetime, including the Cartesian spacetime and the Minkowski spacetime; while $h_{\mu\nu}=h_{\mu\nu}(\eta)$ represents the curved spacetime, reflecting the curved state of gravitational spacetime.

The GOR observational spacetime is the gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η). The spacetime metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$ of $X^{4d}(\eta)$, especially the curved metric $h_{\mu\nu}=h_{\mu\nu}(\eta)$ of it, has the profound implication. The solutions of GOR field equation, including the approximate and the exact, reveal the profound implication of the metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$ and $h_{\mu\nu}=h_{\mu\nu}(\eta)$ of the gravitational spacetime $X^{4d}(\eta)$ of OA(η).

15.5.1 Generalizing the Gravitational Spacetime Metrics of Different Observation Agents

The gravitational-field equation of Einstein's theory of general relativity is referred to as the Einstein field equation; the Poisson equation is the gravitational-field equation of Newton's theory of universal gravitation, can be referred to as the Newtonian field equation. The solution of Einstein field equation is the metric of the observational spacetime $X^{4d}(c)$ of the optical agent OA(c): $g_{\mu\nu}=g_{\mu\nu}(c)$; the solution of Newtonian field equation should be the metric of the observational spacetime X^{4d}_{∞} of the idealized agent OA_{∞}: $g_{\mu\nu}=g_{\mu\nu}(\infty)$.

According to the theorem of Cartesian spacetime, the metric $g_{\mu\nu}(\infty)$ of the idealized observational spacetime is exactly the Minkowski metric: $g_{\mu\nu}(\infty)=\eta_{\mu\nu}$.

In Chapter 14, the theory of GOR has clarified that the GOR gravitational-field equation generalizes and unifies Einstein's field equation and Newton's field equation. The metric of GOR observational spacetime is that of the observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η): $g_{\mu\nu}=g_{\mu\nu}(\eta)$, which naturally generalizes the observational spacetime metrics of all observation agents; The solution of GOR field equation naturally generalizes the solutions of Einstein field equation and Newtonian field equation.

Obviously, the approximate solution of GOR field equation not only generalizes the approximate solution of Einstein field equation of the optical agent OA(c), but also generalizes the solution of Newtonian field equation of the idealized agent OA_∞: if $\eta \to c$, then the approximate solution (Eq. (15.17)) of GOR field equation would reduce to the approximate solution (Eq. (15.3)) of Einstein field equation; as $\eta \to \infty$, the approximate solution (Eq. (15.17)) of GOR field equation would reduce to the solution of Newtonian field equation: $g_{\mu\nu}(\infty)=\eta_{\mu\nu}=\text{diag}(+1,-1,-1,-1)$. It is thus clear that the approximate solution of GOR field equation based on the GOR idealized convergence is isomorphically consistent with both the approximate solution of Einstein field equation based on Einstein's weak-field approximation and the solution of Newtonian field equation under the idealized agent OA_∞.

Likewise, the exact solution of GOR field equation not only generalizes the exact solution of Einstein field equation of the optical agent OA(c), but also generalizes the solution of Newtonian field equation of the idealized agent OA_∞: if $\eta \to c$, then the exact solution (Eqs. (15.31-32)) of GOR field equation would reduce to the exact solution (Eq. (15.7-8)) of Einstein field equation; as $\eta \to \infty$, the exact solution (Eqs. (15.32-32)) of GOR field equation would reduce to the solution of Newtonian field equation: $g_{\mu\nu}(\infty)=\eta_{\mu\nu}=\text{diag}(+1,-1,-r^2,-r^2\sin^2\theta)$. It is thus clear that the exact solution of GOR field equation based on the GOR idealized convergence is isomorphically consistent with both the exact solution of Einstein field equation based on Einstein or Schwarzschild's weak-field approximation and the solution of Newtonian field equation under the idealized agent OA_∞.

The generality of the GOR field equation as the gravitational field equation of the general observation agent OA(η), and the corresponding relationships of isomorphic consistency between the solution of GOR field equation and the solution of Einstein field equation as well as between the solution of GOR field equation and the solution of Newton field equation, once again reflect the logical consistency of the GOR field equation with both Einstein's field equations and Newton's field equation, and further confirm the logical self-consistency of the logical system or theoretical system of GOR.

15.5.2 Spacetime is not Really Curved

In Sec. 12.5.2 **Could Spacetime Really be Curved?** of Chapter 12, based on the GOR factor $\Gamma=\Gamma(\eta)$ of spacetime transformation, the theory of GOR has clarified that spacetime could not really be curved; the so-called spacetime curvature is actually just a sort of observational effect that depends on the observation agent OA(η), and the root and essence lie in the observational locality ($\eta<\infty$) of the observation agent OA(η).

Actually, according to the theorem of Cartesian spacetime (see Sec. 13.2 in Chapter 13), under the idealized agent OA_∞, the curved metric $h_{\mu\nu}$ of the idealized observational spacetime $X^{4d}{}_\infty$ is zero:

$$\lim_{\eta\to\infty} g_{\mu\nu}(\eta)=\eta_{\mu\nu} \quad \text{and} \quad \lim_{\eta\to\infty} h_{\mu\nu}(\eta)=0 \tag{15.33}$$

This also suggests that the objective and real spacetime, whether there were gravitational fields or not, would not be curved.

Now, we can take a different perspective to further verify this conclusion based on the solution of GOR field equation, including the approximate (Eqs. (15.17-8)) and the exact (Eqs. (15.31-2)): **Spacetime is not really curved**.

In the observational spacetime $X^{4d}(\eta)$ of an observation agent OA(η), if the metric $g_{\mu\nu}(\eta)$ of $X^{4d}(\eta)$ is a constant tensor (such as the Minkowski metric $\eta_{\mu\nu}$), then $X^{4d}(\eta)$ is a flat spacetime, otherwise, a curved spacetime.

Observing the solutions in Eq. (15.3) and Eq. (15.7) of Einstein field equation, we know that the metric $g_{\mu\nu}=g_{\mu\nu}(x^i,c)$ of the gravitational spacetime $X^{4d}(c)$ of the optical agent OA(c) depends on the spatial coordinate x^i (i=1,2,3): different spatial coordinates have different metrics, and therefore, the gravitational spacetime $X^{4d}(c)$ in optical observation exhibits a sort of curved shape. In Einstein's theory of general relativity, the speed c of light in vacuum is a cosmic constant. Limited by the perspective of the optical observation agent OA(c), Einstein could only attribute the root cause of spacetime curvature to the gravitational potential $\chi=\chi(x^i)$, and to the distribution of matter and energy in gravitational spacetime.

Observing the solutions in Eq. (15.17) and Eq. (15.32) of GOR field equation, we know that the metric $g_{\mu\nu}=g_{\mu\nu}(x^i,\eta)$ of the gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) also depends on the spatial coordinate x^i (i=1,2,3): different spatial coordinates have different metrics, and therefore, the gravitational spacetime $X^{4d}(\eta)$ in GOR observation also exhibits a sort of curved shape.

However, the solutions in Eq. (15.17) and Eq. (15.32) of GOR field equation show that the metric $g_{\mu\nu}=g_{\mu\nu}(x^i,\eta)$ of the gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) in essence depends on the observation agent OA(η), or in other words, depends on the information-wave speed η of OA(η): under different observation agents, the same gravitational scene would exhibit different curvatures.

This fact indicates that, in essence, the curvature of the gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) is not caused by the distribution of matter and energy, but by the observational locality ($\eta<\infty$) of the observation agent OA(η), which is a sort of observational effect and a sort of apparent phenomenon. Correspondingly, the curvature of gravitational spacetime in Einstein's theory of general relativity is the observational effect and apparent phenomena caused by the observational locality ($c<\infty$) of the optical agent OA(c).

As $\eta\to\infty$, the approximate solution of GOR field equation (15.17) reduces to:

$$\begin{cases} \lim\limits_{\eta\to\infty} g_{00}(\eta) = \lim\limits_{\eta\to\infty}\left(1+2\chi/\eta^2\right) = 1 \\ \lim\limits_{\eta\to\infty} g_{ik}(\eta) = \lim\limits_{\eta\to\infty}\left(-\delta_{ik} + \dfrac{2\chi}{\eta^2}\dfrac{x^i x^k}{r^2}\right) = -\delta_{ik} \\ g_{0i}(\eta) = 0 \quad \left(i,k=1,2,3;\ x^1=x, x^2=y, x^3=z\right) \end{cases} \tag{15.34}$$

That is: $\lim\limits_{\eta\to\infty} g_{\mu\nu}(\eta) = \eta_{\mu\nu} = \mathrm{diag}\left(+1,-1,-1,-1\right)$

where $g_{\mu\nu}=g_{\mu\nu}(x^i,\eta)$ ($x^1=x$, $x^2=y$, x^3=z) is the metric form of the Cartesian-coordinate system, and $\eta_{\mu\nu}$=diag(+1,–1,–1,–1) is the Cartesian-coordinate form of the Minkowski metric.

As $\eta\to\infty$, the exact solution of GOR field equation (15.32) reduces to:

$$\begin{cases} \lim\limits_{\eta\to\infty} g_{00}(\eta) = \lim\limits_{\eta\to\infty}\left(1+2\chi/\eta^2\right) = 1 \\ \lim\limits_{\eta\to\infty} g_{11}(\eta) = \lim\limits_{\eta\to\infty}\left(-\left(1+2\chi/\eta^2\right)^{-1}\right) = -1 \\ g_{22}(\eta) = -r^2 \\ g_{22}(\eta) = -r^2\sin^2\theta \\ g_{\mu\nu}(\eta) = 0 \quad \left(\mu\neq\nu;\ x^1=r, x^2=\theta, x^3=\varphi\right) \end{cases} \tag{15.35}$$

$$\lim_{\eta\to\infty} g_{\mu\nu}(\eta) = \eta_{\mu\nu} = \mathrm{diag}\left(+1,-1,-r^2,-r^2\sin^2\theta\right)$$

where $g_{\mu\nu}=g_{\mu\nu}(x^i,\eta)$ ($x^1=r, x^2=\theta, x^3=\varphi$) is the metric form of spherical-coordinate system, and $\eta_{\mu\nu}$=diag(+1,–1,$-r^2$,$-r^2\sin^2\theta$) is the spherical-coordinate form of the Minkowski metric.

So, if we could employ the idealized observation agent OA_∞ to observe gravitational spacetime, $\eta\to\infty$, without observational locality, then the solutions of GOR field equation, regardless of the approximate or the exact, regardless in the Cartesian coordinates or in the spherical coordinates, would converge to the flat metric: $g_{\mu\nu}\to\eta_{\mu\nu}$, and the curved metric tends to zero: $h_{\mu\nu}\to 0$.

With regard to the approximate solution (Eq. (15.17)) and the exact solution (Eq. (15.32)) of GOR field equation, the limit expressions (Eq. (15.34)) and (Eq. (15.35)) under the idealized agent OA_∞ not only verify the theorem of Cartesian spacetime, but also show us that the objectively real spacetime is the Cartesian spacetime X^{4d}_∞, which is originally flat and not curved due to the distribution of matter and energy. The so-called curvature of gravitational spacetime is only an observational effect and an apparent phenomenon, caused by that human being's observation agents are not ideal enough: the information-wave speeds are limited.

So, the objectively real spacetime is not curved.

But, only under the idealized observation agent OA_∞, the objectively real spacetime would present the flat and real appearance of it.

15.5.3 The Curved Metric $h_{\mu\nu}$ does not Represent Gravitational Radiation

Einstein's theory of relativity, whether the special or the general, is associated with light and the speed *c* of light in vacuum. However, the mainstream school of physics has not truly understood why the speed *c* of light in vacuum appears in Einstein's theory of relativity.

In the solutions of Einstein field equation, including the approximate (Eq. (15.3)) and the exact (Eq. (15.7)), the metric $g_{\mu\nu}=g_{\mu\nu}(c)$ of gravitational spacetime, especially the curved metric $h_{\mu\nu}=h_{\mu\nu}(c)$, is not unexpectedly associated with the speed *c* of light in vacuum. However, Einstein failed to correctly explain why the speed of light appeared in the spacetime metric $g_{\mu\nu}$ and the curved metric $h_{\mu\nu}$.

Actually, this is the embodiment of the optical observation agent OA(c).

Indeed, observations and experiments have shown that gravitational spacetime appears to be somewhat curved.

However, Einstein and the mainstream school of physics hailed to truly realize: most of observations and experiments relied on the optical observation agent OA(c); the so-called spacetime curvature is caused by the observational locality ($c<\infty$) of the optical agent OA(c), which is only an observational effect or an apparent phenomenon. In Einstein's view, the curvature of gravitational spacetime is caused by the accumulation of matter and energy, and is the essential characteristic of the objectively physical world. In Einstein's theory of general relativity, the curved metric $h_{\mu\nu}$ represented the gravitational potential χ, and in particular, represented the objective gravitational radiation, i.e., the so-called gravitational wave.

However, the solutions in Eq. (15.17) and Eq. (15.32) of GOR field equation suggest that, in essence, the metric $g_{\mu\nu}=g_{\mu\nu}(\eta)$ and the curved $h_{\mu\nu}=h_{\mu\nu}(\eta)$ of the observational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η) depend on the information-wave speed η of OA(η). Therefore, the curved metric $h_{\mu\nu}=h_{\mu\nu}(\eta)$ does not represent the gravitational potential, let alone the objective gravitational radiation or the so-called gravitational wave.

The curved metric in the approximate solution of GOR field equation (15.17) is:

$$\begin{gathered} h_{00}(\eta)=\frac{2\chi}{\eta^2} \quad h_{ik}(\eta)=\frac{2\chi}{\eta^2}\frac{x^i x^k}{r^2} \quad h_{0i}(\eta)=0 \\ \left(x^0=\eta t, x^1=x, x^2=y, x^3=z; i,k=1,2,3\right) \end{gathered} \tag{15.36}$$

The curved metric in the exact solution of GOR field equation (15.32) is:

$$\begin{gathered} h_{00}(\eta)=\frac{2\chi}{\eta^2} \quad h_{11}(\eta)=1-\left(1+\frac{2\chi}{\eta^2}\right)^{-1} \quad h_{0i}(\eta)=0 \quad h_{ik}(\eta)=0 \\ \left(x^0=\eta t, x^1=r, x^2=\theta, x^3=\varphi; i,k=1,2,3\right) \end{gathered} \tag{15.37}$$

By observing Eq. (15.36) and Eq. (15.37), we know that, regardless of the Cartesian coordinates or the spherical coordinates, regardless of the approximate solution or the exact solution, the nonzero elements of the curved metric $h_{\mu\nu}$ in the solution of GOR field equation contains an important dimensionless factor: $C_W=|\chi|/\eta^2$, i.e., the ratio of the gravitational potential $|\chi|$ to the square of the information-wave speed η; which can be called **the Factor of Carrier Wave**.

If the observation agent OA(η) is the optical agent OA(c) (like the case of the approximate solution (Eq. (15.3)) or the Schwarzschild exact (Eq. (15.7)) of Einstein field equation), then: $C_W=|\chi|/c^2$.

It should be pointed out that, it is exactly the factor $C_W=|\chi|/c^2$ of carrier wave that makes the gravitational spacetime of Einstein's optical observation appear to be somewhat curved.

From Einstein's perspective of optical observation: $C_W=|\chi|/c^2$, where the speed

c of light in vacuum is a cosmic constant. Therefore, Einstein attributed spacetime curvature to the gravitational potential χ: the stronger the gravity, the larger the $|\chi|$, the greater the C_W or the $|h_{\mu\nu}|$, and the more curved the gravitational spacetime is.

However, in the theory of GOR, from the perspective of the general observation agent OA(η): $C_W=|\chi|/\eta^2$, where η is the information-wave speed of the general observation agent OA(η) and different observation agents might have different information-wave speeds. So, the reason why gravitational spacetime appears to be somewhat curved is in essence not due to the distribution of matter and energy (χ), but due to the observational locality ($\eta<\infty$) of the observation agent OA(η). The solutions in Eq. (15.17) and Eq. (15.32) of GOR field equation show that, in the same gravitational scene under different observation agents, the gravitational spacetime exhibits different curvatures: the lower the information-wave speed η of the observation agent OA(η), the greater the carrier-wave factor $C_W=|\chi|/\eta^2$ and the curved metric $|h_{\mu\nu}|=|h_{\mu\nu}(\eta)|$, the more curved the gravitational spacetime would be. Conversely, if $\eta\to\infty$, the carrier-wave factor $C_W=|\chi|/\eta^2\to 0$, and the curved metric in the solution of GOR field equation tends to zero: $h_{\mu\nu}\to 0$.

According to the theory of GOR, the curved metric $h_{\mu\nu}=h_{\mu\nu}(\eta)$ of gravitational spacetime depends on the information-wave speed η of the observation agent OA(η). This fact suggests that the curved metric $h_{\mu\nu}$ does not represent the gravitational potential χ, let alone gravitational radiation or gravitational waves.

It could be imagined that the curved metric $h_{\mu\nu}=h_{\mu\nu}(\chi,\eta)$ is the **carrier wave** of gravitational-interaction information (χ), that is, the information wave (η) of OA(η) modulated by the gravitational radiation signal χ, which transmits the information about gravitational potential χ at the information-wave speed η.

The problem of whether the curved metric $h_{\mu\nu}$ means **gravitational wave** or **information wave** will be specifically discussed in Chapter 19.

16 GOR and Perihelion Precession

The calibration of the coefficient κ_{GOR} of GOR gravitational-field equation means the formal establishment of the GOR field equation and marks the birth of the theory of Gravitationally Observational Relativity (GOR), and at the same time, represents the unity of the two great gravitational theoretical systems of physics, Newton's theory of universal gravitation and Einstein's theory of general relativity.

Chapter 15 has successfully solved the GOR field equation of static spherically symmetric gravitational spacetime. Now, this chapter will test and verify the GOR field equation and even the whole theoretical system of GOR around the problem of the perihelion precession of Mercury, i.e., one of Einstein's famous three scientific predictions. We will explore the two-body problem of celestial bodies based on the theory of GOR, and construct the dynamic model of the two-body system (M,m) of celestial bodies based on the solution of GOR field equation. We attempt to, under the principle of general correspondence (GC), combining PGC logical route 1 and PGC logical route 2, by analogizing or following the logic of Einstein's theory of general relativity, deduce the motion equation of the observed planet m moving around the fixed star M.

The GOR motion equation of planets will be contrasted or analogized with Newton's motion model of planets and Einstein's motion model of planets to test and verify the theory of GOR including the GOR field equation and the GOR motion equation, to examine the gravitational relativistic phenomena of celestial motion, and in particular, to reexamine Einstein's prediction about the perihelion precession of Mercury.

16.1 The Evolution of Celestial Motion Images

It can be imagined and understood that the ancients were full of curiosity about the earth they survived on, as well as the sun, moon, and stars that rose in the east and fell in the west around the earth every day.

The concept of **the globe** naturally was introduced later on.

Early Chinese people referred to the earth as 大地, equivalent to the earth in English. The ancients thought that the earth was like a Persian carpet. Thanks to it, we would not fall into the hell at the bottom. It was very difficult for the ancients to imagine the earth as a sphere or a globe. However, based on the principle of **Seeing is believing**, the ancients naturally believed that the sun, moon, and stars that rose in the east and set in the west revolved around the earth, and their orbits must be circles: 大地 or the earth must be a globe.

The concept of the globe originated from Ptolemy's geocentric theory (as depicted in Fig. 16.1(a)) [141]. The geocentric theory was formed approximately in the 2nd century AD, and the core ideas of it could be summarized as: firstly, the earth was a globe; secondly, the earth was the center of the universe; thirdly, the sun, moon, and stars all revolved around the earth. In the ancients' view, it was reasonable that the orbits of the sun, moon and stars revolving around the earth were all the idealized or standard circles.

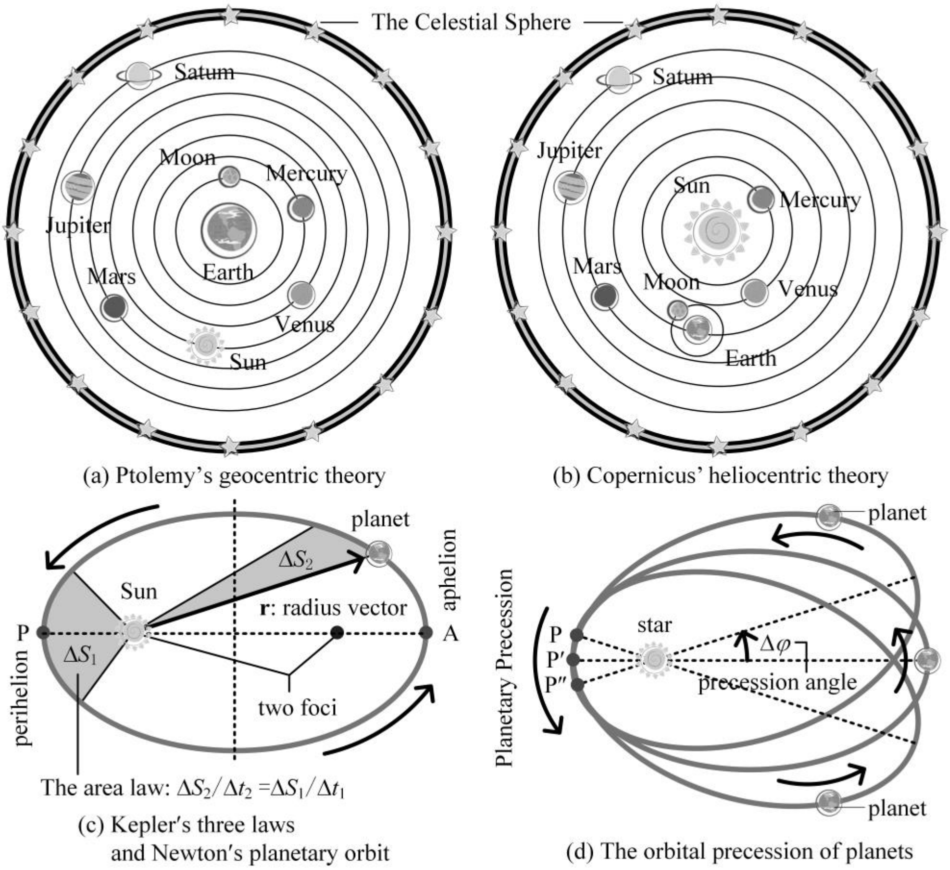

Figure 16.1 The Evolution of Celestial Motion Images: From the Earth to the Globe, from the circles to the ellipses, from the closed ellipses to the precessing ellipses. (a) Ptolemy's geocentric theory: the earth is a ball, that is, the globe; the earth is the center of the universe, the sun, moon, and stars revolve around the earth in the idealized or standard circles. (b) Copernicus' heliocentric theory: the sun is the center of the universe, the moon revolves around the earth in the idealized or standard circle, and the earth and other planets revolve around the sun also in the idealized or standard circle; the orbits of the earth and all other planets form the concentric circles centered around the sun. (c) Kepler's three laws and Newton's planetary orbit: according Kepler, the orbit of a planet is a closed ellipse with the sun at one of its focal points; Newton's motion equation of planets supports Kepler's three laws. (d) The orbital precession of planets: astronomical observation shows that the orbit of a planet is not closed ellipse, and the perihelion of a planet is not fixed, but precesses year after year.

In the 16th century AD, Copernicus created the heliocentric theory (as depicted in Fig. 16.1(b)), in which the center of the universe was moved from the earth to the sun [141,142]. Copernicus' heliocentric theory had made a big step in the right direction to human being's understanding of the universe: the earth spun on its own axis, the moon revolved around the earth, and the earth and other planets revolved around the sun. However, in Copernicus' heliocentric theory, no matter the moon moving around the earth or the earth or the other planets moving around the sun, their orbits still followed the idealized or standard circles, and moreover, all the orbits of the planets were the concentric circles centered on the sun.

In the 17th century AD, Kepler proposed three empirical laws of planetary motion based on the astronomical observation data accumulated by Tycho, known as Kepler's three laws (as depicted in Fig. 16.1(c)) [141,143], including the first: orbit law; the second: area law; the third: harmonic law. The orbit law is also known as the ellipse law: Each planet's orbit is an ellipse with the sun at a focus. Kepler's orbit law shows that the orbit of a planet is an ellipse rather than the idealized or standard circle. However, it is worth noting that Kepler's planetary orbits are closed ellipses, in which there is no the orbital precession or perihelion precession of planets

After Kepler's three law of planetary motion, Newton's three laws as well as Newton's law of universal gravitation were successively born [81].

Based on Newton's laws, Kepler's three laws of planetary motion could be derived theoretically. Since then, Mankind's physics began to entered a new epoch: from phenomenological physics to theoretical physics. Newton's theory of universal gravitation could predict or calculate the more general celestial orbits of conic curves: as a material system moves in a gravitational field, the orbit or trajectory of it could be a circle, an ellipse, a parabola, or a hyperbola, all of which are the idealized or standard conic curves. With respect to the two-body problem of celestial bodies or the two-body system (M,m) of celestial bodies, based on Newton's theory of universal gravitation, one could build up the motion model of the celestial-body m moving in the gravitational field of the star M for calculating the trajectory of the observed object m and predicting the behavior of the observed object m. Newton's motion equation of planets supports Kepler's orbit law: the orbit of a planet is a closed ellipses, and has no orbital precession.

However, astronomical observation tells us that the orbit of a planet is not a closed ellipse, and the perihelion always insistently precesses (as depicted in Fig 16.1(d)). The precession of Mercury's perihelion is particularly prominent, and the observation of optical astronomy shows that the orbital precession rate of Mercury is approximately 5600.73 arc second per 100 years [126].

All theories in physics are only the idealized models of physical reality.

The two-body system (M,m) of the planet m and the star M is itself an extremely idealized system, and is often further idealized as the one-body problem where the two-body system is reduced to the one-body system: the planet m orbits the stationary star M. The corresponding idealized conditions include: (a) the speed of gravitational radiation is infinite; (b) the star M is at rest; (c) both the star M and planet m are particles or mass points, no matter the one-body or the two-body, the (M,m) is an isolated system; (d) it takes no time for the gravitational radiation to cross space; (e) The observer O is based on the stellar perspective of M, and is at rest relative to the star M and located at the zero potential. However, the actual situation of celestial system is that: the speed of gravitational radiation is finite or limited; the star M and the planet m relatively move due to gravitational interaction; the star M and the planet m are not particles, and their material distributions are non-uniformity, asymmetric, and non-static; the (M,m) is surrounded by countless celestial bodies or other matter particles, not an isolated system; the observer O is generally located on the earth, trapped in the gravitational field of the sun and the earth, and moves relative to the star M and the observed planet m.

Therefore, it could be imagined that the moon's orbit around the earth, and the earth, Mercury, Venus, Mars Jupiter, and other planets' orbits around the sun, must not be the idealized or standard closed ellipses, let alone the idealized or standard closed circles. In the objectively physical world, it is natural and reasonable, and even inevitable, for a planet moving around a star to present the orbital precession or the perihelion precession. On the contrary, it is hard for us to imagine that the earth could move round and round the sun in the same closed ellipse or circle without drifting. Actually, as far as the data of astronomical observation are concerned, the objective and real celestial systems conform to our theoretical models so well that it is far beyond our imagination or expectation. As for Mercury, it only precesses at the rate of about 13.5″ per revolution around the sun. It is thus clear that Newton's two-body model of celestial bodies is already quite perfect.

Mercury, as the closest planet to the sun in the solar system, has the most prominent orbital precession. Physicists have conducted the modified calculation based on Newton's theory for the non-idealized factors of the two-body system (*Sun*,*Mercury*). Deducting the precession of the equinoxes caused by the non-inertial geocentric coordinate system (about 90%) and the perturbation made by other planets (especially Venus, Earth, and Jupiter) to Mercury (about 10%), totaling 5557.62 arc second, finally, there is only 5600.73″−5557.62″=43.11″ left unable to find out the attribution, which is a very small amount of the total of 5600.73″. The gravitational locality, the irregular shape of the sun, the sun's spin, and so on could also exacerbate the precession of Mercury's perihelion.

In 1915, after his theory of special relativity [7] of 1905, Einstein established his theory of general relativity [8]. Einstein applied his general relativity to the two-body problem of celestial bodies, and constructed the dynamic model of the two-body system (M,m) of celestial bodies. Einstein's motion equation of the planet m contains the information of the orbital precession of the planet m: the orbital precession term of the planet m that Newton's motion equation has no. Thus, the miracle happened: Einstein's motion equation of Mercury showed that the perihelion precession rate of Mercury's orbit around the sun was 43.03″ per 100 years, which was extremely consistent with the 43.11″ in the total of 5600.73″ that had not yet found out the attribution. Although many physicists have doubts about Einstein's prediction of Mercury precession [68], the mainstream school of physics believes that this calculation conclusion is the support for Einstein's theory of general relativity.

However, the problem is that: whether the 43.11″ or the 43.03″ is a very small fraction of the actual observed value 5600.73″ of Mercury's precession, less than 0.8%; then, why could Einstein's theory of general relativity only predict the 0.8% of the actual observed value 5600.73″ of Mercury's precession, but could not predict the rest 99.2%? In addition, another problem is that: there are many factors that could lead to the precession of Mercury's perihelion; then, why could not physicists employ Einstein's theory of general relativity to deduct the precession of the equinoxes caused by the non-inertial geocentric coordinate system and the perturbation of other celestial bodies to Mercury?

In summary: (i) the actual observed value of Mercury's precession rate reaches 5600.73 arc second per 100 years; (ii), the planetary precession predicted by Newton's dynamic model of the two-body system is null, but based on Newton's theory, one could deduct or calculate the orbital precession rate (532″ per 100 years) of Mercury caused by the perturbation of other celestial bodies to Mercury; (iii), the orbital precession rate (43.03″ per 100 years) of Mercury predicted by Einstein's theory of general relativity is far from the actual observed value of 5600.73″, and there is no comparability between them.

Actually, Einstein's motion equation of planets based on general relativity implies almost all the idealized conditions implied in Newton's motion equation of planets, including the action at a distance of gravitational interaction (see Sec. 12.1 in Chapter 12). The only difference is that Einstein's theoretical model implies the condition of the observational locality of optical observation: in Einstein's theory, the observation medium for transmitting the spacetime information of observed objects is light. As stated repeatedly by the theory of OR (including IOR and GOR), Einstein's theory is the theory of optical observation, belonging to the optical agent OA(c), whose information-wave speed is finite or limited by the speed c of light, and naturally, OA(c) has the observational locality ($c<\infty$).

Both Einstein's motion equation of planets and Newton's motion equation of planets have no the prior information about the orbital precession of planets, for instance, the precession of the equinoxes and the perturbation of other celestial bodies, as well as, the non-idealized factors of the sun. Like Newton's planetary mode, Einstein's planetary motion has no the prior information about the Mercury precession of 5557.62″. So, it is impossible for Einstein to predict or calculate the Mercury perihelion-precession rate of 5557.62″ per 100 years.

The theory of GOR will clarify that the orbital precession rate of Mercury predicted by Einstein's theory of general relativity: 43.03 arc second per 100 years does not represent the real precession of Mercury's perihelion, but the observational effect or apparent phenomenon caused by the observational locality ($c<\infty$) of the optical observation agent OA(c).

It should be pointed out that the data of Mercury precession of the 5600.73 arc second per 100 years were observed by the optical observation agent OA(c). Suppose that the orbital precession of planets as the observational effects of observation agents could be reflected and recorded in the data of astronomical observation, then the Mercury precession of the 43.03″ predicted by Einstein's theory of general relativity might indeed be employed to explain the 43.11″ in the actual observed value of 5600.73″ that has not yet found out the attribution. Thus, this is not only the support for Einstein's theory of general relativity but more importantly the support for the theory of GOR.

As far as the perihelion precession of Mercury is concerned, astronomical observation seems more inclined to support Einstein's theory of general relativity than Newton's theory of universal gravitation. This is not surprising, nor does it mean that Einstein's gravitational theory is more correct than Newton's gravitational theory. Human being's astronomy, including optical astronomy and radio astronomy, is the astronomy of the optical observation agent OA(η). The information-wave speed is the speed c of light in vacuum. Naturally, the astronomical observation by means of the optical agent OA(c) is more consistent with Einstein's theory of general relativity rather than Newton's theory of universal gravitation.

This chapter will establish the new theoretical model for the two-body system (M,m) of celestial bodies based on the theory of GOR, and derive the GOR motion equation of the observed planet m. According to the principle of GC, the GOR motion equation of the planet m must be isomorphically consistent with Einstein' motion equation of the observed planet m., which naturally will contain the orbital precession term of the planet m, and then can predict and calculate the orbital precession rate of the planet m or Mercury. Just as Einstein's motion equation of the planet m depends on the optical agent OA(c) and the light speed c, the GOR motion equation of the planet m must depend on the general observation agent OA(η) and the information-wave speed η of OA(η): under different observation agents, the same planet would exhibit different degrees of orbital precession. Accordingly, one could make such judgment that both the orbital precession rates of planets predicted by Einstein's theory of general relativity and the orbital precession rates of planets predicted by the theory of GOR do not represent the real perihelion precessions of planets, but is just observational effects and apparent phenomena.

16.2 Newton's Celestial-Body Model

Kepler's orbit law suggests that [141], a planet moves around a star in a closed elliptical orbit, and the star is located on a focus of the ellipse, which is mainly based on Tycho's observation data of the Mars [143].

The establishment of Kepler's three laws greatly promoted human being's understanding of the operation laws of celestial motion and the exploration of the motive forces behind celestial motion. Then, Galileo proposed the concept of **Central Force**; Newton established the law of universal gravitation [81]. Thus, the orbit of a planet around the sun could theoretically be derived. However, the motion equations of celestial bodies derived from Newton's law of universal gravitation are more general conic curves, and the elliptic orbit is only one form of them.

Newton's celestial-body model are naturally based on classical mechanics and Newton's laws, including Newton's second law and Newton's law of universal gravitation. Newton's celestial-body model belong to the category of the textbooks of general physics. However, as a special case of the GOR celestial-body model, it is of important significance for us to understand the theory of GOR.

16.2.1 Newton's Two-Body Problem of Celestial Bodies

Both Newton's celestial-body model and Einstein's celestial-body model belong to the two-body problem of celestial bodies, that is, the theoretical model of the two-body system (M,m) of celestial bodies, which is extremely idealized and can be described as follows.

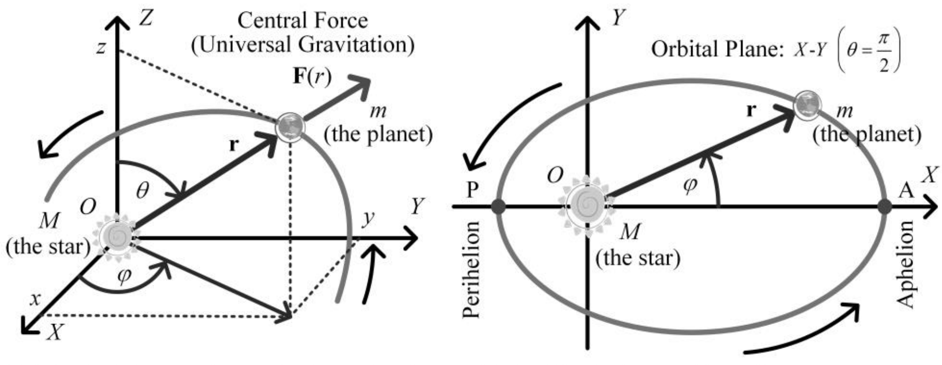

Figure 16.2 The Formalized Coordinates of Celestial Two-Body Problem. (a) Central force and spherical coordinates: the two-body system (*M*,*m*) of celestial bodies is idealized as an isolated system in the space of spherical-coordinate $O(r,\theta,\varphi)$, where the large mass celestial-body *M* (the sun or a star) is located at the coordinate origin of *O*, and the small mass celestial-body *m* (a planet or comet or satellite or even photon) moves in the gravitational field of *M*; universal gravitation is a sort of central force that always points towards the large mass celestial-body *M* (at the coordinate origin *O*). (b) The orbital plane of the small celestial-body *m*: according to the property of central force, the motion of the particle *m* is limited to a fixed plane.

Newton's Two-Body System of Celestial Bodies: (*M*,*m*), the star *M* and the planet *m* interact through the universal gravitation between them, *M* produces the gravitational field, and *m* moves in the gravitational field.

The Idealized Conditions for Newton's Two-Body System: Newton's two-body system (*M*,*m*) implies the following idealized conditions.

(i) The action at a distance of gravitational interaction: gravitational radiation is action at a distance; the speed of gravitational radiation is infinite.

(ii) The isolated system: (*M*,*m*) is an isolated system, not affected by the matter or energy outside (*M*,*m*); both the star *M* and the planet *m* could be regarded as particles or mass points, *M* is at rest and the planet *m* moves in the gravitational field of *M*.

(iii) The idealized observation agent: Newton's observation agent is the idealized observation agent OA_∞, the information-wave speed of OA_∞ is idealized to be infinity, the spacetime information of the observed planet *m* takes no time to cross space.

(iv) The idealized observer: the observer *O* employs the idealized observation agent OA_∞ to observe the planet *m* from the perspective of *M*; in theory, *O* is at rest relative to *M* and located at the position of zero potential.

The Formalized Coordinates of Newton's Two-Body System: as depicted in Fig. 16.2, the motion of Newton's two-body system (*M*,*m*) is described in both the 3d Cartesian-coordinate $O(x,y,z)$ and the corresponding the 3d spherical-coordinate $O(r,\theta,\varphi)$ (Fig. 16.2(a); Eq. (15.6)), the large mass celestial-body *M* is located at the coordinate origin *O*, and the small mass celestial-body *m* moves in the plane of *X*-*Y*

($\theta=\pi/2$) (Fig. 16.2(b)).

Actually, the two-body problem of celestial bodies explored by both Newton and Einstein could be further idealized and reduced to the one-body problem, only examining the motion of the planet *m*: the star *M* is at rest; the observer *O* observe the planet *m* from the perspective of the star *M*.

16.2.2 Newton's Theory of Universal Gravitation and The Two-Body System of Celestial Bodies

In the idealized two-body system (*M*,*m*) of celestial bodies, the matter particle *M* and the matter particle *m* interact through universal gravitation; gravity or gravitation is a sort of central force, and the force acting on the matter particle *m* always points the matter particle *M*. As depicted in Fig. 16.2(a), $\mathbf{F}(r)$ represents the central force exerted on *m*; $\mathbf{F}(r)$ is a function of *r*: $\mathbf{F}(r)=F_r\mathbf{r}/r$. Here, $\mathbf{F}(r)$ belongs to universal gravitation and always points towards the matter particle *M* (located at the coordinate origin *O*), so it follows that $F_r<0$.

Newton's Second Law and Planetary Motion

According to Newton's second law:

$$\mathbf{F}(r)=m\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\quad\left(\mathbf{F}(r)=F_r\frac{\mathbf{r}}{r}\right)\tag{16.1}$$

where $\mathbf{r}$ ($r=|\mathbf{r}|$) is the radius vector of *M* pointing to *m*, $\mathbf{F}(r)$ is the gravitational force or central force exerted by *M* to *m*, *M* and *m* also denote the masses of celestial bodies ($m<<M$).

In the Cartesian-coordinate $O(x,y,z)$:

$$F_x=m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=F_r\frac{x}{r}\quad\text{and}\quad F_y=m\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=F_r\frac{y}{r}\tag{16.2}$$

In the corresponding spherical-coordinate $O(r,\theta,\varphi)$:

$$\begin{cases}F_r=m\left(\dfrac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}-r\left(\dfrac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^2\right)\\ h_K=r^2\dfrac{\mathrm{d}\varphi}{\mathrm{d}t}=r^2\dfrac{\mathrm{d}\varphi}{\mathrm{d}\tau}=const\end{cases}\tag{16.3}$$

where $h_K=rv=L/m$ is a constant, known as **the velocity moment**; $L=rmv$ is the angular momentum, or **the momentum moment**; in the Cartesian spacetime with the idealized observation agent OA_∞, $\mathrm{d}t=\mathrm{d}\tau$.

Equation (16.3) proves Kepler's area law (Fig. 16.1(*c*)): the area swept out by the radius vector $\mathbf{r}$ ($r=|\mathbf{r}|$) of the planet *m* per unit time is the same or equal. And moreover, Eq. (16.3) proves the conservation law of angular momentum: $L=mr^2\mathrm{d}\varphi/\mathrm{d}t$ is a constant.

As a central force, the gravitational force $\mathbf{F}(r)$ always points towards the center of force and mass, i.e., *O* or the star *M*. According to Kepler's laws or classical mechanics, the planet *m* subjected to the gravitational force (a central force) of the star M must always remains in the plane of X-Y ($\theta=\pi/2$).

Binet Equation

The universal gravitation $\mathbf{F}(r)$ is a kind of conservative force. Based on the conservation law and Eq. (16.3), one can get the Binet equation of the planet m:

$$h_K^2 u^2 \left(\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u \right) = -\frac{F_r}{m} \quad \left(u = \frac{1}{r} \right) \tag{16.4}$$

Newton's Law of Universal Gravitation and Planetary Motion

Substituting Newton's law of universal gravitation into Binet equation (16.4), one can get Newton's motion equation of the planet m:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = \frac{GM}{h_K^2} \quad \left(F_r = -\frac{GMm}{r^2} \right) \tag{16.5}$$

where, G is the gravitational constant, and M is the mass of the star ($M >> m$).

The Forms of Newton's Celestial Orbits

Solving the differential equation (16.5), one has

$$r = \frac{1}{u} = \frac{p}{1 + e\cos(\varphi - \varphi_0)} \quad \left(p = \frac{h_K^2}{GM}, e = C_S p \right) \tag{16.6}$$

This is the standard conic equation, where the celestial body M is located at a focus of the conic curse, e is the orbital eccentricity of the celestial body m, and C_S and φ_0 are the integration constants.

By adjusting the zero point of time, or rotating the X-Y plane around the Z-axis one could set the initial angle φ_0 of the orbit of the observed planet m to a specific value, or set it to $\varphi_0=0$. In Eq. (16.6), C_S depends on the initial angular momentum L of m and the initial mechanical energy E of m.

According to Eq. (16.6), in the two-body system (M,m) of celestial bodies, the orbital eccentricity $e=C_S p$ of the small celestial body m (which could be a planet or comet or satellite or even photon) depends on the gravitational constant G and the gravitational field source M, as well as, on the initial angular momentum L of m and the initial mechanical energy E of m.

According to the classical formula of celestial mechanics:

$$e = \sqrt{1 + \frac{2EL^2}{G^2 M^2 m^3}} \quad (E = K + V; L = mvr) \tag{16.7}$$

where the total mechanical energy $E=K+V$ of m is the sum of the kinetic energy K of m and the potential energy V of m; v is the speed of m, $\mathbf{r}$ ($r=|\mathbf{r}|$) is the radius vector of m, and m also denotes the mass of the celestial body m; L is the angular momentum of the celestial body m: $L=mrv$.

According to Eq. (16.7), in the two-body system (M,m) of celestial bodies, the orbital eccentricity e of the celestial body m is a constant.

The eccentricity e determines the form of the orbit of the celestial body m:

(i) $e=0$: a circle;

(ii) $1>e>0$: an ellipse;

(iii) $e=1$: a parabola;

(iv) $e>1$: a hyperbola.

Naturally, the orbit of a planet bound to a star must be an ellipse, and therefore, its orbital eccentricity e must be greater than zero and less than one: $1>e>0$. Taking Mercury and the earth as examples, the eccentricity e of Mercury's orbit around the sun is $e=0.2056$; while the eccentricity e of the earth's orbit around the sun is only 0.0167, closer to a circle than Mercury's orbit.

Newton's motion equation (Eq. (16.5)) of celestial bodies proves Kepler's first law, i.e., the ellipse law: the orbit of a planet is an ellipse.

16.2.3 Newton's Motion Equation of Planets and The Orbital Precession of Planets

Let the initial angle φ_0 of the orbit of the observed planet m be zero: $\varphi_0=0$, then one has the solution of Newton's motion equation (Eq. (16.5)) of the planet m:

$$u=\frac{GM}{h_K^2}(1+e\cos\varphi) \quad \text{and} \quad \frac{\mathrm{d}u}{\mathrm{d}\varphi}=-\frac{GM}{h_K^2}e\sin\varphi \tag{16.8}$$

where the gravitational constant G and the mass M of the star, as well as, the velocity moment h_K and orbital eccentricity e of the planet m, are all constant.

At the perihelion of the planet m, naturally, $\mathrm{d}u/\mathrm{d}\varphi=0$.

Let $\Delta\varphi$ be the orbital precession angle of the planet m per revolution. Let $k=1$, that is, the planet m orbits the star M for one cycle, as depicted in Fig. 16.1(d): the planet m starts from the perihelion P and travels to the next perihelion P′. Then, the scanning angle of the planet m should be $\varphi=2\pi+\Delta\varphi$. Substituting $\varphi=2\pi+\Delta\varphi$ into Eq. (16.8), it follows that $\Delta\varphi=0$.

This means that, according to Newton's planetary models (Eq. (16.5) and Eq. (16.6)), there is no the orbital precession of planets.

Actually, Newton's planetary models (Eqs. (16.5-6)) means that the orbit of the planet m around the star M is the idealized or standard ellipse, which is closed and has no the orbital precession or the perihelion precession. Therefore, Newton's planetary models of the celestial two-body system could not predict or calculate the orbital precession of planets or the perihelion precession of Mercury.

So, why could not Newton's planetary model of the celestial two-body system predict the perihelion precession of planets or Mercury?

In the solar system, the precession of Mercury's orbit is particularly prominent. Perhaps, it is because Mercury is closest to the sun that the non-idealized factors of the two-body system $(Sun, Mercury)$ are the most prominent. Astronomical observation shows that Mercury's perihelion precesses at the rate of 5600.73″ per century. The correction calculation after considering the non-idealized factors shows that the 5557.62″ of it are rooted from the precession of the equinoxes and the perturbation of other celestial bodies. However, Newton's planetary model of the idealized two-body system has no the prior knowledge or information about the precession of the equinoxes and the perturbation of other celestial bodies. Therefore, Newton's planetary model (Eqs. (16.5-6)) could not predict and calculate the Mercury's precession of the 5557.62″. The rest, i.e., the precession of 43.11″, might be attributed to observational residuals or other unknown factors, that needs to be further examined.

It will contribute to our understanding of Einstein's gravitational theory and the gravitational theory of OR to review Newton's planetary model of the celestial two-body system. It will contribute to our understanding of Einstein's prediction of Mercury's precession of the 43.03″ per century to analogize Newton's planetary model with Einstein's planetary model and the GOR planetary model.

16.3 Einstein's Celestial-Body Model

Newton's planetary mode of the idealized celestial two-body system (*M*,*m*) failed to predict or calculated the orbital precession of Mercury, so how could Einstein's planetary model predict the orbital precession of Mercury and calculate the precession angle of 43.03 arc second per 100 years?

Newton's planetary model is derived from classical mechanics and Newton's laws, and is the product of Newton's theory of universal gravitation; Einstein's planetary model is the product of Einstein's theory of general relativity. However, they are both the celestial motion model of the two-body system, describing the motion of celestial bodies in gravitational field. Actually, with respect to the idealized two-body problem of celestial bodies, there is no prior knowledge or information about the orbital precession of planets or Mercury. Therefore, no matter Newton's theory of universal gravitation, or Einstein's theory of general relativity, or even the theory of GOR, could not predict or calculate the objectively orbital precession of planets or Mercury based on the idealized two-body model.

Based on Einstein's theory of general relativity, one could build the Einstein field equation and motion equation for the two-body problem (*M*,*m*) of celestial bodies. Like Newton's motion equation of celestial bodies, Einstein's motion equation of celestial bodies is also a special case of the GOR motion equation of celestial bodies. Under the principle of general correspondence (GC), the logic of Einstein's planetary model will serve as a reference for the GOR planetary model, including the deduction of the motion equation of the planet *m* and the calculation of the orbital precession of the planet *m*.

16.3.1 Einstein's Two-Body Problem of Celestial Bodies

Like Newton's theoretical model of the celestial two-body system, Einstein's theoretical model of the celestial two-body system (*M*,*m*) also contains idealized conditions, which can be described as follows.

Einstein's Two-Body System of Celestial Bodies: (*M*,*m*), the star *M* and the planet *m* interact through the universal gravitation between them, *M* produces the gravitational field, and m moves in the gravitational field.

The Idealized Conditions for Einstein's Two-Body System: Einstein's two-body system (M,m) implies the following idealized conditions.

(i) The action at a distance of gravitational interaction: gravitational radiation is action at a distance; the speed of gravitational radiation is infinite.

(ii) The isolated system: (M,m) is an isolated system, not affected by the matter or energy outside (M,m); both the star M and the planet m could be regarded as particles or mass points, M is at rest and the planet m moves in the gravitational field of M.

The above idealized conditions are the same as that of the idealize conditions of Newton's two-body system. However, Einstein's celestial two-body problem does not contain the conditions of the idealized observation agent and the idealized observer, but instead of the condition of the optical observation agent and the condition of the optical observer.

The Optical Agent and the Conditions of Optical Observation: In Einstein's celestial two-body system (M,m), the observation agent is realistic and non-idealized, and there are the implied observation conditions as follows.

(i) The optical observation agent: Einstein's observation agent is the optical observation agent OA(c), the information-wave speed of OA(c) is the speed c of light, it takes time for OA(c) to transmit the spacetime information of the observed planet m to observers.

(ii) The optical observer: the observer O employs the optical observation agent OA(c) to observe the planet m from the perspective of M; in theory, O is at rest relative to M and located at the position of zero potential.

Newton did not realize that his theory is the theory of the idealized agent OA_∞; likewise, Einstein also did not realize that his theory is the theory of the optical agent OA(c). The theory of OR (including IOR and GOR) has clarified that Einstein's theory of relativity, including the special and the general, is that of optical observation. Naturally, the observation agent in Einstein's planetary model is no longer the idealized agent OA_∞ of Newton's theory of universal gravitation, but the optical agent OA(c) of Einstein's theory of general relativity. The optical agent OA(c) has the observational locality ($c<\infty$), which is the fundamental difference between Newton's two-body system of celestial bodies and Einstein's two-body system of celestial bodies.

The Formalized Coordinates of Einstein's Two-Body System: as depicted in Fig. 16.2, like Newton's formalized coordinates, the motion of Einstein's two-body system (M,m) is also described in both the 3d Cartesian-coordinate $O(x,y,z)$ and the corresponding 3d spherical-coordinate $O(r,\theta,\varphi)$ (Fig. 16.2(a); Eq. (15.6)), the large mass celestial-body M is located at the coordinate origin O, and the small mass celestial-body m moves in the plane of X-Y ($\theta=\pi/2$) (Fig. 16.2(b)).

Like Newton's two-body problem of celestial bodies, Einstein's two-body problem of celestial bodies could also be idealized and reduced to the one-body problem, only examining the motion of the planet m: the star M is at rest; the observer O observe the planet m from the perspective of the star M.

16.3.2 Einstein's Theory of General Relativity and The Two-Body System of Celestial Bodies

After the establishment of his theory of general relativity, Einstein applied it to the two-body problem (M,m) of celestial bodies. Based on his weak-field approximate solution of the gravitational-field equation of the star M, Einstein built the motion equation of the planet m, and calculated that the orbital precession rate of Mercury was 43.03 arc second per 100 years [8]. Later, Schwarzschild obtained the exact solution of Einstein field equation of the static spherically-symmetric gravitational field in the trenches on the front line of World War I [80]. Based on the Schwarzschild solution, one could build the more credible celestial model of the two-body system (M,m), and more accurately predict celestial motion.

For the two-body problem of celestial bodies, based on Einstein's theory of general relativity, by substituting Schwarzschild solution (see Eqs. (15.7-8) in Chapter 15) into the line-element formula of general relativity or Einstein's motion equation (the geodesics), one could build the planetary model of the two-body system (M,m), i.e., the motion equation of the planet m.

Schwarzschild Line-Element Formula

As shown in Eqs. (15.7-8), the Schwarzschild solution is the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta)$ in the spherical-coordinate $O(r,\theta,\varphi)$ form:

$$\begin{aligned} & g_{\mu\nu} = \mathrm{diag}\left(g_{00}, g_{11}, g_{22}, g_{33}\right) = \mathrm{diag}\left(\mathrm{e}^{\xi}, -\mathrm{e}^{\zeta}, -r^2, -r^2\sin^2\theta\right) \\ & \left(\mathrm{e}^{\xi} = 1 - \frac{2GM}{c^2 r}, \mathrm{e}^{\zeta} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1}; \xi + \zeta = 0\right) \end{aligned} \quad (16.9)$$

Then, the Schwarzschild line-element formula can be expressed as:

$$\begin{aligned} \mathrm{d}s^2 &= g_{00}\mathrm{d}x^0\mathrm{d}x^0 + g_{11}\mathrm{d}x^1\mathrm{d}x^1 + g_{22}\mathrm{d}x^2\mathrm{d}x^2 + g_{33}\mathrm{d}x^3\mathrm{d}x^3 \\ &= \mathrm{e}^{\xi}c^2\mathrm{d}t^2 - \mathrm{e}^{\zeta}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \\ & \left(x^0 = ct, x^1 = r, x^2 = \theta, x^3 = \varphi; \mu = 0,1,2,3\right) \end{aligned} \quad (16.10)$$

where both r=0 and r=$2GM/c^2$ are the two singularities of the Schwarzschild line-element formula

The Geodesics of the Planet *m*

By substituting the Schwarzschild solution into Einstein's motion equation of general relativity, one could get the motion equation of the two-body system of celestial bodies, i.e., the geodesics of the observed planet m:

$$\begin{cases} \dfrac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu_{\alpha\beta}\dfrac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\dfrac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0 \\ \Gamma^\mu_{\alpha\beta} = \dfrac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta} + g_{\nu\beta,\alpha} - g_{\beta\alpha,\nu}\right) \end{cases} \quad (16.11)$$

$$\left(x^0 = ct, x^1 = r, x^2 = \theta, x^3 = \varphi; \mu = 0,1,2,3\right)$$

where t is the observational time (Einstein called it the coordinate time), and τ is the intrinsic time (Einstein called it the standard time).

Equation (16.11) has four relations (μ=0,1,2,3): (i) $t=t(\tau)$; (ii) $r=r(\tau)$; (iii) $\theta=\theta(\tau)$; and (iv) $\varphi=\varphi(\tau)$. Examining each relation in Eq. (16.11) will contribute to our understanding of Einstein's theoretical model of the celestial two-body system (M,m) and to our understanding of the planetary orbits under the optical observation agent OA(c). In particular, it will provide the analogy and reference for the deduction of the GOR motion equation of the planet m.

Relation $t=t(\tau)$ and the Factor of Spacetime Transformation

Based on the Schwarzschild metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta)$ [68]:

$$\begin{cases} \varGamma_{00}^{0}=\varGamma_{11}^{0}=\varGamma_{22}^{0}=\varGamma_{33}^{0}=0 \\ \varGamma_{02}^{0}=\varGamma_{03}^{0}=\varGamma_{12}^{0}=\varGamma_{13}^{0}=\varGamma_{23}^{0}=0 \\ \varGamma_{01}^{0}=\dfrac{1}{2}\dfrac{\mathrm{d}\xi}{\mathrm{d}r} \quad \left(\mathrm{e}^{\xi}=1-\dfrac{2GM}{rc^2}\right) \end{cases} \tag{16.12}$$

Let $\mu=0$, then $x^\mu=x^0=ct$. According to Eq. (16.11):

$$\begin{aligned} \frac{\mathrm{d}^2x^0}{\mathrm{d}\tau^2} &= -\varGamma_{\alpha\beta}^{0}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} \\ &= -\varGamma_{00}^{0}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}-2\varGamma_{0i}^{0}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}-\varGamma_{ik}^{0}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^k}{\mathrm{d}\tau} \\ &= -2\varGamma_{01}^{0}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^1}{\mathrm{d}\tau}-\varGamma_{ik}^{0}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^k}{\mathrm{d}\tau} \\ &= -2\varGamma_{01}^{0}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^1}{\mathrm{d}\tau} \end{aligned} \tag{16.13}$$

By contrasting Eq. (16.11) and Eqs. (16.12-13), one could have the second-order differential equation of t:

$$\frac{\mathrm{d}^2t}{\mathrm{d}\tau^2}+\frac{\mathrm{d}\xi}{\mathrm{d}\tau}\frac{\mathrm{d}t}{\mathrm{d}\tau}=0 \tag{16.14}$$

By integrating Eq. (16.14) of t, one could get:

$$\gamma=\frac{\mathrm{d}t}{\mathrm{d}\tau}\equiv\frac{C_t}{1-2GM/c^2r} \tag{16.15}$$

where C_t is a constant, and particularly, $\gamma=\mathrm{d}t/\mathrm{d}\tau$ is exactly the factor of spacetime transformation in Einstein's theory of general relativity.

According to the concepts of Einstein's general relativity, the differential equation (16.14) of t describes the relationship between the coordinate time t and the standard time τ, which means that the coordinate time t of the observed planet m is different from the standard time τ. Actually, the differential equation (16.14) of t has the more profound significance: the observation agent of Einstein's general relativity is the optical agent OA(c), and the observational time t of OA(c) is different from the intrinsic time or the proper time τ, i.e., the objectively real time.

Relation $\theta=\theta(\tau)$ and Kepler's Orbit Law

Based on the Schwarzschild metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta)$ [68]:

$$\begin{cases} \Gamma_{00}^{2}=\Gamma_{11}^{2}=\Gamma_{22}^{2}=0 \\ \Gamma_{01}^{2}=\Gamma_{02}^{2}=\Gamma_{03}^{2}=\Gamma_{13}^{2}=\Gamma_{23}^{2}=0 \\ \Gamma_{12}^{2}=1/r \\ \Gamma_{33}^{2}=-\sin\theta\cos\theta \end{cases} \tag{16.16}$$

Let $\mu=2$, then $x^{\mu}=x^{2}=\theta$. According to Eq. (16.11):

$$\begin{aligned} \frac{\mathrm{d}^2 x^2}{\mathrm{d}\tau^2} &= -\Gamma_{\alpha\beta}^{2}\frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{\beta}}{\mathrm{d}\tau} \\ &= -\Gamma_{00}^{2}\frac{\mathrm{d}x^{0}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{0}}{\mathrm{d}\tau}-2\Gamma_{0i}^{2}\frac{\mathrm{d}x^{0}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{i}}{\mathrm{d}\tau}-\Gamma_{ik}^{2}\frac{\mathrm{d}x^{i}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{k}}{\mathrm{d}\tau}=-\Gamma_{ik}^{2}\frac{\mathrm{d}x^{i}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{k}}{\mathrm{d}\tau} \\ &= -\Gamma_{11}^{2}\frac{\mathrm{d}x^{1}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{1}}{\mathrm{d}\tau}-\Gamma_{22}^{2}\frac{\mathrm{d}x^{2}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{2}}{\mathrm{d}\tau}-\Gamma_{33}^{2}\frac{\mathrm{d}x^{3}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{3}}{\mathrm{d}\tau} \\ &\quad -2\Gamma_{12}^{2}\frac{\mathrm{d}x^{1}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{2}}{\mathrm{d}\tau}-2\Gamma_{13}^{2}\frac{\mathrm{d}x^{1}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{3}}{\mathrm{d}\tau}-2\Gamma_{23}^{2}\frac{\mathrm{d}x^{2}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{3}}{\mathrm{d}\tau} \\ &= -\Gamma_{33}^{2}\frac{\mathrm{d}x^{3}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{3}}{\mathrm{d}\tau}-2\Gamma_{12}^{2}\frac{\mathrm{d}x^{1}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{2}}{\mathrm{d}\tau} \end{aligned} \tag{16.17}$$

By contrasting Eq. (16.11) and Eqs. (16.16-17), one could have the second-order differential equation of θ:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\tau^2}-\sin\theta\cos\theta\left(\frac{\mathrm{d}\varphi}{\mathrm{d}\tau^2}\right)^2+\frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau}=0 \tag{16.18}$$

Without loss of generality, suppose that, at the initial time $t=0$, the observed planet m runs in the plane of X-Y ($\theta=\pi/2$), then:

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right|_0=0 \quad \cos\theta|_0=0 \quad \left.\frac{\mathrm{d}^2\theta}{\mathrm{d}\tau^2}\right|_0=0 \tag{16.19}$$

Equations (16.18) and (16.19) about θ mean that the orbital plane of the planet m is fixed and constant in Einstein's theoretical model (Eq. (16.11)) of the two-body system. This is consistent with Kepler's orbit law and Newton's planetary model.

Relation $\varphi=\varphi(\tau)$ and Kepler's Area Law

Based on the Schwarzschild metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta)$ [68]:

$$\begin{cases} \Gamma_{00}^3 = \Gamma_{11}^3 = \Gamma_{22}^3 = \Gamma_{33}^3 = 0 \\ \Gamma_{01}^3 = \Gamma_{02}^3 = \Gamma_{03}^3 = \Gamma_{12}^3 = 0 \\ \Gamma_{13}^3 = 1/r \\ \Gamma_{23}^3 = \cot\theta \end{cases} \tag{16.20}$$

Let $\mu=3$, then $x^{\mu}=x^3=\varphi$. According to Eq. (16.11):

$$\begin{aligned} \frac{\mathrm{d}^2 x^3}{\mathrm{d}\tau^2} &= -\Gamma_{\alpha\beta}^3 \frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{\beta}}{\mathrm{d}\tau} \\ &= -\Gamma_{00}^3 \frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau} - 2\Gamma_{0i}^3 \frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^i}{\mathrm{d}\tau} - \Gamma_{ik}^3 \frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^k}{\mathrm{d}\tau} = -\Gamma_{ik}^3 \frac{\mathrm{d}x^i}{\mathrm{d}s}\frac{\mathrm{d}x^k}{\mathrm{d}s} \\ &= -\Gamma_{11}^3 \frac{\mathrm{d}x^1}{\mathrm{d}\tau}\frac{\mathrm{d}x^1}{\mathrm{d}\tau} - \Gamma_{22}^3 \frac{\mathrm{d}x^2}{\mathrm{d}\tau}\frac{\mathrm{d}x^2}{\mathrm{d}\tau} - \Gamma_{33}^3 \frac{\mathrm{d}x^3}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} \\ &\quad -2\Gamma_{12}^3 \frac{\mathrm{d}x^1}{\mathrm{d}\tau}\frac{\mathrm{d}x^2}{\mathrm{d}\tau} - 2\Gamma_{13}^3 \frac{\mathrm{d}x^1}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} - 2\Gamma_{23}^3 \frac{\mathrm{d}x^2}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} \\ &= -2\Gamma_{13}^3 \frac{\mathrm{d}x^1}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} - 2\Gamma_{23}^3 \frac{\mathrm{d}x^2}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} \end{aligned} \tag{16.21}$$

By contrasting Eq. (16.11) and Eqs. (16.20-21), one could have the second-order differential equation of φ:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}\tau^2} + \frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} + 2\cot\theta\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} = 0 \tag{16.22}$$

Since the orbital plane of a planet is fixed and constant, without loss of generality, suppose that the planet m runs in the plane of X-Y ($\theta=\pi/2$), then the Eq. (16.22) of φ could be written as follows:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}\tau^2} + \frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} = 0 \tag{16.23}$$

By integrating Eq. (16.23) of φ, one could get:

$$r^2\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \equiv h_K = const \tag{16.24}$$

where the velocity moment $h_K=L/m$ of the planet m is a constant, and naturally, the angular momentum $L=mh_K$ of the planet m is also a constant.

Equation (16.24) is exactly the area law (Eq. (16.3)) of Kepler's three laws as shown in Fig. 16.1(c): the area swept out by the radius vector **r** ($r=|\mathbf{r}|$) of the planet m per unit time is the same or equal. Essentially, Eq. (16.24) is the conservation law of angular momentum: $L=rmv$ is constant. (It is worth noting that here the mass m of the planet is the gravitational mass, or the intrinsic mass of the planet m, rather than the relativistic mass.)

It should be pointed out that, in Newton's motion equation (Eq. (16.3) and Eq. (16.5)) of the planet m, Kepler's area law is expressed as $r^2\mathrm{d}\varphi/\mathrm{d}t\equiv h_K$, where the time t seems to be the observational time or coordinate time, rather than the intrinsic time or standard time τ. Some physicists believe that there is a slight difference between the velocity moment constant $h_K \equiv r^2 \mathrm{d}\varphi/\mathrm{d}\tau$ in Kepler's laws or Newton's classical mechanics and the velocity moment constant $h_K \equiv r^2 \mathrm{d}\varphi/\mathrm{d}t$ in Einstein's theory of generalized relativity [68]. However, as a matter of fact, Kepler's three laws of planetary motion, like Newton's laws, are the laws of the idealized observation agent OA_∞. According to the Lemma 13.1 of the theorem of Cartesian spacetime in Chapter 13, the observational time t of OA_∞ is just the intrinsic time τ: $\mathrm{d}t=\mathrm{d}\tau$. Therefore, Kepler's area law and that ($r^2\mathrm{d}\varphi/\mathrm{d}t \equiv h_K$ in Eq. (16.3)) derived from Newton's laws, as well as that ($r^2\mathrm{d}\varphi/\mathrm{d}\tau \equiv h_K$ in Eq. (16.24)) derived from Einstein's theory of general relativity, are the same or equivalent.

Relation $r=r(\tau)$ and the Orbital Equation of the Planet

Based on the Schwarzschild metric $g_{\mu\nu}=g_{\mu\nu}(r,\theta)$ [68]:

$$\begin{cases} \Gamma^1_{0i} = \Gamma^1_{12} = \Gamma^1_{13} = \Gamma^1_{23} = 0 \\ \Gamma^1_{00} = \dfrac{1}{2}\mathrm{e}^{\xi-\zeta}\dfrac{\mathrm{d}\xi}{\mathrm{d}r} \quad \left(\mathrm{e}^{\xi} = g_{00},\ \mathrm{e}^{\zeta} = -g_{11}\right) \\ \Gamma^1_{11} = \dfrac{1}{2}\dfrac{\mathrm{d}\zeta}{\mathrm{d}r} \quad (\zeta = -\xi) \\ \Gamma^1_{22} = -r\mathrm{e}^{-\zeta} \\ \Gamma^1_{33} = -r\sin^2\theta\,\mathrm{e}^{-\zeta} \end{cases} \tag{16.25}$$

Let $\mu=1$, then $x^\mu=x^1=r$. According to Eq. (16.11):

$$\begin{aligned} \frac{\mathrm{d}^2x^1}{\mathrm{d}\tau^2} &= -\Gamma^1_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} \\ &= -\Gamma^1_{00}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau} - 2\Gamma^1_{0i}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^i}{\mathrm{d}\tau} - \Gamma^1_{ik}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\mathrm{d}x^k}{\mathrm{d}\tau} \\ &= -\Gamma^1_{00}\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau} - \Gamma^1_{11}\frac{\mathrm{d}x^1}{\mathrm{d}\tau}\frac{\mathrm{d}x^1}{\mathrm{d}\tau} \\ &\quad -\Gamma^1_{22}\frac{\mathrm{d}x^2}{\mathrm{d}\tau}\frac{\mathrm{d}x^2}{\mathrm{d}\tau} - \Gamma^1_{33}\frac{\mathrm{d}x^3}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} \end{aligned} \tag{16.26}$$

By contrasting Eq. (16.11) and Eqs. (16.25-26), one could have the second-order differential equation of r:

$$\begin{aligned} \frac{\mathrm{d}^2r}{\mathrm{d}\tau^2} = -\frac{1}{2}\mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r}\left(c^2\mathrm{e}^{\xi}\frac{\mathrm{d}t}{\mathrm{d}\tau}\frac{\mathrm{d}t}{\mathrm{d}\tau} - \mathrm{e}^{\zeta}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}r}{\mathrm{d}\tau}\right) \\ +r\mathrm{e}^{\xi}\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + r\sin^2\theta\,\mathrm{e}^{\xi}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \end{aligned} \tag{16.27}$$

According to the Schwarzschild line-element formula (16.10):

$$c^2 = c^2 \mathrm{e}^{\xi} \frac{\mathrm{d}t}{\mathrm{d}\tau}\frac{\mathrm{d}t}{\mathrm{d}\tau} - \mathrm{e}^{\zeta} \frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}r}{\mathrm{d}\tau} - r^2 \frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} - r^2 \sin^2\theta \frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \tag{16.28}$$

Thus, Eq. (16.27) could be rewritten as

$$\begin{aligned} \frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = &-\frac{c^2}{2}\mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r} + \left(r\mathrm{e}^{\xi} - \frac{r^2}{2}\mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r} \right)\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} \\ &+ \sin^2\theta \left(r\mathrm{e}^{\xi} - \frac{r^2}{2}\mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r} \right)\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \end{aligned} \tag{16.29}$$

Since the orbital plane of the planet m is fixed and constant, without loss of generality, suppose that the planet m runs in the plane of X-Y ($\theta=\pi/2$), then:

$$\begin{aligned} &\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\frac{GM}{r^2} + \left(r - \frac{3GM}{c^2} \right)\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \\ &\qquad \left(\mathrm{e}^{\xi} = 1 - \frac{2GM}{c^2 r},\ \mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r} = \frac{2GM}{c^2 r^2} \right) \end{aligned} \tag{16.30}$$

Equation (16.30) is the theoretical model of the celestial two-body system (M,m) based on the Einstein's theory of general relativity, i.e., Einstein's motion equation of the planet m, which is the second-order nonlinear differential equation of the radius vector $\mathbf{r}$ ($r=|\mathbf{r}|$) and angle φ of the planet m with respect to the standard time τ, where suppose the orbital plane of the planet m is: X-Y ($\theta=\pi/2$).

16.3.3 The Binet form of Einstein's motion Equation of the Planet

Usually, the 3d-space motion trajectory of an object can be expressed by a set of spherical-coordinate equations: (i) $r=r(t)$; (ii) $\theta=\theta(t)$; and (iii) $\varphi=\varphi(t)$. However, for a central force (such as gravity or universal gravitation), the motion plane of the object is fixed or constant, and the motion equation can be reduced to: $r=r(t)$ and $\varphi=\varphi(t)$; and moreover, the angular momentum $L=h_K m$ is conserved, and hence, the differential of φ with respect to the time t can be eliminated by $\mathrm{d}\varphi/\mathrm{d}t=h_K/r^2$. Then, one can transform Eq. (16.30) to the second-order differential equation of the reciprocal u (=1/r): $\mathrm{d}^2r/\mathrm{d}\varphi^2=u(\varphi)$, that is, the Binet form of Einstein's motion equation of the planet m, like Eq. (16.4).

According to Eq. (16.24): $r^2\mathrm{d}\varphi/\mathrm{d}\tau \equiv h_K$, Eq. (16.30) could be rewritten as

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\frac{GM}{r^2} + \frac{1}{r^3}h_K^2 - \frac{3GM}{c^2 r^4}h_K^2 \tag{16.31}$$

By following the logic of Binet equation, let $u=1/r$, then $\mathrm{d}\varphi/\mathrm{d}\tau=h_K/r^2=h_K u^2$, and from Eq. (16.30), it follows that

$$\frac{\mathrm{d}r}{\mathrm{d}\tau} = -h_K \frac{\mathrm{d}u}{\mathrm{d}\varphi} \quad \text{and} \quad \frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -h_K^2 u^2 \frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} \tag{16.32}$$

By substituting Eq. (16.32) into Eq. (16.31), one could get that

$$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{c^2}u^2\right) \quad \left(u=\frac{1}{r}\right) \tag{16.33}$$

This is the Binet form of Einstein's equation of planetary motion.

Equation (16.33) is a second-order nonlinear differential equation of u (=1/r) with respect to φ, which contains both the information of the radius vector **r** (r=|**r**|) of the planet orbit and the angle φ of the planet orbit.

By contrasting with the Binet form of Newton's equation (Eq. (16.5)) of planetary motion, we know that the Binet form of Einstein's equation (Eq. (16.33)) of planetary motion has one more term on the right: $3GM/c^2r^2$.

This suggests that Einstein's equation (16.33) of planetary motion:

(i) is a nonlinear differential equation;

(ii) is a non-standard conic curve, in which $3GM/c^2r^2$ is the precession term of planetary orbit, so the orbit of the observed planet m is no longer a standard or closed ellipse, and the planetary orbit would precess slowly.

16.3.4 Einstein's motion Equation of the Planet and the Orbital Precession of the Planet

Based on Einstein's motion equation of the planet, one could predict or calculate the orbital precession of the planet by solving Einstein's planetary equation (16.33).

As far as the general planets in the solar system are concerned, the term of $3h_K^2/c^2r^2$ (<<1) in Einstein's planetary equation is a small quantity. Regardless of the term of $3h_K^2/c^2r^2$, Einstein's planetary equation (16.33) reduces to Newton's planetary equation (16.5). As a result, one could get the planetary orbit of the conic curve as shown in Eq. (16.6), in which, for the planet bound by the star, the orbital eccentricity of it is $0<e<1$.

The orbital eccentricity of Mercury is 0.206, which is the largest among all the planets in the solar system.

Since $3h_K^2/c^2r^2$ (<<1) is a small quantity, one could employ the progressive approximation method to solve Einstein's planetary equation (16.33) of motion [68].

Substituting Eq. (16.6) into Eq. (16.33):

$$\begin{aligned}\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u&=\frac{GM}{h_K^2}+\frac{3GM}{c^2}u^2\\&=\frac{GM}{h_K^2}+\frac{3GM}{c^2}\left(\frac{GM}{h_K^2}\right)^2(1+e\cos\varphi)^2\\&=\frac{GM}{h_K^2}+\frac{3G^3M^3}{c^2h_K^4}\left(1+\frac{e^2}{2}+2e\cos\varphi+\frac{e^2}{2}\cos2\varphi\right)\end{aligned} \tag{16.34}$$

Solving the differential equation (16.34), one has:

$$u=\left\{\frac{GM}{h_K^2}+\frac{3G^3M^3}{c^2h_K^4}\left(1+\frac{e^2}{2}\right)\right\}(1+e\cos\varphi)$$
$$+\frac{3G^3M^3e}{c^2h_K^4}\left(\varphi\sin\varphi-\frac{e}{6}\cos2\varphi\right) \tag{16.35}$$

At the perihelion of the planet *m*, naturally, $du/d\varphi=0$. Therefore, by taking the derivative of *u* with respect to φ at both ends of Eq. (16.35), one has:

$$0=\frac{3G^2M^2}{c^2h_K^2}\left(\varphi\cos\varphi+\frac{e}{3}\sin2\varphi-\frac{e^2}{2}\sin\varphi\right)-\sin\varphi \tag{16.36}$$

Regardless of $\varphi\cos\varphi$, then

$$\left\{\frac{3G^2M^2}{c^2h_K^2}\left(\frac{2e}{3}\cos\varphi-\frac{e^2}{2}\right)-1\right\}\sin\varphi=0 \tag{16.37}$$

So, $\sin\varphi=0$, or $\varphi=\arcsin\varphi=2k\pi\,(k=0,1,2,\ldots)$.

This means that, without $\varphi\cos\varphi$ like Newton's motion equation of the planet, the orbit of the planet would be a closed ellipse and have no the orbital precession.

Consider $\varphi\cos\varphi$. Let $\Delta\varphi$ be the orbital precession angle of the planet *m* per revolution. Let k=1, that is, the planet *m* orbits the star *M* for one cycle, as depicted in Fig. 16.1(d): the planet *m* starts from the perihelion P and travels to the next perihelion P′. Then, the scanning angle of the planet *m* should be $\varphi=2\pi+\Delta\varphi$.

Substituting $\varphi=2\pi+\Delta\varphi$ into Eq. (16.36) and ignoring high-order small quantities, then it follows that:

$$\Delta\varphi=\frac{6\pi G^2M^2}{c^2h_K^2}(\text{rad per revolution}) \tag{16.38}$$

According to the recommendations of the International Standards Organization:

The speed of light: $c=2.9979245\times10^8\ \text{m}\cdot\text{s}^{-1}$

The universal gravitational constant: $G=6.67430\times10^{-11}\ \text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$

The mass of the sun: $M=1.98847\times10^{30}$ kg

The mass of Mercury: $m=3.301\times10^{23}$ kg

The orbital angular momentum of Mercury: $L=8.9825\times10^{38}\ \text{kg}\cdot\text{m}^2\cdot\text{s}^{-1}$

The velocity moment of Mercury: $h_K=L/m=2.7211\times10^{15}\ \text{m}^2\cdot\text{s}^{-1}$

According to Eq. (16.38), one could calculate the orbital precession of Mercury during one revolution:

$$\Delta\varphi=3.888\times10^6\times\frac{G^2M^2}{c^2h_K^2}(\text{arc sec})=0.1029\,(\text{arc sec}) \tag{16.39}$$

Mercury's orbital period is $T_M=87.961$ day; the earth's orbital period is $T_E=365.24219$ day. So, every 100 earth years, Mercury's orbit would precess $\varphi=100\times\Delta\varphi\times T_E/T_M=42.77$ arc seconds.

The precession angle of Mercury's perihelion calculated by Einstein at that time was 43.03″, which was extremely consistent with the 43.11″ that has not yet found out the attribution. In a letter to a friend, Einstein said: "The equation gives the correct numbers for Mercury's perihelion. You could imagine how happy I am. I couldn't help but be happy for several days."

However, astronomical observation shows that the orbital precession rate of Mercury is 5600.73 arc second per 100 years, which is far greater than that predicted by Einstein's theory of general relativity. Einstein's theory of general relativity could only predict the Mercury's precession of about 43″, which is far from the actual observation, less than 1%. So, such a 43.03″ is far from being employed to determine that Einstein's theory of general relativity can correctly predict the orbital precession of planets.

16.3.5 What does Einstein's 43.03″ Mean?

There are two problems worth pondering about Einstein's prediction of the orbital precession of Mercury:

(i) Now that Einstein's theory of general relativity can predict the 43.11″ that has not yet found out the attribution in the actual orbital precession of Mercury's 5600.73″, why cannot Einstein predict the rest 5557.62″?

(ii) It is known that there is the 532″ per 100 years of Mercury's perihelion procession caused by the perturbation of other celestial bodies to Mercury. Then, what factors should be responsible for the 43.03″ of Mercury's perihelion precession predicted by Einstein's theory of general relativity?

First of all, no matter Newton's motion equation (16.5) or Einstein's motion equation (16.33), it is impossible to predict the orbital precession of planets caused by the precession of the equinoxes and the perturbation of other celestial bodies, for both Newton's planetary model and Einstein's planetary model have no prior information about the precession of the equinoxes and the perturbation of other celestial bodies.

Secondly, Einstein's planetary model (Eq. (16.33)) has the orbital precession term $3h_K^2/c^2r^2$ of planets; while Newton's planetary model (Eq. (16.5)) has no the perihelion precession term of planets. This difference could only be attributed to the different prerequisites of Newton's celestial two-body problem and Einstein's celestial two-body problem. As a matter of fact, the difference between Newton's two-body problem (M,m) and Einstein's two-body problem (M,m) lies only in the observation agents: Newton's observation agent is the idealized agent OA_∞ without the observational locality, it takes no time for the spacetime information of the planet m to cross space; Einstein's observation agent is the optical agent OA(c) with the observational locality ($c<\infty$), and therefore, it takes time for the spacetime information of the planet m to cross space. As shown in Eq. (16.33) and Eq. (16.38), the orbital precession term $3h_K^2/c^2r^2$ of the planet m in Einstein's planetary model (Eq. (16.33)) and the orbital precession angle $\Delta\varphi=\Delta\varphi(c)$ (Eq. (16.38)) predicted by Einstein's planetary model depend both on the speed c of light.

Perhaps, the 43.03″ precession of Mercury's perihelion predicted by Einstein's is exactly the 43.11″ that has not yet found out the attribution in the astronomical observation of Mercury's 5600.73″ precession. So, that is not so much the support for Einstein's theory of general relativity as the support for the theory of GOR: Einstein's 43.03″ is only an observational effect or an apparent phenomenon rooted from the observational locality ($c<\infty$) of the optical agent OA(c).

The theory of GOR will tell us that, in Einstein's planetary model (Eq. (16.33)), the orbital precession term $3h_K^2/c^2r^2$ does not represent the objective and real orbital precession of the observed planet, but the apparent phenomenon rooted from the observational locality ($\eta<\infty$) of the observation agent OA(η).

16.4 The GOR Celestial-Body Model

Like Newton's celestial-body model and Einstein's celestial-body model, the GOR celestial-body model also belongs to the two-body problem of celestial bodies, and is the idealized model of the celestial two-body system (M,m).

The theoretical model of the GOR celestial two-body system (M,m) will be deduced from the GOR field equation and GOR motion equation under the principle of general correspondence (GC). Naturally, in the sense of the principle of GC, the GOR motion equation of the observed planet m must be isomorphically consistent with Einstein's motion equation of the observed planet m.

However, it is somewhat surprising that the GOR motion equation of the observed planet m also is isomorphically consistent with Newton's motion equation of the observed planet m, which is based on Newton's laws and classical mechanics.

16.4.1 The GOR Two-Body Problem of Celestial Bodies

The GOR planetary model, like Newton's and Einstein's, also belong to the two-body problem (M,m) of celestial bodies, and could be reduced to the celestial one-body problem. like Newton and Einstein's planetary models, the GOR planetary model also contains the idealized conditions, and has no prior information that contributes to predicting or calculating the orbital precession of planets, such as the precession of the equinoxes and the perturbation of other celestial bodies.

Like Newton and Einstein's theoretical models of the celestial two-body system, the idealized conditions of the GOR theoretical model of the celestial two-body system (M,m) can also be described as follows.

The GOR Two-Body System of Celestial Bodies: (M,m), the star M and the planet m interact through the universal gravitation between them, M produces the gravitational field, and m moves in the gravitational field.

The Idealized Conditions for the GOR Two-Body System: The GOR celestial two-body system (M,m) implies the following idealized conditions.

(i) The action at a distance of gravitational interaction: gravitational radiation is action at a distance; the speed of gravitational radiation is infinite.

(ii) The isolated system: (M,m) is an isolated system, not affected by the matter or energy outside (M,m); both the star M and the planet m could be regarded as particles or mass points, M is at rest and the planet m moves in the gravitational field of M.

The above idealized conditions are the same as that of the idealize conditions of Newton and Einstein's two-body systems. The difference is that, in the GOR celestial two-body system (M,m), the observation agent is the general observation agent OA(η) ($0<\eta<\infty$).

The General Agent and the Conditions of General Observation: In the GOR celestial two-body system (M,m), the observation agent is the general observation agent OA(η), rather than a specific observation agent.

(i) The general observation agent: The GOR observation agent is the general observation agent OA(η) ($\eta\geq v$), in theory, any form of matter motion could be employed as the observation medium, the information-wave speed η of OA(η) should be higher than or equal to the speed v of the observed celestial body m.

(ii) The general observer: the observer O employs the general observation agent OA(η) ($\eta\geq v$) to observe the planet m from the perspective of M; in theory, O is at rest relative to M and located at the position of zero potential.

The GOR hypotheses of observation agents are reasonable: as stressed repeatedly in the theory of OR, the observation medium through which human beings perceive or observe the objective world may not necessarily be light alone.

The Formalized Coordinates of the GOR Two-Body System: as depicted in Fig. 16.2, like Newton and Einstein's formalized coordinates, the motion of the GOR two-body system (M,m) is also described in both the 3d Cartesian-coordinate $O(x,y,z)$ and the corresponding 3d spherical-coordinate $O(r,\theta,\varphi)$ (Fig. 16.2(a); Eq. (15.6)), the large mass celestial-body M is located at the coordinate origin O, and the small mass celestial-body m moves in the plane of X-Y ($\theta=\pi/2$) (Fig. 16.2(b)).

Based on the above idealized conditions or hypotheses of the GOR two-body problem (M,m) of celestial bodies, under the principle of GC, through both PGC logic route 1 and PGC logic route 2, we will deduce the GOR celestial-body model or the GOR motion equation of the observed planet m by analogizing or following the logic of Einstein's celestial-body mode.

16.4.2 The Theory of GOR and The Two-Body System of Celestial Bodies

In Sec. 15.3 and Sec. 15.4 of Chapter 15, we have obtained the vacuum solution of GOR field equation for the static spherically-symmetric gravitational field, including the approximate and the exact. For the two-body problem (M,m) of celestial bodies, based on the theory of GOR, by substituting the exact solution (see Eqs. (15.31-32) in Chapter 15) of GOR field equation into the GOR line-element formula or the GOR motion equation (the geodesics), we can build the planetary model of the two-body system (M,m), i.e., the motion equation of the planet m.

The GOR Line-Element Formula of the Planet

As shown in Eqs. (15.31-32), the exact solution of GOR field equation is the spacetime metric $g_{\mu\nu}=g_{\mu\nu}(\eta,r,\theta)$ in the spherical-coordinate $O(r,\theta,\varphi)$ form:

$$g_{\mu\nu}(\eta)=\mathrm{diag}\left(g_{00},g_{11},g_{22},g_{33}\right)=\mathrm{diag}\left(\mathrm{e}^{\xi},-\mathrm{e}^{\zeta},-r^2,-r^2\sin^2\theta\right)$$
$$\left(\mathrm{e}^{\xi}=1-\frac{2GM}{\eta^2 r},\ \mathrm{e}^{\zeta}=\left(1-\frac{2GM}{\eta^2 r}\right)^{-1};\ \xi+\zeta=0\right) \tag{16.40}$$

Then, the GOR line-element formula can be expressed as:

$$\begin{aligned}\mathrm{d}s^2&=g_{00}\mathrm{d}x^0\mathrm{d}x^0+g_{11}\mathrm{d}x^1\mathrm{d}x^1+g_{22}\mathrm{d}x^2\mathrm{d}x^2+g_{33}\mathrm{d}x^3\mathrm{d}x^3\\&=\mathrm{e}^{\xi}\eta^2\mathrm{d}t^2-\mathrm{e}^{\zeta}\mathrm{d}r^2-r^2\mathrm{d}\theta^2-r^2\sin^2\theta\mathrm{d}\varphi^2\\&\qquad\left(x^0=\eta t,\ x^1=r,\ x^2=\theta,\ x^3=\varphi;\ \mu=0,1,2,3\right)\end{aligned} \tag{16.41}$$

where r=0 and r=$2GM/c^2$ are the singularities of the GOR line-element formula

The GOR Geodesics of the Planet *m*

By substituting the exact solution of GOR field equation into the GOR motion equation, we get the GOR motion equation of the two-body system (M,m) of celestial bodies, i.e., the geodesics of the observed planet m:

$$\begin{cases}\dfrac{\mathrm{d}^2x^{\mu}}{\mathrm{d}\tau^2}+\Gamma^{\mu}_{\alpha\beta}(\eta)\dfrac{\mathrm{d}x^{\alpha}}{\mathrm{d}\tau}\dfrac{\mathrm{d}x^{\beta}}{\mathrm{d}\tau}=0\\ \Gamma^{\mu}_{\alpha\beta}(\eta)=\dfrac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta}+g_{\nu\beta,\alpha}-g_{\beta\alpha,\nu}\right)\end{cases} \tag{16.42}$$
$$\left(x^0=\eta t,\ x^1=r,\ x^2=\theta,\ x^3=\varphi;\ \mu=0,1,2,3\right)$$

where t is the observational time, and τ is the intrinsic time (proper time).

Like Einstein's motion equation (Eq. (16.11)) of the planet m, Eq. (16.42) also has four relations (μ=0,1,2,3): (i) $t=t(\tau)$, (ii) $r=r(\tau)$, (iii) $\theta=\theta(\tau)$, and (iv) $\varphi=\varphi(\tau)$, in which $t=t(\tau)$ is the time equation, while $r=r(\tau)$, $\theta=\theta(\tau)$, and $\varphi=\varphi(\tau)$, are the space equations.

16.4.3 Relation $t=t(\tau)$ and the Factor of Spacetime Transformation

Based on the GOR metric $g_{\mu\nu}=g_{\mu\nu}(\eta,r,\theta)$ in Eq. (16.40) [68]:

$$\begin{cases}\Gamma^0_{00}=\Gamma^0_{11}=\Gamma^0_{22}=\Gamma^0_{33}=0\\ \Gamma^0_{02}=\Gamma^0_{03}=\Gamma^0_{12}=\Gamma^0_{13}=\Gamma^0_{23}=0\\ \Gamma^0_{01}=\dfrac{1}{2}\dfrac{\mathrm{d}\xi}{\mathrm{d}r}\quad\left(\mathrm{e}^{\xi}=1-\dfrac{2GM}{r\eta^2}\right)\end{cases} \tag{16.43}$$

Let μ=0, then $x^{\mu}=x^0=\eta t$. According to Eq. (16.42):

$$\frac{\mathrm{d}^2x^0}{\mathrm{d}\tau^2}=-2\Gamma^0_{01}(\eta)\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^1}{\mathrm{d}\tau} \tag{16.44}$$

By contrasting Eq. (16.42) and Eqs. (16.43-44), we have the second-order differential equation of t:

$$\frac{\mathrm{d}^2 t}{\mathrm{d}\tau^2}+\frac{\mathrm{d}\xi}{\mathrm{d}\tau}\frac{\mathrm{d}t}{\mathrm{d}\tau}=0 \tag{16.45}$$

By integrating Eq. (16.45) of t, we get:

$$\Gamma(\eta)=\frac{\mathrm{d}t}{\mathrm{d}\tau}\equiv\frac{C_t}{1-2GM/\eta^2 r} \tag{16.46}$$

where C_t is a constant, and particularly, $\Gamma(\eta)=\mathrm{d}t/\mathrm{d}\tau$ is exactly the GOR factor of spacetime transformation in the theory of GOR.

According to the concepts of GOR theory, the differential equation (16.45) of t describes the relationship between the observational time t and the intrinsic time τ. Equation (16.45) suggests that the observational time t of OA(η) is different from the intrinsic time or the proper time τ, i.e., the objectively real time.

16.4.4 Relation $\theta=\theta(\tau)$ and Kepler's Orbit Law

Based on the GOR metric $g_{\mu\nu}=g_{\mu\nu}(\eta,r,\theta)$ in Eq. (16.40) [68]:

$$\begin{cases}\Gamma_{00}^2=\Gamma_{11}^2=\Gamma_{22}^2=0\\ \Gamma_{01}^2=\Gamma_{02}^2=\Gamma_{03}^2=\Gamma_{13}^2=\Gamma_{23}^2=0\\ \Gamma_{12}^2=1/r\\ \Gamma_{33}^2=-\sin\theta\cos\theta\end{cases} \tag{16.47}$$

Let $\mu=2$, then $x^\mu=x^2=\theta$. According to Eq. (16.42):

$$\frac{\mathrm{d}^2 x^2}{\mathrm{d}\tau^2}=-\Gamma_{33}^2(\eta)\frac{\mathrm{d}x^3}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau}-2\Gamma_{12}^2(\eta)\frac{\mathrm{d}x^1}{\mathrm{d}\tau}\frac{\mathrm{d}x^2}{\mathrm{d}\tau} \tag{16.48}$$

By contrasting Eq. (16.42) and Eqs. (16.47-48), we have the second-order differential equation of θ:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\tau^2}-\sin\theta\cos\theta\left(\frac{\mathrm{d}\varphi}{\mathrm{d}\tau^2}\right)^2+\frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau}=0 \tag{16.49}$$

Without loss of generality, suppose that, at the initial time $t=0$, the observed planet m runs in the plane of X-Y ($\theta=\pi/2$), then:

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right|_0=0 \quad \cos\theta\big|_0=0 \quad \left.\frac{\mathrm{d}^2\theta}{\mathrm{d}\tau^2}\right|_0=0 \tag{16.50}$$

Equations (16.49) and (16.50) about θ mean that the orbital plane of the planet m is fixed and constant in Einstein's theoretical model (Eq. (16.42)) of the two-body system. This is consistent with Kepler's orbit law, as well as, with Newton's planetary model and Einstein's planetary model.

16.4.5 Relation $\varphi=\varphi(\tau)$ and Kepler's Area Law

Based on the GOR metric $g_{\mu\nu}=g_{\mu\nu}(\eta,r,\theta)$ in Eq. (16.40) [68]:

$$
\begin{cases}
\Gamma_{00}^3 = \Gamma_{11}^3 = \Gamma_{22}^3 = \Gamma_{33}^3 = 0 \\
\Gamma_{01}^3 = \Gamma_{02}^3 = \Gamma_{03}^3 = \Gamma_{12}^3 = 0 \\
\Gamma_{13}^3 = 1/r \\
\Gamma_{23}^3 = \cot\theta
\end{cases}
\tag{16.51}
$$

Let $\mu=3$, then $x^\mu=x^3=\varphi$. According to Eq. (16.42):

$$
\frac{\mathrm{d}^2 x^3}{\mathrm{d}\tau^2} = -2\Gamma_{13}^3(\eta)\frac{\mathrm{d}x^1}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} - 2\Gamma_{23}^3(\eta)\frac{\mathrm{d}x^2}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} \tag{16.52}
$$

By contrasting Eq. (16.42) and Eqs. (16.51-52), we have the second-order differential equation of φ:

$$
\frac{\mathrm{d}^2\varphi}{\mathrm{d}\tau^2} + \frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} + 2\cot\theta\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} = 0 \tag{16.53}
$$

Since the orbital plane of a planet is fixed and constant, without loss of generality, suppose that the planet m runs in the plane of X-Y ($\theta=\pi/2$), then the Eq. (16.53) of φ could be written as follows:

$$
\frac{\mathrm{d}^2\varphi}{\mathrm{d}\tau^2} + \frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} = 0 \tag{16.54}
$$

By integrating Eq. (16.54) of φ, we get:

$$
r^2\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \equiv h_K = const \tag{16.55}
$$

where the velocity moment $h_K=L/m$ of the planet m is a constant, and naturally, the angular momentum $L=mh_K$ of the planet m is also a constant.

Equation (16.55) is exactly the area law (Eq. (16.3)) of Kepler's three laws as shown in Fig. 16.1(c): the area swept out by the radius vector $\mathbf{r}$ ($r=|\mathbf{r}|$) of the planet m per unit time is the same or equal. Essentially, Eq. (16.55) is the conservation law of angular momentum: $L=rmv$ is constant. (It is worth noting that here the mass m of the planet is the gravitational mass, or the intrinsic mass of the planet m, rather than the relativistic mass.)

This is consistent with the conclusion of Newton's planetary model and Einstein's planetary model. In particular, it suggests that, like Newton's planetary model and Einstein's planetary model, the GOR planetary model can also deduce Kepler's area law.

16.4.6 Relation $r=r(\tau)$ and the Planetary Orbit

Based on the GOR metric $g_{\mu\nu}=g_{\mu\nu}(\eta,r,\theta)$ in Eq. (16.40) [68]:

$$\begin{cases} \Gamma_{0i}^1 = \Gamma_{12}^1 = \Gamma_{13}^1 = \Gamma_{23}^1 = 0 \\ \Gamma_{00}^1 = \frac{1}{2}\mathrm{e}^{\xi-\zeta}\frac{\mathrm{d}\xi}{\mathrm{d}r} \quad \left(\mathrm{e}^{\xi} = g_{00}, \mathrm{e}^{\zeta} = -g_{11}\right) \\ \Gamma_{11}^1 = \frac{1}{2}\frac{\mathrm{d}\zeta}{\mathrm{d}r} \quad (\zeta = -\xi) \\ \Gamma_{22}^1 = -r\mathrm{e}^{-\zeta} \\ \Gamma_{33}^1 = -r\sin^2\theta\mathrm{e}^{-\zeta} \end{cases} \tag{16.56}$$

Let $\mu=1$, then $x^{\mu}=x^1=r$. According to Eq. (16.42):

$$\frac{\mathrm{d}^2 x^1}{\mathrm{d}\tau^2} = -\Gamma_{00}^1\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\frac{\mathrm{d}x^0}{\mathrm{d}\tau} - \Gamma_{11}^1\frac{\mathrm{d}x^1}{\mathrm{d}\tau}\frac{\mathrm{d}x^1}{\mathrm{d}\tau} - \Gamma_{22}^1\frac{\mathrm{d}x^2}{\mathrm{d}\tau}\frac{\mathrm{d}x^2}{\mathrm{d}\tau} - \Gamma_{33}^1\frac{\mathrm{d}x^3}{\mathrm{d}\tau}\frac{\mathrm{d}x^3}{\mathrm{d}\tau} \tag{16.57}$$

By contrasting Eq. (16.42) and Eqs. (16.56-57), we have the second-order differential equation of r:

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\frac{1}{2}\mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r}\left(\eta^2\mathrm{e}^{\xi}\frac{\mathrm{d}t}{\mathrm{d}\tau}\frac{\mathrm{d}t}{\mathrm{d}\tau} - \mathrm{e}^{\zeta}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}r}{\mathrm{d}\tau}\right) + r\mathrm{e}^{\xi}\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + r\sin^2\theta\mathrm{e}^{\xi}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \tag{16.58}$$

According to the GOR line-element formula (16.41):

$$\eta^2 = \eta^2\mathrm{e}^{\xi}\frac{\mathrm{d}t}{\mathrm{d}\tau}\frac{\mathrm{d}t}{\mathrm{d}\tau} - \mathrm{e}^{\zeta}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}r}{\mathrm{d}\tau} - r^2\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} - r^2\sin^2\theta\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \tag{16.59}$$

Thus, Eq. (16.58) can be rewritten as

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\frac{\eta^2}{2}\mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r} + \left(r\mathrm{e}^{\xi} - \frac{r^2}{2}\mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r}\right)\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + \sin^2\theta\left(r\mathrm{e}^{\xi} - \frac{r^2}{2}\mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r}\right)\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \tag{16.60}$$

Since the orbital plane of the planet m is fixed and constant, without loss of generality, suppose that the planet m runs in the plane of X-Y ($\theta=\pi/2$), then:

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\frac{GM}{r^2} + \left(r - \frac{3GM}{\eta^2}\right)\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \quad \left(\mathrm{e}^{\xi} = 1 - \frac{2GM}{\eta^2 r}, \mathrm{e}^{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}r} = \frac{2GM}{\eta^2 r^2}\right) \tag{16.61}$$

Equation (16.61) is the GOR theoretical model of the celestial two-body system (M,m) based on the theory of GOR, i.e., the GOR motion equation of the planet m, which is the second-order nonlinear differential equation of the radius vector $\mathbf{r}$ ($r=|\mathbf{r}|$) and angle φ of of the planet m with respect to the standard time τ, where suppose the orbital plane of the planet m is: X-Y ($\theta=\pi/2$).

16.4.7 The Binet form of the GOR motion Equation of the Planet

According to Eq. (16.55): $r^2\mathrm{d}\varphi/\mathrm{d}\tau \equiv h_K$, Eq. (16.61) can be rewritten as

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\frac{GM}{r^2} + \frac{1}{r^3} h_K^2 - \frac{3GM}{\eta^2 r^4} h_K^2 \tag{16.62}$$

Actually, the GOR theoretical model (Eq. (16.62) of the two-body system (M,m) of celestial bodies is the contraction of the spatial equations $r=r(\tau)$, $\theta=\theta(\tau)$ and $\varphi=\varphi(\tau)$ in the GOR motion equation (16.42) of the planet m, in which $\theta=\pi/2$ and $\mathrm{d}\varphi/\mathrm{d}\tau \equiv h_K/r^2$.

However, Eq. (16.62) is a second-order nonlinear differential equation of the radius vector $\mathbf{r}$ ($r=|\mathbf{r}|$) with respect to the proper time τ, which lacks the information about the angle φ of the planet orbit, could not be independently employed as the theoretical model of the celestial body two-body system (M,m), and could not independently describe the motion of the planet m. Generally, including classical celestial mechanics and Einstein's theory of general relativity, the theoretical model of celestial two-body system tends to be expressed in the form of Binet equation.

By following the logic of Binet equation, let $u=1/r$, then $\mathrm{d}\varphi/\mathrm{d}\tau=h_K/r^2=h_K u^2$, and from Eq. (16.61), it follows that

$$\frac{\mathrm{d}r}{\mathrm{d}\tau} = -h_K \frac{\mathrm{d}u}{\mathrm{d}\varphi} \quad \text{and} \quad \frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -h_K^2 u^2 \frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} \tag{16.63}$$

By substituting Eq. (16.63) into Eq. (16.62), we get that

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = \frac{GM}{h_K^2}\left(1 + \frac{3h_K^2}{\eta^2} u^2\right) \quad \left(u = \frac{1}{r}\right) \tag{16.64}$$

where G is the gravitational constant, M is the mass of the star, $\mathbf{r}$ ($r=|\mathbf{r}|$) is the radius vector of the star M pointing to the planet m, $h_K=r^2\mathrm{d}\varphi/\mathrm{d}\tau$ is the velocity moment of the planet m running around the star M, η is the information-wave speed of the general observation agent OA(η), and $u=u(\varphi)$ can be regarded as the running trajectory of the planet m.

This is the Binet form of the GOR equation of planetary motion.

By contrasting with the Binet form of Einstein's equation (Eq. (16.33)) of planetary motion, we know that the Binet form of the GOR equation (Eq. (16.64)) of planetary motion also has one more term on the right: $3GM/\eta^2 r^2$.

This suggests that the GOR equation (16.64) of planetary motion:

(i) is a nonlinear differential equation;

(ii) is a non-standard conic curve, in which $3GM/\eta^2 r^2$ is the precession term of planetary orbit, so the orbit of the observed planet m is also a non-standard and non-closed ellipse, and the planetary orbit would precess slowly.

It is worth pointing out that the objectively physical world is unique. So, the objectively real orbit of a planet must be definite: either precessing or not.

However, the orbital precession term $3GM/\eta^2 r^2$ in Eq. (16.64) completely depends on the observation agent OA(η) and the information-wave speed η of

OA(η): under different observation agents, the same planet presents different degrees of orbital precession. This fact suggests that the orbital precession term $3GM/\eta^2r^2$ in Eq. (16.64) does not represent the objective and real precession of the planet's perihelion, but the observational effect of OA(η) due to $\eta<\infty$.

It is thus clear that what is observed may not necessarily be objective and real, unless the observer O could employ the idealized observation agent OA_∞ ($\eta\to\infty$) to observe the objectively physical world.

16.5 The GOR Prediction for Planetary Precession

Newton's motion equation (16.5) of the planet m has no the orbital precession term. Einstein's motion equation (16.33) of the planet m contains the orbital precession term: $3GM/c^2r^2$; the GOR motion equation (16.64) contains the orbital precession term: $3GM/\eta^2r^2$. Einstein's planetary precession depends on the speed light c, while the GOR planetary precession depends on the information-wave speed η of the observation agent OA(η). Here OA(η) is the general observation agent. In theory, the observation medium of OA(η) could be any form of matter motion or any matter wave, and the speed η of transmitting observed information could be any speed of matter motion or matter wave.

Under the principle of GC, through PGC logic route 1 or through PGC logic route 2, by analogizing or following Einstein's logic for solving Einstein's motion equation (16.33) of the planet m (see Sec. 16.3.4), we can solve the differential equation (16.64) of the GOR motion equation of the planet m, and get that:

$$u=\left\{\frac{GM}{h_K^2}+\frac{3G^3M^3}{\eta^2h_K^4}\left(1+\frac{e^2}{2}\right)\right\}(1+e\cos\varphi)$$
$$+\frac{3G^3M^3e}{\eta^2h_K^4}\left(\varphi\sin\varphi-\frac{e}{6}\cos 2\varphi\right) \tag{16.65}$$

At the perihelion of the planet m, naturally, $du/d\varphi=0$. Therefore, by taking the derivative of u with respect to φ at both ends of Eq. (16.65), we have:

$$0=\frac{3G^2M^2}{\eta^2h_K^2}\left(\varphi\cos\varphi+\frac{e}{3}\sin 2\varphi-\frac{e^2}{2}\sin\varphi\right)-\sin\varphi \tag{16.66}$$

Regardless of $\varphi\cos\varphi$, then

$$\left\{\frac{3G^2M^2}{\eta^2h_K^2}\left(\frac{2e}{3}\cos\varphi-\frac{e^2}{2}\right)-1\right\}\sin\varphi=0 \tag{16.67}$$

So, $\sin\varphi=0$, or $\varphi=\arcsin\varphi=2k\pi\,(k=0,1,2,\ldots)$.

This means that, if without $\varphi\cos\varphi$, then, like Newton's motion equation of the planet, the GOR orbit of the planet would also be a closed ellipse and have no the orbital precession.

Consider $\varphi\cos\varphi$. Let $\Delta\varphi$ be the orbital precession angle of the planet m per revolution. Let k=1, that is, the planet m orbits the star M for one cycle, as depicted in Fig. 16.1(d): the planet m starts from the perihelion P and travels to the next perihelion P′. Then, the scanning angle of the planet m should be $\varphi=2\pi+\Delta\varphi$.

Substituting $\varphi=2\pi+\Delta\varphi$ into Eq. (16.66) and ignoring high-order small quantities, then it follows that:

$$\Delta\varphi(\eta)=\frac{6\pi G^2M^2}{\eta^2h_K^2}(\text{rad per revolution}) \tag{16.68}$$

where η is the information-wave speed of the general observation agent OA(η), in theory, can be any speed, not necessarily the speed light c.

It is possible for the GOR equation (16.68) in the theory of GOR to predict or calculate both Newton's planetary orbit precession and Einstein's planetary orbit precession. According to the GOR planetary-precession equation (16.68):

$$\begin{cases}\Delta\varphi_\infty=\lim\limits_{\eta\to\infty}\Delta\varphi(\eta)=\lim\limits_{\eta\to\infty}\dfrac{6\pi G^2M^2}{\eta^2h_K^2}=0\\ \Delta\varphi(c)=\lim\limits_{\eta\to c}\Delta\varphi(\eta)=\dfrac{6\pi G^2M^2}{c^2h_K^2}\ (\text{rad per revolution})\end{cases} \tag{16.69}$$

As shown in Eq. (16.69), under the idealized agent OA$_\infty$, as $\eta\to\infty$, $\Delta\varphi_\infty=0$: the planet has no the orbital precession, which is consistent with the conclusion of Newton's planetary mode; under the optical agent OA(c), as $\eta\to c$, the GOR planetary-precession equation (16.68) reduces to Einstein's planetary-precession equation (16.38), $\Delta\varphi(c)>0$: Mercury's orbital precession is 0.1029 arc second per revolution, or 42.77 arc second per century, which is consistent with the conclusion of Einstein's planetary model.

Observing the orbital precession term $3GM/\eta^2r^2$ of the GOR motion equation (16.64), we know that the precession of a planet's perihelion presented by the theoretical model of the GOR celestial two-body system depends on the observation agent OA(η) and the speed of observation medium transmitting the spacetime information about planetary motion: under the different observation agents, the same planet would exhibit different degrees of orbital precession. This fact suggests that the planetary orbital precession predicted by the idealized model of GOR celestial two-body system, for instance, that predicted by the idealized model of Einstein's celestial two-body system, is not the objective and real planetary precession. In essence, it is the observation effect of the observation agent OA(η): an apparent phenomenon caused by the observational locality ($\eta<\infty$) of the observation agent OA(η), which would disappear under the idealized agent OA$_\infty$.

The theory of Observational Relativity (OR, including IOR and GOR) has clarified that all relativistic effects, including the special (inertial) and the general (gravitational), are observational effects and apparent phenomena, the root and essence lie in the observational locality ($\eta<\infty$) of the observation agent OA(η). The t**ime dilation** in Einstein's special relativity is an optical observation effect, and the **planetary precession** in Einstein's general relativity is also an optical observation effect, rooted from the observational locality ($c<\infty$) of the optical agent OA(c). In terms of the observational effects caused by the observational locality ($\eta<\infty$) of the observation agent OA(η), the root and essence of the orbital precession in the planetary models of the idealized two-body system is the same as that of the time dilation as a relativistic effect in the theory of relativity. The observational or observed time would dilate; likewise, the observational or observed precession rate of planetary orbits would also dilate.

The theory of GOR does not doubt the existence of the actual orbital precession of planets. Actually, restricted by various non-idealized factors, the orbit of any celestial body could not be the idealized or standard conic curve, and even could not be fixed to a specific plane.

However, as stressed repeatedly in the theory of OR, the theoretical model of the idealized two-body system (M,m) of celestial bodies, no matter Newton's or Einstein's, or that of the theory of GOR, has no the prior information about the orbital precession of planets, for example, the precession of the equinoxes and the perturbation of other celestial bodies, as well as, the non-idealized factors of the sun. So, no matter based on Newton's theory of universal gravitation, or on Einstein's theory of general relativity, or on the theory of GOR, the theoretical model of the idealized two-body system (M,m) of celestial bodies could not predict or calculate the actual orbital precession of planets.

16.6 The Unity of Celestial-Motion Theories

The motion of celestial bodies in the universe has stimulated human infinite curiosity. The theory of celestial motion explores how celestial bodies move, what laws celestial bodies follow to move, and what forces drive celestial bodies to move.

The motion of celestial bodies in the universe is rooted from the gravitational interaction rather than from the so-called spacetime curvature: it is the universal gravitation that drives the celestial bodies in the universe to move.

As far as gravitational interaction is concerned, there are two great theoretical systems in physics: the first is Newton's theory of universal gravitation [81]; the second is Einstein's the theory of general relativity [8]. Therefore, the theory of celestial motion could also be divided into two major theoretical systems: Newton's theory of celestial motion and Einstein's theory of celestial motion. Newton's theory of celestial motion is naturally the product of classical mechanics and Newton's theory of universal gravitation; while Einstein's theory of celestial motion is naturally the product of Einstein's theory of general relativity. Beyond doubt, for humanity's physics, the unification of Newton's classical theory of celestial motion and Einstein's relativistic theory of celestial motion must be of great significance.

The theory of Observational Relativity (OR, including IOR and GOR) has generalized and unified the physical theories or models of all observational agents, including Newton's classical mechanics of the idealized agent OA_∞ and Einstein's relativity theory of the optical agent OA(c). As the theoretical system of OR gradually unfolds, the characteristics of IOR and GOR generalizing and unifying the physical theories or models of all observational agents will be more and more fully displayed. In the theory of IOR, the IOR spacetime transformation (i.e., so-called the general Lorentz transformation) has generalized and unified the Galilean transformation and the Lorentz transformation. In the theory of GOR, the GOR gravitational-field equation has generalized and unified Newton's field equation (i.e. the Poisson-equation form of Newton's law of universal gravitation) and Einstein's field equation; the GOR motion equation has generalized and unified Newton's motion equation (i.e., the Newton second-law form of Newton's law of universal gravitation) and Einstein's motion equation. Now, the GOR theory of celestial motion has generalized and unified Newton's classical theory of celestial motion and Einstein's relativistic theory of celestial motion.

The universe is the sum of all celestial bodies or everything, which is beyond doubt enormous and complex. So, both Newton's theory of universal gravitation and Einstein's theory of general relativity do their best to idealize and simplify the problem of celestial motion, and reduce it to the Many-body problem of celestial bodies, the three-body problem, the two-body problem, or even the one-body problem, as stated in Sec. 16.2 and Sec. 16.3 of this chapter.

According to the theory of GOR, this chapter builds up the theoretical model of the GOR two-body system (*M*,*m*) of celestial bodies (Eq. (16.64)), that is, the GOR motion equation of the observed planet *m*. The GOR theoretical model of the celestial two-body system represents the GOR theory of celestial motion, that is, a new theory of celestial motion, and provides new insights into astrophysics, including new interpretations for Newton's theory of celestial motion and Einstein's theory of celestial motion, as well as, the new understanding of astronomical phenomena such as planetary precession.

More importantly, The GOR theoretical model of the celestial two-body system (Eq. (16.64)) has generalized and unified Newton's theoretical model of the celestial two-body system (Eq. (16.5)) and Einstein's theoretical model of the celestial two-body system (Eq. (16.33)).

As shown in equation (16.64):

$$\mathrm{OA}(\eta):\quad \frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{\eta^2}u^2\right)$$

$$\begin{cases}\mathrm{OA}(c):\quad \lim\limits_{\eta\to c}\dfrac{GM}{h_K^2}\left(1+\dfrac{3h_K^2u^2}{\eta^2}\right)=\dfrac{GM}{h_K^2}\left(1+\dfrac{3h_K^2u^2}{c^2}\right)\\ \mathrm{OA}_\infty:\quad \lim\limits_{\eta\to\infty}\left\{\dfrac{GM}{h_K^2}\left(1+\dfrac{3h_K^2}{\eta^2}u^2\right)\right\}=\dfrac{GM}{h_K^2}\end{cases} \tag{16.70}$$

under the optical agent OA(c), as $\eta\to c$, the GOR motion equation (16.64) of the planet strictly reduces to Einstein's motion equation of the planet (16.33); under the idealized agent OA_∞, as $\eta\to\infty$, the GOR motion equation (16.64) of the planet strictly reduces to Newton's motion equation (16.5) of the planet.

As stated above, Newton's planetary model is derived from classical mechanics and Newton's laws, especially from Newton's law of universal gravitation; while Einstein's planetary model is derived from Einstein's theory of general relativity, especially based on Einstein's field equation and Einstein's motion equation. However, both Newton's planetary model and Einstein's planetary model have the strict corresponding relationship of isomorphic consistency with the GOR planetary model, and therefore, they have both been generalized and unified into the theoretical model of the GOR two-body system of celestial bodies. Thus, both Newton's planetary model from Newton's classical mechanics and Einstein's planetary model from Einstein's relativity theory are only special cases of the GOR planetary model, serving the respective observation agents: Einstein's theory of celestial motion is the product of the optical agent OA(c), under the optical agent OA(c), the astronomical observation must be more consistent with Einstein's theory of celestial motion; Newton's theory of celestial motion is the product of the idealized agent OA_∞, under the idealized agent OA_∞, the astronomical observation would be more consistent with Newton's theory of celestial motion, and particularly, more consistent with the objectively physical reality.

The theoretical model of the GOR two-body system of celestial bodies has generalized and unified Newton's theoretical model of celestial two-body system and Einstein's theoretical model of celestial two-body system. This means that: the GOR planetary model is logically consistent with both Newton's planetary model and Einstein's planetary model; and at the same time, confirms the theoretical validity of the GOR planetary mode and even the theory of GOR.

17 GOR and Gravitational Deflection

This chapter continues to examine and test the theory of GOR with Einstein's three major scientific predictions.

This time it the problem of the gravitational redshift of light.

The problem of the gravitational deflection of light, like the problem of the orbital precession of planets stated in Chapter 16, can be reduced to the two-body problem of celestial bodies (M,m): the sun M produces the gravitational field; the photon m moves in the gravitational field of the sun M.

Aimed at the problem of the gravitational deflection of light and Einstein's prediction of the gravitational deflection of light, this chapter will apply the GOR field equation and the GOR motion equation of to build the theoretical model of the two-body system (M,m), that is, the GOR motion equation of the observed photon m. Under the principle of general correspondence (GC), through both PGC logical route 1 and PGC logical routs2, by analogizing or following the logic of Einstein's theory of general relativity, the theory of GOR attempts to deduce the GOR motion equation of the photon (m) sweeping over the surface of the sun (M).

The GOR motion equation of the photon m sweeping over the surface of the star M will be analogized or contrasted with the Newton and Einstein's models of photons in gravitation field, so that we can test or verify the theory of GOR and the GOR motion equation of photons in gravitation field, reexamine the gravitational deflection of light and the corresponding gravitational-relativistic effect, and then, reveal why Newton's prediction gravitational deflection is different from Einstein's prediction of gravitational deflection.

17.1 On the Gravitational Deflection of Light

The gravitational deflection of light is one of the three famous predictions made by Einstein for testing and verifying his theory of general relativity, which means that, since gravitational spacetime is curved, the light must be curved or deflected in a gravitational field: the light beam sweeping past the sun must be curved.

Originally, the prediction of the gravitational deflection of light was proposed by Einstein based on the principle of equivalence before the formal establishment of his theory of general relativity.

The Principle of Equivalence: Inertial force and gravitational force, or inertial field and gravitational field, are locally equivalent, and have local indiscernibility for all physical observations and experiments.

As depicted in Fig. 17.1(a), a spacecraft is flying in space, and a beam of light is perpendicular to the longitudinal axis Y of the intrinsic-coordinate $O(X,Y)$ of the spacecraft and enters from the left window of the spacecraft.

The kinematical equation of the light beam or photons in $O(X,Y)$ is:

$$\begin{cases} x = X_0 + ct \\ y = Y_0 - v_0 t - \frac{1}{2} a t^2 \end{cases} \tag{17.1}$$

where $t=0$ when the light passes through the window aperture, (X_0,Y_0) is the coordinate of the window aperture, v_0 is the initial speed of the spacecraft, $a=|\mathbf{a}|$ is the acceleration of the spacecraft, and c is the speed of light in vacuum.

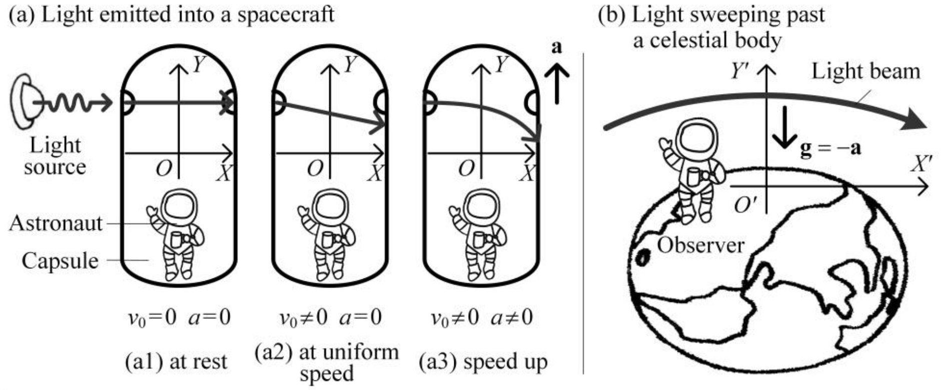

Figure 17.1 Equivalence Principle and Gravitational Deflection: According to Einstein's equivalence principle, the astronaut in the space capsule could not distinguish whether the spacecraft is in an accelerated state **a** or in a gravitational field **g** (=–**a**). (a) Light emitted into a spacecraft: The light or photons enters from the left window aperture into the spacecraft; in the astronaut's view, if the spacecraft at rest relative to the light source, then the light beam is horizontal and straight; if the spacecraft at a uniform speed, then the light beam is still straight, but slightly tilted; if the spacecraft accelerated, then the light beam appears somewhat curved. (b) Light sweeping past of a celestial body: According to Einstein's equivalence principle, if the astronaut is located on the surface of the celestial body (with the gravitational field **g** of it), then in his or her view, the light beam sweeping over the surface of the celestial body must be curved, just like the case of the spacecraft in an accelerating state.

According to Eq. (17.1), generally, in the intrinsic-coordinate $O(X,Y)$ of the spacecraft, that is, in the view of the astronaut in capsule, the trajectory the light or photons is a quadratic curve like a projectile flying on the earth's surfaces:

$$y = Ax^2 + Bx + C \quad \begin{pmatrix} A = -a/2c^2 \\ B = aX_0/c^2 - v_0/c \\ C = Y_0 + v_0 X_0/c - aX_0^2/2c^2 \end{pmatrix} \tag{17.2}$$

According to Eqs. (17.1-2), as depicted in Fig. 17.1(a), in the astronaut's view: (i) if the spacecraft at rest relative to the light source (Fig. 17.1(a1)), the light beam is a horizontal straight line; (ii) if the spacecraft at a uniform speed (Fig. 17.1(a2)), the light beam is still a straight line, but slightly tilted; (iii) if the spacecraft accelerated (Fig. 17.1(a3)), the light beam appears to be slightly curved.

According to Einstein's equivalence principle, inertial force is locally equivalent to gravitational force, and inertial field is locally equivalent to gravitational field. So, the astronaut could not determine whether the spacecraft is in an accelerated state (**a**) or in a gravitational field (**g**=−**a**).

Therefore, as depicted in Fig. 16.1(b), if the astronaut is located on the surface of a celestial body or in a gravitational field (**g**), then in the astronaut's view, a light beam that sweeps over the surface of the celestial body must be curved, just like the case of the spacecraft in an accelerating state (**a**). This is the gravitational deflection of light predicted by Einstein based on the principle of equivalence.

It should be pointed out that Einstein believed that the gravitational deflection of light was caused by the curvature of gravitational spacetime, and that the curvature of gravitational spacetime was caused by the accumulation of matter and energy.

As a matter of fact, Newton's law of universal gravitation can also predict the gravitational deflection of light: as particles of matter, photons are no different from projectiles. In a gravitational field, the motion trajectory of photons emitted from a light source, like projectiles launched from an artillery, must be curved due to the action of gravity or universal gravitation. Moreover, classical mechanics and Newton's theory of universal gravitation can also make the quantitative calculation for the gravitational deflection of light. Of course, according to Newton's theory of universal gravitation, the bending of light is not due to the so-called spacetime curvature, but the gravitational interaction between matter and matter.

Before the formal establishment of general relativity, Einstein's prediction of the gravitational deflection of light based on the principle of equivalence was only qualitative: a light beam would bend in a gravitational field. Einstein tried to quantitatively predict and calculate the gravitational deflection of light, but the calculation model he had to employ could only be the Kinematics model described by Eqs. (17.1-2). In particular, it is worth noting that the Kinematics equations (17.1-2) are the product of classical mechanics, that is, the product of the idealized observation agent OA_∞. Therefore, as expected, at that time, Einstein's calculation for the gravitational deflection angle of light was the same as that based on Newton's theory of universal gravitation.

After the formal establishment of general relativity, Einstein had gotton the field equation and the motion equation. Einstein employed his approximate solution (Eq. (15.3)) of the field equation to build up the motion equation of photons in gravitation field, and obtain the theoretical value of the gravitational deflection angle of light. What is particularly striking is that the predictive value of Einstein's theory of general relativity is twice that of Newton's theory of universal gravitation. In order to test his prediction of the gravitational deflection of light and verify his equivalence principle and general relativity, Einstein conceived the experiment to determine the bending angle of the starlight sweeping past the sun when total solar eclipses were occurring.

Total Solar Eclipses and the Starlight Sweeping past the Sun: As depicted in Fig. 17.2, the light or photons emitted by the star *S* located at the deep sky A sweeps past the sun and flies towards the earth; according to the principle of equivalence and Einstein's theory of general relativity, the spacetime around the sun is curved, so the flight path of the light or photons must be curved. In the view of the earth's observers, the star *S* is located in the direction of B, and there must be a deflection angle δ between the directions of A and B. Einstein proposed that such a phenomenon of the bending of light in gravitational fields could be observed by the earth's observers when total solar eclipses were occurring.

Observing the starlight sweeping over the surface of the sun through total solar eclipses is a good idea, otherwise the starlight would be submerged in the light of the sun and could not be observed.

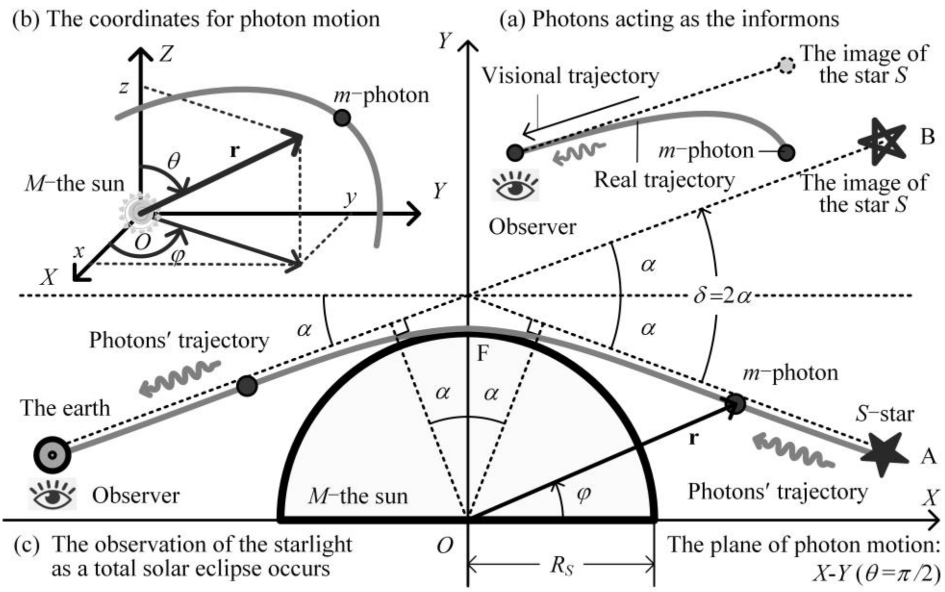

Figure 17.2 The Prediction and Test of Gravitational Deflection. (a) Photons acting as the informons: In optical observation, the spacetime information of photons is carried and transmitted by photons themselves. Before photons reach, the observer cannot observe them and cannot determine their trajectory; only when a photon arrives at the observation point, can the observer perceive or observe the photon and determines the visional direction of its image, that is, in theory the tangent direction of the photon trajectory at the observation point. (b) The coordinates for photon motion: In order to easily describe photon motion in the gravitational field of the sun, take the sun as the coordinate origin *O*, establish the spherical-coordinate $O(r,\theta,\varphi)$ and the corresponding Cartesian-coordinate $O(x,y,z)$. (c) The observation of the starlight as a total solar eclipse occurs: The starlight or the photon *m* comes from the distant star *S* located at the point A; suppose that the photon *m* moves in the plane *X-Y* ($\theta=\pi/2$), sweeps over the surface of the sun, and then, flies towards the earth. Einstein envisioned that: the spacetime around the sun was curved, and hence, the trajectory of the photon *m* would also be curved; thus, in the view of the earth's observers, the star *S* would be located at the point B rather than A, and there was a deflection angle $\delta=2\alpha$ between the directions of A and B. Einstein proposed to determine the gravitational deflection angle δ by means of total solar eclipses.

The Theoretical Value of the Bending angle of Starlight: The predictive value δ_N of the star deflection angle calculated with Newton's theory of universal gravitation is $\delta_N=0.875''$; while the predictive value δ_E of the star deflection angle calculated with Einstein's theory of general relativity is $\delta_E=1.75''=2\delta_N$, which is twice Newton's predictive value.

So, the physics community and even the whole world were eagerly anticipating the historical moment of testing and verifying the predictions of the gravitational deflection of starlight, looking forward to the confrontation between Einstein's theory of general relativity and Newton's theory of universal gravitation.

On May 29, 1919, this historical moment came [145]: on this day, the earth could observe a total solar eclipse. In order to verify Einstein's prediction about the gravitational deflection, a team led by British astronomer Eddington set out from Britain in Mar, 1919 and went to the island of Príncipe along the coast of West Africa to carry out the most important observation of total solar eclipses in human history. Taking advantage of the total solar eclipse, Eddington's team had observed the deflection angle of the starlight sweeping past the sun: $\delta = 1.61'' \pm 0.40''$. Almost at the same time, in Sobral, Brazil, a team led by another British astronomer, Dyson, observed the deflection angle of starlight: $\delta = 1.98'' \pm 0.16''$.

Eddington and Dyson's observations of the total solar eclipse tend to support Einstein's theory of general relativity [146].

After the announcement of Eddington and Dyson's observation conclusions, the scientific community generally accepted Einstein's theory of general relativity. The mainstream school of physics believed that the observation of the total solar eclipse meant that Einstein's gravitational theory defeated Newton's gravitational theory: Newton was wrong; Einstein is right! The Times of London published a full-page news (on Nov 7, 1919): **Revolution in Scientific, New Theory of the Universe, Newtonian Ideas Overthrown**.

It was the observation of the 1919 total solar eclipse that established the sacred position of Einstein's general relativity in physics, while Newton's theory of universal gravitation was regarded as the approximation of Einstein's general relativity: only valid under the cases of macrography, slow-speed, and weak-field.

Is Einstein's theory of general relativity really right?

Is Newton's theory of universal gravitation really wrong?

Or, is Newton's theory of universal gravitation really a transitional approximate theory of Einstein's theory of general relativity?

After the observation of the 1919 total solar eclipse, Eddington wrote a poem to express his insights:

Oh, leave the Wise our measures to collate
One thing at least is certain, light has weight
One thing is certain and the rest debate
Light rays, when near the Sun, do not go straight.

Eddington's insights in the poem is rational and discreet.

According to the observation of the 1919 total solar eclipse, Eddington affirmed that light would be curved in gravitational fields. However, it is worth noting that, in Eddington's view, the observation of the 1919 total solar eclipse suggests that: "Light has weight." Here, Eddington had conveyed two important messages to us:

(i) A photon has its own **weight** or mass (we prefer to believe that Eddington referred to the intrinsic mass of photons, that is, the rest mass of photons);

(ii) The bending of light is caused by photons' **weight**, in other words, by the universal gravitation, rather than by the so-call spacetime curvature.

Eddington's knowledge and understanding of the weight of light or photons is simple and plain, and consistent with the basic thoughts of the theory of OR. According to the theory of OR [26-28], photons, and even all matter particles, have their own **intrinsic mass** (Einstein called it **rest mass**), that is, the mass with the effect of gravity or universal gravitation. Eddington's understanding of the essence of light's bending conforms to Newton's theory of universal gravitation, and to the gravitational theory of OR, i.e., the theory of GOR.

Actually, the theory of OR (including IOR and GOR) has already clarified in previous chapters that the objectively real spacetime is flat, which could not be curved by any thing, nor by the distribution of matter or energy. Therefore, in terms of the essence of the gravitational deflection of light, Newton's theory of universal gravitation is right: the gravitational deflection of light is caused by the action of gravity or universal gravitation, not by spacetime curvature.

As far as the phenomenon of the gravitational deflection of light in optical observation is concerned, the prediction of Einstein's theory of general relativity seems to be closer to the actual observation of total solar eclipses. The development of astronomical observation technology, including **Radio Frequency Measurement** and **Astrometric Satellite Measurement**, has further improved the observational accuracy of the gravitational deflection of the light [130,147]. On Aug 21, 2017, a total solar eclipse occurred across the United States; Observers observed it in Wyoming and obtained the most accurate result of the gravitational deflection of light in history [148]: the deflection angle $\delta = 1.7512''$ of starlight, with the uncertainty of only 3.4%. This observation is extremely consistent with Einstein's prediction, which naturally represents the support for Einstein's the theory of general relativity.

However, according to the theory of OR, observation does not represent the objective reality: what is observed may not necessarily be objective or real.

As far as the current technical level is concerned, human astronomical observation, no matter the optical or the radio belongs to that of the optical observation agent OA(c), which employs light or electromagnetic interaction as the medium for transmitting astronomical information at the speed c of light. Einstein's theory of general relativity is the theory of the optical observation agent OA(c). Therefore, it is understandable and even inevitable that the observation conclusions of total solar eclipses are more consistent with Einstein's prediction, or with Einstein's theory of general relativity.

The theory of OR has clarified that the optical agent OA(c) has the observational locality ($c < \infty$) of its own, and its observations, including the observations of total solar eclipses, contain apparent phenomena, that is, the observational effects of OA(c), which are not completely objective and real. Therefore, the validity of Einstein's theory of general relativity is only the validity in the sense of optical observation, that is, the observational or phenomenal validity, which can be referred to as **the phenomenalistic validity**.

It is worth noting that the test of the gravitational deflection of light conceived by Einstein based on his theory of general relativity was specifically designed for the optical observation agent OA(c), where the photons of starlight are both the observed object and the informons of OA(c). As depicted in Fig. 17.2, the observer with OA(c) could not observe the motion trajectory of starlight, but could only feel or infer that the star S is located in the direction the sun as the photons of starlight reach the observation point (for example, human retina or an observation equipment). However, what the observers on the earth see or observe in this way is not the objective and real star S, but an image of it.

Newton's theory of universal gravitation is the theory of the idealized agent OA_∞. For the gravitational deflection of light, the observed object is still the starlight or the photons of it. However, the informons of OA_∞ that transmit the information of the starlight or photons are not the photons themselves, but the idealized informons of OA_∞ with infinitesimal momentum and infinite speed. Therefor, OA_∞ has no observational perturbation ($h_\eta \to 0$) and has no observational locality ($\eta \to \infty$).

The theory of GOR will clarify that if we could observe the gravitational deflection of light by means of the idealized observation agent OA_∞, then the starlight in total solar eclipses would be consistent with the gravitational deflection of light predicted by Newton's theory of universal gravitation, and would tend to support Newton's theory of universal gravitation rather than Einstein's theory of general relativity. So, Newton's theory of universal gravitation is really the right gravitational theory that is more in line with the objectively physical reality.

This chapter will calculate the deflection angle of the starlight sweeping over the solar surface based on the vacuum solution of the GOR field equation for the static spherically-symmetric gravitational spacetime in Chapter 15 and the theoretical model of the GOR two-body system (M,m) of celestial bodies, where M is the sun producing the gravitational field and m is the starlight or photons as the observed object P. We will find out that: different observation agents would observe different starlight deflection angles. Therefore, by analogizing Newton's theory of universal gravitation and Einstein's theory of general relativity, the theory of GOR will interpret the phenomenon and essence of the gravitational deflection of light.

17.2 Newton and Gravitational Deflection

Newton's theory of universal gravitation can also predict and calculate the gravitational deflection of light.

According to Newton's theory of universal gravitation, the reason of the gravitational deflection of light is quite simple: in a gravitational field, a flying photon is just like a projectile launched from an artillery; naturally, the flying trajectory of it must be curved due to the action of universal gravitation.

Originally, the test of the gravitational deflection of the starlight sweeping past the sun when a total solar eclipse occurs was conceived by Einstein for verifying his theory of general relativity, but it can also be employed to test or verify Newton's theory of universal gravitation and the theory of GOR.

17.2.1 Newton's Gravitational-Deflection Problem

As depicted in Fig. 17.2, in the test of total solar eclipses conceived by Einstein according to his theory of general relativity, the gravitational-deflection problem of the starlight sweeping past the sun could also be reduced to the two-body problem (*M*,*m*) of celestial bodies, where *m* could be a planet or comet or satellite or even photon, like (*star*,*planet*) or (*Sun*,*Mercury*): the two-body problem of (*Sun*,*photon*). Like the theoretical model of the two-body system of (*star*,*planet*), one could also build the theoretical model of the two-body system of (*Sun*,*photon*), and then, predict and calculate the gravitational deflection agent of the starlight sweeping over the solar surface.

In the two-body problem of (*Sun*,*Mercury*), the observed object *m* is Mercury; in the two-body problem of (*Sun*,*photon*), the observed object is starlight or photons. Like his the two-body system of (*star*,*planet*), Newton's the two-body system of (*Sun*,*photon*) is also extremely idealized and can be described as follows.

Newton's Two-Body System of (*Sun*,*photon*): (*M*,*m*), where *M* is the sun and *m* is the observed photon of the starlight sweeping past the sun, the sun *M* and the photon *m* interact through the universal gravitation between the sun *M* and the observed photon *m*, the sun *M* produces the gravitational field, and the photon *m* moves in the gravitational field.

The Idealized Conditions for Newton's Two-Body System of (*Sun*,*photon*): Newton's (*Sun*,*photon*) system implies the following idealized conditions.

(v) The action at a distance of gravitational interaction: gravitational radiation is action at a distance; the speed of gravitational radiation is infinite.

(vi) The isolated system: (the Sun *M*, the photon *m*) is an isolated system, not affected by the matter or energy outside (*M*,*m*); both the sun *M* and the photon *m* could be regarded as particles or mass points, the sun *M* is at rest and the photon *m* moves in the gravitational field of *M*.

(vii)The idealized observation agent: Newton's observation agent is the idealized observation agent OA_∞, the information-wave speed of OA_∞ is idealized to be infinity, the spacetime information of the observed photon *m* takes no time to cross space.

(viii) The idealized observer: the observer *O* employs the idealized observation agent OA_∞ to observe the photon *m* from the perspective of the earth; in theory, *O* is at rest on the earth and located at the position of zero potential.

The Formalized Coordinates of Newton's Two-Body System of (*Sun*, *photon*): as depicted in Fig. 17.2, Newton's two-body system of (*Sun*,*photon*) could be described in both the 3d Cartesian-coordinate $O(x,y,z)$ and the corresponding the 3d spherical-coordinate $O(r,\theta,\varphi)$ (Fig. 17.2(b); Eq. (15.6)), the sun *M* is located at the coordinate origin *O*, and the photon *m* moves in the plane of *X*-*Y* ($\theta=\pi/2$) (Fig. 17.2(c)) of the curved spacetime of the sun *M*..

Actually, the two-body problem of (*Sun*,*photon*) explored by both Newton and Einstein could be further idealized as and reduced to the one-body problem, only examining the motion of the photon *m*.

It should be pointed out that, in Newton's gravitational-deflection problem of light, the starlight or photons is the observed object, and the transmission of the observed information depends on the idealized informons (with infinitesimal momentum and infinite speed) of the idealized agent OA_∞.

17.2.2 Newton's Motion Equation of Photons

As stated earlier, the gravitational-deflection problem of the starlight sweeping past the sun could also be reduced to the two-body problem (*M*,*m*) of celestial bodies, like that of (*Sun*,*Mercury*): the two-body problem of (*Sun*,*photon*). Moreover, as depicted in Fig. 17.2, the two-body system of (*Sun*,*photon*) also employs the same coordinate system as that of the two-body problem of (*star*,*planet*) in Fig. 16.2. Therefore, by analogizing or following the logic route in Sec. 16.2 **Newton's Celestial-Body Model** of Chapter 16, based on classical mechanics and Newton's theory of universal gravitation, one could in the spherical-coordinate $O(r,\theta,\varphi)$ build the two-body model of (*Sun*,*photon*), i.e., **Newton's motion equation of the photon**, which has the same form as Newton's motion equation (16.5) of the planet:

$$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}\quad\left(u=\frac{1}{r}\right) \tag{17.3}$$

where, G is the gravitational constant, and M is the mass of the sun, $h_K=r^2\mathrm{d}\varphi/\mathrm{d}\tau=R_Sc$ is the velocity moment of the photon m, R_S is the solar radius, $\mathbf{r}$ ($r=|\mathbf{r}|$) is the radius vector of the sun M pointing to the photon m, and c is the speed of light.

On the Velocity Moment of the Photon h_K

In Eq. (17.3), the velocity moment h_K of the photon is $h_K=L/m$, where $L=mvr$ is the momentum moment of the photon m (i.e. angular momentum), and m slso represents the photon mass (in classical mechanics, m must be the Newtonian mass); v is the speed of photon m in the gravitational field of the sun M.

Suppose that the photon m sweeps over the surface of the sun (the point F in Fig. 17.2(c) is the perihelion of the photon m), then the flight speed v of the photon is $v=r\mathrm{d}\varphi/\mathrm{d}\tau\approx c$, and for the radius vector $\mathbf{r}$ ($r=|\mathbf{r}|$), $r\approx R_S$.

Therefore, the velocity moment h_K of the photon m in Eq. (17.3) is:

$$h_K=r^2\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}=rv\approx R_Sc \tag{17.4}$$

17.2.3 Newton's Flight Path of Photons

As stated in Sec. 16.2 of Chapter 16, bound to the star M, the differential equation (16.5) or Eq. (16.6) representing the orbit of the planet m must be a circle or an ellipse. However, the mass of the photon m is so small and the speed of the photon m is so high, that the sun M could not bind the photon m. So, the flight path of the photon m must not be a circle or an ellipse.

The solution of the motion equation (17.3) of the photon m must be the same in form as the solution of the motion equation (16.5) of the planet m. By analogizing or following the logic route in Sec. 16.2 **Newton's Celestial-Body Model** of Chapter 16, one could get the solution of Eq. (17.3):

$$u=\frac{1}{r}=\frac{GM}{R_S^2c^2}\left(1+e\cos\left(\varphi-\varphi_0\right)\right)\quad\left(e=C_S\,\frac{R_S^2c^2}{GM}\right) \tag{17.5}$$

where the integration constant C_S depends on the initial angular momentum L and mechanical energy E of the photon m; the integration constant φ_0 is the initial angle of the photon orbit, which can be set to a specific value by adjusting the zero point of the time or by rotating the plane X-Y around the Z-axis; e is the orbital eccentricity of the photon m.

Equation (17.5) suggests that Newton's motion equation of photon m is a standard conic curve. According to Eq. (17.5), if $\varphi=\pi/2$ (the photon m is sweeping past the perihelion F on the solar surface), then $u=1/r=1/R_S$ and $GM/R_S^2c^2<<1$.

Thus, one could get that:

$$C_S\cos\left(\frac{\pi}{2}-\varphi_0\right)=\frac{1}{R_S}\left(1-\frac{GM}{R_Sc^2}\right)\approx\frac{1}{R_S} \tag{17.6}$$

Let $\varphi_0=\pi/2$, then $C_S=1/R_S$.

Thus, according to Fig. 17.2(c), the starlight trajectory (Eq. (17.5)) is:

$$\begin{cases} u=\dfrac{GM}{R_S^2c^2}\left(1+e\cos\left(\varphi-\dfrac{\pi}{2}\right)\right) \\ e=\dfrac{R_Sc^2}{GM}\gg 1\quad\left(C_S=\dfrac{1}{R_S},\,\varphi_0=\dfrac{\pi}{2}\right)\end{cases} \tag{17.7}$$

where e is Newton's eccentricity of the photon orbit

As stated in Sec. 16.2 of Chapter 16, in the two-body system (M,m) of celestial bodies, the orbital eccentricity e of the celestial body m (which could be a planet or comet or satellite or even photon) depends on the gravitational constant G and the gravitational source M, as well as, on the initial angular momentum L of the celestial body m and the initial mechanical energy E of the celestial body m.

Therefore, the orbital eccentricity e of the photon m in Eq. (17.3) could also be calculated according to the classical celestial mechanics formula (16.7).

Both the mechanical energy E and the angular momentum L of the photon m could be regarded as conserved quantities. As shown in Eq. (17.4), considering that the photon m sweeps over the surface of the sun (the point F in Fig. 17.2(c) is the photon's perihelion), then with $v\approx c$ and $r\approx R_S$, one could get that:

$$E=\frac{1}{2}mc^2-\frac{GMm}{R_S}\quad\text{and}\quad L=R_Scm \tag{17.8}$$

Thus, the orbital eccentricity e of the photon m is:

$$e=\sqrt{1+\frac{2EL^2}{G^2M^2m^3}}=\sqrt{1+\left(c^2-\frac{2GM}{R_S}\right)\frac{R_S^2c^2}{G^2M^2}}\quad\left(c^2\gg\frac{2GM}{R_S}\right)$$
$$=\sqrt{1+\frac{R_S^2c^4}{G^2M^2}}=\frac{R_Sc^2}{GM}\quad\left(\frac{R_S^2c^4}{G^2M^2}\gg 1\right)\tag{17.9}$$

This result is the same as that in Eq. (17.7).

According to the recommendations of the International Standards Organization:

The speed of light: $c=2.9979245\times10^8\ \mathrm{m\cdot s^{-1}}$

The universal gravitational constant: $G=6.67430\times10^{-11}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}$

The mass of the sun: $M=1.98847\times10^{30}$ kg

The radius of the sun: $R_S=6.96\times10^8$ m

based on Eq. (17.7), one could calculate the orbital eccentricity *e* of the photon *m*:

$$e=\frac{R_Sc^2}{GM}=4.7133\times10^5\tag{17.10}$$

where the eccentricity of the photon *m* is *e* (>>1), and hence, the orbit of the starlight or photons sweeping over the solar surface is a hyperbola.

According to the solution (Eq. (17.7)) of Newton's motion equation (17.3) of the photon *m* and the eccentricity *e* in Eq (17.10), we know that, as far as the theoretical model of the idealized two-body system of (*Sun,photon*) based on Newton's theory of universal gravitation is concerned, the orbit of photons sweeping over the solar surface is a idealized and standard conic curve.

It is thus clear that, even according to Newton's theory of universal gravitation, light would also be curved in gravitational fields.

However, the gravitational deflection angle of starlight predicted by Newton's theory of universal gravitation is different from that predicted by Einstein's theory of general relativity.

17.2.4 Newton's Deflection Angle of Starlight

Now, based on the solution (Eq. (17.5)) of Newton's motion equation (17.3) of the photon *m* and the eccentricity equation (Eq. (17.7)) of photon orbit, one could calculate the gravitational deflection angle of the starlight or photons sweeping past the sun *M*, denoted as δ_N, and could be called **Newton's gravitational deflection angle of starlight**.

As depicted in Fig. 17.2(c), suppose that the photon *m* is emitted from the distant star *S*, sweeps past the sun *M*, and reaches the earth faraway from the sun *M*, then one could think that: $r\to\infty$, $u\to0$, and $\varphi\to\pi+\alpha$.

Thus, from Eq. (17.5) or Eq. (17.7), it follows that:

$$0=\frac{GM}{R_K^2c^2}\left(1+e\cos\left(\frac{\pi}{2}+\alpha\right)\right) \tag{17.11}$$

Obviously, α is a small quantity: $\sin\alpha\approx\alpha$. Therefore, based on Newton's theory of universal gravitation, Newton's gravitational deflection angle of starlight or photons sweeping past the sun is

$$\begin{aligned}\delta_N &= 2\alpha=\frac{2}{e}=\frac{2GM}{R_Sc^2}\,(\mathrm{rad})\\ &=4.2433\times10^{-6}\,(\mathrm{rad})=0.87525\,(\mathrm{arc\ sec})\end{aligned} \tag{17.12}$$

Newton's theory of universal gravitation predicts that: the bending angle δ_N of the starlight sweeping past the sun is 0.87525″. (This prediction represents the objectively gravitational deflection of light, which implies the hypothetical condition: Newton's observation agent is the idealized agent OA_∞.)

Indeed, the gravitational deflection angle of the starlight sweeping past the sun predicted by Newton's theory of universal gravitation is not quite consistent with the actual observation values of the optical observers on the earth: the prediction of 0.87525″ seems to be only half of the actual observation value of the 1.7512″ observed by means of the total solar eclipse in Wyoming [148].

It is worth noting that our observations of total solar eclipses, including the Eddington's and the Dyson's [145-147], as well as that in Wyoming [148], are all the astronomical observation of the optical agent OA(c). To test or verify Newton's gravitational deflection of light, we must take advantage of the idealized observation agent OA_∞, rather than the optical observation agent OA(c).

Later on, the theory of GOR will clarify that: it is not Newton's fault, but the observation's fault; it is not due to the deviation of Newton's theory of universal gravitation, but the deviation of the optical observation agent OA(c). Newton's prediction represents the objective and real gravitational deflection of light; while Einstein's prediction represents the gravitational deflection of light presented by the optical observation agent OA(c). The gravitational deflection of light presented by optical agent OA(c) contains the observational effects and apparent phenomena of OA(c), does not represent the objectively gravitational deflection.

Actually, both Newton's bending angle δ_N of starlight and Einstein's bending angle δ_E of starlight are the special cases of the GOR bending angle δ_{GOR}.

17.3 Einstein and Gravitational Deflection

Einstein's prediction of the gravitational deflection of light originated from his thought of spacetime curvature: gravitational spacetime is curved, so the flight path of light or photons in gravitational fields must also be curved.

Einstein's concept and ideology of spacetime curvature is extremely mysterious and abstruse.

Perhaps, you could understand the curvature of space, but anyway, you could not understand the curvature of spacetime: it is extremely difficult for you to imagine the bending time. Perhaps Einstein's original intention was not to prove the gravitational deflection of light by observation or experiment, but to prove that gravitational spacetime is curved by the gravitational deflection of light.

However, the gravitational deflection of light does not mean spacetime curvature, or the bending of space and time, for Newton's theory of universal gravitation could also interpret the gravitational deflection of light.

Before the formal establishment of his general relativity, in order to interpret his view of spacetime curvature, Einstein speculated the gravitational deflection of light based on the principle of equivalence in the way depicted in Fig. 17.1. After the formal establishment of his general relativity, Einstein proposed the test of the gravitational deflection of the starlight sweeping past the sun, and calculated the theoretical value of the bending angle of the starlight based on his field equation. Any case, Einstein's theory of general relativity gave us the ideas different from Newton's theory of universal gravitation.

In particular, Einstein's theoretical value of the starlight bending angle is different from Newton's theoretical value of the starlight bending angle.

Here, we review Einstein's prediction of the gravitational deflection of light and the theoretical calculation based on general relativity, and reexamine Einstein's problem of the gravitational deflection of light, which will be contributed to our recognition and understanding of the theory of GOR.

17.3.1 Einstein's Gravitational-Deflection Problem

As stated in Sec. 17.2.1, the gravitational-deflection problem of light could be reduced to the two-body problem (M,m) of celestial bodies, where m could be a planet or comet or satellite or even photon, like (*star*,*planet*) or (*Sun*,*Mercury*): the two-body problem of (*Sun*,*photon*). Therefore, based on Einstein's theory of general relativity, one could build the theoretical model of the two-body system of (*Sun*,*photon*), and then, predict and calculate the gravitational deflection agent of the starlight sweeping over the solar surface.

In Einstein's gravitational-deflection problem, like Einstein's two-body system of (*star*,*planet*) or (*Sun*,*Mercury*), the theoretical model of Einstein's two-body system of (*Sun*,*photon*) also contains the idealized conditions, which could be described as follows.

Einstein's Two-Body System of (*Sun*,*photon*): (M,m), where M is the sun and m is the observed photon of the starlight sweeping past the sun, the sun M and the photon m interact through the universal gravitation between the sun M and the observed photon m, the sun M produces the gravitational field, and the photon m moves in the gravitational field.

The Idealized Conditions for Einstein's Two-Body System of (*Sun*,*photon*): Einstein's (*Sun*,*photon*) system implies the following idealized conditions.

(i) The action at a distance of gravitational interaction: gravitational radiation is action at a distance; the speed of gravitational radiation is infinite.

(ii) The isolated system: (the Sun M, the photon m) is an isolated system, not affected by the matter or energy outside (M,m); both the sun M and the photon *m* could be regarded as particles or mass points, the sun *M* is at rest and the photon *m* moves in the gravitational field of *M*.

The above idealized conditions are the same as that of Newton's (*Sun*,*photon*) two-body problem. However, Einstein's (*Sun*,*photon*) two-body problem does not contain the conditions of the idealized observation agent and idealized observer, but instead of the condition of the optical observation agent and the condition of the optical observer.

The Optical Agent and the Conditions of Optical Observation: In Einstein's gravitational-deflection problem of the two-body system (the Sun *M*, the photon *m*), the observation agent is realistic and non-idealized, and there are the implied observation conditions as follows.

(iii) The optical observation agent: Einstein's observation agent in the test of the gravitational deflection of light is the optical observation agent OA(c), the information-wave speed of OA(c) is the speed c of light, it takes time for OA(c) to transmit the spacetime information of the observed starlight or the observed photon *m* to the observers on the earth.

(iv) The optical observer: the observer *O* employs the optical observation agent OA(c) to observe the observed starlight or the observed photon *m* from the perspective of the earth; in theory, *O* is at rest on the earth and located at the position of zero potential.

The Formalized Coordinates of Einstein's Two-Body System of (*Sun*, *photon*): Like Newton's two-body problem of (*Sun*,*photon*) stated in Sec. 17.2, as depicted in Fig. 17.2, Einstein's two-body system of (*Sun*,*photon*) could also be described in both the 3d Cartesian-coordinate $O(x,y,z)$ and the corresponding the 3d spherical-coordinate $O(r, \theta, \varphi)$ (Fig. 17.2(b); Eq. (15.6)), the sun *M* is located at the coordinate origin *O*, and the photon *m* moves in the plane of *X*-*Y* ($\theta=\pi/2$) (Fig. 17.2(c)) of the gravitational field of the sun *M*.

As stated in Sec. 17.2, the two-body problem of (*Sun*,*photon*) explored by both Newton and Einstein could be further idealized as and reduced to the one-body problem, only examining the motion of the photon *m*.

It should be pointed out that, in Einstein's gravitational-deflection problem of (*Sun*,*photon*), the photon *m* is not only the observed object, but also the informon of the optical agent OA(c): the spacetime information of the observed photon *m* is carried and transmitted by the photon itself.

In this way, the motion of photons does not have the problem of being perturbed by the informons of observation agents. However, observers could only observe the arrival position and direction of starlight and photons, but could not observe the flight path of starlight or photons.

17.3.2 Einstein's Motion Equation of Photons

After the formal establishment of his general relativity, Einstein applied the approximate solution (Eq. (15.3)) of his field equation and the line-element equation (15.4) to calculate the gravitational deflection angle of the starlight sweeping past the sun. After Schwarzschild obtained the exact solution (Eq. (15.7)) of Einstein field equation [80], one could build the motion equation of photons based on Einstein's theory of general relativity, and then, predict or calculate the gravitational deflection angle of the starlight [68].

Einstein's theory of general relativity could also reduce the problem of the gravitational deflection of light to the two-body problem (M,m) of celestial bodies, like that of (*Sun*,*Mercury*): the two-body problem of (*Sun*,*photon*). Moreover, as depicted in Fig. 17.2, the two-body system of (*Sun*,*photon*) also employs the same coordinate system as that of the two-body problem of (*star*,*planet*) in Fig. 16.2. By analogizing or following the logic route in Sec. 16.3 **Einstein's Celestial-Body Model** of Chapter 16, based on Einstein's theory of general relativity, one could in the spherical-coordinate $O(r,\theta,\varphi)$ build the two-body model of (*Sun*,*photon*), i.e., **Einstein's motion equation of photons**, which has the same form as Einstein's motion equation (16.33) of planets:

$$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{c^2}u^2\right)\quad\left(u=\frac{1}{r}\right) \tag{17.13}$$

where, G is the gravitational constant, and M is the mass of the sun, $h_K=r^2\mathrm{d}\varphi/\mathrm{d}\tau=R_Sc$ is the velocity moment of the photon m, R_S is the solar radius, $\mathbf{r}$ (r=|$\mathbf{r}$|) is the radius vector of the sun M pointing to the photon m, and c is the speed of light.

It is worth noting that, as the theory of the optical agent OA(c), in Einstein's theory of general relativity, the line-element ds=0 as the speed of the observed object P (such as the photon m in the two-body system (*Sun*,*photon*)) reaches the speed c of light. Actually, more generally, under the principle of general correspondence (GC), according to the line-element equation $\mathrm{d}s=g_{\mu\nu}(\eta)\mathrm{d}x^\mu\mathrm{d}x^\nu$ of the general observation agent OA(η) in the theory of GOR, the line-element $\mathrm{d}s\to0$ as the speed $v\to\eta$ of the observed object P.

In a strict sense, the case of $\mathrm{d}s\to0$ could only hold when m is both the observed object and the informon, for under this case, the speed v of the observed object m is strictly equal to the information-wave speed η of the observation agent OA(η).

In the definition of observation agent (Def. 10.1) in the theory of GOR, the information-wave speed η should be the real-time speed of the information wave or informons of the observation agent OA(η). For the optical observation agent OA(c), the c in the time axis $x^0=ct$ is the speed of light in vacuum. In Einstein's gravitational deflection experiment of observing the starlight passing over the solar surface, the speed of the starlight may not be necessarily the speed c of light in vacuum; however, the spacetime information of the starlight is carried and transmitted by the starlight photons: the photons are both the observed object and the messenger of starlight information. At this time, the speed of the photons as the observed object and the speed of the photons as the informons is naturally the same, regardless of whether the speed of the photon decays in the gravitational field. In this way, the spacetime line-element ds of the photon m must be zero: ds=0.

Therefore, in Eq. (17.13), in the case where the photon is both the observed object and the informon, the velocity moment h_K of the photon m is:

$$h_K = r^2 \frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \to \infty \quad \left(\mathrm{d}\tau = \frac{\mathrm{d}s}{c} \to 0\right) \tag{17.14}$$

Thus, Einstein's motion equation (17.13) of the photon m is reduced to:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = 3\frac{GM}{c^2}u^2 \quad \left(u = \frac{1}{r}\right) \tag{17.15}$$

Unlike Newton's motion equation (17.3) of photons, Einstein's motion equation (17.15) of photons is not a linear equation, but a nonlinear differential equation.

17.3.3 Einstein's Flight Path of Photons

Solving Einstein's motion equation (17.15) of photons, one could get the flight path of the photon m from the perspective of the optical observation agent OA(c). Einstein's motion equation (17.15) of photons is a nonlinear differential equation, which is difficult to solve. Fortunately, one could have the following approximate solution (see the reference [68]).

Let the solar mass M=0, then Eq. (17.15) could be reduced to

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = 0 \tag{17.16}$$

which have the solution

$$u = C_S \cos\left(\varphi - \varphi_0\right) \tag{17.17}$$

where both C_S and φ_0 are the integration constants.

According to Fig. 17.2(c), if $\varphi=\pi/2$ (the photon m is sweeping past the perihelion F on the solar surface), then $u=1/r=1/R_S$, where R_S is the solar radius. Substituting into Eq. (17.17), one could have that:

$$C_S \cos\left(\frac{\pi}{2} - \varphi_0\right) = \frac{1}{R_S} \tag{17.18}$$

By adjusting the zero point of the time, φ_0 could be set to $\varphi_0=\pi/2$, and then $C_S=1/R_S$. Substituting into Eq. (17.17), one could get the solution of Eq. (17.16), that is, the general solution u_g of Eq. (17.15):

$$u_g = \frac{1}{R_S}\cos\left(\varphi - \frac{\pi}{2}\right) = \frac{1}{R_S}\sin\varphi \tag{17.19}$$

Equation (17.19) is the situation where the photon m sweeps through the point F (the perihelion of photon m) on the solar surface in the direction parallel to the X-axis. It is thus clear that, in the spacetime without gravitational field (M=0), or in other world, in the inertial spacetime, the flight path of photons is a straight line. This is in line with our basic knowledge.

Obviously, the right end $3GMu^2/c^2 < 3GM/c^2R_S{}^2 \ll 1$ ($r=1/u>R_S$) of Eq. (17.15) is a small quantity. Therefore, one could employ the progressive approximation method to solve Einstein's motion equation (17.15) of photons [68]: substitute Eq.

(17.19) into Eq. (17.16) to get

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = 3\frac{GM}{R_S^2 c^2}\sin^2\varphi \tag{17.20}$$

Equation (17.20) has the following solution, that is, the special solution u_s of Einstein's motion equation (17.15) of photons:

$$u_s = \frac{GM}{R_s^2 c^2}\left(1+\cos^2\varphi\right) \tag{17.21}$$

Thus, the solution u of Einstein's motion equation (Eq. (17.15)) of photons should be the sum of the general solution u_g (Eq. (17.19)) and the special solution u_s (Eq. (17.21)):

$$u = \frac{1}{R_S}\sin\varphi + \frac{GM}{R_s^2 c^2}\left(1+\cos^2\varphi\right) \tag{17.22}$$

As depicted in Fig. 17.2(c), φ could be set to $\varphi=\pi+\phi$ ($\alpha\geq\phi\geq-\alpha$), where both α and $|\phi|$ are small quantities, and then $\cos^2(\pi+\phi)=1$. Substituting into the right end of Eq. (17.22), one could have that:

$$\begin{aligned} u &\approx \frac{1}{R_S}\sin\varphi + 2\frac{GM}{R_s^2 c^2} \\ &= \frac{2GM}{R_s^2 c^2}\left(1+\hat{e}\cos\left(\varphi-\frac{\pi}{2}\right)\right) \quad \left(e=\frac{R_S c^2}{GM},\ \hat{e}=\frac{e}{2}>1\right) \end{aligned} \tag{17.23}$$

where $e=R_S c^2/GM$ is the Newton's eccentricity of photon orbit; $\hat{e}=e/2=R_S c^2/GM$ is the Einstein's eccentricity of photon orbit.

Equation (17.23) is also a standard hyperbola. However, Eq. (17.22) is the approximate solution of Einstein's motion equation (17.15) of photons, and Eq. (17.23) is the approximate expression of equation (17.22). Therefore, Einstein's motion equation (17.15) of photons is actually not a standard hyperbola: in Einstein's theoretical model of the two-body system of (*Sun,photon*), the flight path of the photon *m* is an approximate hyperbola. According to Eq. (17.23), the orbital eccentricity $\hat{e}=R_S c^2/2GM$ of Einstein's motion equation of photons is half of the orbital eccentricity $e=R_S c^2/GM$ of Newton's motion equation of photons. This means that the flight path of the starlight in a gravitational field presented by the optical agent OA(c) is more curved than that by the idealized agent OA_∞. So, Einstein's gravitational deflection angle δ_E of light would naturally be larger than Newton's gravitational deflection angle δ_N of light.

17.3.4 Einstein's Deflection Angle of Starlight

Now, based on the solution (Eq. (17.22)) of Einstein's motion equation (17.15) of the photon *m*, one could calculate the gravitational deflection angle of the starlight or photons sweeping past the sun *M*, denoted as δ_E, and could be called **Einstein's gravitational deflection angle of starlight**.

As depicted in Fig. 17.2(c), suppose that the photon m is emitted from the distant star S, sweeps past the sun M, and reaches the earth faraway from the sun M, then we think that: $r\to\infty$, $u\to 0$, and $\varphi\to\pi+\alpha$.

Thus, from Eq. (17.22), it follows that:

$$0=\frac{1}{R_S}\sin(\pi+\alpha)+\frac{GM}{R_s^2c^2}\left(1+\cos^2(\pi+\alpha)\right) \tag{17.24}$$

Obviously, α is a small quantity: $\sin(\pi+\alpha)\approx-\alpha$ and $\cos(\pi+\alpha)\approx-1$. Therefore, based on Einstein's theory of general relativity, Einstein's gravitational deflection angle δ_E of starlight or photons sweeping past the sun is

$$\begin{aligned}\delta_E &= 2\alpha=\frac{4GM}{R_S c^2}\,(\text{rad})\\ &=8.4866\times10^{-6}\,(\text{rad})=1.7505\,(\text{arc sec})\end{aligned} \tag{17.25}$$

Einstein's theory of general relativity predicts that: in the view of the earth's observers, the bending angle δ_E of the starlight sweeping past the sun is 1.7505″. (This prediction implies the hypothetical condition: Einstein's observation agent is the optical agent OA(c).)

By contrasting Eq. (17.25) and Eq. (17.12), we know that Einstein's gravitational deflection angle of starlight is twice Newton's gravitational deflection angle: $\delta_E=2\delta_N$, which is in line with the actual observation value, and especially, in line with the starlight deflection angle 1.7512″ of the total solar eclipse in Wyoming, the United States [148].

As mentioned earlier, our observations of total solar eclipses, including the Eddington's and the Dyson's [145-147], as well as that in Wyoming [148], are all the astronomical observation of the optical agent OA(c). Naturally, as the optical observation theory, Einstein's gravitational deflection angle δ_E of starlight should be more in line with the observation conclusion of the optical agent OA(c).

It is worth noting that, like Newton's bending angle δ_N of starlight, Einstein's bending angle δ_E of starlight are also a special case of the GOR bending angle δ_{GOR}.

17.4 GOR and Gravitational Deflection

The theory of GOR can also predict the gravitational deflection of light.

The theory of GOR is the theory of the general observation agent OA(η). It could be expected that the prediction of the theory of GOR for the gravitational deflection of the starlight sweeping past the sun depends on the information-wave speed η of OA(η): different observation agents would have different bending angles of light, which is denoted by δ_{GOR} or $\delta_{\text{GOR}}(\eta)$. This means that, for the same gravitational scene, different observation agents would present different degrees of the gravitational deflection of light.

In particular, both Newton's deflection angle δ_N and Einstein's deflection angle δ_E will become special cases of the GOR deflection angle δ_{GOR}.

17.4.1 The GOR Gravitational-Deflection Problem

For the theory of GOR, the gravitational-deflection problem of light can also be reduced to the two-body problem (M,m) of celestial bodies, where m could be a planet or comet or satellite or even photon, like (*star*,*planet*) or (*Sun*,*Mercury*): the two-body problem of (*Sun*,*photon*). Therefore, based on the theory of GOR, we can build the theoretical model of the two-body system of (*Sun*,*photon*), and then, predict and calculate the gravitational deflection agent of the starlight sweeping over the solar surface.

In the GOR gravitational-deflection problem, like the GOR two-body system of (*star*,*planet*) or (*Sun*,*Mercury*), the theoretical model of the GOR two-body system of (*Sun*,*photon*) also contains the idealized conditions described as follows.

The GOR Two-Body System of (*Sun*,*photon*): (M,m), where M is the sun and m is the observed photon of the starlight sweeping past the sun, the sun M and the photon m interact through the universal gravitation between the sun M and the observed photon m, the sun M produces the gravitational field, and the photon m moves in the gravitational field.

The Idealized Conditions for the GOR Two-Body System of (*Sun*,*photon*): The GOR (*Sun*,*photon*) system implies the following idealized conditions.

(i) The action at a distance of gravitational interaction: gravitational radiation is action at a distance; the speed of gravitational radiation is infinite.

(ii) The isolated system: (the Sun M, the photon m) is an isolated system, not affected by the matter or energy outside (M,m); both the sun M and the photon m could be regarded as particles or mass points, the sun M is at rest and the photon m moves in the gravitational field of M.

The above idealized conditions are the same as that of the idealized conditions of Newton and Einstein's two-body systems of (*Sun*,*photon*). However, the GOR two-body problem of (*Sun*,*photon*) does not contain the condition of the idealized observation agent or the optical agent, but instead of the condition of the general observation agent and the condition of the general observer.

The General Agent and the Conditions of General Observation: In the GOR gravitational-deflection problem of the two-body system (the Sun M, the photon m), the observation agent is not the specific observation agent, and there are the implied observation conditions as follows.

(i) The general observation agent: The GOR observation agent in the test of the gravitational deflection of light is the general observation agent OA(η) ($\eta \geq v$), in theory, any form of matter motion could be employed as the observation medium, the information-wave speed η of OA(η) should be higher than or equal to the speed v ($\approx c$) of the observed photon m.

(ii) The general observer: the observer O employs the general observation agent OA(η) ($\eta \geq v$) to observe the observed starlight or the observed photon m from the perspective of the earth; in theory, O is at rest on the earth and located at the position of zero potential.

The Formalized Coordinates of the GOR Two-Body System of (*Sun*,

***photon*)**: Like Newton and Einstein's two-body problems of (*Sun*,*photon*) stated in Sec. 17.2 and Sec. 17.3, as depicted in Fig. 17.2, the GOR two-body system of (*Sun*,*photon*) could also be described in both the 3d Cartesian-coordinate $O(x,y,z)$ and the corresponding the 3d spherical-coordinate $O(r,\theta,\varphi)$ (Fig. 17.2(b); Eq. (15.6)), the sun M is located at the coordinate origin O, and the photon m moves in the plane of X-Y ($\theta=\pi/2$) (Fig. 17.2(c)) of the gravitational field of the sun M.

17.4.2 The GOR Motion Equation of Photons

The theory of GOR could also reduce the problem of the gravitational deflection of light to the two-body problem (M,m) of celestial bodies, like that of (*Sun*,*Mercury*): the two-body problem of (*Sun*,*photon*). Moreover, as depicted in Fig. 17.2, the two-body system of (*Sun*,*photon*) also employs the same coordinate system as that of the two-body problem of (*star*,*planet*) in Fig. 16.2.

Based on the theory of GOR, by analogizing or following the logic route in Sec. 16.4 **The GOR Celestial-Body Model** of Chapter 16, we can build the two-body model of (*Sun*,*photon*) in the spherical-coordinate $O(r,\theta,\varphi)$, i.e., **the GOR motion equation of photons**, which has the same form as the GOR motion equation (16.64) of planets:

$$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}+\frac{3GM}{\eta^2}u^2 \quad \left(u=\frac{1}{r}\right) \tag{17.26}$$

where, G is the gravitational constant, and M is the mass of the sun, $h_K=r^2\mathrm{d}\varphi/\mathrm{d}\tau=R_Sc$ is the velocity moment of the photon m sweeping past the solar surface, R_S is the solar radius, $\mathbf{r}$ ($r=|\mathbf{r}|$) is the radius vector of the sun M pointing to the photon m, and η is the information-wave speed of the general observation OA(η), $u=u(\varphi)=1/r(\varphi)$ could be regarded as the trajectory of the photon m.

The GOR motion equation (17.26) of photons generalizes Newton's motion equation (17.3) of photons, Einstein's motion equation (17.15) of photons, and even the motion equation (17.26) of any observation agent OA(η) ($\eta\in[c,+\infty)$).

Newton's Motion Equation of Photons: $\eta\to\infty$

Considering the perihelion of the photon m, i.e., the case that the observed photon m is sweeping past the point F on the solar surface as depicted in Fig. 17.2(c), then the speed of the photon m is $v=r\mathrm{d}\varphi/\mathrm{d}\tau\approx c$, and for the radius vector $\mathbf{r}$ ($r=|\mathbf{r}|$) of the sun M pointing to the photon m, $r\approx R_S$. Thus, as shown in Eq. (17.4), the velocity moment of photon m is $h_K=r^2\mathrm{d}\varphi/\mathrm{d}\tau=rv\approx R_Sc$.

As $\eta\to\infty$, the observation agent OA(η) would be the idealized observation agent OA_∞. Under this case, $u=u_\infty$ in Eq. (17.26) would be the flight path of the photon m presented by the idealized agent OA_∞, the right end of Eq. (17.26) would be:

$$\lim_{\eta\to\infty}\left\{\frac{GM}{h_K^2}+\frac{3GM}{\eta^2}u^2\right\}=\frac{GM}{h_K^2} \quad \left(h_K=R_Sc\right) \tag{17.27}$$

In this way, the GOR motion equation (17.26) of photons would be reduced to Newton's motion equation of photons:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = \frac{GM}{h_K^2} \quad \left(h_K = R_S c\right) \tag{17.28}$$

Einstein's Motion Equation of Photons: $\eta \to c$

As $\eta \to c$, the observation agent OA(η) would be the optical observation agent OA(c). Under this case, $u = u(c)$ in Eq. (17.26) would be the flight path of the photon m presented by the optical agent OA(c), in which the photon m is both the observed object and the informon of OA(c): the speed v of the photon m as the observed object and the speed η of the photon m as the informon of OA(η) are the same as the speed of light. Therefore, as stated in Sec. 17.3.2, under this case, the line-element of the photon m is zero: $\mathrm{d}s = 0$.

Thus, the right end of Eq. (17.26) would be:

$$\begin{aligned} &\lim_{\eta \to c} h_K = \lim_{\eta \to c} r^2 \frac{\mathrm{d}\varphi}{\mathrm{d}\tau} = \infty \quad \left(\mathrm{d}\tau = \frac{\mathrm{d}s}{c} \to 0\right) \\ &\lim_{\eta \to c} \left\{ \frac{GM}{h_K^2} + \frac{3GM}{\eta^2} u^2 \right\} = 3\frac{GM}{c^2} u^2 \end{aligned} \tag{17.29}$$

In this way, the GOR motion equation (17.26) of photons would be reduced to Einstein's motion equation of photons:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = 3\frac{GM}{c^2} u^2 \tag{17.30}$$

The Motion Equation of Photons under OA(η): $\infty > \eta > c$

According to the theory of OR, the information-wave η of an observation agent OA(η) have to be greater than or equal to the speed v of the observed object P. In Eq. (17.26), the observed object P is the photon m in the gravitational field of the sun M, with the speed $v \approx c$. Therefore, it is required that $\eta \geq c$; in a strict sense, it is required that either the photon m acts as the informon of OA(η) or the information-wave speed η to be greater or even far greater than the speed the photon m: $\eta >> c$.

Considering the general observation agent OA(η) ($\infty > \eta > c$), then $u = u(\eta)$ is the flight path of the photon m presented by the general observation agent OA(η). Substituting the velocity moment $h_K = R_S c$ of the photon m into Eq. (17.26), then the GOR motion equation (17.26) of the photon m is:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = \frac{GM}{R_S^2 c^2} + \frac{3GM}{\eta^2} u^2 \tag{17.31}$$

To sum up, whether Newton's motion equation (17.3) or (17,28) of photons, Einstein's motion equation (17.15) or (17,30) of photons, or the motion equation (17.31) of photons under the general observation agent OA(η) ($\infty > \eta > c$), is generalized and unified in the GOR motion equation (17.26) of the photon m.

It is worth noting that in the GOR motion equation (17.26) of the photon m, $u = u(\eta)$: the flight path of the photon m depends on the observation agent OA(η), or depends on the information-wave speed η of OA(η). Different observation agents present different flight paths of the photon *m* and different degrees of the gravitational deflection of light.

However, the objectively physical world must be unique. In the two-body system of (*Sun*,*photon*), the objective and real flight path of the photon *m* must also be unique. It is thus clear that, the flight path of the photon *m* observed by an observer with an observation agent OA(η) does not represent the objective reality of the gravitational deflection of light, unless the observer could take advantage of the idealized observation agent OA_∞.

17.4.3 The GOR Trajectory of Starlight

Obviously, the first term GM/h_K^2 at the right end of the GOR motion equation (17.26) of the photon *m* represents the objective and real physical information of the photon *m* as the observed object, involving the gravitational constant *G* and the solar mass *M*, as well as the velocity moment h_K of the photon *m*; the second term $3GMu/\eta^2$ depends on the information-wave η of the observation agent OA(η), representing the observational effect of the observation agent OA(η). This observational effect is caused by the observational locality ($\eta<\infty$) of the observation agent OA(η): the larger the information-wave speed η, the smaller the observational effect; if $\eta\to\infty$, then $3GMu/\eta^2\to0$, accordingly, the observational effect of OA(η) or apparent phenomenon would disappear.

Based on the GOR motion equation (17.26) of the photon *m*, it could be expected that, for different observation agents or different information-wave speeds, the flight path of the photon *m* would exhibit different degrees of the gravitational deflection of light. If the GOR motion equation (17.26) of the photon *m* could be solved, one could get the flight path of the photon *m* and calculate the gravitational deflection angle $\delta_{GOR}=\delta_{GOR}(\eta)$ of a specific agent OA(η).

However, the GOR motion equation (17.26) of the photon *m* is a nonlinear differential equation, which is difficult to solve, unless Eq. (17.26) is reduced to Newton's motion equation (17.3) or (17.28) of the photon *m*. In the GOR motion equation (17.26) of the photon *m*, the information-wave speed η of the observation agent OA(η) covers the $[c,+\infty)$. In particular, $\eta=c$ may be the discontinuous point of the gravitational-deflection model of Eq. (17.26).

Fortunately, on the $[c,+\infty)$, Eq. (17.26) has approximate solutions.

The Photon's Trajectory under the Idealized Agent OA_∞: $\eta\to\infty$

As $\eta\to\infty$, the observation agent OA(η) would become the idealized observation agent OA_∞, the GOR motion equation (17.26) of the photon *m* reduces to Newton's motion equation (17.3) or (17.28) of the photon *m*. Thus, we can get the exact solution; as shown in Eq. (17.5) in Sec. 17.2.3, the flight path of the photon *m* is an idealized or standard hyperbola.

Naturally, as $\eta\to\infty$, the gravitational deflection angle $\delta_{GOR}=\delta_{GOR}(\infty)$ of the starlight based on the GOR motion equation (17.26) of the photon *m* would be exactly Newton's deflection angle: $\delta_N=2GM/R_Sc^2$.

According to the theory of GOR, the idealized observation agent OA_∞ has no observational locality, and therefore, it presents observers with the objective and real scene of photon motion: $\delta_N=\delta_{GOR}(\infty)=2GM/R_Sc^2$ is the objectively gravitational deflection angle of the starlight sweeping the solar surface.

The Photon's Trajectory under the Optical Agent OA(*c*): $\eta\to c$

As $\eta\to c$, the observation agent OA(η) would naturally become the optical observation agent OA(c), the GOR motion equation (17.26) of the photon *m* reduces to Einstein's motion equation (17.15) or (17.30) of the photon *m*. Thus, we can get the approximate solution as shown in Eqs. (17.22-23) in Sec. 17.3.3: the flight path of the photon *m* is an approximate hyperbola.

Naturally, as $\eta\to c$, the gravitational deflection angle $\delta_{GOR}=\delta_{GOR}(c)$ of the starlight based on the GOR motion equation (17.26) of the photon *m* is exactly Einstein's deflection angle: $\delta_E=\delta_{GOR}(c)=4GM/R_Sc^2$.

It should be pointed out that the optical observation agent OA(η) has the observational locality ($c<\infty$), what it presents to observers does not represent the objective and real scene of photon motion.

Although it is quite consistent with the observational conclusion of the optical agent OA(η), the gravitational deflection angle $\delta_E=4GM/R_Sc^2$ calculated by Einstein's theory of general relativity does not represent the objectively bending angle of the starlight sweeping over the solar surface.

The Photon's Trajectory under the General Agent OA(η): $\infty>\eta>c$

As $\infty>\eta>c$, the GOR motion equation (17.26) of the photon *m* could be rewritten as Eq. (17.31), in which neither the first term $GM/h_K^2=GM/R_S^2c^2$ nor the second term $3GMu/\eta^2$ of the right end is zero. Therefore, Eq. (17.31) is quite difficult to solve. The existence of the second term $3GMu/\eta^2$ means that the photon motion scene presented by the observation agent OA(η) ($\infty>\eta>c$) to observers depends on the information-wave speed η of OA(η), and contains the observational effects of OA(η) or apparent phenomena.

For the specific observation agent OA(η) ($\infty>\eta>c$), in order to obtain the approximate solution of the GOR motion equation (17.26) of the photon *m*, it is necessary to suppose that the information-wave speed η is far greater than the speed c of light: $\eta>>c$. In the problem of the gravitational deflection of the starlight sweeping past the sun as depicted in Fig. 17.2(c), the velocity moment of the photon *m* is $h_K\approx R_Sc$ and $r=1/u\geq R_S$. If $\eta>>c$, then the second term $3GMu^2/\eta^2$ at the right end of Eq. (17.31) would be a small quantity relative to the first term $GM/R_S^2c^2$:

$$\frac{3GM}{\eta^2}u^2\bigg/\frac{GM}{R_S^2c^2}=3R_S^2u^2\frac{c^2}{\eta^2}\leq 3\frac{c^2}{\eta^2}\ll 1 \tag{17.32}$$

Thus, by following the logic route of the reference [68], we can employ the progressive approximation method to solve the GOR motion equation (17.31) of the photon *m* and get the approximate solution. Excluding the second term $3GMu^2/\eta^2$ at the right end, the GOR motion equation (17.31) of the photon *m* is reduced to Eq.

(17.28), that is, Newton's motion equation (17.3) of the photon *m*, and the solution is exactly Eq. (17.7) in Sec. 17.2.3:

$$u=\frac{GM}{R_S^2c^2}\left(1+e\cos\left(\varphi-\frac{\pi}{2}\right)\right)=\frac{GM}{R_S^2c^2}\left(1+e\sin\left(\varphi\right)\right)\quad\left(e=\frac{R_Sc^2}{GM}\right)\tag{17.33}$$

where $e=R_Sc^2/GM$ is Newton's orbital eccentricity of the starlight or the photon *m*.

Since $3GMu^2/\eta^2$ is a small quantity relative to $GM/R_S^2c^2$, one could further employ the progressive approximation method to substitute Eq. (17.33) into the right end of the GOR motion equation (17.31) of the photon *m*, and get that:

$$\begin{aligned}\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u&=\frac{GM}{R_S^2c^2}+\frac{3GM}{\eta^2}\left(\frac{GM}{R_S^2c^2}\right)^2\left(1+e\sin\varphi\right)^2\\&=\frac{GM}{R_S^2c^2}+3\frac{G^3M^3}{R_S^4c^4\eta^2}\left(1+2e\sin\varphi+e^2\sin^2\varphi\right)\end{aligned}\tag{17.34}$$

Equation (17.34) has the following solution:

$$\begin{aligned}u=&\left\{\frac{GM}{R_S^2c^2}+\frac{3G^3M^3}{R_S^4c^4\eta^2}\left(1+\frac{e^2}{2}\right)\right\}\left(1+e\sin\varphi\right)\\&+\frac{3G^3M^3}{R_S^4c^4\eta^2}e\left\{\left(\frac{\pi}{2}-\varphi\right)\cos\varphi+\frac{e}{6}\cos2\varphi\right\}\end{aligned}\tag{17.35}$$

As depicted in Fig.17.2(c), let $\varphi=\pi+\phi$ ($\alpha\geq\phi\geq-\alpha$), where both α and $|\phi|$ are small quantities, $\cos(\pi+\phi)\approx-1$ and $\cos2(\pi+\phi)\approx1$. Since $e>>1$ and $\eta>>c$, Eq. (17.35) could be approximated as:

$$\begin{aligned}u&\approx\left\{\frac{GM}{R_S^2c^2}+\frac{3GM}{2R_S^2\eta^2}\right\}\left(1+e\sin\varphi\right)+\frac{GM}{2R_S^2\eta^2}\\&=\frac{GM}{R_S^2}\left\{\frac{1}{c^2}+\frac{2}{\eta^2}\right\}\left(1+\hat{e}\sin\varphi\right)\quad\left(\hat{e}=\left(2\eta^2+3c^2\right)e/2\left(\eta^2+2c^2\right)\right)\end{aligned}\tag{17.36}$$

where $\hat{e}$ is the eccentricity of the photon orbit of the observation agent OA(η) ($\eta>>c$); in particular, $\hat{e}\to e$ as $\eta\to\infty$, that is, exactly the eccentricity of the photon orbit of the idealized observation agent OA_∞ (also known as Newton's eccentricity of the photon orbit).

Equation (17.36) is also a standard hyperbola. However, Eq. (17.35) is the approximate solution of the GOR motion equation (17.31) of photons, and Eq. (17.36) is the approximate expression of equation (17.35). Therefore, the GOR photon orbit of the observation agent OA(η) ($\eta>c$) is actually not a standard hyperbola: as shown in Eq. (17.36), in GOR theoretical model of the two-body system of (*Sun,photon*) under the observation agent OA(η) ($\eta>c$), the flight path of the photon *m* is only an approximate hyperbola.

By observing Eq. (17.36), we know that: as $\eta\to\infty$, the GOR orbital eccentricity $\hat{e}$ of the photon *m* under the observation agent OA(η) ($\eta>>c$) converges to

Newton's orbital eccentricity e of the photon m under the idealized agent OA_∞. Naturally, this means that the orbital eccentricity of GOR photon Equations of motion is logical consistent with the orbital eccentricity of Newton photon motion; or that, as $\eta\to\infty$, the GOR photon trajectory (Eq. (17.35)) converges to the idealized or standard hyperbola.

Here, for the problem of the gravitational deflection of light, the GOR motion equation (17.26) of the photon m has not only generalized Newton's motion equation (17.3) of the photon m but also Einstein's motion equation (17.15) of the photon m. The theory of GOR once again has unified Newton's theory and Einstein's theory. This once again reflects the logical consistency not only between the theory of GOR and Newton's theory of universal gravitation but also between the theory of GOR and Einstein's theory of general relativity, and moreover, reflects the logical self consistency and the theoretical validity of the theory of GOR.

17.4.4 The GOR Deflection Angle of Starlight

The GOR motion equation (17.26) of the photon m shows that: for different observation agents, the flight path of the photon m would exhibit different degrees of gravitational deflection or different bending angles.

Now, both Newton's motion equation (17.3) of the photon m and Einstein's motion equation (17.15) of the photon m have become the special cases of the GOR motion equation (17.26) of the photon m.

The Bending Angle under the Idealized Agent OA_∞: $\eta\to\infty$

By analogizing and following the logic route in Sec. 17.2 to solve the GOR motion equation (17.3) or (17.28) of the photon m, we can get the gravitational deflection angle of the starlight under the idealized agent OA_∞: $\delta_{\text{GOR}}=2GM/R_Sc^2$.

This is the Newton's gravitational deflection angle $\delta_N=\delta_{\text{GOR}}(\infty)$ of light.

The Bending Angle under the Optical Agent OA(c): $\eta\to c$

By analogizing and following the logic route in Sec. 17.3 to solve the GOR motion equation (17.15) or (17.30) of the photon m, we can get the gravitational deflection angle of the starlight under the optical agent OA(c): $\delta_{\text{GOR}}=4GM/R_Sc^2$.

This is the Einstein's gravitational deflection angle $\delta_E=\delta_{\text{GOR}}(c)$ of light.

The Bending Angle under the General Agent OA(c): $\infty>\eta>c$

In regard to the continuity and monotonicity of the solution of the GOR motion equation (17.31) of the photon m with respect to $\eta\in(c,\infty)$, the GOR gravitational deflection angle δ_{GOR} of the starlight sweeping past the sun should be:

$$\frac{4GM_S}{R_Sc^2}=\delta_E\geq\delta_{\text{GOR}}(\eta)\geq\delta_N=\frac{2GM_S}{R_Sc^2} \tag{17.37}$$

where $\delta_{\text{GOR}}=\delta_{\text{GOR}}(\eta)$ depends on the observation agent OA(η): different observation agents must present different degrees of the gravitational deflection of light.

As depicted in Fig. 17.2(c), the photon m starts from distant the star S at the point A, sweeps over the surface of the sun M and reaches the earth faraway from the sun M, then we think that: $r\to\infty$, $u\to 0$, and $\varphi\to\pi+\alpha$, where α is a small quantity. Thus, according to Eq. (17.35), we have

$$\begin{aligned}0=&\left\{\frac{GM}{R_S^2c^2}+\frac{3G^3M^3}{R_S^4c^4\eta^2}\left(1+\frac{e^2}{2}\right)\right\}\left(1-e\sin\alpha\right)\\&+\frac{3G^3M^3}{R_S^4c^4\eta^2}e\left\{-\left(\frac{\pi}{2}+\alpha\right)\sin\alpha+\frac{e}{6}\cos 2\alpha\right\}\end{aligned} \tag{17.38}$$

According to Eq. (17.10), Newton's orbital eccentricity e of the starlight sweeping past the sun is $e=R_Sc^2/GM>>1$.

Substituting Newton's eccentricity e into Eq. (17.38), we get

$$\alpha(\eta)=\frac{GM}{R_Sc^2}\left\{1+2\frac{c^2}{\eta^2}\right\}\Big/\left\{1+\frac{3c^2}{2\eta^2}\right\} \tag{17.39}$$

Thus, the GOR gravitational deflection angle of the starlight is:

$$\delta_{\text{GOR}}(\eta)=2\alpha(\eta)=\frac{2GM}{R_Sc^2}\left(1+\frac{c^2}{3c^2+2\eta^2}\right) \tag{17.40}$$

where the speed c of light represents the speed $v\approx c$ of the observed photon m and involves the velocity moment $h_K=R_Sc$ of the photon m as the observed object; the information-wave speed η reflects the observational locality ($\eta<\infty$) of the GOR observation agent OA(η).

The gravitational deflection angle of the starlight δ_{GOR} in Eq. (17.40) or α in Eq. (17.39) is only the approximate solution of the GOR motion equation (17.31) of the photon m, which requires that the information-wave speed η of OA(η) is large enough or far greater than the speed c of light: $\eta>>c$.

Nevertheless, Eq. (17.40) still provides us with the following insights:

(i) The first term $2GM/R_Sc^2$ in Eq. (17.40) is exactly Newton's gravitational deflection angle δ_N of the starlight, independent of the observation agent OA(η), representing the objective and real gravitational deflection of light.

(ii) The second term $2GM/R_S(3c^2+2\eta^2)$ in Eq. (17.40) depends on the observation agent OA(η), which means that different observation agents exhibit different degrees of the gravitational deflection of light, and hence, contains the observation effects of OA(η) and apparent phenomena.

(iii) The observation effects or apparent phenomena are rooted from the observational locality ($\eta<\infty$) of the observation agent OA(η): under the idealized agent OA_∞ ($\eta\to\infty$), the GOR gravitational deflection angle δ_{GOR} would converge to Newton's gravitational deflection angle δ_N: $\delta_{\text{GOR}}\to\delta_N$.

According to Eq. (17.40), in terms of an observation agent OA(η) ($\infty>\eta>c$):

$$\frac{\mathrm{d}\delta_{\text{GOR}}(\eta)}{\mathrm{d}\eta}=-\frac{8GM\eta}{R_S\left(2\eta^2+3c^2\right)^2}<0 \tag{17.41}$$

Equation (17.41) means that the larger the information-wave speed η of the observation agent OA(η), the smaller the GOR gravitational deflection angle $\delta_{\mathrm{GOR}}(\eta)$ of the starlight presented by OA(η) to an observer, which is closer to the objective and real Newton's gravitational deflection angle δ_N.

This conclusion is consistent with Eq. (17.37).

As expected earlier in this chapter, both Newton's gravitational deflection angle δ_N and Einstein's gravitational deflection angle δ_E are only special cases of the GOR gravitational deflection angle δ_{GOR} of the starlight sweeping past the sun.

17.5 The Gravitational Deflection of Light: Phenomenon and Essence

The test of the gravitational deflection of light provides a paradigm example for us to test and verify the theory of GOR. In this way, we could further recognize and understand the role of observation and observation agents in physics, and therefore, and we could interpret the phenomena and essence in physical observations and physical experiments.

Aimed at the problem of the gravitational deflection of light, the physics community have endlessly been struggling to determine whether Einstein's prediction or Newton's prediction is right.

Now, the theory of GOR tells us that: for the gravitational deflection of light, Einstein's prediction and Newton's prediction are based on different theoretical systems with different observation agents. What Einstein's theory of general relativity predicts is the phenomena of optical observation; while what Newton's theory of universal gravitation predicts is the essence of the physical world.

As far as the phenomenon is concerned, under optical observation, Einstein's prediction of gravitational deflection is right, supported by the optical observation agent OA(c), consistent with the phenomenon in optical observation. However, in essence, under ideal observation, Newton's prediction of gravitational deflection is right, supported by the idealized observation agent OA_∞, consistent with the gravitational deflection of light in the objectively physical world.

Different perspectives, different phenomena.

However, the essence remains the same.

17.5.1 Galilean-Newtonian Perspective

As stressed repeatedly by the theory of OR (including IOR and GOR), Galileo's doctrine and Newton's theory is the theory of ideal observation and the true portrayal of the objectively physical world.

The theory of OR, as stated in Sec. 17.2.1, has already clarified that Galileo's doctrine and Newton's theory implies the extremely idealized conditions of observation: taking advantage of the idealized observation agent OA_∞, in which the information-wave speed of OA_∞ is infinite, it takes no time to transmit the information of observed objects, and there is no observational locality and hence no relativistic effects; the momentum of the informons is infinitesimal, and there is no perturbation to observed objects and hence no quantum effects.

Only under such idealized observation conditions could the natural world present us with the objectively original outlook of it. Of course, the idealized condition agent OA_∞ can only be imagined but cannot be acquired.

The idealized agent OA_∞ can be called **Galilean-Newtonian Perspective**.

Once, I interpreted the theory of observational relativity (OR) for youngsters and talked about the concept of **the idealized observation agent**, a young girl suddenly said: "That's God's perspective!" She was absolutely right: the completely objective and real physical world could only be appreciated by God himself. Restricted by the observational locality of realistic observation agents, human beings will never be able to perceive or observe the completely objective and real physical world.

However, human reason could touch it.

It is because the information-wave speed of the idealized agent OA_∞ is infinite that Galileo's doctrine and Newton's theory have no relativistic effects; it is because the informon momentum of the idealized agent OA_∞ is infinitesimal that Galileo's doctrine and Newton's theory have no quantum effects. Based on God's perspective, from the perspective of the idealized observation agent OA_∞, Galileo and Newton could be able to touch the objectively physical world.

In the two-body problem of (the star M, the planet m) as stated in Chapter 16, the observed object P is the planet m. In general, the informon mass of observation agents is far smaller than the mass of the planet m and would not produce the significant perturbation to the planet. Therefore, we do not have to take into account the problem of the informon momentum of observation agents in the two-body problem of (*star,planet*). However, in the two-body problem of (the sun M, the photon m), the observed object P is the photon m with extremely small mass; the motion could be easily perturbed by the informons of observation agents, leading to quantum effects. Therefore, the two-body problem of (*Sun,photon*) has to involve the problem of the informon momentum of observation agents. Fortunately, the informons of the idealized observation agent OA_∞ has no momentum.

It is thus clear that, observing a photon flying in the sky with the idealized observation agent OA_∞ is just like observing a bird or an airplane flying in the air with the optical observation agent $OA(c)$. Aimed at the problem of the gravitational deflection of light, the motion trajectory of photons presented by the idealized agent OA_∞ to observers is the objectively gravitational deflection of light. In other words, the gravitational deflection of light predicted by Newton's theory of universal gravitation represents the objectively gravitational deflection of light.

So, Newton's gravitational deflection angle $\delta_N=2GM/R_Sc^2$ of the starlight sweeping over the solar surface calculated from Newton's motion equation (17.3) of photons is the gravitational deflection scene of the starlight under God's perspective, representing the objectively gravitational deflection.

17.5.2 Einstein's Perspective

As stressed repeatedly by the theory of OR (including IOR and GOR), Einstein's theory of relativity is the theory of optical observation, and the relativistic effects described by Einstein are observational effects and apparent phenomena of optical observation.

As stated in Sec. 17.3.1, Einstein's theory of general relativity implies the conditions of optical observation, belonging to the optical observation agent OA(c): the information wave of OA(c) is light wave, and the information-wave speed of OA(c) is the light speed c, it takes time to transmit the information of observed objects, there is the observational locality ($c<\infty$) of OA(c), presenting relativistic effects; the momentum of the informons of OA(c) is the momentum of photons, there is the perturbation to observed objects, presenting quantum effects.

The optical agent OA(c) can be called **Einstein's Perspective**.

Einstein did not really realize that his theory was only a partial theory, a theory of optical observation which is valid only under the optical observation agent OA(c); in particular, Einstein did not realize that there is the observational locality ($c<\infty$) in optical observation, and the observational locality ($c<\infty$) of OA(c) is the root and essence of relativistic effects.

As far as the problem of the gravitational deflection of light is concerned, Einstein observes the gravitational deflection of light under the optical agent OA(c), in which photons are both the observed object of OA(c) and the informons of OA(c): the carrying and transmission of the spacetime information of photons depends on photons themselves. Therefore, as depicted in Fig. 17.2(c), the optical agent OA(c) cannot observe the real motion trajectory of the photon m, and only as the photon m reaches our retina or observation devices can we perceive the existence and position of the photon m and take the illusory image (at the point B) of the star S as the real star S or the direction of the star S.

As clarified in Sec. 17.3, the observational locality ($c<\infty$) of the optical agent OA(c) leads to the observational effects and apparent phenomena of OA(c): Einstein's gravitational deflection angle $\delta_E=4GM/R_Sc^2$ of the starlight sweeping over the solar surface calculated from Einstein's motion equation (17.15) of photons is the gravitational deflection scene presented by the optical agent OA(c) to observers, and does not represent the objectively gravitational deflection. However, as far as the phenomenon is concerned, the gravitational deflection of light predicted by Einstein's theory of general relativity is quite consistent with the observational conclusion of the optical agent OA(c), and supported by the optical observation of total solar eclipses.

As a matter of fact, the observations of total solar eclipses, including the Eddington's and the Dyson's [145-147], as well as, that in Wyoming [148], the United States, are the optical observations made by the optical agent OA(c), and the gravitational deflection phenomena of light presented by OA(c) naturally supports Einstein's prediction for the gravitational deflection of light.

17.5.3 GOR Perspective

Human perception of the objective world must rely on certain observation media or observation agents. A realistic observation agent OA(η) must have the observational locality ($\eta<\infty$) of it, and what OA(η) presents to observers must only be an image of the objective world rather than the objective world itself. We would never be able to perceive or observe the completely real objective-world. Different observation agents present different observational images to observers.

However, the objectively physical world must be one-of-a-kind.

The theory of OR (including IOR and GOR) has already clarified that, in theory, all forms of matter motion could serve as observation media for transmitting the spacetime information of observed objects to observers. In the theory of GOR, the optical observation agent OA(c) no longer holds the special status: as stated in the principle of general correspondence (GC) (Chapter 11), **All observation agents are equal**; light is not the only observation medium that human beings could utilize.

So, we could observe the objective world from the broader perspective.

This is the so-called **GOR perspective**.

Aimed at the problem of the gravitational deflection of light, based on the GOR field equation and the GOR motion equation, the theory of GOR has built up the GOR theoretical model of the two-body system of (the sun M, the photon m), i.e., the GOR motion equation (17.26) of photons, which generalizes and unifies Newton's motion equation (17.3) of photons and Einstein's motion equation (17.15) of photons. The GOR motion equation (17.26) of photons is the theoretical mode of the general observation agent OA(η) ($\eta \in [c, +\infty)$), naturally depends on OA(η) and the information-wave speed η of OA(η), contains the observational effects of OA(η). Only as $\eta \to \infty$, the GOR motion equation (17.26) of photons reduces to Newton's motion equation (17.3) of photons, and is independent of the observation agent OA(η) and the information-wave speed η of OA(η). In this case, the GOR motion equation (17.26) of photons, i.e., Newton's motion equation (17.3) of photons, depicts the objectively gravitational deflection of light.

Naturally, the GOR deflection angle $\delta_{\text{GOR}}=\delta_{\text{GOR}}(\eta)$ of the starlight calculated from the GOR motion equation (17.26) of photons depends on OA(η) and the information-wave speed η of OA(η). As calculated and stated in Sec. 17.4: as $\eta \to c$, $\delta_{\text{GOR}}(\eta)=\delta_E$, that is, Einstein's deflection angle; the larger the information-wave speed η of OA(η), the closer OA(η) is to the idealized agent OA_∞, and the closer the GOR deflection agent $\delta_{\text{GOR}}(\eta)$ is to Newton's deflection angle δ_N; as $\eta \to \infty$, $\delta_{\text{GOR}}(\eta)=\delta_N$, i.e., Newton's deflection angle, that is, the objectively gravitational deflection angle $\delta_{\text{GOR}}(\infty)$ of the starlight. Sweeping past the sun M.

The theory of GOR has clarified that, although it is quite consistent with the phenomenon of optical observation, the prediction of Einstein's theory of general relativity does not represents the objectively physical reality; on the contrary, although it is not quite consistent with the phenomenon of optical observation, the prediction of Newton's theory of universal gravitation is exactly the objectively physical realistic.

Based on the theory of GOR, from the broad perspective of GOR theory, we have finally discovered that what we perceive or observe is only phenomenal, may not necessarily be essential or objective.

18 GOR and Gravitational Redshift

This chapter continues to examine and test the theory of GOR with Einstein's three major scientific predictions.

This time it the problem of the gravitational redshift of light.

The problem of the gravitational redshift of light, like the problem of the gravitational deflection of light stated in Chapter 17, can be reduced to the two-body problem of celestial bodies (the sun *M*, the photon *m*): the sun *M* produces the gravitational field; the photon *m* moves in the gravitational field.

Under the principle of general correspondence (GC), by analogizing or following the logic of Einstein's theory of general relativity, the theory of GOR attempts to reexamine the phenomenon of the gravitational redshift of light, and based on the invariance of time-frequency ratio, deduces the gravitational-redshift equation of light. The theory of GOR will redefine the concept of **gravitational redshift** based on the principle of conservation of energy, examining the gravitational redshift of photons and informons under different observation agents. In particular, the theory of GOR will reinterpret the theory of gravitational redshift that is based on the classical mechanics and Newton's theory of universal gravitation.

The GOR gravitational-redshift equation, like all theoretical models in the theory of OR (including IOR and GOR), has the high generalization, which will generalize and unify Newton's gravitational-redshift equation and Einstein's gravitational-redshift equation, and moreover, provide new insight into the theory of gravitational redshift.

18.1 On the Gravitational Redshift of Light

The prediction of the gravitational redshift of light, like the prediction of the gravitational deflection of light, was originally one of Einstein's three famous predictions to test and verify his general relativity and was proposed by Einstein based on the principle of equivalence before the formal establishment of Einstein's theory of general relativity.

The gravitational redshift of light is a kind of phenomenon that the frequency of light in a gravitational field would decay with the variation of gravitational potential. Of course, in gravitational fields, there are not only the phenomenon of gravitational redshift (frequency decay) but also the phenomenon of gravitational blueshift (frequency growth).

As depicted in Fig. 18.1(a1-2), a spacecraft is sailing in space, the light source at the rear of the spacecraft emits light or photons with a frequency of f towards the front of the spacecraft, and the frequency of the light or photons observed by the astronaut at the front of the spacecraft is f_O. It can be determined that: $f_O=f$ as the spacecraft flies at uniform speed (as shown in Fig. 18.1(a1): $a=|\mathbf{a}|=0$); $f_O<f$ as the spacecraft accelerates (as shown in Fig. 18.1(a2): $a=|\mathbf{a}|>0$).

According to Einstein's equivalence principle: inertial force is equivalent to gravitational force, and inertial field is equivalent to gravitational field; so, the astronauts in the spacecraft could not determine whether the spacecraft is in an accelerated state (**a**) or in a gravitational state (**g**=−**a**). Therefore, as depicted in Fig. 18.1(b1), suppose that there is a celestial body (a gravitational field: **g**=−**a**), then the observer would find that the frequency f of the light flying from the celestial body towards the inertial space must present decay or redshift ($f_O<f$).

This is namely the **gravitational redshift** of light.

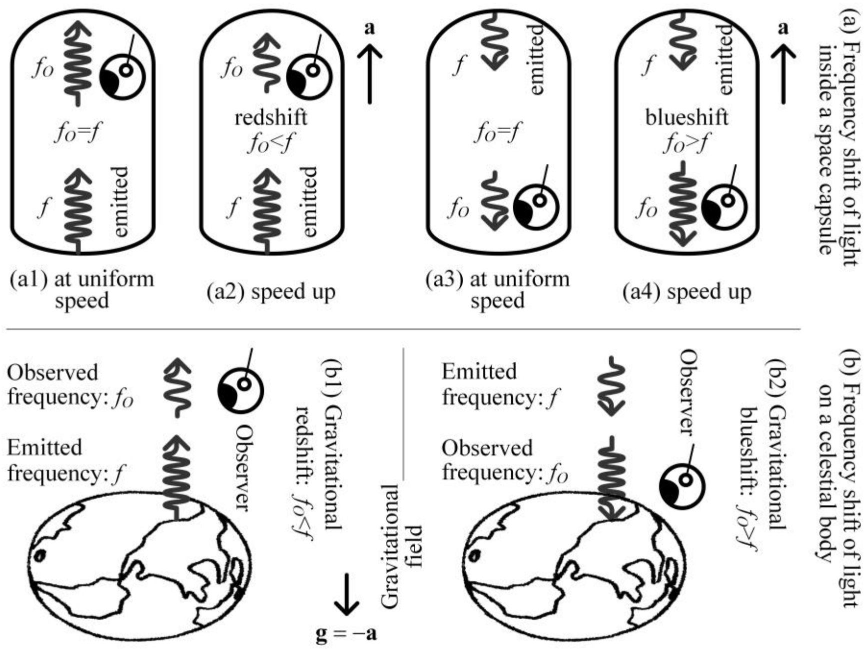

Figure 18.1 Equivalence Principle and Gravitational Redshift. (a) The frequency shift of light inside a spacecraft: As the spacecraft travels at uniform speed, the light source stationary in the spacecraft emits light with the frequency f which is constant relative to the astronauts inside the spacecraft ($f_O=f$ in (a1) and (a3)); as the spacecraft accelerates, the astronaut at the front finds that the frequency f of the light emitted by the light source at the rear presents decay or redshifts ($f_O<f$ in (a2)), while the astronaut at the rear finds that the frequency f of the light emitted by the light source at the front presents growth or blueshift ($f_O>f$ in (a4)). (b) Frequency shift of light on a celestial body: According to Einstein's equivalence principle, the astronauts inside the spacecraft could not distinguish whether the spacecraft is in an accelerated state or in a gravitational field, and therefore, for a massive celestial body (a gravitational field **g**=−**a** in (b)), the observer would find that the frequency f of the light flying from the celestial body towards the inertial space must present decay or redshift ($f_O<f$ in (b1)), while the observer would find that the frequency f of the light flying from the inertial space towards the celestial body must present growth or blueshift ($f_O>f$ in (b2)).

On the contrary, as depicted in Fig. 18.1(a3-4), the light source at the front of the spacecraft emits light or photons with a frequency of f towards the rear of the spacecraft, and the frequency of the light or photons observed by the astronaut at the rear of the spacecraft is f_O. It can be determined that: $f_O=f$ as the spacecraft flies at uniform speed (as shown in Fig. 18.1(a3): $a=|\mathbf{a}|=0$); $f_O>f$ as the spacecraft accelerates (as shown in Fig. 18.1(a4): $a=|\mathbf{a}|>0$).

According to Einstein's equivalence principle: as depicted in Fig. 18.1(b2), suppose that there is a celestial body (a gravitational field: $\mathbf{g}=-\mathbf{a}$), then the observer would find that the frequency f of the light flying from the inertial space towards the celestial body must present growth or blueshift ($f_O>f$).

This is namely the **gravitational blueshift** of light.

Different from the gravitational deflection of light, the gravitational redshift of light is usually interpreted based on Einstein's dilation doctrine of gravitational time, rather than based on Einstein's curvature doctrine of gravitational spacetime. It is based on Einstein's dilation doctrine of gravitational time that Einstein was able to predict and calculate the gravitational redshift of light. Actually, in Einstein's theory of general relativity, the gravitational redshift of light and the gravitational dilation of time are the same or equivalent physical effects [149-151].

The easiest way to test or verify the gravitational redshift of light is for the observers on the earth to observe the optical spectrum from the sun. The sun radiates light or photon to the earth: let f be the frequency of the sunlight on the solar surface and f_O the frequency observed by the observer on the earth. Whether based on Einstein's prediction of the gravitational redshift of light or Newton's prediction of the gravitational redshift of light, the optical spectrum of the sun would inevitably exhibit redshift as the sunlight from the sun to the earth: $f_O<f$.

Einstein's prediction of the gravitational redshift of light based on the principle of equivalence is qualitative.

Before the formal establishment of his theory of general relativity, Einstein could not quantitatively predict and calculate the gravitational redshift of light. After the formal establishment of his theory of general relativity, Einstein solved the field equation of general relativity to obtain the spacetime metric $g_{\mu\nu}$, and based on the spacetime line-element equation of photons $\mathrm{d}s=g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu}$ and the factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ of spacetime transformation in general relativity, derived the gravitational-redshift equation of the solar spectrum. In this way, Einstein calculated the theoretical value of the gravitational redshift of the solar spectrum:

$$Z_E=\frac{\Delta f}{f}=1-\frac{\sqrt{g_{00}\left(R_S+D_{SE}\right)}}{\sqrt{g_{00}\left(R_S\right)}}\quad\left(D_{SE}\gg R_S\right)$$
$$\approx-\frac{GM_S}{R_S c^2}\approx-2.12\times10^{-6}\quad\left(g_{00}=1-\frac{2GM_S}{rc^2}\right)\tag{18.1}$$

where G is the gravitational constant, M_S is the solar mass, R_S is the solar radius, D_{SE} is the distance between the sun and the earth, and $g_{00}(r)$ is the metric 00-element at the distance of r from the solar centroid; Z_E is Einstein's relative frequency-shift ($Z_E<0$ means redshift; $Z_E>0$ means blueshift), Δf is the absolute frequency-shift, and f is the reference frequency.

Adam (1959 [152]) provided the actual observed values of the redshift of the solar spectrum: $Z=-2\times10^{-6}$. Blamont and Roddier (1961 [153]) and Brault (1963 [154]) provided the actual observed value of the red shift of the solar spectrum: $Z=-2.12\times10^{-6}$, which conforms to the theoretical value predicted by Einstein within the accuracy range of 5%.

The actual observations of the solar spectrum seem to support Einstein's prediction of gravitational redshift. However, there are also views that [68]: the gravitational redshift of light in Eq. (18.1) is so small that the turbulence in the solar chromosphere and Stark effect may have the uncertain impact on the accuracy of the observation or measurement of gravitational redshift.

In addition to the sun, the spectra of other stars could also be employed to test Einstein's prediction of gravitational redshift; In particular, high-density stars could exhibit more significant gravitational redshifts.

In 1954, Popper observed the spectrum of 40 Eridani B and found that the redshift of its optical spectrum was $Z=-5.6\times10^{-5}$ [155]; the theoretical value predicted by Einstein's theory of general relativity was $Z_E=-7\times10^{-5}$. In 1971, Greenstein and et al observed the spectrum of Sirius B and found that the redshift of its optical spectrum was $Z=-(3.0\pm0.05)\times10^{-4}$ [156]; the theoretical value predicted by Einstein's theory of general relativity was $Z_E=-(2.8\pm1)\times10^{-4}$. These observations are generally consistent with Einstein's prediction of gravitational redshift of light. However, those stars are so far away from the earth that their masses and radii are difficult to accurately be determined, and hence, there are some doubts about the accuracy of these observations or measurements.

In 1958, Mössbauer discovered Mössbauer Effect [157]: the effect of recoil-free gamma-ray resonance absorption, which has the extremely high energy-resolution of up to 10^{-13}, and created the conditions for precisely measuring the gravitational redshift of light on the earth's surface. Thus, on the surface of the earth, one could measure the gravitational redshift of gamma ray with the height difference ΔH of only 20 meters.

Suppose that the generator of light signal on the earth's surface emits light or photons with frequency f from bottom to top (from top to bottom), the receiver of light signal at a distance of $\Delta H>0$ (<0) from the earth's surface measures the light signal with frequency f_O, then the frequency difference is that $\Delta f=f_O-f$.

According to Einstein's theory of general relativity:

$$Z_E=\frac{\Delta f}{f}=1-\frac{\sqrt{g_{00}\left(R_E+\Delta H\right)}}{\sqrt{g_{00}\left(R_E\right)}}\approx-\frac{g}{c^2}\Delta H \qquad (18.2)$$

$$\left(\Delta H\ll R_E,\ g=\frac{GM_E}{R_E^2}\right)$$

where Z_E is Einstein's relative frequency-shift ($Z_E<0$ means redshift; $Z_E>0$ means blueshift), M_E is the earth's mass, R_E is the earth's radius, ΔH is the height difference between the receiver and the generator, $g_{00}(R_E)$ is the metric 00-element at the distance of R_E from the earth's centroid, $g_{00}(R_E+\Delta H)$ is the metric 00-element at the distance of $R_E+\Delta H$ from the earth's centroid, and g is the gravitational acceleration of the earth's surface.

The harder light or photons could obtain stronger Mössbauer effect and more significant gravitational redshift, such as the gamma ray emitted by ^{57}Fe nuclei. In 1960, Pound and Rebka employed the gamma ray emitted by ^{57}Fe nuclei as the radiation source and set ΔH=22.5m to observed the gravitational redshift of the gamma ray. According to the theoretical value calculated from Eq. (18.2), $Z_E = -2.46\times10^{-15}$. Although Z_E is very small, it could be determined by Mössbauer effect. The observed value of Pound and Rebka was $Z = -(2.57\pm0.26)\times10^{-15}$: the ratio of the actual Z to the theoretical Z_E is Z/Z_E=1.05±0.10; the observed value Z is consistent with Einstein's theoretical value Z_E within the accuracy range of 10% [158]. In 1964, Pound and Snider repeated the 1960 experiment and improved the observation accuracy. This time the ratio of the actual Z to the theoretical Z_E is 0.990±0.00760: the observed value Z is quite consistent with Einstein's theoretical value Z_E within the accuracy range of 1% [159].

The observation and experiment of the gravitational redshift of light are generally consistent with the theoretical prediction and calculation of Einstein's theory of general relativity. However, unlike the case of the gravitational deflection of light, this time Einstein's prediction of gravitational redshift seems to have lost its challengers and competitors: Newton's theory of universal gravitation could also predict and calculate the gravitational redshift of light, and the theoretical value seems to be the same as Einstein's theoretical value, with no the observational distinguishability.

Newton's prediction of gravitational redshift is not based on Einstein's equivalence principle nor on Einstein's dilation doctrine of gravitational time.

Newton's prediction of gravitational redshift is based on the most concise principle of physics: the principle of conservation of energy. The frequency of light represents the kinetic energy of photons; the decay of kinetic energy of photons must lead to the decay or redshift of the frequency of light or photons. According to the principle of conservation of energy, based on classical mechanics and Newton's theory of universal gravitation, one could calculate the gravitational redshift of light, in which it is necessary to calculate the potential energy of photons in the gravitational field that depends on the classical mass of photons, especially on the gravitational mass of photons. According to Einstein's relativity theory, photons have no rest mass, which means that photons have no classical mass or gravitational mass. Without the gravitational mass of photons, it is impossible for Newton's classical mechanics to quantitatively predict or theoretically calculate the gravitational redshift of light.

The current Newtonian formula of the gravitational redshift of light takes Einstein's relativistic mass as the gravitational mass of Newton's classical mechanics to calculate the potential-energy difference ΔV of photons and deduce the pseudo Newtonian formula of the gravitational redshift of light:

$$Z_{PN} = \frac{\Delta f}{f} = \frac{h\Delta f}{hf} = \frac{\Delta E}{E} = -\frac{\Delta V}{mc^2} \quad \left(E = mc^2 = hf\,;\, \Delta V = V_O - V\right) \qquad (18.3)$$

where Z_{PN} is the frequency-shift (Z_{PN}<0 means redshift; Z_{PN}>0 means blueshift) in the pseudo Newtonian formula of the gravitational redshift of light, $E=mc^2$ could be regarded as the intrinsic kinetic energy (i.e., the energy of the photon *m* in vacuum), *m* represents the relativistic mass of a photon (both the relativistic inertial mass and the relativistic gravitational mass), ΔE is the increment of energy of the photon from the emitter to the observer, *V* is the potential energy of the photon as it is emitted, and V_O is the potential energy of the photon as it is observed.

Suppose that the gravitational mass *m* of the photon in Eq. (18.3) is constant, then one could calculate the difference ΔV of potential energy of the photon and the pseudo Newtonian gravitational redshift Z_{PN}.

For the observation of the gravitational redshift of the solar spectrum in Eq. (18.1), based on Newton's classical mechanics, you could employ Eq. (18.3) which mixes with relativity theory and quantum theory to calculate the absolute frequency-shift Δf and the relative frequency-shift $Z_{PN}=\Delta f/f$ of the solar spectrum:

$$Z_{PN}=\frac{\Delta f}{f}=-\frac{GM_S}{R_S c^2}\frac{D_{SE}}{R_S+D_{SE}}\approx-\frac{GM_S}{R_S c^2} \tag{18.4}$$

$$\begin{cases} f=\dfrac{E}{h}=\dfrac{mc^2}{h} \\ \Delta f=\dfrac{\Delta E}{h}=-\dfrac{\Delta V}{h}=-\dfrac{V_O-V}{h}=\dfrac{GM_S m}{h}\left(\dfrac{1}{R_S+D_{SE}}-\dfrac{1}{R_S}\right) \\ \quad =-\dfrac{GM_S m}{h}\dfrac{D_{SE}}{R_S\left(R_S+D_{SE}\right)}\approx-\dfrac{GM_S m}{R_S h} \quad \left(D_{SE}\gg R_S\right) \end{cases}$$

For the observation of the gravitational redshift of the gamma ray on the earth's surface in Eq. (18.2), based on Newton's classical mechanics, you could employ Eq. (18.3) which mixes with relativity theory and quantum theory to calculate the absolute frequency-shift Δf and the relative frequency-shift $Z_{PN}=\Delta f/f$ of the gamma ray emitted by ^{57}Fe nuclei:

$$\begin{aligned} &\Delta f=\frac{\Delta E}{h}=-\frac{\Delta V}{h}=-\frac{mg\Delta H}{h} \quad \left(\Delta V=mg\Delta H\right) \\ &Z_{PN}=\frac{\Delta f}{f}=\frac{\Delta E}{E}=-\frac{\Delta V}{mc^2}=-\frac{g}{c^2}\Delta H \quad \left(g=GM_E/R_E^2\right) \end{aligned} \tag{18.5}$$

By contrasting Eqs. (18.4-5) and Eqs. (18.1-2), we know that, as far as the observation of the gravitational redshifts of the solar spectrum and Mössbauer effect are concerned, Newton's prediction and Einstein's prediction are nearly identical: $Z_{PN}\approx Z_E$, and difficult to be distinguished in observation. Thus, people are no longer curious about who is right, Newton or Einstein, but why Einstein and Newton are so harmonious and consistent this time.

It should be pointed out that, as shown in Eq. (18.3), the current Newtonian formula of gravitational redshift not only relies on Einstein formula $E=mc^2$ involving relativity theory but also relies on Planck equation $E=hf$ involving quantum theory. Accordingly, the current Newtonian formula of gravitational redshift is not the product of pure classical mechanics and Newton's theory of universal gravitation, but the mixture of Newton's theory of universal gravitation and Einstein's theory of general relativity, and even involves quantum theory.

Later on, we will deduce Newton's gravitational-redshift equation purely based on classical mechanics and Newton's theory of universal gravitation, which will be generalized and unified into the theoretical system of GOR.

18.2 Einstein and Gravitational Redshift

Einstein's prediction of the gravitational redshift of light, like his prediction of the gravitational deflection of light, is based on his equivalence principle.

Before the formal establishment of his general relativity, Einstein could only employ his equivalence principle to qualitatively interpret the logical thought of the gravitational redshift of light in the way as depicted in Fig. 18.1, but could not quantitatively calculate of the gravitational redshift of light. After the formal establishment of his general relativity, Einstein derived the gravitational-redshift equation of light based on the approximate solution (Eq. (15.3) in Sec. 15.2.1 of Chapter 15) of Einstein field equation and calculated the theoretical value of the gravitational redshift of the solar spectrum, providing the basis for testing and verifying his general relativity and the gravitational redshift of light.

It is of important significance to explore the problem of the gravitational redshift of light and carry out the observation and experiment of the gravitational redshift of light: verifying Einstein's equivalence principle, testing Einstein's doctrine of the gravitational dilation of time, testing Einstein's doctrine of the gravitational redshift of light, and testing Einstein's theory of general relativity.

Here, we review Einstein's prediction of the gravitational redshift of light and the theoretical calculation based on general relativity, and reexamine Einstein's theory of the gravitational redshift of light, which will be contributed to our recognition and understanding of the theory of GOR.

18.2.1 Einstein's Gravitational-Redshift Equation

Einstein's deduction of the gravitational-redshift equation does not directly derive and calculate the frequency shift of light or photons moving in gravitational spacetime, but based on the equivalence between the gravitational redshift of light and the gravitational dilation of time, indirectly derive and calculate the gravitational redshift of light.

The Idealized Static Spherically-Symmetric Gravitational Spacetime

As depicted in Fig. 18.2, the celestial body *M*, as the gravitational source, produces an idealized static spherically-symmetric gravitational spacetime, which satisfies the idealized conditions of **static spacetime**:

$$\frac{\partial}{\partial t} g_{\mu\nu} = 0 \quad \text{and} \quad g_{i0} = 0 \quad (i = 1, 2, 3) \tag{18.6}$$

where the first term is the condition of **stationary spacetime**, the second term is the condition of **orthogonal spacetime**.

The gravitational spacetime that satisfies the two idealized conditions in Eq. (18.6) is referred to as static spacetime.

Einstein's approximate solution (Eq. (15.3) in Sec. 15.2.1 of Chapter 15) of Einstein field equation and Schwarzschild exact solution (Eq. (15.7) in Sec. 15.2.2 of Chapter 15) of Einstein field equation are both the static spherically-symmetric spacetime metrics, and can be employed to deduce the gravitational-redshift equation of light.

The Deduction of Einstein's Gravitational-Redshift Equation

According to the logic of Einstein's theory of general relativity (see Zhao's literature [160]), as depicted in Fig. 18.2, suppose that, in the gravitational spacetime with the celestial body M as the gravitational source and gravitational center O, the light source or optical clock T_P is stationary at the point A (at the distance of r_A from the gravitational center) and emits light or photos to the point B (at the distance of r_B from the gravitational center O): the optical clock T_P at the point A emits light or photons with the frequency of $f_A(t_1)$ at the coordinate time t_1, and the observer at the point B receives the light or photons with the frequency of $f_B(t_2)$ at the coordinate time t_2; then T_P at A emits light or photons with the frequency of $f_A(t_1')$ at the coordinate time t_1', and the observer at B receives the light or photons with the frequency of $f_B(t_2')$ at the coordinate time t_2'.

According to the idealized conditions in Eq. (18.6), the gravitational field of M is stationary spacetime and does not change with time. Suppose the frequency of the optical signals emitted by the light source or optical clock T_P at the coordinate times t_1 and t_1' is the same: $f_A(t_1)=f_A(t_1')=f_A$, then the observer at the point B should also receive the two optical signals with the same frequency at the coordinate times t_2 and t_2': $f_B(t_2)=f_B(t_2')=f_B$. Moreover, the time interval between the transmission of two optical signals from the point A to the point B should also be the same:

$$\begin{aligned} & t_2 - t_1 = t_2' - t_1' \\ \text{or}\quad & t_1' - t_1 = t_2' - t_2 \quad \left(t_1' - t_1 \equiv \mathrm{d}t_A,\, t_2' - t_2 \equiv \mathrm{d}t_B\right) \end{aligned} \tag{18.7}$$

Equation (18.7) suggests that the time difference between the two optical signals emitted by the light source or optical clock T_P at the point A is equal to the time difference between the two optical signals received by the observer at the point B: $\mathrm{d}t_A=\mathrm{d}t_B$.

According to the reference [160]: "t is the coordinate time, not the intrinsic time τ that the observer actually measured." The literature [160] claims that the intrinsic times experienced by the observers at the points A and B are $\mathrm{d}\tau_A$ and $\mathrm{d}\tau_B$, respectively, and should be different.

According to the factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ (Eq. (12.11) in Sec. 12.2.4 of Chapter 12) of spacetime transformation in Einstein's theory of general relativity:

$$\begin{aligned} & \mathrm{d}\tau_A = \sqrt{g_{00}(r_A)}\mathrm{d}t_A \quad \text{and} \quad \mathrm{d}\tau_B = \sqrt{g_{00}(r_B)}\mathrm{d}t_B \\ \text{so that}\quad & \frac{\mathrm{d}\tau_B}{\mathrm{d}\tau_A} = \frac{\sqrt{g_{00}(r_B)}}{\sqrt{g_{00}(r_A)}} \quad \left(\mathrm{d}t_B = \mathrm{d}t_A\right) \end{aligned} \tag{18.8}$$

where, as depicted in Fig. 18.2, $g_{00}(r_A)$ and $g_{00}(r_B)$ are respectively the metric 00-elements of the point A and the point B in the gravitational field of M.

Equation (18.8) suggests that $\mathrm{d}\tau_A \neq \mathrm{d}\tau_B$ or $\mathrm{d}\tau_A > \mathrm{d}\tau_B$.

This is the phenomenon of the time dilation of gravitational spacetime or the effect of potential-clock running slower.

As stated by the reference [160], Einstein ever suggested testing the effect of potential-clock running slower by means of the spectral frequency shift (redshift or blueshift). The intrinsic frequency of the atomic radiation spectrum reflects the intrinsic oscillation frequency of the atom: $f=\mathrm{d}N/\mathrm{d}\tau$, where N is the intrinsic number of atomic oscillations. In the gravitational spacetime of M, when the number of atomic oscillations observed by the observers at the points A and B is the same ($\mathrm{d}N_B=\mathrm{d}N_A$), it could be inferred from the formula $f=\mathrm{d}N/\mathrm{d}\tau$ that:

$$f_A \mathrm{d}\tau_A = f_B \mathrm{d}\tau_B \quad \text{or} \quad \frac{f_A}{f_B} = \frac{\mathrm{d}\tau_B}{\mathrm{d}\tau_A} \tag{18.9}$$

By substituting Eq. (18.9) into Eq. (18.8), Einstein had the frequency shift:

$$\begin{aligned} Z_E &= \frac{\Delta f}{f} = \frac{f_B - f_A}{f_B} = 1 - \frac{f_A}{f_B} = 1 - \frac{\mathrm{d}\tau_B}{\mathrm{d}\tau_A} \\ &= 1 - \frac{\sqrt{g_{00}(r_B)}}{\sqrt{g_{00}(r_A)}} \quad \left(g_{00} = 1 + \frac{2\chi}{c^2};\ \chi = -\frac{GM}{r} \right) \end{aligned} \tag{18.10}$$

where G is the gravitational constant, M is the mass of the celestial body, r_A is the distance of the point A (where the light source or optical clock T_P is at rest) from the gravitational center O of M, and r_B is the distance of the point B from the gravitational center O of M.

This is Einstein's gravitational-redshift equation.

The simplest and easiest way to test the gravitational redshift of light is to observe the gravitational redshift of the spectrum of the sun. The corresponding theoretical calculation can be based on Eq. (18.10): let $M=M_S$ be the solar mass, the point A be located on the solar surface ($r_A=R_S$), and $g_{00}(r_B)\to 1$ as $r_B\to\infty$.

By substituting Schwarzschild metric $g_{00}(R_S)=1+2GM_S/R_Sc^2$ into Eq. (18.10), one could have that:

$$Z_E = 1 - \frac{1}{\sqrt{g_{00}(R_S)}} \approx -\frac{GM_S}{R_S c^2} = -2.12\times10^{-6} \tag{18.11}$$

This is the theoretical gravitational redshift of the solar spectrum observed by the observers in the free spacetime S_F.

Let B be the earth. Since the distance between the sun and the earth is far greater than the solar radius: $r_B >> R_S$, the gravitational redshift of the solar spectrum observed by the observers on the earth should be approximate to Einstein's predictive value of $Z_E\approx -2.12\times10^{-6}$ in Eq. (18.11).

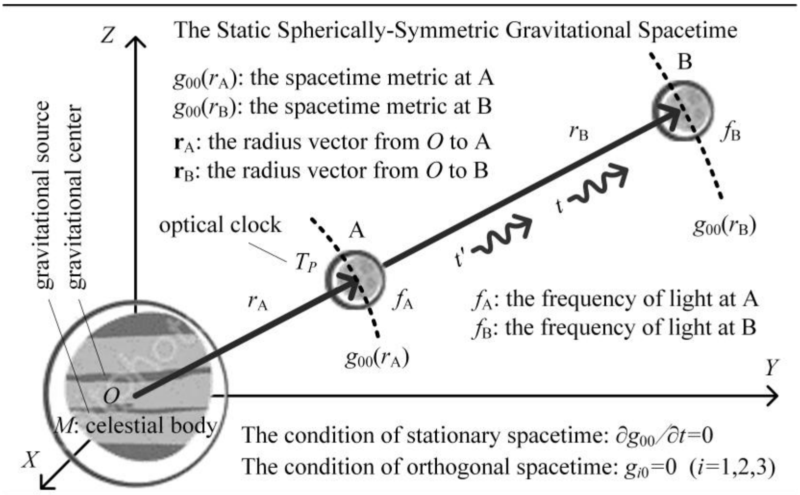

Figure 18.2 The Gravitational Redshift of Light in A Static Spherically-Symmetric Gravitational Spacetime: The celestial body *M* is the gravitational source and center of the spherically-symmetric gravitational spacetime. The light source or optical clock T_P at the point A stationary in the gravitational spacetime of *M* emits the light signal with the frequency f_A to the point B. The observer at the point B receives the light signal with the frequency f_B. According to Einstein's equivalence principle and Einstein's theory of general relativity: $f_B < f_A$, which means that the frequency of the light signal exhibits decay in gravitational spacetime, showing the phenomenon of gravitational shift.

18.2.2 The Invariance of Time-Frequency Ratio and the Gravitational Redshift of Light

There are some doubts about Einstein's gravitational-redshift equation (18.10) of light and about the deduction of Eq. (18.10), which is worth a discussion.

According to the theory of OR (see Sec. 12.3 in Chapter 12), the intrinsic time $\mathrm{d}\tau$ is the objective and real time, does not depend on observers, essentially but not mathematically, nor depends on observation agents. We are not able to understand why the intrinsic times $\mathrm{d}\tau_A$ and $\mathrm{d}\tau_B$ are different in Eq. (18.8), and moreover, we are able to understand why the atomic-oscillation numbers $\mathrm{d}N_A$ and $\mathrm{d}N_B$ of different observers are the same in Eq. (18.9).

Logically, the deduction of Einstein's gravitational-redshift equation (18.10) of light is not very clear or definite.

According to the theory of OR, $\mathrm{d}\tau_A=\mathrm{d}\tau_B=\mathrm{d}\tau$. Therefore, based on Einstein's factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ (Eq. (12.11) in Sec. 12.2 of Chapter 12) of spacetime transformation in general relativity, Eq. (18.8) needs to be transformed into:

$$\mathrm{d}\tau=\sqrt{g_{00}(r_A)}\mathrm{d}t_A \quad \text{and} \quad \mathrm{d}\tau=\sqrt{g_{00}(r_B)}\mathrm{d}t_B$$
$$\text{so that} \quad \frac{\mathrm{d}t_A}{\mathrm{d}t_B}=\frac{\sqrt{g_{00}(r_B)}}{\sqrt{g_{00}(r_A)}} \tag{18.12}$$

The theory of OR has an important theorem or even a principle (see Eq. (2.3) in

Sec. 2.3.2 of Chapter 2): **the invariance of time-frequency ratio**, which is the important relationship between the time t and the clock frequency f.

According to the invariance of time-frequency ratio, it follows that:

$$\frac{\mathrm{d}t_{\mathrm{A}}}{f_{\mathrm{A}}}=\frac{\mathrm{d}t_{\mathrm{B}}}{f_{\mathrm{B}}} \quad \text{or} \quad \frac{f_{\mathrm{A}}}{f_{\mathrm{B}}}=\frac{\mathrm{d}t_{\mathrm{A}}}{\mathrm{d}t_{\mathrm{B}}} \tag{18.13}$$

where both $\mathrm{d}t_{\mathrm{A}}$ and $\mathrm{d}t_{\mathrm{B}}$ are the observational times of the optical clock T_P, while f_{A} and f_{B} are the observational frequencies of the optical clock T_P.

In this way, you could also get Einstein's gravitational-redshift equation of light:

$$\begin{aligned} Z_E &= \frac{\Delta f}{f}=\frac{f_{\mathrm{B}}-f_{\mathrm{A}}}{f_{\mathrm{B}}}=1-\frac{f_{\mathrm{A}}}{f_{\mathrm{B}}}=1-\frac{\mathrm{d}t_{\mathrm{A}}}{\mathrm{d}t_{\mathrm{B}}} \\ &=1-\frac{\sqrt{g_{00}(r_{\mathrm{B}})}}{\sqrt{g_{00}(r_{\mathrm{A}})}} \quad \left(g_{00}=1+\frac{2\chi}{c^2};\, \chi=-\frac{GM}{r}\right) \end{aligned} \tag{18.14}$$

Equation (18.14) is the same as Einstein's equation (18.10).

However, Eq. (18.14) is based on the invariance of time frequency ratio from the theory of OR, with clear and concise logic. This result confirms the principle of the invariance of time frequency ratio. As a matter of fact, as stated in Sec. 2.3 of Chapter 2, the invariance of time-frequency ratio is the logical consequence of the time definition (Def. 2.2) of OR theory, that is, a theorem of the theory of OR.

18.2.3 The Gravitational Redshift of the Optical Observation Agent

According to the theory of OR, Einstein's theory of general relativity is the theory of optical observation, and the observation agent of it is the optical agent OA(c), where the observation medium or the information wave that transmits the observed information of observed objects to observers is light or electromagnetic interaction.

By examining Fig. 18.2, you might have some doubts.

Firstly, as far as Einstein's theory of general relativity is concerned, in the precession problem of Mercury's perihelion, the observed object is Mercury, and it is light that transmits the motion information of Mercury to observers; in the gravitational-deflection problem of light, the observed object is light, and it is the light itself that transmit the motion information of light to observers.

So, in the gravitational-redshift problem of light, what is the observed object? Is it the moving photon or the stationary light source (optical clock) T_P at the point A?

It is worth noting that, whether Eq. (18.10) or Eq. (18.14), the deduction of Einstein's gravitational-redshift equation needs to make use of the factor $\gamma=1/\sqrt{(g_{00})}$ of spacetime transformation. However, in general relativity, the general expression of Einstein's factor of spacetime transformation is $\gamma=1/\sqrt{((\sqrt{(g_{00})}-\gamma_i v^i/c)^2-v^2/c^2)}$ (Eq. (12.16) in Sec. 12.2.4 of Chapter 12). Under the condition of orthogonal spacetime in Eq. (18.6), $\gamma_i=0$, and therefore, $\gamma=1/\sqrt{(g_{00}-v^2/c^2)}$ (Eq. (12.12) in Sec. 12.2.4 of

Chapter 12). Thus, whether Eq. (12.12) or Eq. (12.16)), $\gamma=1/\sqrt{(g_{00})}$ requires the observed object to be stationary in gravitational spacetime: $v=0$.

It follows that, in the problem of the gravitational redshift of light depicted in Fig. 18.2, the observed object is not light or photons, but can only be the light source (optical clock) T_P stationary at the point A that emits light signals, and the light or photons emitted by T_P is only the observation medium of Einstein's optical agent OA(c), i.e., the information wave or informons of OA(c) that carries and transmits the spacetime information of T_P. (It should be noted that, according to Einstein's factor (Eq. (12.16)) of spacetime transformation, there is no need for the condition of orthogonal spacetime ($g_{i0}=0$ (i=1,2,3)) if $v=0$.)

Secondly, another issue is: In the problem of the gravitational redshift depicted in Fig. 18.2, whether the clock T_P as the observed object must be a light source or must be an optical clock?

Actually, as stated earlier, Einstein's gravitational-redshift equation of light does not directly calculate the gravitational redshift of light, but indirectly derives and calculates the gravitational redshift of light based on the equivalence between the gravitational redshift of light and the gravitational dilation of time. Whether Eq. (18.10) or Eq. (18.14), the so-called gravitational redshift Δf or $Z_E=\Delta f/f$ of light is actually the gravitational dilation of time in Einstein's factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ of spacetime transformation: $\gamma=\mathrm{d}t/\mathrm{d}\tau=1/\sqrt{(g_{00})}$ means the dilation of gravitational time dt.

Equation (18.11) visually shows the relationship between the gravitational redshift Z_E of light and Einstein's spacetime-transformation factor $\gamma=1/\sqrt{(g_{00})}$: $Z_E=1-\gamma$. In other words, as an observer of the free spacetime S_F observes the solar spectrum, the observed gravitational-redshift Z_E of light is the frequency-shift or dilation of the time in the gravitational spacetime of the sun:

$$Z_E = 1-\gamma = 1-\frac{\mathrm{d}t}{\mathrm{d}\tau} = 1-\frac{1}{\sqrt{g_{00}(r)}} \quad (r \ge R_S) \tag{18.15}$$

where dt is the observational (observed) time of the solar gravitational spacetime, and may be called the solar gravitational time.

Accordingly, the clock T_P in Eq. (18.15) (as shown in Fig. 18.2) is a potential clock at rest in the gravitational spacetime of the celestial body M or at rest on the surface of celestial body M.

In theory, all matter waves or periodic physical phenomena could be employed to define clock or time.

According to de Broglie's theory of matter waves, all matter particles or matter systems are matter waves, and could be employed to define clock or time. So, as the observed object of gravitational redshift, the clock T_P can be any matter system or any periodic physical phenomenon.

However, in Einstein's theory of general relativity, it is the optical observation agent OA(c) that transmits the information of T_P to observers. Therefore, in Fig. 18.2, the signals emitted by T_P from the point A to the point B must be light or photons, which is not the observed object, but the observation medium of OA(c): the information wave and informons of OA(c), for carrying and transmitting the information of T_P, including the temporal and spatial information of T_P, and at the same time, including the observational effect caused by the observational locality ($c<\infty$) of the optical observation OA(c).

At this point, it can be concluded that Einstein's gravitational-redshift equation of light is the gravitational-redshift equation of the information wave and informons of the optical observation agent OA(c); Einstein's gravitational-redshift theory of light is the gravitational-redshift theory of the optical observation agent OA(c). In Einstein's gravitational-redshift equation (18.10) of light, Z_E is the gravitational redshift of the optical agent OA(c), representing the gravitational redshift of the whole optical observation system or optical agent OA(c): not only the gravitational redshift of information waves and informons of OA(c), but also the gravitational redshift of the observational time of OA(c), and even the gravitational redshift of de Broglie's matter waves of all matter systems.

18.3 Newton and Gravitational Redshift

Newton's theory of universal gravitation can also interpret and calculate the gravitational redshift of light.

However, unlike Einstein's theory of general relativity, Newton's theory of universal gravitation does not need the principle of equivalence for interpreting the gravitational redshift of light, nor does it need the gravitational effect of time dilation or the gravitational effect of potential-clock running slower.

Actually, as stated in Sec. 18.1 of this chapter, the gravitational redshift of light does not mean the time dilation of gravitational spacetime, nor the gravitational effect of potential-clock running slower. In essence, the effect of gravitational redshift lies in the transformation of energy forms. Newton's theory of the gravitational redshift of light is based on the most concise principle of physics: the principle of conservation of energy.

18.3.1 Pseudo Newtonian Formula of the Gravitational Redshift of Light

Newton's theory of universal gravitation interprets the gravitational redshift of light based on the principle of conservation of energy, which needs to calculate the classical kinetic-energy K and the classical potential-energy V of photons in gravitational spacetime, and therefore, needs the classical mass m_∞ of photons: K needs the classical inertia-mass of m_i; V needs the classical gravitational-mass of m_g.

However, Newton's classical mechanics has no knowledge about the classical mass m_∞ (including m_i and m_g) of photons.

Therefore, the current strategy has to borrow Einstein formula $E=mc^2$ and the Planck equation $E=hf$ to calculate the photon mass: $m=E/c^2$ or $m=hf/c^2$; moreover, by employing Einstein's relativistic mass m as Newton's classical mass to deduce the so-called Newtonian gravitational-redshift equation.

The following derivation originates from Zhao's literature [160]: the current derivation of the so-called Newtonian gravitational-redshift equation of light is generally the same. (However, we could not determine whether this approach or strategy began with Einstein.)

As depicted in Fig. 18.2, suppose that, in the static spherically-symmetric gravitational spacetime of the celestial body *M*, a photon flies from the point A to point B, then according to Newton's classical mechanics, the loss of the photon's energy (kinetic energy) should be:

$$\Delta E=\int_{r_\mathrm{A}}^{r_\mathrm{B}}\left(-\frac{GMm_g}{r^2}\right)\mathrm{d}r=-GM\int_{r_\mathrm{A}}^{r_\mathrm{B}}\frac{m_g}{r^2}\mathrm{d}r \tag{18.16}$$

where, G is the gravitational constant, M is the mass of the celestial body as gravitational source, and m_g is the gravitational mass of the photon.

Suppose that the gravitational mass m_g and the inertial mass m_i of a photon are the same, and that the photon mass is lossless during motion, then:

$$\Delta E=-GM\int_{r_\mathrm{A}}^{r_\mathrm{B}}\frac{m_i}{r^2}\mathrm{d}r=GMm_i\left.\frac{1}{r}\right|_{r_\mathrm{A}}^{r_\mathrm{B}}=GMm_i\left(\frac{1}{r_\mathrm{B}}-\frac{1}{r_\mathrm{A}}\right) \tag{18.17}$$

It is worth noting that, in order to deduce Newton's gravitational-redshift equation of light, the literature [160] specifically lists the following relations of non-Newtonian classical mechanics:

$$\begin{cases} E=m_i c^2=hf \\ m_i=\dfrac{hf}{c^2} \\ \Delta E=h\Delta f \end{cases} \tag{18.18}$$

where m is the relativistic mass in Einstein formula $E=mc^2$, used as both the classical inertial-mass m_i of the photon and the classical gravitational-mass m_g of the photon. By substituted Eq. (18.17), one could have the absolute redshift of photons:

$$\Delta f=\frac{\Delta E}{h}=\frac{GM}{c^2}\left(\frac{1}{r_\mathrm{B}}-\frac{1}{r_\mathrm{A}}\right)f \tag{18.19}$$

Then, the relative redshift Z_{PN} of photons is:

$$Z_{PN}=\frac{\Delta f}{f}=\frac{GM}{c^2}\left(\frac{1}{r_\mathrm{B}}-\frac{1}{r_\mathrm{A}}\right) \tag{18.20}$$

where Z_{PN} is referred to as the pseudo Newtonian redshift.

As stated earlier in this chapter, the current Newtonian gravitational-redshift equation (18.20) of light borrows both Einstein formula $E=mc^2$ and Planck equation $E=hf$ for deducing the so-called Newtonian redshift, involving both relativity theory and quantum theory. This means that the current Newtonian gravitational-redshift equation of light is not the product of the pure Newtonian classical mechanics, but the mixture of Newton's classical mechanics and Einstein's relativity theory, as well as, even quantum theory, which can be called **the pseudo Newtonian formula** of the gravitational redshift of light.

Generally, GM/rc^2 (<<1) is a small quantity. Therefore, by contrasting Einstein's gravitational-redshift equation (18.10) and the pseudo Newtonian gravitational-redshift equation (18.20), one could find out that Einstein's redshift Z_E and the pseudo Newtonian redshift Z_{PN} are approximate without the observational distinguishability:

$$Z_E = 1 - \frac{\sqrt{g_{00}(r_\mathrm{B})}}{\sqrt{g_{00}(r_\mathrm{A})}} \approx \frac{GM}{c^2}\left(\frac{1}{r_\mathrm{B}} - \frac{1}{r_\mathrm{A}}\right) = Z_{PN} \tag{18.21}$$

It should be pointed out that the observational indistinguishability between Z_E and Z_{PN} is exactly because Z_{PN} is only the pseudo Newtonian.

We expect the real Newtonian theory of gravitational redshift, that is, Newton's gravitational-redshift equation of light which is purely based on classical mechanics and Newton's theory of universal gravitation.

18.3.2 Real Newtonian Formula of the Gravitational Redshift of Light

According to the theory of OR: the essence of the gravitational redshift of light is not the gravitational dilation of time; both the gravitational effect of time dilation and the gravitational effect of potential clock running slower are the observational effects, rooted from the observational locality ($\eta<\infty$) of the observation agent OA(η). According to the theorem of Cartesian spacetime, under the idealized observation agent OA_∞, both the gravitational effect of time dilation and the gravitational effect of potential clock running slower would disappear.

The essence of the gravitational redshift of light is the conservation of energy and the transformation of energy forms, following the principle of conservation of energy. Therefore, the definition and calculation of the gravitational redshift of light should be based on the viewpoint of energy and the idea of conservation of energy, rather than the dilative effect of time in gravitational spacetime.

The frequency f of a photon represents the kinetic energy K of the photon. Under the optical observation agent OA(c), the kinetic energy K of the photon (or one of the informons of OA(c)) is $K=E=mc^2=hf$, where m is the relativistic mass of the photon; Under the idealized observation agent OA_∞, the kinetic energy K of the photon as the observed object P of OA_∞ is $K=m_\infty c^2/2=m_o c^2/2$ (Eq. (5.26) and Eq. (5.8)), where m_∞ is the classical mass of the photon and m_o is the rest mass of the photon. According to the theory of OR, a photon has different frequencies under different observation agents; in particular, under the idealized agent OA_∞, the spectrum of light or photons would be unobservable. Thus, the definition $Z=\Delta f/f$ of the gravitational redshift of light under the optical agent OA(c) would no longer applicable to the idealized agent OA_∞, and naturally, nor to classical mechanics and Newton's theory of universal gravitation.

According to Newton's theory of universal gravitation, in the gravitational field, there is a relation of **as one falls another rise** between the kinetic energy K and potential energy V of a matter particle, keeping conservation of total energy.

As in the case of ordinary matter particles, for a photon moving in gravitational spacetime, if its potential energy V increases, then its kinetic energy K decreases, and vice versa. In essence, the gravitational redshift of light means the kinetic-energy decay of photons. Therefore, the gravitational redshift of light, whether under the idealized agent OA_∞ or under the optical agent OA(c), can be defined as the decay or redshift of photons' kinetic energy.

Einstein's optical agent OA(c) defines the gravitational redshift of light as $Z_E=\Delta f/f$ based on the observational frequency f and the observational frequency-shift Δf of photons in gravitational spacetime. Actually, under the principle of conservation of energy, it could be equivalently transformed into the gravitational redshift definition $Z_E=\Delta K/K$ based on the observational kinetic-energy K and the observational kinetic-energy shift ΔK of photons in gravitational spacetime:

$$Z_E = \frac{\Delta f_c}{f_c} = \frac{h\Delta f_c}{hf_c} = \frac{\Delta K_c}{K_c} \tag{18.22}$$

where Z_E is Einstein's gravitational redshift of light, f_c and Δf_c are respectively the relativistic frequency and relativistic frequency-shift of photons under the optical agent OA(c), while K_c and ΔK_c are respectively the relativistic kinetic-energy and relativistic kinetic-energy shift of photons under the optical agent OA(c).

In this way, based on the idea of energy, under the principle of conservation of energy, by following the logic of Eq. (18.22), Newton's gravitational redshift of light could be defined as follows:

$$Z_N = \frac{\Delta K_\infty}{K_\infty} \tag{18.23}$$

where Z_N is Newton's gravitational redshift of light, K_∞ and ΔK_∞ are respectively the classical kinetic-energy and classical kinetic-energy shift of photons under the idealized observation agent OA_∞.

It should be pointed out that: if Newton's gravitational redshift Z_N of light is defined with Eq. (18.23), then the observed object of gravitational redshift of light in Fig. 18.2 would no longer the light source (potential clock) T_P stationary at the point A, but the light or photons; the observation agent would no longer the optical agent OA(c), but the idealized agent OA_∞, and therefore, the informons transmitting gravitational-redshift information of light would no longer photons themselves, but the idealized informons of OA_∞ with the infinite speed and the infinitesimal momentum. In classical mechanics, photons lose their special status as informons and are no different from ordinary matter particles. Thus, according to Newton's classical mechanics, the classical kinetic-energy K_∞ and the classical potential-energy V_∞ of a photon in gravitational spacetime should be:

$$\begin{cases} K_{F\infty} = m_\infty c^2/2 \\ V_\infty(r) = -GMm_\infty/r \\ K_\infty(r) = K_{F\infty} - V_\infty(r) = m_\infty c^2/2 + GMm_\infty/r \end{cases} \tag{18.24}$$

where $K_{F\infty}=m_\infty c^2/2$ is the kinetic energy of the photon in vacuum or the free spacetime S_F, $K_\infty(r)$ and $V_\infty(r)$ are respectively the classical kinetic-energy and potential-energy of the photon at the distance r from the gravitational center O, and m_∞ is the classical mass of the photon.

According to the principle of conservation of energy: $\Delta K=-\Delta V$.

For the gravitational-redshift scene of light described in Fig. 18.2, from Eq. (18.23) and Eq. (18.24), it is follows that:

$$Z_N=\frac{\Delta K_\infty}{K_\infty}=-\frac{\Delta V_\infty}{K_\infty}=-\frac{V_\infty(r_{\rm B})-V_\infty(r_{\rm A})}{K_\infty(r_{\rm B})}\quad \left(\Delta V_\infty=V_\infty(r_{\rm B})-V_\infty(r_{\rm A})\right)$$
$$=-\frac{\left(-GMm_\infty/r_{\rm B}\right)-\left(-GMm_\infty/r_{\rm A}\right)}{\left(m_\infty c^2/2\right)-\left(-GMm_\infty/r_{\rm B}\right)}=\frac{2GMr_{\rm B}}{r_{\rm B}c^2+2GM}\left(\frac{1}{r_{\rm B}}-\frac{1}{r_{\rm A}}\right) \qquad (18.25)$$

where $K_\infty(r_{\rm B})$ is the classical kinetic-energy of the photon at the distance $r_{\rm B}$ from the gravitational center O, and $V_\infty(r_{\rm A})$ and $V_\infty(r_{\rm B})$ are the classical potential-energies of the photon at the distances $r_{\rm A}$ and $r_{\rm B}$ from the gravitational center O, respectively.

This is truly Newton's gravitational-redshift equation of light.

Equation (18.25) is purely based on Newton's classical mechanics and Newton's law of universal gravitation, independent of relativity theory and quantum theory, no involving Einstein formula $E=mc^2$ and Planck equation $E=hf$, and different from Einstein's gravitational-redshift equation (18.10) and the pseudo Newtonian gravitational-redshift equation (18.20).

Suppose that an observer in the free spacetime S_F observes the solar spectrum: $M=M_S$, $r_{\rm A}=R_S$ and $r_{\rm B}\to\infty$, then Newton's gravitational-redshift equation (18.25) of light reduces to:

$$Z_N=\frac{2GMr_{\rm B}}{r_{\rm B}c^2}\left(-\frac{1}{r_{\rm A}}\right)=-\frac{2GM_S}{R_Sc^2} \qquad (18.26)$$

Where, G is the gravitational constant, M_S is the solar mass, R_S is the solar radius, and Z_N is Newton's gravitational redshift of the solar spectrum.

This is different from our previous knowledge and understanding.

the conclusion Eq. (18.26) is different from that of the pseudo Newtonian gravitational-redshift equation (18.20): as far as the solar spectrum is concerned, Newton's gravitational-redshift Z_N of the solar spectrum in Eq. (18.26) is not the same as or approximate to Einstein's gravitational-redshift Z_E of the solar spectrum in Eq. (18.11), but is twice Einstein's Z_E: $Z_N=2Z_E$.

Later on, the correctness and validity of Newton's gravitational redshift equation (18.25) will be confirmed by the gravitational-redshift theory of GOR and the GOR gravitational-redshift equation of light.

18.4 GOR and Gravitational Redshift

The theory of GOR can also predict or calculate gravitational redshift.

However, the theory of GOR is the theory of the general observation agent OA(η), and the gravitational redshift of it may not necessarily be the gravitational redshift of light. The gravitational redshift of GOR can be the gravitational redshift of the informons of the general observation agent OA(η) and the gravitational redshift of the observed object *P* (including general matter particles and photons) under OA(η). It can be expected that, as far as light or photons is concerned, the gravitational redshift of light predicted by the theory of GOR depends on the observation agent OA(η) and the information-wave speed η of OA(η): under different observation agents, the light or photons in the same gravitational scene will exhibits different degrees of gravitational redshift.

As a matter of fact, no matter Einstein's gravitational-redshift equation of light or Newton's gravitational-redshift equation of light is destined to be generalized and unified into the gravitational-redshift theory of GOR.

18.4.1 The Gravitational Redshift of the General Observation Agent OA(η)

Under the principle of general correspondence (GC), by analogizing the logic of Einstein's gravitational-redshift equation (18.10), no matter following PGC logical route 1 or PGC logical route 2, the theory of GOR can derive the GOR gravitational-redshift equation that must be isomorphically consistent with Einstein's equation (18.10). What is surprising is that it will still be isomorphically consistent with Newton's equation (18.25).

It should be pointed out that: Einstein's gravitational-redshift theory originally intended to explore the decay or redshift of the frequency of light or photons in gravitational spacetime; but actually, Einstein's gravitational-redshift equation (Eq. (18.10) or Eq. (18.14)) was the gravitational-redshift equation for the observation time of the optical agent OA(c), and the logical deduction of it was based on the gravitational-dilation effect of the observational time of the optical agent OA(c). As stated in Sec. 18.2.3, Einstein's gravitational-redshift theory is the theory of gravitational redshift of the optical observation agent OA(c), representing the gravitational redshift of the whole optical observation system.

Therefore, if the GOR gravitational-redshift equation is derived based on the principle of GC, then it will be the gravitational-redshift of the information waves and informons of the general observation agent OA(η), rather than the gravitational redshift of light or photons, unless the observation agent OA(η) is the optical observation agent OA(c).

Here, by analogizing or following the logic stated in Sec. 18.2.2, the theory of GOR will deduce the GOR gravitational-redshift equation of the information wave or informons of the general observation agent OA(η) based on the invariance of time-frequency ratio.

Similar to the case of the optical observation agent OA(c), for the general observation agent OA(η) in the theory of GOR, in theory, the clock T_P in Fig. 18.2 could be any wave or any periodic physical phenomenon. However, the signal radiated by T_P from the point A to the point B is not light or photons, but the information wave or informons of OA(η), that is, the observation medium of OA(η) carrying and transmitting the spacetime information of T_P, including the temporal and spatial information of T_P.

According to the theory of GOR, for the general observation agent OA(η), the factor GOR of spacetime transformation is $\Gamma(\eta)=\mathrm{d}t_\eta/\mathrm{d}\tau$, or:

$$\begin{gathered}\mathrm{d}\tau=\sqrt{g_{00}(r_\mathrm{A})}\mathrm{d}t_\eta(r_\mathrm{A}) \quad \text{and} \quad \mathrm{d}\tau=\sqrt{g_{00}(r_\mathrm{B})}\mathrm{d}t_\eta(r_\mathrm{B}) \\ \text{so that} \quad \frac{\mathrm{d}t_\eta(r_\mathrm{A})}{\mathrm{d}t_\eta(r_\mathrm{B})}=\frac{\sqrt{g_{00}(r_\mathrm{B},\eta)}}{\sqrt{g_{00}(r_\mathrm{A},\eta)}} \quad \left(g_{00}(r)=1+\frac{2GM}{r\eta^2}\right)\end{gathered} \tag{18.27}$$

where η is the information-wave speed of OA(η), t_η is the observational (observed) time of OA(η) depending OA(η) on and η, $\mathrm{d}\tau$ is the objective and true time (i.e., the intrinsic time or proper time), $\mathrm{d}t_\eta(r_\mathrm{A})$ and $\mathrm{d}t_\eta(r_\mathrm{B})$ are the coordinate time-elements at the spatial points A and B, respectively.

According to the theory of OR, the information waves or informons of a specific observation agent OA(η) has its specific frequency f_η. However, all observation agents follow the invariance of time-frequency ratio:

$$\frac{\mathrm{d}t_\eta(r_\mathrm{A})}{f_\eta(r_\mathrm{A})}=\frac{\mathrm{d}t_\eta(r_\mathrm{B})}{f_\eta(r_\mathrm{B})} \quad \text{or} \quad \frac{f_\eta(r_\mathrm{A})}{f_\eta(r_\mathrm{B})}=\frac{\mathrm{d}t_\eta(r_\mathrm{A})}{\mathrm{d}t_\eta(r_\mathrm{B})} \tag{18.28}$$

where, it is worth noting that, $\mathrm{d}t_\eta(r_\mathrm{A})$ and $\mathrm{d}t_\eta(r_\mathrm{B})$ are respectively the observational time-elements of T_P at the spatial points A and B, $f_\eta(r_\mathrm{A})$ and $f_\eta(r_\mathrm{B})$ are respectively the observational frequencies of T_P at the spatial points A and B.

The gravitational redshift of the information wave or informons of the general observation agent OA(η) should be defined as: $Z_{\mathrm{OA}(\eta)}=\Delta f_\eta/f_\eta$. For the gravitational redshift scene in Fig. 18.2, the gravitational redshift equation of the information wave or informons of the general observation agent OA(η) can be derived based on the invariance of time-frequency ratio:

$$\begin{aligned}Z_{\mathrm{OA}(\eta)}&=\frac{\Delta f_\eta}{f_\eta}=\frac{f_\eta(r_\mathrm{B})-f_\eta(r_\mathrm{A})}{f_\eta(r_\mathrm{B})}=1-\frac{f_\eta(r_\mathrm{A})}{f_\eta(r_\mathrm{B})}=1-\frac{\mathrm{d}t_\eta(r_\mathrm{A})}{\mathrm{d}t_\eta(r_\mathrm{B})} \\ &=1-\frac{\sqrt{g_{00}(r_\mathrm{B},\eta)}}{\sqrt{g_{00}(r_\mathrm{A},\eta)}} \quad \left(g_{00}=1+\frac{2\chi}{\eta^2};\ \chi=-\frac{GM}{r}\right)\end{aligned} \tag{18.29}$$

This is the GOR gravitational-redshift equation of the general observation agent OA(η), which is isomorphically consistent with Einstein's gravitational-redshift equation (Eq. (18.10) or Eq. (18.14)) of light.

Here, it is stressed again that the GOR gravitational-redshift equation (18.29)) is not the gravitational-redshift equation of light, but the gravitational-redshift equation of the information-wave and informons of the general observation agent OA(η).

$Z_{\mathrm{OA}(\eta)}$ represents the gravitational redshift of the whole observation system of OA(η): $Z_{\mathrm{OA}(\eta)}$ is not only the gravitational redshift of information-wave and informons of OA(η), but also the gravitational redshift of observational time of OA(η), and even the gravitational redshift of the general de Broglie's matter waves of all matter systems; moreover, in theory, OA(η) could be any observation agent, while η could be any speed of matter motion.

Equation (18.29) suggest that: for different observation systems or different observation agents with different information-wave speeds, the observational times of them would exhibit different degrees of gravitational dilation, the observational frequencies of them would exhibit different degrees of gravitational redshift. In particular, suppose that an observer in the free spacetime S_F observes the frequency spectrum of the information waves or informons of a specific observation agent OA(η): $r_{\mathrm{B}}\to\infty$ and $g_{00}(r_{\mathrm{B}})\to 1$, then Eq. (18.29) reduces to:

$$Z_{\mathrm{OA}(\eta)} = 1 - \frac{1}{\sqrt{g_{00}\left(r_{\mathrm{A}}\right)}} = 1 - \frac{1}{\sqrt{1 - \dfrac{2GM}{r_{\mathrm{A}}\eta^2}}} \tag{18.30}$$

Equations (18.29) and (18.30) can be applied to calculate the gravitational redshift $Z_{\mathrm{OA}(\eta)}$ of the information-wave spectrum of any observation agent OA(η) (including the idealized agent OA_∞ and the optical agent OA(c)).

For the idealized agent OA_∞: $\eta\to\infty$, suppose that T_P in Fig. 18.2 radiates the information wave and informons of OA_∞ from the point A to the point B, then:

$$Z_{\mathrm{OA}_\infty} = \lim_{\eta\to\infty}\left\{1 - \frac{1}{\sqrt{1 - 2GM/r_{\mathrm{A}}\eta^2}}\right\} = 0 \tag{18.31}$$

which means that there is no gravitational redshift in the observational spectrum of the information wave and informons of the idealized agent OA_∞: $Z_{\mathrm{OA}\infty}=0$; the observational time of the idealized agent OA_∞ has no gravitational dilation: $\mathrm{d}t/\mathrm{d}\tau=1$. This is the same as or consistent with the logical conclusion of the theorem of Cartesian spacetime in Chapter 13.

For the optical agent OA(c): $\eta\to c$, suppose that T_P in Fig. 18.2 radiates the information wave (light) and informons (photons) of OA(c) from the point A (solar surface: $M=M_S$ and $r_{\mathrm{A}}=R_S$) to the point B, then:

$$Z_{\mathrm{OA}(c)} = 1 - \frac{1}{\sqrt{1 - 2GM_S/R_S c^2}} \approx -\frac{GM_S}{R_S c^2} \tag{18.32}$$

where $Z_{\mathrm{OA}(c)}$ is exactly the gravitational redshift Z_E of the solar spectrum in the Einstein's gravitational-redshift equation (Eq. (18.1) or Eq. (18.11)).

It is thus clear that, in general, the GOR gravitational-redshift equation (18.29) is not the gravitational-redshift equation of light or photons, but rather the gravitational-redshift equation for the information waves and informons of the observation agent OA(η). Only if OA(η) is OA(c), the GOR gravitational-redshift equation (18.29) reduces to Einstein's gravitational-redshift equation (Eq. (18.10) or Eq. (18.14)). At this case, $Z_{\mathrm{OA}(\eta)}$ is the gravitational redshift of light observed by the optical agent OA(c): $Z_E=Z_{\mathrm{OA}(c)}$.

By contrasting with the conclusion of the gravitational redshift under the optical observation agent OA(c) in Sec. 18.2.3, one could conclude that the GOR gravitational-redshift equation (18.29) is the gravitational-redshift model of the general observation agent OA(η), where $Z_{\mathrm{OA}(\eta)}$ is the gravitational redshift of the observation agent OA(η), representing the gravitational redshift of the whole observation system OA(η): it is not only the gravitational redshift of the information wave and informons of OA(η), but also the gravitational redshift of the observational time of OA(η), and even the gravitational redshift of the general de Broglie's matter waves of all matter systems.

18.4.2 The GOR Gravitational-Redshift Equation of Light

The GOR gravitational-redshift of light is the gravitational redshift of light observed by the general observation agent OA(η), which means that: the observed object P is light or photons, the gravitational redshift refers to the spectroscopic redshift of light or photon in gravitational spacetime; the observation agent of observer is the general observation agent OA(η), which may not necessarily be the optical agent OA(c). For example, Newton's gravitational-redshift equation (18.25) in Sec. 18.3.2 is the theoretical model of the idealized agent OA_∞ observing light, where the photons, like general matter particles in Newton's classical mechanics, are the observed objects of OA_∞.

According to the theory of OR, according to Corol. 3.2 (**The Observational Ultimate Speed $\boldsymbol{\eta}$**) of the theorem of the invariance of information-wave speeds in Chapter 3, the information-wave speed η of the observation agent OA(η) must be greater than or equal to the motion speed v of the observed object P. In the problem of the gravitational redshift of light, the observed object P is light or photons with the speed $v \approx c$, so it requires that: $\eta \geq c$.

In theory, we could observe and measure the gravitational redshift of light under the observation agent OA(η) ($\eta \geq c$). By analogizing or following the logic of defining $Z_E=\Delta K_c/K_c$ (Eq. (18.22)) and $Z_N=\Delta K_\infty/K_\infty$ (Eq. (18.23)) in Sec. 18.3.2 based on the energy shift ΔK, we can deduce the GOR gravitational-redshift equation of light under the general observation agent OA(η) ($\eta \geq c$).

The Definition of the GOR Gravitational-Redshift of Light

As stated in Sec. 18.3.2, the essence of the gravitational redshift of light is the conservation of energy and the transformation of energy forms, following the principle of conservation of energy in physics. Under the principle of general correspondence (GC), based on the idea of conservation of energy, by following the logic of Eq. (18.22) and Eq. (18.23), the theory of GOR defines the gravitational redshift of light under the general observation agent OA(η) ($\eta \geq c$) as:

$$Z_{\mathrm{GOR}} = \frac{\Delta K_\eta}{K_\eta} = -\frac{\Delta V_\eta}{K_\eta} \quad \left(c \leq \eta \in \mathrm{OA}(\eta)\right) \tag{18.33}$$

where Z_{GOR} is the gravitational frequency-shift of light or photons observed with OA(η) ($\eta \geq c$) ($Z_{\mathrm{GOR}}<0$ means redshift), K_η and ΔK_η are respectively the relativistic kinetic-energy and its increment of a photon observed with OA(η) ($\eta \geq c$), V_η and ΔV_η are respectively the relativistic potential-energy and its increment of a photon observed with OA(η) ($\eta \geq c$).

The Energy of a Photon under the General Observation Agent OA(η)

As far as the spherically-symmetric gravitational spacetime under the general observation agent OA(η) ($\eta \geq c$) is concerned, let K_η and V_η be the kinetic energy and potential energy of a photon observed with OA(η), respectively. According to the theory of GOR, the observational kinetic-energy K_η and potential-energy V_η of the photon under the OA(η) ($\eta \geq c$) are:

$$\begin{cases} E_\eta = m_\eta \eta^2 \\ K_{F\eta} = \left(\Gamma|_{\chi=0} - 1\right) m_o \eta^2 = \left(1 - \Gamma^{-1}|_{\chi=0}\right) m_\eta \eta^2 \\ V_\eta(r) = \left(1 - \Gamma|_{v=0}\right) m_o \eta^2 \\ K_\eta(r) = K_{F\eta} - V_\eta(r) = m_\eta \eta^2 - \left(1 - \Gamma|_{v=0}\right) m_o \eta^2 \end{cases} \tag{18.34}$$

$$\left(\Gamma = 1\Big/\sqrt{1 + 2\chi/\eta^2 - v^2/\eta^2}\right)$$

where χ is the Newtonian gravitational potential, m_o is the rest mass of the photon, m_η is the relativistic mass or observational mass of the photon under OA(η), E_η and $K_{F\eta}$ are respectively the observational total energy and kinetic-energy of the photon under OA(η), $K_\eta(r)$ and $V_\eta(r)$ are respectively the observational kinetic-energy and potential-energy of the photon at the distance r from the gravitational center O: $K_{F\eta}=K_\eta(r)+V_\eta(r)$.

It should be noted that, in Eqs. (18.33) and (18.34), the observed object P is a photon, and the speed v of it is the speed of the photon (in a weak gravitational field, $v \approx c$; specifically, if when $\chi=0$ then the photon speed is exactly the speed c of light in vacuum), while η ($\geq c$) is the information-wave speed of the observation agent OA(η); $E_\eta = m_\eta \eta^2$ is the OR mass-energy relation; $\Gamma = \Gamma(\eta)$ is the GOR factor of spacetime transformation (see Eqs. (12.35) and (12.36) in Chapter 12), where $\Gamma|_{\chi=0}$ denotes the inertial spacetime-transformation factor, and $\Gamma|_{v=0}$ denotes as the gravitational spacetime-transformation factor:

$$\begin{cases} \Gamma(\eta)\big|_{\chi=0} = 1\Big/\sqrt{1 - v^2/\eta^2} \\ \Gamma(\eta)\big|_{v=0} = 1\Big/\sqrt{1 + 2\chi/\eta^2} = 1\Big/\sqrt{g_{00}} \end{cases} \tag{18.35}$$

$$\left(g_{00} = 1 + 2\chi/\eta^2 \,;\, \chi = -GM/r\right)$$

where, $\chi=0$ means that the observed object is in an inertial state, and not subjected to gravity or universal gravitation; $v=0$ means that the observed object P is at rest in a gravitational spacetime.

The inertial factor $\Gamma|_{\chi=0}$ of spacetime transformation involves the measurement of the photon's relativistic kinetic-energies $K_{F\eta}$ and $K_\eta(r)$ under OA(η); the gravitational factor $\Gamma|_{v=0}$ of spacetime transformation involves the measurement of the photon's relativistic potential-energy $V_\eta(r)$ under OA(η).

It is worth noting that, the kinetic-energy and potential-energy (in Eq. (18.34)) observed with OA(η) not only generalizes Einstein's relativistic kinetic-energy and relativistic potential-energy (observed with OA(c)), but also generalizes Newton's classical kinetic-energy and classical potential-energy (observed with OA_∞).

As $\eta\to c$, the observation agent OA(η) is the optical agent OA(c), the photon's observational kinetic-energy $K_{F\eta}$ and potential-energy $V_\eta(r)$ in Eq. (18.34) under OA(η) reduce to Einstein's relativistic kinetic-energy and relativistic-potential energy under OA(c):

$$\begin{aligned} K_F\big|_{\mathrm{OA}(c)} &= \lim_{\eta\to c}\left(1-\Gamma^{-1}\big|_{\chi=0}\right)m_\eta\eta^2 = mc^2 \quad \left(\Gamma^{-1}\big|_{\chi=0}=\sqrt{1-c^2/\eta^2}\right) \\ V_r\big|_{\mathrm{OA}(c)} &= \lim_{\eta\to c}\left(1-\Gamma\big|_{v=0}\right)m_o\eta^2 = \left(1-\gamma\big|_{v=0}\right)m_oc^2 \end{aligned} \tag{18.36}$$

where γ is the spacetime-transformation factor of Einstein's relativity theory (see Eq. (12.12) or Eq. (12.16)), m is Einstein's relativistic mass of a photon observed with the optical agent OA(c), and according to Einstein formula $E=mc^2$, the relativistic kinetic-energy of a photon in vacuum is $E=mc^2$.

As $\eta\to\infty$, the observation agent OA(η) is the idealized agent OA_∞, the photon's observational kinetic-energy $K_{F\eta}$ and potential-energy $V_\eta(r)$ in Eq. (18.34) under OA(η) reduce to Newton's classical kinetic-energy and classical potential-energy under the idealized agent OA_∞:

$$\begin{aligned} K_F\big|_{\mathrm{OA}_\infty} &= \lim_{\eta\to\infty}\left(\Gamma\big|_{\chi=0}-1\right)m_o\eta^2 = \lim_{\eta\to\infty}\left(\frac{1}{\sqrt{1-c^2/\eta^2}}-1\right)m_o\eta^2 = \frac{1}{2}m_oc^2 \\ V_r\big|_{\mathrm{OA}_\infty} &= \lim_{\eta\to\infty}\left(1-\Gamma\big|_{v=0}\right)m_o\eta^2 \\ &= \lim_{\eta\to\infty}\left(1-\frac{1}{\sqrt{1+2\chi/\eta^2}}\right)m_o\eta^2 = -\frac{GMm_o}{r} \quad \left(\chi=-GM/r\right) \end{aligned} \tag{18.37}$$

Equations (18.36) and (18.37) confirm the logical self-consistency of the energy formula (Eq. (18.34)) of GOR theory, and moreover, confirm the logical consistency not only between the GOR energy formula and Einstein's relativistic energy formula but also between the GOR energy formula and Newton's classical energy formula.

The Deduction of the GOR Gravitational-Redshift Equation of Light

For the gravitational redshift scene of light depicted in Fig. 18.2: $\Delta V=V_B-V_A$, from Eq. (18.33) and Eq. (18.34), it follows that:

$$
\begin{aligned}
Z_{\text{GOR}} &= \frac{\Delta K_{\eta}}{K_{\eta}} = -\frac{\Delta V_{\eta}}{K_{\eta}} = -\frac{V_{\eta}(r_{\text{B}}) - V_{\eta}(r_{\text{A}})}{K_{\eta}(r_{\text{B}})} = -\frac{V_{\eta}(r_{\text{B}}) - V_{\eta}(r_{\text{A}})}{K_{F\eta} - V_{\eta}(r_{\text{B}})} \\
&= \frac{\left(m_o\eta^2 \big/ \sqrt{g_{00}(r_{\text{B}},\eta)}\right) - \left(m_o\eta^2 \big/ \sqrt{g_{00}(r_{\text{A}},\eta)}\right)}{K_{F\eta} - \left(1 - 1\big/\sqrt{g_{00}(r_{\text{B}},\eta)}\right) m_o\eta^2}
\end{aligned}
\qquad (18.38)
$$

$$
\begin{cases}
K_{F\eta} = \left(\Gamma|_{\chi=0} - 1\right) m_o\eta^2 = \left(1 - \Gamma^{-1}|_{\chi=0}\right) m_{\eta}\eta^2 \\
m_{\eta} = m_o\, \Gamma|_{\chi=0} \quad \left(\Gamma|_{\chi=0} = 1\big/\sqrt{1 - c^2/\eta^2}\right) \\
\Delta V_{\eta} = V_{\eta}(r_{\text{B}}) - V_{\eta}(r_{\text{A}})
\end{cases}
$$

where m_o is the rest mass of the photon, m_{η} is the observational mass of the photon under OA(η), $K_{F\eta}$ is the observational kinetic-energy of the photon in vacuum or the free spacetime S_F under OA(η), $K_{\eta}(r_{\text{B}})$ is the observational kinetic-energy of the photon at the distance r_{B} from the gravitational center O under OA(η), $V_{\eta}(r_{\text{A}})$ and $V_{\eta}(r_{\text{B}})$ are respectively the observational potential-energies of the photon at the distances r_{A} and r_{B} from the gravitational center O under OA(η), $g_{00}(r_{\text{A}},\eta)$ and $g_{00}(r_{\text{B}},\eta)$ are respectively the metric 00-elements of the spacetime metric $g_{\mu\nu}(r_{\text{A}},\eta)$ at the spatial points A and B under OA(η).

This is the GOR gravitational-redshift equation of light.

Equation (18.38) indicates that the gravitational redshift $Z_{\text{GOR}}=Z_{\text{GOR}}(\eta)$ ($\eta \geq c$) of light depends on the observation agent OA(η) and the information-wave speed η: the same photon in the same gravitational scene would exhibit different degrees of gravitation redshift under different observation agents; the higher the information-wave speed η of OA(η), the more significant the relativistic gravitational redshift of light is.

For the gravitational redshift of the solar spectrum ($r_{\text{A}}=R_S$ and $r_{\text{B}}=\infty$):

As $\eta \to c$, $K_{F\eta}=mc^2$ and $Z_{\text{GOR}}(\eta)=-GM_S/R_Sc^2=Z_E$;

As $\eta \to \infty$, $K_{F\eta}=m_{\infty}c^2/2$ and $Z_{\text{GOR}}(\eta)=-2GM_S/R_Sc^2=Z_N$.

Therefore, due to the continuity and monotonicity of $Z_{\text{GOR}}(\eta)$ ($\eta \in [c,\infty)$) in Eq. (18.38), the GOR gravitational-redshift $Z_{\text{GOR}}(\eta)$ of light obeys:

$$
\frac{2GM_s}{R_s c^2} = |Z_N| \geq |Z_{\text{GOR}}(\eta)| \geq |Z_E| = \frac{GM_s}{R_s c^2} \qquad (18.39)
$$

18.5 The Gravitational Redshift and Photon Mass

Besides **Time**, perhaps **Mass** is the most fundamental physical quantity.

In Newton's classical mechanics, the mass of matter particles can be called **classical mass** and denoted as m_{∞}, which has both inertial effect and gravitational effect; in Einstein's relativity theory, the mass of matter particles can be called **relativistic mass** and denoted as m, which includes the rest mass m_o and the mass Δm dilated due to the motion (v) of matter: $\Delta m=m(v)-m_o$. According to the theory of

OR, $\Delta m=\Delta m(v)$ purely depends on the motion speed v of a matter particle, which therefore is not the objective and real mass of material particles and does not have real physical effects (including inertial effect and gravitational effect).

In Einstein's view, photons, as a class of matter particles, seemed to possess some particularity: we have not yet determined the classical mass or rest mass of photons. Einstein believed that photons have no rest mass.

However, according to the theory of OR, based on Def. 1.2 in Chapter 1, the classical mass m_∞ and rest mass m_o of matter particles are the same and equal: $m_\infty=m_o$, which is the objective and true mass of matter particles, that is, the intrinsic mass of matter particles. As stated in Sec. 5.1.5 **The Problem of Photon Rest Mass** of Chapter 5, all matter particles, including photons and even gravitons, have the rest mass m_o or classical mass m_∞ of their own.

Now, the gravitational-redshift theory of GOR has produced a byproduct: the theoretical value of the rest mass of photons.

18.5.1 The Classical Mass m_∞ is exactly the Rest Mass m_o

Originally, **Classical Mass** refers to the mass of matter in Newton's classical mechanics; while **Rest Mass** is the product of Einstein's relativity theory.

Let m be a matter system and at the same denote the mass of it. According to the theory of OR, the mass $m=m(\eta)$ of a matter system under the general observation agent OA(η) depends on the information-wave speed η of OA(η). As $\eta\to\infty$, OA(η) would be the idealized agent OA$_\infty$, and the observational (observed) mass $m(\eta)$ under OA(η) would converge to the classical mass m_∞. According to the IOR mass-speed relation (Eq. (5.5) in Chapter 5), we have that:

$$m_\infty=\lim_{\eta\to\infty} m(\eta)=\lim_{\eta\to\infty}\Gamma(\eta)m_o=\lim_{\eta\to\infty}\frac{m_o}{\sqrt{1-v^2/\eta^2}}=m_o \qquad (18.40)$$

Equation (18.40) suggests that the so-called rest mass m_o is exactly the classical mass m_∞: it turns out that Einstein's rest mass m_o is Newton's classical m_∞.

According to the theory of IOR and GOR, the intrinsic mass m of a matter particle or a matter system is the objective and real mass, which has the objectively inertial effect and the objectively gravitational effect: $m_\infty=m_i=m_g=m_o$. Therefore, whether the classical mass m_∞ of Newton's classical mechanics or the rest mass m_o of Einstein's relativity theory, there must lead to the effect of gravitational redshift.

If photons have the intrinsic mass of their own (no matter the classical mass m_∞ or the rest mass m_o), then photons not only have inertial effect, but also have gravitational effect, and the photons in gravitational spacetime must exhibit the phenomenon of gravitational redshift.

So, do photons have the intrinsic mass of their own?

If so, how much does a photon weigh?

18.5.2 Does a Photon Has Rest Mass?

Einstein's theory of special relativity claimed that photons have no rest mass.

This means that photons have no classical mass, and even have no the intrinsic mass of their own, which is contrary to human being's simple and plain view of nature.

The readers familiar with Einstein's theory of special relativity know that there is a Lorentz singularity in the Lorentz transformation: as the speed v of the observed object P reaches the speed c of light in vacuum, the Lorentz factor γ would reach infinity: $\gamma=1/\sqrt{(1-v^2/c^2)}\rightarrow\infty$. Therefore, at the Lorentz singularity, the relativistic mass m of the observed object P would also be infinity: $m=\gamma m_o=\infty$, unless photons have no the rest mass m_o of their own: $m_o=0$.

Thus, in special relativity, Einstein had to set the rest mass m_o of photons to zero. Observations and experiments seem to be quite compatible: until today, the rest mass of photons has not yet been detected by observation or experiment, but the upper limit value is seemingly decreasing and tends to zero [37,82-84].

In 2014, the upper limit value of photon mass recommended by the particle data group (PDG) was 1.5×10^{-54} kg [85].

Actually, people have already observed the rest mass of photons, but they just have not realized that it is the rest mass of photons. In 1919, by observing the total solar eclipse at the island of Príncipe, Eddington discovered that the trajectory of the starlight sweeping over the solar surface was curved as predicted by Einstein, affirming that "light has weight" (as remarked in Eddington's poem).

The theory of OR has theoretically clarified [26-28] that photons, and even all particles or systems of matter, have the rest mass or intrinsic mass of their own, that is, the objective and true mass.

The theory of OR provides a detailed discussion on the issue of the rest mass of photons [26-28]. In short, photons, like all matter particles, have the rest mass of their own. As far as the relativistic mass m and rest mass m_o of photons are concerned, to paraphrase Hawking's words [31], the Lorentz transformation and the mass-speed relation in Einstein's special relativity breaks down or fails at the Lorentz singularity. Actually, whether from the view of the singularity of the mathematical model or from the view of observations and experiments, the conclusion of photons having no mass is only a manifestation of the observational locality ($c<\infty$) of the optical agent OA(c): you could not expect light or photons to act as the observation medium for detecting the mass of photons themselves! Alternatively, the rest mass of photons could not be measured and determined by the optical observation agent OA(c).

The theory of OR has clarified [26-28]: if we could observe photons under the superluminal observation agent OA(η) ($\eta>c$), then we would find out that photons have the rest mass of their own.

From to the IOR mass-speed relation (Eq. (5.5)), it follows that:

$$m_o = \Gamma^{-1}(\eta)m = m\sqrt{1-\frac{c^2}{\eta^2}} > 0 \quad (m\neq 0, \eta > c) \tag{18.41}$$

It is thus clear that: photons, and even all matter particles, have their own rest mass m_o. As clarified in Sec. 12.5 **All Relativistic Effects are Observational Effects and Apparent Phenomena** of Chapter 12, the relativistic mass m depends on the observation agent OA(η): $m=\Gamma(\eta)m_o=\Gamma_\infty m_o+\Delta\Gamma(\eta)m_o$ ($\Gamma_\infty\equiv1$), where only the rest mass m_o is the objective and real mass, while $\Delta m=\Delta\Gamma(\eta)m_o$ is purely the observational effect. Both the objectively inertial property and the objectively gravitational property of matter particles are decided only by the rest mass m_o.

Since photons have the rest mass m_o, naturally, photons have classical mass m_∞. Thus, we can calculate the classical mass m_∞ of the photon in Eq. (18.24) based on Newton's classical mechanics and Newton's law of universal gravitation:

$$\begin{cases} K_{F\infty}=m_\infty c^2/2=m_o c^2/2 \\ V_\infty(r)=-GMm_\infty/r=-GMm_o/r \\ K_\infty(r)=K_{F\infty}-V_\infty(r)=m_o c^2/2+GMm_o/r \end{cases} \qquad (m_\infty=m_o) \qquad (18.42)$$

18.5.3 How Much does a Photon Weigh?

The problem of photon rest mass is one of the major problems in physics.

Although Einstein claimed that photons have no rest mass, people, due to their intrinsic view of nature, are subconsciously unwilling to accept the argument of photons having no rest mass. Many physicists, including great de Broglie [32,33], Schrödinger [34,35], and Feynman [36], devote their energy and time to exploring and detecting the rest mass of photons.

The theory of OR has affirmed that photons have the mass of their own, that is, the classical mass m_∞ or the rest mass m_o.

According to the theory of OR, especially based on the OR mass-speed relation (Eq. (5.5)) and the OR mass-energy relation $E=m\eta^2$, and at the same time, by combining observations and experiments, we can infer and determine the classical mass m_∞ or the rest mass m_o of photons.

In Planck's experiment of blackbody radiation [14], the cavity of blackbody radiation is at rest relative to the laboratory and observers. In view of this, the literature [27] ever regarded the experiment of blackbody radiation as the experiment under the idealized observation agent OA_∞, and regarded the photon energy E in Planck equation $E=hf$ as the classical kinetic-energy of photons: $hf=m_\infty c^2/2$; accordingly, it seemed that the rest mass of photons with the frequency f should be $m_o=m_\infty=2hf/c^2$.

However, this speculation is somewhat suspicious.

Actually, the observed object in the experiment of blackbody radiation is light or photons, where the messenger transmitting the information of light is light itself, or, the messengers transmitting the information of photons are photons themselves. So, the observation agent in the experiment of blackbody radiation is the optical agent OA(c): the information wave of OA(c) is light and the informons of OA(c) are photons. Einstein formula $E=mc^2$, the so-called mass-energy relation, is a formula in Einstein's special relativity, belonging to the optical agent OA(c). For photons, it is widely recognized that $E=mc^2=hf$, which suggests that both Einstein formula $E=mc^2$ and Planck equation $E=hf$ are the theoretical models of the optical agent OA(c). Therefore, Planck's photon energy $E=hf$ under the optical agent OA(c) is not equal to Newton's photon kinetic-energy $K=m_\infty c^2/2$ under the idealized agent OA_∞.

The theory of OR has affirmed that a photon has the rest mass of its own: $m_o=m_\infty$. Therefore, according to the GOR energy formulae in Eq. (18.34), the observational kinetic-energy K_c and potential-energy V_c of a photon under the optical observation agent OA(c) should be:

$$\begin{cases} K_{Fc} = E = mc^2 \\ V_c(r) = \lim\limits_{\eta\to c} V_\eta(r) = \left(1 - 1\big/\sqrt{g_{00}(r)}\right) m_o c^2 \\ K_c(r) = \lim\limits_{\eta\to c} K_\eta(r) = mc^2 - \left(1 - 1\big/\sqrt{g_{00}(r)}\right) m_o c^2 \end{cases} \tag{18.43}$$

where m and m_o are respectively the relativistic mass and rest mass of the photon, $K_c(r)$ and $V_c(r)$ are respectively the relativistic kinetic-energy and relativistic potential-energy of the photon at the distance r from the gravitational center O, and $K_{Fc}=mc^2$ is the relativistic kinetic-energy of the photon in vacuum or in the free spacetime S_F, that is, the total energy of the photon under the optical observation agent OA(c): $K_{Fc}=E=K_c(r)+V_c(r)$.

For the gravitational redshift scene of light depicted in Fig. 18.2: $\Delta V=V_B-V_A$, according to the GOR gravitational-redshift equation (18.38) of light, under the optical observation agent OA(c), substituting the observational (observed) energy of OA(c) in Eq. (18.43) into Eq. (18.38), we have:

$$Z_E = \lim_{\eta\to c} Z_{\text{GOR}} = \frac{\Delta K_c}{K_c} = \frac{\left(m_o c^2\big/\sqrt{g_{00}(r_B)}\right) - \left(m_o c^2\big/\sqrt{g_{00}(r_A)}\right)}{mc^2 - \left(1 - 1\big/\sqrt{g_{00}(r_B)}\right) m_o c^2} \tag{18.44}$$

where $K(r_B)$ is the observational kinetic-energy of OA(c) as the photon is at the distance r_B from the gravitational center O, $V(r_A)$ and $V(r_B)$ are respectively the observational potential-energy of OA(c) as the photon is at the distances r_A and r_B from the gravitational center O, and $g_{00}(r_A)$ and $g_{00}(r_B)$ are respectively the metric 00-elements of the points A and B in the gravitational spacetime of OA(c).

Suppose that observers of the free spacetime S_F observe the solar spectrum with the optical observation agent OA(c): $M=M_S$, $r_A=R_S$, and $g_{00}(r_A)=1-2GM/R_Sc^2$; $r_B\to\infty$, and $g_{00}(r_B)=1-2GM/r_Bc^2\to 1$. Then, Eq. (18.44) would be reduced to:

$$Z_E = \frac{m_o c^2 - m_o c^2\big/\sqrt{g_{00}(r_A)}}{mc^2} \approx -\frac{GM_S}{R_S c^2}\frac{m_o}{m} \qquad \left(g_{00}(r_A) = 1 - \frac{2GM_S}{R_S c^2}\right) \tag{18.45}$$

where G is the Gravitational constant, M_S is the solar mass, R_S is the solar radius, and Z_E is the gravitational redshift of the solar spectrum observed with the optical agent OA(c), i.e., Einstein's gravitational red shift.

Equation (18.45) is the gravitational-redshift equation of the solar spectrum which is based on the energy-shift definition of $Z_E=\Delta K/K$ and derived from the principle of conservation of energy; while Eq. (18.1) is the gravitational-redshift equation of the solar spectrum which is based on the frequency-shift definition of $Z_E=\Delta f/f$ and derived from Einstein's factor $\gamma=\mathrm{d}t/\mathrm{d}\tau$ of spacetime transformation under the optical agent OA(c). In Eq. (18.1), the theoretical value of the gravitational redshift of the solar spectrum is $Z_E=-GM_S/R_Sc^2=-2.12\times10^{-6}$. According to the work of Adam, Blamont, Roddier, and Brault et al [152-154], this theoretical value has already been verified by observations and experiments.

So, the contrasting Eq. (18.45) and Eq. (18.1), we get that:

$$\frac{GM_S}{R_Sc^2}\frac{m_o}{m}=\frac{GM_S}{R_Sc^2}$$
$$\text{that is}\quad m_o=m=\frac{hf}{c^2}\quad\left(E=mc^2=hf\right)\tag{18.46}$$

where m_o is the rest mass of the photon; m and f are respectively the observational mass and observational frequency of the photon as an informon of OA(c).

Equation (18.46) suggests that: photons do indeed have the rest mass m_o; in particular, photons with different frequencies have different rest masses.

Substituting the conclusion $m_o=m$ from Eq. (18.46) into Eq. (18.44), we have:

$$Z_E=\frac{\left(1\big/\sqrt{g_{00}(r_\mathrm{B})}\right)-\left(1\big/\sqrt{g_{00}(r_\mathrm{A})}\right)}{1\big/\sqrt{g_{00}(r_\mathrm{B})}}=1-\frac{\sqrt{g_{00}(r_\mathrm{B})}}{\sqrt{g_{00}(r_\mathrm{A})}}\tag{18.47}$$

This is exactly Einstein's gravitational-redshift equation (Eq. (18.10) or Eq. (18.14)) of light. Equation (18.47) not only confirms the GOR gravitational-redshift equation (18.38) of light based on the definition $Z_{\mathrm{GOR}}=\Delta K_\eta/K_\eta$ of energy shift, but also confirms Einstein's gravitational-redshift equation (18.10) of light based on the definition $Z_E=\Delta f/f$ of frequency shift and the gravitational-redshift equation (18.14) of light based on the invariance of time-frequency ratio. At the same time, it confirms the important conclusion of photon rest mass: $m_o=m=hf/c^2$.

Equations (18.46) and (18.47) mean that, whether based on the observation of the gravitational redshift of the solar spectrum, or based on the equivalence between the gravitational-redshift definitions of energy shift and frequency shift, the relativistic mass or observational mass m presented by photons as informons of the optical agent OA(c) is exactly the rest mass m_o or classical mass m_∞ of photons; moreover, the rest mass m_o of photons can be calculated from the observed frequency f presented by photons as informons of the optical agent OA(c).

Photons with different frequencies have different masses: $m_o=m_\infty=m=hf/c^2$.

It turns out that, if the observed object P (m) is a photon, then the relativistic mass m in Einstein formula $E=mc^2$ is exactly the rest mass m_o of the photon or the classical mass m_∞ of the photon

In this way, we could finally understand why experimental physicists have been working so hard to find the rest mass of photons but have found nothing. This indeed echoes the line of a Chinese poet: They would never discern the true face of Lushan Mountains, but only for they were in the mountains themselves.

Let us leave the observational or experimental verification for the theoretical value of photon rest mass in Eq. (18.46) to experimental physicists.

18.6 The Unity of Newton and Einstein in the Gravitational-Redshift Theory of GOR

Einstein's prediction of the gravitational redshift of light was based on his equivalence principle, while Einstein's gravitational-redshift equation of light was derived from his theory of general relativity.

People try to derive the gravitational- redshift equation of light from Newton's classical mechanics and Newton's law of universal gravitation, so as to compare and contrast Einstein's theory of gravitational redshift and Newton's theory of gravitational redshift, and then, compare and contrast Einstein's theory of general relativity and Newton' theory of universal gravitation. As a result, the pseudo Newtonian gravitational-redshift equation has been manufactured. As shown in Eq. (18.21), the pseudo Newtonian gravitational-redshift equation (18.20) is the same as or approximate to Einstein's gravitational-redshift equation (18.10): as far as the optical spectrum of the sun is concerned, the pseudo Newtonian gravitational redshift Z_{PN} and Einstein's gravitational redshift Z_E do not have the observational distinguishability.

Actually, as stressed repeatedly by the theory of GOR, Einstein's theory of general relativity and Newton's theory of universal gravitation belong to different observation agents and serve different observation systems. So, they have no the comparability of which is right and which is wrong. For the theoretical models of different observation agents, it is natural and even inevitable that there are the theoretical and observational differences among them; on the contrary, it is abnormal and illogical, just as in the case of the pseudo Newtonian gravitational-redshift equation.

The theory of GOR has deduced the real Newtonian gravitational-redshift equation (18.25) which is completely and purely based on Newton's classical mechanics and Newton's law of universal gravitation. As far as the gravitational redshift of the solar spectrum is concerned, the real Newtonian theoretical-value Z_N (Eq. (18.26)) is twice Einstein's theoretical-value N_E (Eq. 18.1)): Z_N=2Z_E.

Furthermore, the theory of GOR has derived the gravitational-redshift equation of light for the general observation agent OA(η) $\eta \geq c$), the so-called GOR gravitational-redshift equation (18.38) of light.

The gravitational-redshift theory of GOR demonstrates the high generality and universality of the theory of GOR.

According to Sec. 18.5 and Eqs. (18.46-47), the relativistic mass m of photons under the optical observation agent OA(c) is exactly the rest mass m_o of photons. Therefore, we have that:

$$\lim_{\eta \to c} K_{F\eta} = \lim_{\eta \to c} \left(1 - \Gamma^{-1}\big|_{\chi=0}\right) m_\eta \eta^2$$
$$= \lim_{\eta \to c} \left(1 - \sqrt{1 - c^2/\eta^2}\right) m_\eta \eta^2 = mc^2 = m_o c^2 \qquad (18.48)$$

Thus, as $\eta\to c$, under the optical agent OA(η), it holds true that:

$$\lim_{\eta\to c} Z_{\mathrm{GOR}} = \lim_{\eta\to c}\frac{\left(m_o\eta^2\big/\sqrt{g_{00}(r_\mathrm{B})}\right)-\left(m_o\eta^2\big/\sqrt{g_{00}(r_\mathrm{A})}\right)}{K_{F\eta}-\left(1-1\big/\sqrt{g_{00}(r_\mathrm{B})}\right)m_o\eta^2}$$

$$=\frac{m_oc^2\big/\sqrt{g_{00}(r_\mathrm{B})}-m_oc^2\big/\sqrt{g_{00}(r_\mathrm{A})}}{m_oc^2-\left(1-1\big/\sqrt{g_{00}(r_\mathrm{B})}\right)m_oc^2}=1-\frac{\sqrt{g_{00}(r_\mathrm{B})}}{\sqrt{g_{00}(r_\mathrm{A})}}=Z_E \tag{18.49}$$

This is exactly Einstein' gravitational-redshift equation (18.10) of light.

It is thus clear that, as the observation agent OA(η) is the optical agent OA(c), the GOR gravitational-redshift equation (18.38) of light strictly reduces to Einstein's the gravitational-redshift equation (18.10) of light.

Under the idealized agent OA_∞, photon momentum is classical momentum:

$$\lim_{\eta\to\infty} K_{F\eta} = \lim_{\eta\to\infty}\left(\Gamma|_{\chi=0}-1\right)m_o\eta^2$$

$$=\lim_{\eta\to\infty}\left(\frac{1}{\sqrt{1-c^2/\eta^2}}-1\right)m_o\eta^2=\frac{1}{2}m_oc^2 \tag{18.50}$$

Thus, as $\eta\to\infty$, under the idealized agent OA_∞, it holds true that:

$$\lim_{\eta\to\infty} Z_{\mathrm{GOR}} = \lim_{\eta\to\infty}\frac{\left(m_o\eta^2\big/\sqrt{g_{00}(r_\mathrm{B})}\right)-\left(m_o\eta^2\big/\sqrt{g_{00}(r_\mathrm{A})}\right)}{K_{F\eta}-\left(1-1\big/\sqrt{g_{00}(r_\mathrm{B})}\right)m_o\eta^2}$$

$$=\lim_{\eta\to\infty}\frac{\left(1+GM/r_\mathrm{B}\eta^2\right)-\left(1+GM/r_\mathrm{A}\eta^2\right)}{c^2/2\eta^2+GM/r_\mathrm{B}\eta^2}$$

$$=\frac{2GMr_\mathrm{B}}{r_\mathrm{B}c^2+2GM}\left(\frac{1}{r_\mathrm{B}}-\frac{1}{r_\mathrm{A}}\right)=Z_N \tag{18.51}$$

This is exactly the real Newtonian gravitational-redshift equation (18.25) of light, i.e., Newton's gravitational-redshift equation of light.

It is thus clear that, as the observation agent OA(η) is the idealized agent OA_∞, the GOR gravitational-redshift equation (18.38) of light strictly reduces to Newton's gravitational-redshift equation (18.25) of light, which is really based on Newton's classical mechanics and Newton's law of universal gravitation.

So, the gravitational-redshift theory of GOR has generalized and unified Newton's theory of gravitational redshift and Einstein's theory of gravitational redshift, indicating that the GOR gravitational-redshift equation of light is logically consistent with both Einstein's gravitational-redshift equation of light and Newton's gravitational-redshift equation of light. This confirms the logical self-consistency and theoretical correctness of the GOR gravitational-redshift equation, and also, confirm the logical rationality and theoretical correctness of the real Newtonian gravitational-redshift equation (18.25).

19 GOR and Gravitational Waves

Perhaps, compared with Einstein's three major predictions, including (i) the gravitational redshift of light, (ii) the gravitational deflection of light, and (iii) the abnormal precession of Mercury's perihelion, the **Gravitational Wave** is the most attractive and specious prediction of Einstein's theory of general relativity.

The theory of Gravitationally Observational Relativity (GOR) has no doubt about the existence of gravitational waves.

Actually, in a sense, just as electromagnetic field means electromagnetic waves, gravitational field means gravitational wave; just as electromagnetic interaction employs photons as the mediated meson to transmit electromagnetic force, gravitational interaction employs gravitons as the mediated meson to transmit gravitational force.

However, this does not mean that Einstein correctly predicted gravitational waves, nor that the Laser Interferometer Gravitational-Wave Observatory (LIGO) of the United States really have detected gravitational waves [161,162].

As a matter fact, both Newton's theory of universal gravitation and Einstein's the general relativity has no the prior information or knowledge about the speed of radiation in their axiom systems or logical premises. As stated in Sec. 12.1.1 **The Gravitational Locality** of Chapter 12, both Newton's gravitational theory and Einstein's gravitational theory imply an important idealized hypothesis: gravity is action at a distance; the speed of gravitational radiation is infinite, it takes no time for gravity to cross space.

Newton realized that such an idealized hypothesis was not in line with the objectively physical reality [163]: "… is to me such an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it." Einstein also did not believe that the physical world has action at a distance. However, with no prior information or knowledge about the speed of gravitational radiation, Newton had to take gravity or gravitational force as action at a distance in his law of universal gravitation. Likewise, Einstein's theory of general relativity also had no prior information or knowledge about the speed of gravitational radiation. So, both Newton and Einstein failed to take into account the locality problem of gravitational interaction in their respective gravitational theories. According to general relativity, in the case of weak gravitational fields, Einstein's gravitational-field equation is reduced to the Poisson-equation form of Newton's law of universal gravitation. We could not imagine that the speed of gravitational radiation would depend on the strength of gravitational fields: Is the weaker the gravity, the faster and even infinite the speed of gravity?

Actually, as stated in Sec. 12.1 **The Problem of the Locality of Gravitational Spacetime** of Chapter 12, the locality in Einstein's theory of general relativity is only the observational locality, not the locality of gravitational interaction.

So how did Einstein predict **Gravitational Waves**?

Based on his theory of general relativity, Einstein derived a wave equation, in which the wave function $\bar{h}_{\mu\nu}$ is defined with the curved metric $h_{\mu\nu}$, and $h_{\mu\nu}$ involves the speed c of light in vacuum and Newton's gravitational potential χ: $h_{\mu\nu} \sim \chi/c^2$. Accordingly, Einstein believed that $\bar{h}_{\mu\nu}$ must represent a gravitational wave, and the gravitational wave must propagate at the speed c of light.

This is Einstein's prediction of gravitational waves.

Einstein's prediction of gravitational waves has two problems:

(i) Does the wave function $\bar{h}_{\mu\nu}$ or the curved metric $h_{\mu\nu}$ really represent the gravitational potential χ or gravitational waves?

(ii) Is the speed of gravitational radiation or gravitational wave really the speed c of light in vacuum?

The theory of GOR will analyze Einstein's prediction of **Gravitational Waves**: to examine the true or false of Einstein's gravitational-wave function; to examine the true or false of LIGO's gravitational-wave detection.

19.1 Einstein's Prediction of Gravitational Waves

It is said that the concept of **Gravitational Wave** was first proposed by Einstein and Eddington. Of course, the prediction of gravitational waves was originated from the so-called gravitational-wave equation derived by Einstein based on his theory of general relativity [164,1655].

As for the prediction of gravitational waves, Einstein as always took the method of weak-field approximation as the linearization method for his theory of general relativity, by which the nonlinear gravitational-spacetime problem was simplified into the linear problem of weak-field approximation. By following the logic of weak-field approximation, as the metric of gravitational spacetime $g_{\mu\nu}(x^\alpha,c)$ is approximate to the Minkowski metric $\eta_{\mu\nu}$, the nonlinear Einstein field equation is approximate to the linear gravitational-field equation. In this way, Einstein could deduce his gravitational-wave equation.

As stated in Sec. 13.1.2 of Chapter 13, according to the factor γ (Eq. (13.16)) of spacetime transformation in Einstein's theory of general relativity, in order to make the metric $g_{\mu\nu}(x^\alpha,c)$ of gravitational spacetime under optical observation approximate the Minkowski metric $\eta_{\mu\nu}$: $g_{\mu\nu}(x^\alpha,c) \approx \eta_{\mu\nu}$, Einstein had to create the scene of weak gravitational field: $|\chi| << c^2$ and $|\gamma_i v^i| << c$.

In this way, the nonlinear metric $g_{\mu\nu}(x^\alpha,c)$ of gravitational spacetime could be linearized as the linear equation (13.1):

$$|\chi| \ll c^2 \text{ and } |\gamma_i v^i| \ll c:$$

$$g_{\mu\nu}(x^\alpha,c) = \eta_{\mu\nu} + h_{\mu\nu}(x^\alpha,c) \quad (|h_{\mu\nu}| \ll |\eta_{\mu\nu}|)$$

This is the condition of weak field in Einstein's method of weak-field approximation, where $\eta_{\mu\nu}$=diag(+1,–1,–1,–1) is referred to as the flat metric, and $h_{\mu\nu}$ is referred to as the curved metric ($h_{\mu\nu}$ and its derivative of each order are small quantities).

It is worth noting that the curved metric $h_{\mu\nu}$ in the condition of weak field was regarded by Einstein as the weak gravitational potential, that is, the so-called **subtle ripples** rising in the (nearly) flat spacetime ($g_{\mu\nu}(x^{\alpha},c)\approx\eta_{\mu\nu}$) due to the perturbation of weak gravitational radiation.

As stated in Sec. 13.1.3 **The Conditions of Weak-Field Approximation** of Chapter 13, Einstein's method of weak-field approximation has five hypothetical conditions: (i) the weak field, (ii) the slow speed, (iii) the static field, (iv) the spacetime orthogonality, and (v) the harmonic coordinates.

Reviewing Einstein's logical deduction of gravitational-wave equation based on the method of weak-field approximation will contribute to our recognition and understanding of Einstein's equation of gravitational waves and Einstein's prediction of gravitational waves, so that the theory of GOR could analogize and follow Einstein's logic of deducing the GOR information-wave equation. The theory of GOR will take Einstein's gravitational-wave equation as the analogical object of the GOR information-wave equation.

Most of the books or literature introducing Einstein's theory of general relativity has the content of Einstein's equation of gravitational waves and Einstein's prediction of gravitational waves. The following introduction and description about Einstein's equation of gravitational waves is in line with the original appearance of Einstein's theory of general relativity, which mainly refers to the literature of Zhao and Liu Liao [130,166], and the literature of Henry [167].

The left end of Einstein field equation (14.2) is usually defined or marked as the tensor $G_{\mu\nu}(c)$, i.e., the so-called Einstein tensor, and then Einstein's field equation can be simply written as:

$$G_{\mu\nu}(c) = -\kappa_E T_{\mu\nu}$$
$$\left(G_{\mu\nu}(c) \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R,\ \kappa_E = \frac{8\pi G}{c^4}\right) \tag{19.1}$$

where $R_{\mu\nu}$ is the Ricci tensor (i.e. the curvature of spacetime), R is the Gaussian curvature, $g_{\mu\nu}$ is the spacetime metric, $T_{\mu\nu}$ is the energy-momentum tensor, and κ_E is the coefficient of Einstein field equation.

According to Einstein's logic of weak-field approximation, under the conditions of weak-field approximation listed in Sec. 13.1.3 of Chapter 13, the Ricci tensor $R_{\mu\nu}$ and the Gaussian curvature R are approximated as:

$$\begin{cases} R_{\mu\nu} = \dfrac{1}{2}\Box h_{\mu\nu} \\ R = \dfrac{1}{2}\Box h \end{cases} \quad \left(R \equiv R^{\mu}_{\mu} = \eta^{\mu\nu}R_{\mu\nu}\right) \tag{19.2}$$

Define the **Metric-Perturbation** tensor:

$$h^{-}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h \quad \left(h \equiv h^{\mu}_{\mu} = \eta^{\mu\nu}h_{\mu\nu}\right) \tag{19.3}$$

where the tensor $h^{-}_{\mu\nu}$ of metric-perturbation is defined with the curved metric $h_{\mu\nu}$ in the condition (Eq. (13.1)) of weak field: $h_{\mu\nu}\sim\chi$.

In Einstein's theory of general relativity, the so-called metric perturbation means that, under the condition of weak field, $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ **(**$|h_{\mu\nu}|<<1$**)**, the curved metric $h_{\mu\nu}$ represents the Newtonian gravitational-potential χ and is a small quantity; the Minkowski metric $\eta_{\mu\nu}$ represents the flat spacetime perturbed by a weak Newtonian gravitational potential, rising subtle ripples.

Thus, under the conditions of weak-field approximation, the Einstein tensor $G_{\mu\nu}(c)$ can be defined with the metric-perturbation tensor $h^{-}_{\mu\nu}$, and Einstein's field equations (19.1) can be expressed with the metric-perturbation tensor $h^{-}_{\mu\nu}$:

$$G_{\mu\nu}\equiv\frac{1}{2}\Box h^{-}_{\mu\nu}=-\kappa_E T_{\mu\nu} \tag{19.4}$$

Then, the corresponding condition of harmonic coordinates is reduced to:

$$h^{-}{}_{\mu\alpha}{}^{,\alpha}=0 \tag{19.5}$$

According to Einstein's theory of general relativity, the metric $g_{\mu\nu}$ of gravitational spacetime under the optical observation agent OA(c) depends on the spacetime coordinate x^{α} (α=0,1,2,3) and the speed c of light in vacuum: $g_{\mu\nu}=g_{\mu\nu}(x^{\alpha},c)=g_{\mu\nu}(t,x^{i},c)$ (i=1,2,3), where the time axis $x^0=ct$ (t is the time coordinate). Likewise, the curved metric $h_{\mu\nu}$ and the metric-perturbation tensor $h^{-}_{\mu\nu}$, as well as, the energy-momentum tensor $T_{\mu\nu}$, also depend on the spacetime coordinate x^{α} (α=0,1,2,3) and the speed c of light in vacuum: $h_{\mu\nu}=h_{\mu\nu}(t,x^{i},c)$, $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(t,x^{i},c)$, and $T_{\mu\nu}=T_{\mu\nu}(t,x^{i},c)$.

The field equation (19.4) that meets the condition of harmonic coordinates in Eq. (19.5) has the following solution:

$$h^{-}_{\mu\nu}\left(t,x^{i},c\right)=-\frac{\kappa_E}{2\pi}\int\frac{T_{\mu\nu}\left(t-\frac{|x^{i}-x'^{i}|}{c},x'^{i},c\right)}{|x^{i}-x'^{i}|}\mathrm{d}^3x'^{i} \tag{19.6}$$

where the integral domain is the 3d space occupied by the gravitational source, x^{i} (i=1,2,3) is the coordinate of the observer, and x'^{i} (i=1,2,3) is the coordinate of matter distribution, $|x^{i}-x'^{i}|$ is the distance between the observation coordinate x^{i} and the coordinate x'^{i} of gravitational matter.

Equation (19.6) is the retarded integral formula of the metric-perturbation tensor $h^{-}_{\mu\nu}$, also known as the delayed solution of $h^{-}_{\mu\nu}$. Here, the so-called **retard** or **delay** has important implications and is the manifestation of the principle of locality: it takes time for energy or information to cross space. Therefore, for the object being acted on, energy is only a retarded or delayed force or matter interaction; for the object being observed, information is only a retarded or delayed physical signal.

In the solution of Einstein field equation, whether in Einstein's approximate solution (Eq. (15.3)) or in Schwarzschild's exact solution (Eq. (15.7)), the curved metric $h_{\mu\nu}$ is related with the Newtonian gravitational potential χ: $h_{\mu\nu}\sim\chi$. As Zhao observed [166]: "In essence, $h^{-}_{\mu\nu}$ is $h_{\mu\nu}$. In Einstein's view, the metric-perturbation tensor $h^{-}_{\mu\nu}$ is just gravitational radiation, representing the gravitational field.

Accordingly, Einstein had reason to believe: the retarded or delayed integral formula (Eq. (19.6)) of the metric-perturbation tensor $h^{-}_{\mu\nu}$ meant that, at the coordinate x^i of time t, the gravitational field $h^{-}_{\mu\nu}(t,x^i,c)$ or gravitational potential $\chi(t,x^i,c)$ is decided by the distribution and motion of the gravitational matter ($T_{\mu\nu}$) at the time $t-|x^i-x'^i|/c$; it followed that, the gravitational radiation propagates at the speed c of light in vacuum. At this point, the speed of gravitational radiation seemed to be affirmed, but the concept of Gravitational Wave had not yet formed.

Under the conditions of weak-field approximation, the energy-momentum tensor $T_{\mu\nu}$, which serves as the gravitational source, is approximately zero: $T_{\mu\nu}\approx 0$, and thus, the field equation (19.4) could be reduced to the form of vacuum field equation: $\Box h^{-}_{\mu\nu}(c)=0$. From the definition of the d' Alembert operator "□" in the optical agent OA(c) and the definition of the Laplace operator "∇^2" (see Sec. 5.5 in Chapter 5 and Eq. (5.29)), it follows that:

$$\begin{aligned} &\Box h^{-}_{\mu\nu}(c)=0 \qquad \left(\Box=\partial^2/c^2\partial t^2-\nabla^2\right) \\ &\text{or} \quad \nabla^2 h^{-}_{\mu\nu}(c)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}h^{-}_{\mu\nu}(c)=0 \end{aligned} \tag{19.7}$$

where, under the case of weak gravitational field, $\Box\approx-\nabla^2$.

This is Einstein's wave equation.

Hendry observed [167]: "This is a key result."

Equation (19.7) is the mathematical form of waves: a typical **wave equation** where the metric-perturbation tensor $h^{-}_{\mu\nu}$ is the wave function, and the speed c of light in vacuum is exactly the wave velocity of $h^{-}_{\mu\nu}(c)$. Similar to the case of retarded or delayed integral formula (Eq. (19.6)), since the connection between $h^{-}_{\mu\nu}(c)$ and the Newtonian gravitational potential $\chi=-GM/r$, according to Eq. (19.7), Einstein took it for granted that the wave of $h^{-}_{\mu\nu}(c)$ represented the gravitational wave travelling at the speed c of light in vacuum.

Thus, the concept and prediction of **Gravitational Wave** were born.

Later, by analogy with the quantization of electromagnetic wave, gravitational wave is also quantized; by analogy with the photon as the mediated meson of electromagnetic interaction, physicists have conceived the concept of **Graviton**, and assume that gravitons are the mediated mesons transmitting gravitational force or gravitational interaction. According to Einstein's theory of special relativity, the rest mass of a matter particle moving at the speed c of light should be zero. So, the graviton is hypothetically an elementary particle with no rest mass. At the same time, the graviton must be a spin-2 boson for the gravitational source is the stress-energy tensor (a second-order tensor).

However, Einstein's theoretical prediction of gravitational waves seems to have been wrong from the beginning.

19.2 LIGO Project: Detecting Gravitational Waves

Based on the principle of positivism in physics, after Einstein made the prediction of gravitational waves in theory, physics has launched a labor-intensive, time-consuming and money-consuming campaign of detecting gravitational waves.

The gravitational wave is regarded as the last missing piece of the puzzle in the demonstration of Einstein's theory of general relativity.

In 2015, LIGO Scientific Collaboration announced that LIGO had detected gravitational waves for the very first time.

In 2017, the main members of LIGO team, Weiss, Barish and Thorne, won the Nobel Prize in Physics for detecting gravitational waves. At this point, LIGO seemed to be brought to successful completion of verifying Einstein's prediction of gravitational waves. Therefore, the sacred and inviolable status of Einstein's theory of general relativity and even the whole theoretical system of Einstein's theory of relativity was further consolidated.

Reviewing the history of gravitational-wave detection and reexamining LIGO's project of detecting gravitational waves will contribute to our recognition and understanding of LIGO's principle and scheme of gravitational-wave detection as well as LIGO's conclusion of gravitational-wave detection. In this way, the theory of GOR will reveal the true or false of LIGO's gravitational waves.

19.2.1 The Early Phase: the Principle Scheme of Gravitational-Wave Detection

Originally, detecting gravitational waves seems to be extremely easy.

We are detecting and utilizing gravitational waves every day. Electromagnetic field means electromagnetic radiation, which is also known as electromagnetic waves; gravitational field means gravitational radiation, which is now known as gravitational wave. According to the retarded integral formula (19.6) and wave equation (19.7) of the metric-perturbation tensor $h^{-}_{\mu\nu}(c)$, gravitational radiation or gravitational wave is produced by the energy-momentum tensor $T_{\mu\nu}$: matter or energy must radiate gravity or gravitational waves. The distribution of the earth's matter leads to form a nearly spherically-symmetric gravitational field. All matter objects on the surface of the earth are affected by gravity or gravitational waves of the earth, exhibiting the corresponding gravitational effects. As we weigh an apple with a scale, we are detecting the gravity radiated by the earth. In this way, we could claim: "We have detected the gravitational wave from the earth!" It is more realistic than the gravitational waves detected by LIGO.

However, detecting gravitational waves also seems to be extremely difficult.

Compared with the observers of the earth, such as LIGO, the gravitational field of the earth seems to be too quiet and lack flows or ripples. The gravitational radiation or gravitational wave of the earth acting on the apple is invisible and intangible, which is difficult to image or directly detect. Perhaps, fast-moving dense stars or black holes could produce the gravitational radiation or gravitational wave extremely strong enough to perturb or disturb the quiet gravitational field of the earth. Thus, we could devise ways to detect the gravitational radiation or gravitational wave from outer space.

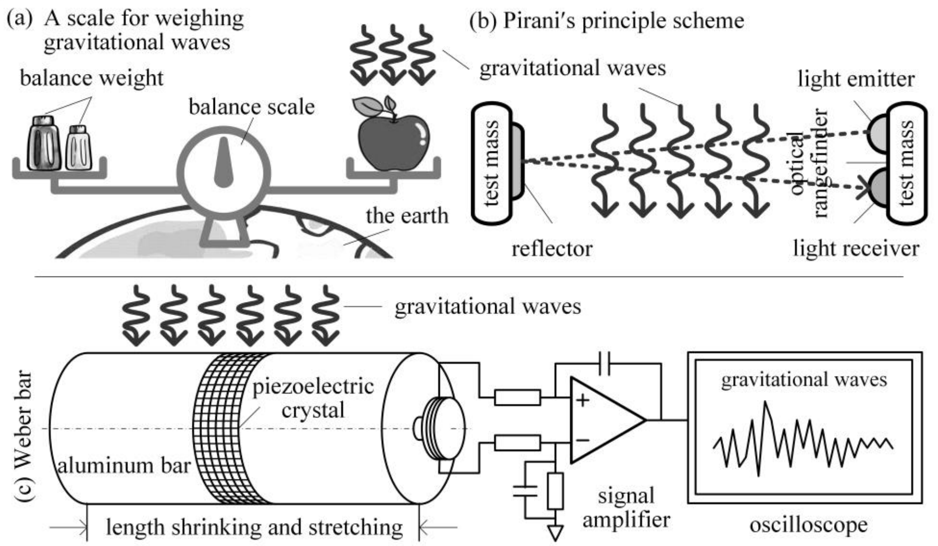

Figure 19.1 The Principle Scheme of Gravitational-Wave Deflection. (a) A scale for weighing gravitational waves: Perhaps, by making use of a scale, you could weigh gravitational waves just like your weighing an apple. (b) Pirani's principle scheme: Based on Einstein's theory of general relativity, Pirani proved that gravitational wave could shrink and stretch space; therefore, as long as you have an optical rangefinder, you could measure the shrinking and stretching of Pirani length, and in this way, you could detect gravitational waves. (c) Weber bar: Inspired by Pirani's theory, Weber made the so-called Weber bar, an antenna for detecting gravitational waves; Weber believed that the shrinking or stretching of space would shrink or stretch the Weber bar, which could be detected by the piezoelectric sensors around the Weber bar and converted into electrical signals, so that you could detect and discover gravitational waves.

As depicted in Fig. 19.1(a), put an apple at one end of the balance scale and the balance weight with the mass equal to the apple at the other end of the scale. Then, in the nearly static gravitational field of the earth, the scale keeps balance. To improve resolution and sensitivity, it is assumed that the arms of the scale are long enough. If the gravitational radiation or gravitational wave from outer space swept past the apple from top to bottom, then the weight of the apple must be changed slightly, and the scale would lose its balance, and even exhibit vibrations. In this way, one could observe and detect the waves of gravitational radiation that is, gravitational waves.

It is a good idea to make use of a balance scale to **weigh** gravitational wave – Russian scientist Mikhail Lomonosov ever made use of a balance scale to **weigh** the law of conservation of matter. However, experimental physicists would think it foolish to use a balance scale to **weigh** gravitational waves: the gravitational interaction between matter is too weak, only the $1/10^{36}$ of electromagnetic force; even though there are extremely strong gravitational-radiation sources in outer space, due to the distance far from the earth and the law of inverse square, they could have minimal perturbations on the apple on the scale. So, it would be impossible to weigh gravitational radiation or gravitational wave with a scale.

However, LIGO's Gravitational-Wave Observatory is exactly a balance scale.

Einstein's specious gravitation waves has exacerbated the complexity of detecting gravitational waves. Since the establishment of Einstein's theory of general relativity and the birth of the prediction of gravitational waves in 1916 [168], physicists had not put forward a specific scheme for detecting gravitational waves in over 30 years. Until 1955, British physicist Pirani proposed [168] that the detection of gravitational radiation or gravitational wave should be based on Riemann tensor: although it is difficult to detect gravitational waves with a single test object (such as an apple on a balance scale as shown in Fig. 19.1(a)), gravitational radiation or gravitational wave might be detected or discovered by observing the change of the spatial distance between the two small-mass objects as depicted in Fig. 19.1(b) (the interaction between them might be negligible). Based on Einstein's theory of general relativity, Pirani proved that the spatial distance between two objects would change as gravitational radiation or gravitational wave was sweeping past: the waves of gravitational radiation in different directions would lead to either expansion or contraction of the spatial distance.

It should be pointed out that, until today, Pirani's scheme has still been the principle scheme of gravitational-wave detection system, and is even being adopted by LIGO's detection system.

Inspired by Pirani's idea and scheme of detecting gravitational waves, Weber of the University of Maryland in the United States carried out the actual detection of gravitational waves [169]. Weber designed and made the so-called gravitational-wave antenna for detecting gravitational waves, which was later known as **resonant bar detector**, or **Weber bar**. A Weber bar is a solid cylinder made from aluminum, about 1 to 2 m in length and 0.2 to 1 m in diameter. According to Pirani's theory, Weber believes that, as depicted in Fig. 19.1(c), as gravitational radiation or gravitational wave disturbed the gravitational-wave antenna, the Weber bar would shrink or stretch along the lengthwise direction of its own; as the gravitational wave frequency is consistent with the resonant frequency of the Weber bar, such shrinking or stretching effect could be detected by the piezoelectric sensors around the Weber bar and converted into electrical signals. After being amplified by an electronic amplifier, the images of gravitational wave could be displayed on the oscilloscope.

It should be pointed out that, there was the difference between Weber's view of gravitational-wave detection and Pirani's.

According to Einstein's theory of general relativity [8], gravitational spacetime would shrink or stretch: space might shrink or stretch; time might also shrink or stretch, and the shrinking or stretching of time likely meant the change of motion speed or direction of matter (including light or photons) in gravitational spacetime. Pirani believed that [168], gravitational radiation or gravitational wave would lead to the shrinking or stretching of space; Weber believed that [169], gravitational radiation or gravitational wave would lead to the shrinking or stretching of material objects. Of course, you can think that the shrinking or stretching of space is equivalent to the shrinking or stretching of material objects: space shrinking (stretching) shrinks (stretches) material objects. That was exactly what Weber thought.

In 1968, Weber announced that his Weber bar had detected gravitational wave [170,171]. However, many later repeated experiments with copied Weber bar failed to detect gravitational waves. The final conclusion was that the sensitivity of Weber bars was not high enough to detect the gravitational radiation or gravitational wave that Weber claimed to have detected.

Weber bars were too short and not easy to lengthen, and therefore, the shrinking and stretching effects were extremely limited, which was the fatal disadvantage of Weber's resonant bar detectors.

19.2.2 Binary Pulsars and Gravitational Waves

The development of radio astronomy has broadened the horizon of human beings, which has extended the electromagnetic-wave spectrum of human perceiving the objective world from the frequency band of visible light to almost the whole band of electromagnetic waves. The four major discoveries of astronomy in the 20th century: quasars, pulsars, interstellar molecules, and the cosmic microwave background radiation, are exactly the achievements of radio astronomy.

However, unlike optical astronomy that employs eyes to see or observe visually, radio astronomy relies half on listening and half on guessing, which is often specious, and makes one half believe and half doubt.

From Little Green Men to Pulsars

To detect gravitational waves, we must have massive compact stars acting as the radiation source of gravitational waves.

In 1967, Jocelyn Bell, a PhD student in the Cavendish Laboratory of the University of Cambridge in the United Kindom employed a radio telescope to have observed a star in the constellation Vulpecula that continuously emitted periodic electromagnetic-pulse signals, once every 1.33 s [172]. After calculation and analysis, Bell and her supervisor, Antony Hewish, decided that it is an unknown celestial body, and named it **Pulsar** [173]. In this way, Bell and Hewish discovered the first pulsar: PSR1919+21. In 1974, Hewish became the first astronomer to win the Nobel Prize in Physics for discovering pulsars [174].

Astronomers believe that, a pulsar is a highly magnetized and rotating neutron star that emits a beam of electromagnetic radiation, whose mass is enormous, second only to a back hole. Thus, pulsars are imagined as the ideal source of gravitational waves for testing and verifying Einstein's prediction.

However, it is still worth thinking about: Do pulsars really exist?

Both optical telescopes and radio telescopes can not directly and visually identify whether a star is a neutron star, let alone to identify its rotating at a high speed. Astronomers can only guess what the periodic electromagnetic-pulse signals detected by radio telescopes represent. However, such kind of guesswork is difficult to be verified and seems to only stay at the level of guesswork, which can only be indirectly verified by the matching of the observational data and the mathematical model of pulsars.

The universe is vast and complex with all kinds of possibilities.

At the beginning, astronomers even believed that those electromagnetic pulses were the signals sent by aliens to the earth, and therefore, the PSR1919+21 was nicknamed **LGM-1** (for **Little Green Men**).

Before the PSR1919+21, Neutron stars were just a class of hypothetical celestial bodies, which existed in the form of mathematical model in the computer database. After the PSR1919+21, the calculation of computer simulation showed that the host of the periodic electromagnetic-pulse signals conforms to the hypothetical neutron star, which is small in size, high in density, large in mass, and rotates at a high speed. Thus, pulsars become the product of the hypothesis of neutron stars, in turn, become the evidence of the hypothesis of neutron stars. The hypothesis of pulsars and the hypothesis of neutron stars constitute circular reasoning that seems to be quite suspicious logically.

In any case, pulsars, or neutron stars rotating as high speeds, provide an option for interpreting the periodic electromagnetic-pulse signals detected by radio telescopes. It is said that the observatories around the world have already found more than 2000 pulsars. In 2016, China built the world's largest radio telescope: the Five-hundred-meter Aperture Spherical radio Telescope (FAST). It is reported that FAST has already found over 800 pulsars up to Jul, 2023. Perhaps, more strictly, FAST has detected more than 800 periodic Electromagnetic-pulse signals, not necessarily neutron stars or pulsars.

From Pulsars to Binary Pulsars

A **Binary Pulsar** is a binary-star system: a pulsar with a binary companion, often a white dwarf or neutron star, or even another pulsar. A binary-star system with two pulsars is referred to as a **Double Pulsar**.

In 1974, the year when Bell's supervisor Hewish won the Nobel Prize in physics, Hulse, a PhD student at the University of Massachusetts in the United States, was assigned by his supervisor Taylor to participate in the pulsar detection at Arecibo Observatory in Puerto Rico, where there was the radio telescope with the largest aperture (305 m) in the world at that time. One day, Hulse detected a weak periodic electromagnetic-pulse signal. It was certain that a new pulsar had been discovered.

However, unlike the general pulsars, the pulse period of this pulsar presented periodic variation. Hulse conjectured that this might be because the pulsar was moving around a companion star to have formed Doppler effect. Hulse reported his findings and conjecture to his supervisor Taylor. So, Taylor flew to Arecibo observatory, and together with Hulse, built the mathematical model of the binary pulsar, it was calculated that [175]: the period of the pulsar's orbit around the companion star was about 7.75 hours, the maximum speed of the pulsar was about 300 km/s, the average speed of the companion star was about 200 km/s, and the average distance between the pulsar and the companion star was equivalent to the solar radius. In this way, Hulse and Taylor discovered the first case of binary pulsars [176]: PSR1913+16.

At first, Bell and Hewish speculated that the periodic pulse signal they detected came from a rotating celestial body or star. Now, Hulse and Taylor speculated that the periodic variation of the pulse period was due to the revolution of a pulsar around a companion star, which had the similar logic to Bell and Hewish's.

Later on, according to Einstein's theory of general relativity [177-179], it was asserted that, in theory, a binary pulsar would lose its energy due to gravitational radiation, and therefore, the PSR1913+16's orbit would gradually precess, and the orbital semi-major axis and pulse period of it would be gradually shortened. Thus, professor Taylor has been continuously observing PSR1913+16 for decades [180]. It is said that the observed values are consistent exactly with the theoretical values, with the difference of only 0.4%. It seems that Hulse and Taylor not only discovered binary pulsars, but also indirectly verified the existence of gravitational waves [181,182]. Everything seems to be developing in the direction of supporting Einstein's theory of general relativity and Einstein's prediction of gravitational waves.

In 1993, Hulse and Taylor won the Nobel Prize in Physics for PSR1913+16.

It is reported that the observatories around the world have already detected more than 150 cases of binary pulsars so far: the PSR1913+16 is the first case and has been continuously observing for decades [180]. Some believe that the PSR1913+16 provides the most accurate test for Einstein's theory of general relativity so far.

Likewise, it is still worth thinking about: Do binary pulsars really exist?

Just as both neutron stars and pulsars are only conjectures in essence, binary pulsars are also a sort of conjecture in essence. Whether with optical telescopes or with radio telescopes, it is impossible to visually identify a binary pulsar. Hulse and Taylor could only guess why the pulse period of PSR1913+16 changed periodically. No matter how well the observed data of PSR1913+16 matches the mathematical model, it does not mean that binary pulsars really exist. Actually, according to the theory of GOR, it is the **exact match** between the observed data of PSR1913+16 and the theoretical values of mathematical model that shows the PSR1913+16 as a binary pulsar has many doubts about the precession of its orbit, the shortening of its semi-major axis, and the periodic variation of its pulse period.

Of course, in any case, binary pulsars provide an option for interpreting the periodic variation of the PSR1913+16's pulse period.

19.2.3 LIGO Detector and the Principle of Detection

LIGO, the Laser Interferometer Gravitational-Wave Observatory of the United States, has the mission of detecting gravitational radiation or gravitational wave, intended to verify Einstein's prediction of gravitational waves.

The prototype of LIGO is made by professor Rainer Wesis at the Massachusetts Institute of Technology in the United States.

In the 1970s, Wesis developed a laser interferometry to detect gravitational waves. Actually, Weiss and LIGO's detectors are similar to the Michelson Interferometer that was used in the Michelson-Morley experiment to detect **ether** in 1887 [2]. Although Michelson and Morley did not find the so-called ether, the Michelson-Morley experiment led to the establishment of the principle of the invariance of light speed, and finally, led to the establishment of Einstein's theory of relativity, including the special and the general.

The basic structure of LIGO detection system is depicted in Fig. 19.2(a), where the basic principle of detecting gravitational waves was originated from Pirani's idea and scheme [108]: according to Einstein's theory of general relativity, Pirani believed that gravitational radiation or gravitational wave would shrink and stretch space, and therefore, testing the shrinking and stretching of the spatial distance between two objects (test masses) could detect and discover the gravitational radiation or gravitational wave sweeping past the test objects.

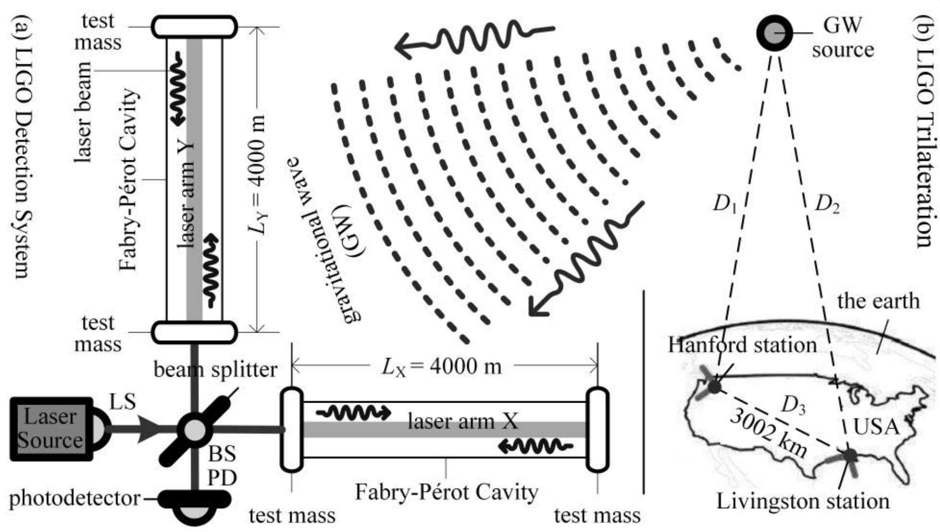

Figure 19.2 The Principle and Structure of LIGO Detector. (a) LIGO Detection System: (i) The laser source LS emits monochromatic laser with stable frequency; (ii) The beam splitter BS divides the laser into two beams, one entering the arm-X and the other entering the arm-Y; (iii) Each laser arm is 4000m long and has two test-mass bodies respectively placed at the both ends; (iv) There is the Fabry-Pérot cavity between the two test-mass bodies of each laser arm; (v) The two laser beams move back and forth repeatedly in the respective Fabry-Pérot cavity, so that the equivalent length of the laser arms could reach 1.12×10^6 m; (vi) Then, the two laser beams in the arm-X and arm-Y return to the beam splitter BS to form interference; (vii) The photodetector PD records and reports the interference effects. (b) LIGO Trilateration: LIGO has set up two base stations in USA, one is the LIGO Livingston and the other is the LIGO Hanford, so that LIGO could locate the source of gravitational waves based on trilateration method and then determine the speed of gravitational waves.

Actually, the basic principle of Weber's detection system, the Weber bar, was also originated from Pirani's idea and scheme. However, Weber bars were too short, and the physical effect of shrinking and stretching of space was extremely limited. So, it was very difficult for Weber bars to sense or detect the gravitational radiation or gravitational wave sweeping past Weber bars.

In structure, as depicted in Fig. 19.2(a), the LIGO detector imitates the Michelson interferometer: it has two perpendicular laser arms (X and Y), each of which has two test-mass bodies 4000 m apart from; a beam of laser moves back and forth repeatedly in the Fabry-Pérot Cavity between the two test-mass bodies, with the equivalent arm-length of 1.12×10^6 m, which is 5×10^5 times that of Weber bar. So, the sensitivity of LIGO detector to detect gravitational radiation and gravitational wave is unmatched by Weber's resonant bar detectors.

In working process, as depicted in Fig. 19.2(a), the LIGO detector also imitates the Michelson interferometer: the laser source LS emits a beam of monochromatic laser with stable frequency, which is divided by the beam splitter BS into two beams of the same intensity, one enters the arm-X and the other enters the arm-Y; after traveling 1.12×10^6 m in their respective Fabry-Pérot Cavity, the two laser beams return to the beam splitter BS to converge and form interference, either constructive interference or destructive interference, and the effect of laser interference will be recorded by the photodetector PD. According to Einstein's theory of general relativity, if gravitational radiation or gravitational wave sweeps past the LIGO detector, then one of the laser arm X or Y will shrink and the other with stretch, which makes the two laser beams exhibit an optical-path difference, and thus, the interference fringes will display on the screen.

In this way, the LIGO detector or detection system will record the relevant information about gravitational radiation or gravitational wave.

In 1991, the Massachusetts Institute of Technology and the California Institute of Technology, with the support of the National Science Foundation of the United States, officially started the joint construction of LIGO detection system. In order to improve the reliability of gravitational-wave detection, LIGO has built two base stations to detect gravitational radiation or gravitational wave: one is the LIGO Livingston, located in Livingston, Louisiana, USA, the other is the LIGO Hanford, located in Hanford, Washington, USA, with the distance of about 3002 km (equivalent to the optical path of about 10ms). Actually, as depicted in Fig 19.2(b), the real intention of LIGO to build the two base stations of gravitational-wave detection is to locate the source of gravitational waves based on trilateration method and then determine the speed of gravitational waves.

LIGO detection system was completed at the end of 1999. Perhaps because LIGO detection system after completion had failed to detect gravitational waves, LIGO detection system was upgraded from 2005 to 2007. The upgraded LIGO is called the Advanced LIGO, or aLIGO.

LIGO has gradually developed into an international scientific commonwealth for detecting gravitational waves: **LIGO Scientific Collaboration** (LSC). To accurately locate the gravitational wave source, at least three base stations of gravitational-wave detection have to be built on the earth surface. In 2007, the Virgo base station established by the European Gravitational Observatory (EGO) and the two base stations of LIGO began to be put into grid-connected operation. At the same time, all observatories around the world, including the optical and the radio, have participated in LIGO's gravitational-wave detection, that is, the so-called omni-directional and multi messenger detection of gravitational waves.

19.2.4 LIGO's Detection Targets

Naturally, detecting gravitational waves needs the source of gravitational radiation or gravitational wave to be the detection target.

We do not need to prove the existence of gravitational radiation or gravitational wave. The sea is a natural gravitational-wave observatory. As a gravitational-wave observatory, the sea's detection targets are the matter systems which are massive enough to radiate sufficiently strong gravitational waves, such as the moon and the sun. The tides of the sea have already told us that all celestial bodies, including stars, planets and satellites, are radiating gravity or gravitational waves.

According to Newton's law of universal gravitation, all matter bodies attract each other: any matter body, such as the sun, the earth, the moon, an apple, or even a photon, is the source of gravitational radiation or gravitational wave.

The mass of an apple is small and cannot be compared to a massive celestial object. However, an apple can be unlimitedly close to the LIGO detector.

Theoretically, according to the law of inverse square, if an apple is sufficiently close to the LIGO detector, then the intensity of gravitational radiation or gravitational wave of it would be fully amplified, and thus, the LIGO detector would be able to detect the gravitational wave radiated by the apple. For the LIGO detector, the mass of apples may be too small, which might be replaced by a train. Suppose that a train is traveling closely against one of the LIGO laser arms, then the gravitational wave radiated by the train might be strong enough to perturb or disturb the spacetime around LIGO. So, could LIGO detect the gravitational radiation or gravitational wave emitted by the train?

All in all, there are so many gravitational-wave sources on the earth's surface for LIGO to test or detect. However, LIGO seeks far and neglects what lies close at hand: It has been doing its best to detect the gravitational radiation or gravitational wave from the distance stars in outer space. So, what kind of stars are suitable for LIGO detector to employ as its detection targets?

Naturally, LIGO must have made a lot of calculations on candidate targets.

Let us identify them one by one together with LIGO.

The first is the earth. Any matter system has the gravitational field of its own which radiates gravity or gravitational waves. Naturally, the earth is no exception: all matter objects on the earth's surface, including LIGO, are affected by the gravity or gravitational waves of the earth. LIGO detector is located on the earth's surface, in the gravitational field of the earth. So, why could not the earth's gravitational radiation or gravitational wave trigger the LIGO detector? The earth's gravitational field is approximate to a static spherically-symmetric gravitational field, which is approximately uniform, symmetrical, and equipotential everywhere on the earth's surface. As far as the spacetime around LIGO is concerned, the gravitational field of the earth is stationary or static. Therefore, if there is no the invasion of external gravitational radiation or gravitational wave, then there will be no the optical path difference between the two laser beams in the laser arms X and Y of the LIGO detection system, and thus, no interference fringe will be recorded by the photodetector PD of the LIGO laser interferometer. So, the gravitational radiation of or gravitational wave of the earth cannot trigger the LIGO detector.

The second is the moon, the celestial body closest to the earth. The mass of the moon is far greater than that of apples and trains. The moon's gravitational radiation or gravitational wave can trigger the tides of the sea. However, although the moon's gravitational field sweeps past and past the LIGO detection system, LIGO has not detected the gravitational radiation or gravitational wave of the moon. It seems that the strength of the gravitational field of the moon at a distance of 3.844×10^5 km meters apart from the earth is not enough to trigger the LIGO detector. Admittedly, compared to the apple and the train, the moon is too far from the earth.

The third is the sun, the fixed star closest to the earth. The mass of the sun is far greater than that of the moon, and therefore, the sun can radiate the gravity or gravitational waves that is far stronger than that of the moon and can also trigger the tides of the sea. However, the distance between the sun and the earth is much greater than the distance between the moon and the earth, and therefore, for the earth's oceans, the tidal force of the sun is only half that of the moon. So, compared to the moon, the sun is even less likely to trigger the LIGO detector.

The remaining candidates can only be the stars deep in outer space, which are far from the earth and must have far greater mass than the sun.

Perhaps, pulsars can be the detection targets of LIGO or the candidates of gravitational-wave sources. Pulsars are neutron stars with high density, large mass, and high-speed rotation. Theoretically, pulsars could radiate high-energy gravitational waves. However, pulsars seem to fail the test of LIGO candidate detection-targets. So, what about binary pulsars and double pulsars? Actually, pulsars, no matter binary pulsars or double pulsars, are too far away from the earth, the intensity of their gravitational radiation decays with distance according to the law of inverse square, and hence, it is impossible for them to trigger the LITO detector on the earth. As a matter of fact, even the gravitational field of black holes cannot trigger the LIGO detector on the earth's surface.

In addition to the factors of distance and mass, like the gravitational fields of the moon and the sun, relative to the earth or the LIGO detector, the gravitational fields of neutron stars and black holes are stationary and static: too calm or too quiet, it is also difficult for them to trigger the LIGO detector. Unless, as some astronomers have envisioned [177-179], two stars merge or coalesce to violently erupt matter and energy in the forms of gravitational radiation or gravitational wave.

Taylor's continuous observation of the Hulse-Taylor binary pulsar and the computer simulation show that [180], the orbital period of PSR1913+16 around the companion star is reduced by 76.5μs per year, and the semi-major axis of elliptical orbit is shortened by 3.5 m per year, indicating that the Hulse-Taylor binary pulsar will eventually merge or coalesce. LIGO must have established a lot of dynamical models of binary stars based on Einstein's theory of general relativity, and must have simulated the evolution of binary-star systems by means of supercomputers [183-186], which involves a new technology of computer application: **Numerical Relativity**. The so-called numerical relativity, based on Einstein's theory of relativity (including the special and the general), makes use of the numerical simulation on computer to deduce the motion of celestial bodies, for example, the merging or coalescing of binary stars. LIGO's computer simulations based on numerical relativity show that: as it merges or coalesces, a binary-star system, especially a double-blackhole system, would emit high-energy gravitational waves that could be strong enough to trigger the LIGO detector.

So, the LIGO team targeted the detection target of LIGO detector towards binary-star systems, especially double-blackhole systems, looking forward to their merging and violently erupting.

However, the problem is that:

(i) Are there really binary-blackhole systems in the universe?

(ii) Even if there are binary-blackhole systems in the universe, even if a binary-blackhole system would merge or coalesce on day, could the erupted matter or mass really be all transformed into the energy of gravitational radiation? Could the erupted gravitational wave really trigger the LIGO detector on the distant earth?

Relevant issues will be discussed in Sec.19.4.

19.2.5 LIGO Discovers Gravitational Waves

On Feb 11, 2016, LIGO officially announced that: at 5:51 (EST) on Sep 14, 2015, LIGO detected a gravitational-wave signal for the first time erupted by a binary-blackhole system during its merging or coalescing, which was named the GW150914 [161].

It is worth noting that, from the discovery of GW150914 by LIGO detector to the official announcement of this discovery, LIGO had experienced 150 days of silence. So, what was LIGO doing during those 150 days? Of course, LIGO was contemplating and guessing: what did the signal from the LIGO photodetector PD mean? LIGO needed time to identify the GW150914, and needed time to locate the signal source of the GW150914. So, how did LIGO affirm that the GW150914 was a gravitational wave, and how did LIGO affirm that the GW150914 erupted from a binary-blackhole system during its merging or coalescing?

The announcement of discovering GW150914 has drawn some questioning voices. There are extreme views that [187-189]: it is impossible for LIGO to detect the real gravitational wave, the GW150914 is just the noise that appears on the LIGO laser interferometer. The reason for questioning is not necessarily be sufficient, but listen to both sides and you will be enlightened. Whether the GW150914 does represent the gravitational wave is indeed worth discussing. At least, we should not aim it at the merging or coalescing of a binary-blackhole system from the beginning.

Optical telescopes represent optical astronomy, and radio telescopes represent radio astronomy. Now, some astronomers believe that LIGO detector has opened gravitational wave astronomy [42]. Of course, like radio astronomy, gravitational wave astronomy could not intuitively observe celestial phenomena, or visually see celestial bodies or stars with eyes like optical astronomy.

Therefore, as depicted in Fig. 19.3, LIGO’s gravitational-wave detection has to rely half on listening and half on guessing.

It could be affirmed that the twin detection systems, the LIGO Livingston and the LIGO Hanford, did hear the chirp from outer space at 9:50 (UTC) on Sep 14, 2015 [190,191]: GW150914. That should not be any noise: the probability of the same noise appearing successively in two detectors is extremely low.

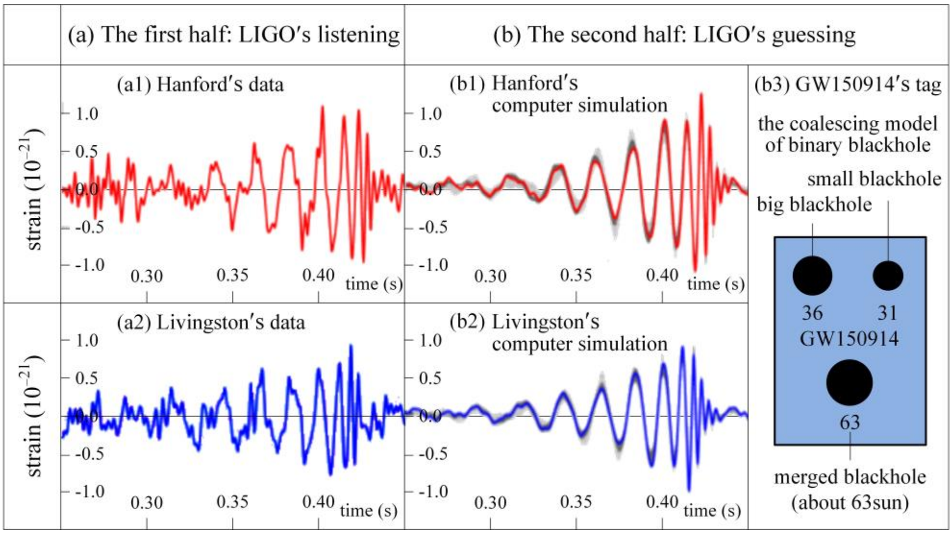

Figure 19.3 GW150914: Half Listening and Half Guessing. (a) The First Half: LIGO's listening, i.e., all the objective and real observation data that LIGO can provide – (a1) the observation data of LIGO Hanford; (a2) the observation data of LIGO Livingston. (b) The Second Half: LIGO's guessing, i.e., to locate the signal source of GW150914 based on LIGO's listening, to build the coalescing model of binary-blackhole systems based on Einstein's theory of general relativity, and to match the GW150914 with the appropriate model by computer simulation – (b1) the computer simulation based on the data of LIGO's Hanford; (b2) the computer simulation based on the data of LIGO Livingston; (b3) GW150914's tag: GW150914 had successfully been matched with a coalescing mode of binary-blackhole system located at about 1.3 billion light-years away from the earth. It is the basis for LIGO's conclusion that the observation data of LIGO is in good agreement with computer simulation. As a matter of fact, no matter how close the observation data is to the model or computer simulation, the conclusion might be specious: One false step will make a great difference.

Figure 19.3(a) is the first half of LIGO detection: **LIGO's listening**.

The data and curves in Fig. 19.3(a) are the **chirp** of the gravitational wave GW150914 heard by LIGO detector [192,193], where the horizontal axis represents time, and the vertical axis represents the vibration amplitude of gravitational wave: the strain of space distance distortion; the peak value of the strain of gravitational wave is 1×10^{-21}. Figure 19.3(a1) shows the strain curve of gravitational wave recorded by LIGO Hanford; figure 19.3(a2) shows the strain curve of gravitational wave recorded by LIGO Livingston.

The data and curves in Fig. 19.3(a) are relatively objective and real observation values of LIGO detector, and in a sense, they are the total of the empirical evidences of GW150914 that LIGO can provide.

The signal detected by LIGO Hanford is 7ms later than that detected by LIGO Livingston. LIGO believes that this is in line with Einstein's prediction that gravitational waves propagate at the speed of light.

However, LIGO still lacks sufficient empirical evidences to affirm that the GW150914 is a gravitational wave, and at the same time, still lacks sufficient empirical evidences to identify which bird chirped and where the bird chirped.

So, in addition to listening, LIGO has to rely on guessing.

Figure 19.3(b) is the second half of LIGO detection: **LIGO's guessing**.

Human beings have too many unknowns about the universe. In such a world of infinite possibilities, GW150914 means infinite options. However, LIGO aimed the GW150914 at a gravitational wave from the beginning, and envisioned it erupted from a binary-stars system, especially a binary-blackhole system due to its merging or coalescing [183-186]. LIGO spent 150 days to guess what GW150914 meant: based on the dynamical model of binary-star systems that derived from Einstein's theory of general relativity [194,195], started up the supercomputer of LIGO to carry out the numerical simulation on binary-star systems, and expected the GW50914 to be matched with a certain binary-star model.

This is much like a **ring toss** game on the streets: the GW150914 is the ring, and the countless binary-star modes in the computer database of LIGO are the dolls.

Table 19.1 The Binary-Blackhole Coalescing Mode Matched with GW150914 [192,193]

Physical Quantity	Guess Value	Error Upper-Limit	Error Lower-Limit
Primary black hole mass	36.2 sun	+5.2	–3.8
Secondary black hole mass	29.1 sun	+3.7	–4.4
Final black hole mass	62.3 sun	+3.7	–3.1
Final black hole spin	0.68	+0.05	–0.06
Luminosity distance	420 Mpc	+150	–180
Source redshift z	0.09	+0.03	–0.04
Energy radiated	3.0 sun	+0.5	–0.5

where **sun** is the unit of one solar mass, and **Mpc** is the unit of luminosity distance.

Notes: (i) Both the primary and secondary stars are black holes. (ii) Luminosity distance: calculate the distance of a matter object based on the law of inverse square and the observed luminous flux and intrinsic luminosity, which needs relativistic correction due to the redshift of light required; this means that LIGO relies on the optical agent OA(c) to calculate the distance of gravitational-wave sources. (iii) The radiated energy of the merging or coalescing of binary-blackhole systems takes the unit of mass, which means that LIGO applies Einstein formula $E=mc^2$ to calculate the energy of gravitational wave; actually, the theory of OR has already clarified that the energy of matter cannot be simply converted by the mass m and the light speed c, just as the nuclear energy released by an atomic bomb is the release of atomic bound energy, rather than the conversion of atomic mass. (iv) It should be pointed out that even if a merging or coalescing event of binary-blackhole system really occurs, the eruption of its mass and energy could take various forms, including electromagnetic radiation, strong radiation and weak radiation, and even pure matter ejection, rather than just gravitational radiation or in the form of gravitational waves; in particular, the energy of gravitational radiation energy or gravitational waves might be only a tiny part of it.

As a matter of fact, before the GW150914 was detected, LIGO had already prepared and stored countless binary-star dolls in the computer database of its own: there must be one suitable for the GW150914. In the end, the GW150914 fell over a coalescing model of binary-blackhole system, which consisted of two blackholes, with the total mass of 67sun, located 1.3 billion light years away from the earth (as depicted in Fig. 19.3(b)).

As shown in Tab. 19.1, according to the coalescing model of binary-blackhole system matched with GW150914, the signal GW150914 detected by LIGO detector originated from an event of binary-blackhole coalescing, $1.3_{-0.55}^{+0.60}$ (0.75-1.9) billion light-years away from the earth, in which the primary star has the mass of 36.2sun, the secondary star has the mass of 29.1sun, the merged black hole has the mass of 62.3sun, and the erupted energy is about 3.0sun. As depicted in Fig. 19.3(b), the expected strain curves could be calculated by computer simulation based on numerical relativity [192,193]. LIGO believes that the expected strain curves predicted by the coalescing models of binary-blackhole system are in good agreement with the strain curves of GW150914 recorded by LIGO Livingston and LIGO Hanford. However, we are not very clear about the criteria for such agreement. Actually, no matter how close the observational data is to the model or computer simulation, the conclusion might be specious: One false step will make a great difference.

After the GW150914, LIGO successively detected the second signal of the gravitational wave GW151226 (03:38:53 UTC on Dec 26, 2015) and the third signal of the gravitational wave GW170104 (10:11:58.6 UTC on Jan 4, 2017). Like the GW150914, the GW151226 and GW170104 were matched with the respective coalescing model of binary-blackhole system: the GW151226 is about 1.4 billion light-years away from the earth and has the total mass of 22sun; the GW170104 is about 3.0 billion light-years away from the earth and has the total mass of 51sun.

For their contributions to LIGO and the gravitational-wave detection, Weiss (an honorary professor of MIT), Thorne and Barish (professors of California Institute of Technology) jointly won the Nobel Prize in Physics in 2017.

At 10:30:43 UTC on Aug 14, 2017, for the first time, LIGO Hanford and LIGO Livingston, as well as the Virgo base station, jointly detected the gravitational-wave signal [196]: GW170817, matching the coalescing model of binary-blackhole system about 1.8 billion light-years away from the earth and has the total mass of 56sun.

At 12:41:04 UTC on Aug 17, 2017, LIGO and Virgo jointly detected the gravitational-wave signal [197]: GW170817, successfully matching the coalescing model of binary neutron stars for the first time, about 85 million light-years away from the earth, with about the total mass of 3sun.

From GW150914 to GW200322 (no later data can be found), LIGO and LSC announced that 91 gravitational-wave signals had been detected in four and a half years, and claimed that these signals were all from double-star coalescing events, including 84 cases of binary blackhole coalescing, 2 cases of binary neutron star coalescing, 5 cases of black hole and neutron star coalescing, and 1 case of either black hole and neutron star coalescing or binary neutron star coalescing. On average, LIGO can detect one coalescing event of two blackholes every 20 days.

The binary-blackhole systems matched by LIGO gravitational-wave signals are mostly located within the range of 1-5 billion light-years away from the earth. According to observation and simulation calculation [180], the Hulse-Taylor binary pulsar PSR1913+16 is precessing and will merge or coalesce in about 300 million years. If a binary-blackhole system had the lifespan of 300 million years, then there should be far more existing binary-blackhole systems in the universe than we could imagine, and moreover, new binary-blackhole systems must have been evolving continuously (perhaps, at least one binary-blackhole system per 20 days).

19.2.6 How does LIGO Determine the Speed of Gravitational Waves

According to the retarded integral formula (19.6) and wave equation (19.7) of the metric-perturbation tensor $h_{\bar{\mu}\nu}(c)$ derived from Einstein's theory of general relativity, Einstein made his prediction of gravitational waves: matter systems radiate gravitational waves, and the speed of gravitational waves is exactly the speed c of light in vacuum [164,165].

However, it is one thing to predict the speed of gravitational waves theoretically or mathematically, and another thing to measure the speed of gravitational waves observationally or experimentally.

LIGO announces that gravitational waves have already been detected by LIGO detector. However, LIGO seems to have never explicitly given its conclusion on the speed of gravitational waves. As a matter of fact, LIGO has never officially or really measured the gravitational-wave speed, and LIGO's so-called trilateration method seems to be ineffective and unreliable.

Therefore, LIG has to indirectly calculate and conjecture the gravitational-wave speed κ according to the so-called **multi-messenger** data [198,199].

Originally, LIGO wished to measure the speed of gravitational waves by taking advantage of the trilateration method, as depicted in Fig. 19.2(b). However, all the sources of LIGO gravitational waves are too far away and their directions and distances cannot be accurately determined. Even though the participation of the Virgo base station in Italy and the KAGRA base station in Japan has improved the positioning accuracy of gravitational-wave sources, it is difficult to meet the requirements of accurately measuring the gravitational-wave speed. In particular, if the gravitational-wave speed κ is not the speed c of light predicted by Einstein [8], but the $\kappa > 7\times10^6 c$ predicted by Laplace [43], then the calibration or correction of the time between different base stations will become a major obstacle to accurately determine the gravitational-wave speed κ. After LIGO had detected the GW150914, some LIGO members once said that whether the gravitational-wave speed is the speed of light remains to be further verified.

Later, all observatories around the world participate the LIGO detection of gravitational waves, form the league of so-called multi-messenger astronomy, and have detected the so-called **electromagnetic counterparts** of gravitational waves, which seems to have provided LIGO with a new way to infer and determine the speed of gravitational waves. It is worth noting that such electromagnetic counterparts not only appeared in the events of binary neutron-star coalescing [198,199], but also in the events of binary black-hole coalescing [200].

Only 0.4s after LIGO had detected the first signal of the gravitational-wave GW150914, the Fermi gamma-ray Burst Monitor detected the gamma-ray burst that seemed to come from the same radiation source of GW150914 [201,202]. Although some physicists believe that the merging or coalescing of binary-blackhole systems could not radiate electromagnetic matter, the eruption of electromagnetic matter is actually possible and reasonable. The merging or coalescing of binary-blackhole systems must violently erupt matter and energy outwards. If the universe really has binary-blackhole systems, they must radiate all forms of matter and energy, including to gravitational, electromagnetic, strong and week interactions. Just as nuclear explosions can erupt the protons and neutrons confined within atomic nuclei, we can imagine that, the merging or coalescing of binary-blackhole systems can erupt the quarks confined within protons and neutrons.

Only 1.7s after LIGO and Virgo had detected the first coalescing event of binary neutron-star: GW170817, the Fermi Gamma-ray Burst Monitor (GBM) and the International Gamma-Ray Astrophysics Laboratory (INTEGRAL) detected the gamma-ray burst GRB170817A that seems to come from the same radiation source of GW170817 [198,199]. In the following weeks, observatories around the world successively reported that they had detected electromagnetic matter that seemed to come from the same radiation source of GW170817 and GRB170817A with different frequencies or wavelengths sweeping past the earth. This is regarded as the masterpiece of multi-messenger astronomy.

LIGO believes that [199]: the gravitational-wave signal GW170817 and its electromagnetic counterparts came from the same merging or coalescing event of binary-star system: they departed almost at the same time, after hundreds of millions of light years or even billions of light years, reached the earth almost at the same time. Therefore, the speed of gravitational waves must be the speed c of light.

Due to the better positioning accuracy, GW170817 and GRB170817A are selected by LIGO as the samples to determine the gravitational-wave speed κ.

Suppose that GW170817 does represent the gravitational wave and does come from the coalescing event of a binary-star system with the luminosity distance D_L=26 Mpc, i.e., $D=3.2616\times10^6\times D_L$ light year away from the earth, about 85 million light-years; GBR170817A does represent the gamma-ray burst erupted from the source of GW170817 with the speed v_{EM} of electromagnetic waves, i.e., the speed c of light in vacuum. Then, the relative difference between the speed κ of gravitational waves and the speed c of light can be defined as: $\Delta v/c\approx\Delta t/T$, where $\Delta v=\kappa-c$, $T=D/c$, $\Delta t=T-T_G$, T is the travel time of GBR170817A, and T_G is the travel time of GW170817. If the peak signal of GW170817 and the first gamma photon of GRB170817A departed at the same time, then $\Delta t\approx1.74\pm0.05$s and $\kappa\geq c$, $\Delta v/c$ may be taken as the upper limit value; if GRB170817A departed 10s later than GW170817, then $\Delta t\approx1.74\text{s}-10\text{s}=-8.26$s and $\kappa\geq c$, $\Delta v/c$ may be taken as the lower limit value. Thus, the relative wave-speed difference $\Delta v/c$ is [199]:

$$-3\times10^{-15}\leq\frac{\Delta v}{v_{\mathrm{EM}}}\leq+7\times10^{-16}\quad\left(v_{\mathrm{EM}}\equiv c\right)\tag{19.8}$$

Equation (19.8) originates from the research report jointly submitted by LIGO

Scientific Collaboration (LSC), Virgo Collaboration, Fermi Gamma-ray Burst Monitor (GBM), and INTEGRAL [199]. The speed of light in vacuum recommended by the International Standards Organization (ISO) is: c=2.99792458×10^8 ms^{-1}. Equation (19.8) means that the speed κ of gravitational waves is exactly equal to the speed v_{EM} of gamma-ray bursts. In other words, the speed κ of gravitational waves is exactly the speed c of light in vacuum.

We could understand that, if GW170817 and GRB170817A really originated from the same coalescing event of a binary-star system, and after a long journey of nearly 100 million light-years, they arrived at the earth almost at the same time (only 1.7s difference before and after), then the speed of GW170817 must be exactly the same as the speed of GRB170817A.

However, the problem is that: Is the GW170817 really a gravitational wave erupted by a binary-star system during its merging or coalescing?

The theory of GOR will reveal the mystery behind it for us.

19.3 The GOR Information-Wave Equation

As indicated in Sec. 12.1.1 **The Gravitational Locality** of Chapter 12, both Einstein's theory of general relativity and Newton's theory of universal gravitation imply an important idealized hypothesis: gravity or gravitational interaction is action at a distance, it takes no time to cross space. Whether Einstein's theory of general relativity or Newton's theory of universal gravitation, there is no prior knowledge or information about the speed of gravitational radiation in their axiom systems or logical premises.

Therefore, no matter logically or theoretically, no matter Einstein's theory of general relativity or Newton's theory of universal gravitation, it is impossible to draw the conclusion that the speed of gravitational radiation or the speed of gravitational waves is the speed of light.

Under the principle of general correspondence (GC), following PGC logic route 1 or PGC logic route 2, by analogizing the logic of Einstein's theory of general relativity, the theory of GOR can also deduce or derive the retarded integral formula and the wave equation of the metric-perturbation tensor $\bar{h}_{\mu\nu}(\eta)$. However, in the theory of GOR, the metric-perturbation tensor $\bar{h}_{\mu\nu}(\eta)$ cannot be interpreted as gravitational radiation or gravitational wave. Instead, it is the information wave of the general observation agent OA(η), which transmits the information of observed objects for observers at the speed η.

19.3.1 The Deduction of GOR Wave Equation

Under the principle of GC, the theory of GOR deduces the GOR wave equation by analogizing the deductive logic of Einstein's wave equation. However, the deduction of GOR wave equation does not follow the Einstein's logic of weak-field approximation. Like the establishment of GOR field equation in Chapter 14 and the solution of GOR field equation of static spherically-symmetric gravitational spacetime in Chapter 15, the deduction of GOR wave equation adopts the GOR logic of idealized convergence.

As stated in Sec. 13.3 **The GOR Logical Way of Idealized Convergence** of Chapter 13, according to the theorem of Cartesian spacetime, if $\eta\to\infty$, then the GOR spacetime $X^{4d}{}_\infty$ would converge to the Cartesian spacetime $X^{4d}{}_\infty$, and the GOR spacetime metric $g_{\mu\nu}(x^\alpha, \eta)$ would converge to the Minkowski metric $\eta_{\mu\nu}$=diag(+1,–1,–1,–1). Therefore, if the information-wave speed of the observation agent OA(η) is high enough, then according to Eq. (13.11), the GOR spacetime metric $g_{\mu\nu}(x^\alpha, \eta)$ could be decomposed and linearized as Eq. (13.16):

$$\eta \gg \sqrt{|\chi|} \quad \text{and} \quad \eta \gg \left|\gamma_i v^i\right|:$$

$$g_{\mu\nu}\left(x^\alpha,\eta\right)=\eta_{\mu\nu}+h_{\mu\nu}\left(x^\alpha,\eta\right) \quad \left(\left|h_{\mu\nu}\right| \ll \left|\eta_{\mu\nu}\right| \text{ and } \lim_{\eta\to\infty} h_{\mu\nu}\left(x^\alpha,\eta\right)=\mathbf{0}\right)$$

This is the condition of flat spacetime in the GOR logic of idealized convergence, where the curved metric $h_{\mu\nu}$ and its derivatives of each order are small quantities. According to the theorem of Cartesian of spacetime: $g_{\mu\nu}(x^\alpha, \eta)\to\eta_{\mu\nu}$ as $\eta\to\infty$, or, $h_{\mu\nu}(x^\alpha, \eta)\to 0$ as $\eta\to\infty$.

According to the GOR logical way of idealized convergence (see Sec. 13.3 in Chapter 13), the condition of GOR idealized convergence only requires that: the information-wave speed η of the observation agent OA(η) is large enough or $\eta\to\infty$.

Einstein made the gravitational spacetime $X^{4d}(c)$ of the optical observation agent OA(c) approximately flat by taking advantage of the logical way of weak-field approximation, and then linearized Einstein field equation; while the theory of GOR makes the gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η) tend to be flat by taking advantage of the logical way of idealized convergence, and then linearized the GOR field equation. Therefore, in the theory of GOR, the gravitational field under OA(η) is not necessarily the weak field. Thus, the curved metric $h_{\mu\nu}(\eta)$ in the condition of flat spacetime is no longer the disturbance of weak gravitational radiation on the flat spacetime ($\eta_{\mu\nu}$), and the gravitational wave is also no longer the subtle ripples in the flat spacetime ($\eta_{\mu\nu}$) caused by the disturbance of weak gravitational radiation.

It should be pointed out that the condition of GOR idealized convergence not only meets the condition of flat spacetime, but also meets all the conditions of Einstein's logical way of weak-field approximation, including the weak field, slow speed, static field, spacetime orthogonality, and harmonic coordinates (see Sec. 13.1.3 in Chapter 13). So, by analogizing or following the logic of Einstein's theory of general relativity, the theory of GOR can derive the GOR retarded integral formula of $\bar{h}_{\mu\nu}(\eta)$ which will be isomorphically consistent with Einstein's retarded integral formula (19.6) of $\bar{h}_{\mu\nu}(c)$, derive the GOR wave equation of $\bar{h}_{\mu\nu}(\eta)$ which will be isomorphicall consistent with Einstein's wave equation (19.7) of $\bar{h}_{\mu\nu}(c)$.

Like Einstein field equation (Eq. (14.2)), the left end of the GOR field equation (Eq. (14.32)) can also be defined or marked as the GOR tensor $G_{\mu\nu}(\eta)$, and the GOR field equation can be simply rewritten as:

$$G_{\mu\nu}(\eta) = -\kappa_{\text{GOR}} T_{\mu\nu}$$
$$\left(G_{\mu\nu}(\eta) \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R,\ \kappa_{\text{GOR}} = \frac{8\pi G}{\eta^4}\right) \tag{19.9}$$

where η is the information-wave speed of the general observation agent OA(η), $R_{\mu\nu}=R_{\mu\nu}(\eta)$ is the Ricci tensor of OA(η) (representing the spacetime curvature of OA(η)), $R=R(\eta)$ is the Gaussian curvature of OA(η), $g_{\mu\nu}=g_{\mu\nu}(\eta)$ is the spacetime metric of OA(η), $T_{\mu\nu}=T_{\mu\nu}(\eta)$ is the energy-momentum tensor of OA(η), and $\kappa_{\text{GOR}}=\kappa_{\text{GOR}}(\eta)$ is the coefficient of GOR field equation under OA(η).

Under the condition of GOR idealized convergence: η is large enough or $\eta\to\infty$, according to Sec. 14.6 of Chapter 14 and Eq. (14.59) in Chapter 14, the Ricci tensor $R_{\mu\nu}(\eta)$ and Gaussian curvature $R(\eta)$) approximate or tend to:

$$\begin{cases} R_{\mu\nu}(\eta) = \frac{1}{2}\Box h_{\mu\nu}(\eta) \\ R(\eta) = \frac{1}{2}\Box h(\eta) \end{cases} \quad \left(R \equiv R^{\mu}_{\mu} = \eta^{\mu\nu} R_{\mu\nu}\right) \tag{19.10}$$

Define the metric-perturbation tensor of OA(η):

$$h^{-}_{\mu\nu}(\eta) \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h \quad \left(h \equiv h^{\mu}_{\mu} = \eta^{\mu\nu} h_{\mu\nu}\right) \tag{19.11}$$

where the metric-perturbation tensor $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(\eta)$ is defined with the curved metric $h_{\mu\nu}$ according to the flat-spacetime condition (Eq. (13.16)) of GOR idealized convergence; $h_{\mu\nu}$ relies on the Newtonian gravitational potential χ: $h_{\mu\nu}\sim\chi$.

Thus, under the condition of GOR idealized convergence, the GOR tensor $G_{\mu\nu}(\eta)$ can also be defined with the metric-perturbation tensor $h^{-}_{\mu\nu}$, and the GOR field equation (19.9) can also be expressed with the metric-perturbation tensor $h^{-}_{\mu\nu}$ as:

$$G_{\mu\nu}(\eta) \equiv \frac{1}{2}\Box h^{-}_{\mu\nu}(\eta) = -\kappa_{\text{GOR}} T_{\mu\nu}(\eta) \tag{19.12}$$

The corresponding condition of harmonic coordinates reduces to

$$h^{-\ ,\alpha}_{\mu\alpha}(\eta) = 0 \tag{19.13}$$

According to the theory of GOR, the gravitational-spacetime metric $g_{\mu\nu}$ of the observation agent OA(η) depends on the spacetime coordinates x^{α} (α=0,1,2,3) and the information-wave speed η of OA(η): $g_{\mu\nu}=g_{\mu\nu}(x^{\alpha},\eta)=g_{\mu\nu}(t,x^{i},\eta)$ (i=1,2,3), where the time axis $x^0=\eta t$, t is the observational time of OA(η). Likewise, the curved metric $h_{\mu\nu}$, the metric-perturbation tensor $h^{-}_{\mu\nu}$, and the energy-momentum tensor $T_{\mu\nu}$, all depend on the spacetime coordinates x^{α} and the information-wave speed η of OA(η): $h_{\mu\nu}=h_{\mu\nu}(t,x^{i},\eta)$, $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(t,x^{i},\eta)$, and $T_{\mu\nu}=T_{\mu\nu}(t,x^{i},\eta)$.

By analogizing or following the retarded solution of $h^{-}_{\mu\nu}(c)$ in Einstein's theory of general relativity, the GOR field equation (19.12) satisfying the condition of harmonic coordinate (Eq. (19.13)) has the following retarded integral formula which is isomorphically consistent with Einstein's retarded integral formula (Eq. (19.6)):

$$h^{-}_{\mu\nu}\left(t,x^i,\eta\right)=-\frac{\kappa_{\mathrm{GOR}}}{2\pi}\int\frac{T_{\mu\nu}\left(t-\dfrac{|x^i-x'^i|}{\eta},x'^i,\mu\right)}{|x^i-x'^i|}\mathrm{d}^3x'^i \tag{19.14}$$

where the integral domain is the 3d space occupied by the gravitational source, x^i (i=1,2,3) is the coordinate of the observer, and x'^i (i=1,2,3) is the coordinate of matter distribution, $|x^i-x'^i|$ is the distance between the observation coordinate x^i and the coordinate x'^i of gravitational matter.

Equation (19.14) is the retarded integral formula of the metric-perturbation tensor $h^{-}_{\mu\nu}(\eta)$ in theory GOR, or the retarded solution of the metric-perturbation tensor $h^{-}_{\mu\nu}(\eta)$, or **the GOR retarded integral formula**.

However, it should be pointed out that the so-called **retarded** in Eq. (19.14) depends on the information-wave speed η of the observation agent OA(η), rather than the speed c of light. More specifically, the so-called **retarded** in Eq. (19.14) is not the delay of gravitational interaction, but the delay of observational (observed) information, that is, the delay of the information wave of OA(η), rooted from the observational locality ($\eta<\infty$) of OA(η).

The GOR retarded integral formula (19.14) of the metric-perturbation tensor $h^{-}_{\mu\nu}(\eta)$ suggests that the information received by the observer O at the coordinate x^i at the time t is the information emitted by the observed object P at the coordinate x'^i at the time $t-|x^i-x'^i|/\eta$: in the observational spacetime $X^{4\mathrm{d}}(\eta)$ of OA(η), the observational (observed) information would be delayed by $|x^i-x'^i|/\eta$.

Under the condition of GOR idealized convergence: η is large enough or $\eta\to\infty$, $\kappa_{\mathrm{GOR}}T_{\mu\nu}\to 0$. Thus, the GOR field equation (19.12) would reduced to the form of vacuum field equation: $\Box h^{-}_{\mu\nu}(\eta)=0$. From to the definition of the d' Alembert operator "□" in the general observation agent OA(η) and the definition of the Laplace operator "∇^2" (see Sec. 5.5 **D' Alembert Operator in OR Theory** in Chapter 5 and Eq. (5.31)), it follows that:

$$\begin{aligned}&\Box h^{-}_{\mu\nu}\left(\eta\right)=0 \qquad \left(\Box=\partial^2/\eta^2\partial t^2-\nabla^2\right)\\ &\text{or}\quad \nabla^2h^{-}_{\mu\nu}\left(\eta\right)-\frac{1}{\eta^2}\frac{\partial^2}{\partial t^2}h^{-}_{\mu\nu}\left(\eta\right)=0\end{aligned} \tag{19.15}$$

In this way, we have deduced a wave equation, named as **the GOR wave equation**, in which the wave function is the metric-perturbation tensor $h^{-}_{\mu\nu}(\eta)$ of the observation agent OA(η) and the speed of the wave is the information-wave speed η of the observation agent OA(η).

Now, the problem is that: What does the metric-perturbation tensor $h^{-}_{\mu\nu}(\eta)$ in the GOR wave equation (19.15) mean? Or, what wave is the metric-perturbation tensor $h^{-}_{\mu\nu}(\eta)$ as the wave function in the GOR wave equation (19.15)?

It is worth noting that the GOR wave equation (19.15) has generalized Einstein's wave equation (19.7): as the observation agent OA(η) is the optical agent

OA(c), the GOR wave equation is exactly Einstein's wave equation.

Now, you may have had your own judgment: the GOR wave equation, including Einstein's wave equation, is not the gravitational-wave equation, but the information-wave equation; the wave function $\bar{h}_{\mu\nu}(\eta)$ of GOR wave equation, including the $\bar{h}_{\mu\nu}(c)$ of OA(c), does not represent the wave of gravitational radiation predicted by Einstein – the **Gravitational Wave**, but the **Information Wave** of the observation agent OA(η), which transmits the information of observed objects for observers. Specifically, Einstein's wave equation is exactly the information-wave equation of the optical agent OA(c). Naturally, the information wave of OA(c) is light wave, the information-wave speed of OA(c) is the speed c of light.

Now, you may have finally been able to understand why Einstein's gravitational wave propagates at the speed of light?

The theory of GOR, based on the GOR logical way of idealized convergence, has derived the retarded integral formula (Eq. (19.14)) of GOR field equation which is isomorphically consistent with Einstein's retarded integral formula (Eq. (19.6)) of Einstein field equation, and has derived the GOR wave equation (19.15) which is isomorphically consistent with Einstein's wave equation (19.7). The GOR retarded integral formula and the GOR wave equation has provided a theoretical basis for us to correctly understand Einstein's prediction of gravitational waves.

19.3.2 The Unity of Einstein's Wave Equation and Newton's Wave Equation

Einstein's wave equation is actually the weak-field vacuum form of Einstein field equation; the Poisson vacuum form of Newton field equation, i.e., Laplace's equation: $\nabla^2\chi=0$, may be referred to as **Newton's wave equation**.

Like all the relationships in the theory of OR (including IOR and GOR), the GOR retarded integral formula (Eq. (19.14)) and the GOR wave equation (19.15) has not only generalized Einstein's retarded integral formula and Einstein's wave equation, but also generalized Newton's integral formula and Newton's wave equation. In other words, the GOR wave equation has generalized and unified Einstein's wave equation and Newton's wave equation.

According to the GOR retarded integral formula (Eq. (19.14)):

$$\lim_{\eta\to c} T_{\mu\nu}\left(t-\frac{|x^i-x'^i|}{\eta}, x'^i, \eta\right) = T_{\mu\nu}\left(t-\frac{|x^i-x'^i|}{c}, x'^i, c\right) \quad (19.16a)$$

$$\lim_{\eta\to \infty} T_{\mu\nu}\left(t-\frac{|x^i-x'^i|}{\eta}, x'^i, \eta\right) = T_{\mu\nu}\left(t, x'^i, \infty\right) \quad (19.16b)$$

Equation (19.16a) suggests that: as $\eta\to c$, OA(η) will be the optical agent OA(c), the GOR retarded integral formula (Eq. (19.14)) will reduce to Einstein's retarded integral formula (Eq. (19.6)). Equation (19.16b) suggests that: as $\eta\to\infty$, OA(η) would be the idealized agent OA_∞, the GOR retarded integral formula (Eq. (19.14)) would reduce to Newton's integral formula, the observational locality of OA(η) would disappear, and therefore, the observed information would no longer be retarded or delayed.

According to the GOR wave equation (19.15):

$$\lim_{\eta\to\infty}\left(\nabla^2 h^-_{\mu\nu}(\eta)-\frac{1}{\eta^2}\frac{\partial^2}{\partial t^2}h^-_{\mu\nu}(\eta)\right)=\nabla^2 h^-_{\mu\nu}(c)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}h^-_{\mu\nu}(c) \qquad (19.17a)$$

$$\lim_{\eta\to\infty}\left(\nabla^2 h^-_{\mu\nu}-\frac{1}{\eta^2}\frac{\partial^2}{\partial t^2}h^-_{\mu\nu}\right)=\nabla^2 h^-_{\mu\nu} \ \text{ that is } \ \nabla^2 h^-_{\mu\nu}=0 \ \text{ or } \ \nabla^2\chi=0 \qquad (19.17b)$$

Equation (19.17a) suggests that: as $\eta\to c$, OA(η) will be the optical agent OA(c), the GOR wave equation (19.15) will reduce to Einstein's wave equation (19.7). Equation (19.17b) suggests that: as $\eta\to\infty$, OA(η) would be the idealized agent OA_∞, the GOR wave equation (19.15) would reduce to Newton's wave equation, i.e., Newton's law of universal gravitation in the vacuum form of Poisson equation – Laplace's equation: $\nabla^2\chi=0$, in which the information-wave speed would be infinite and it would take no time for the observed information to cross space.

It is thus clear that both Einstein's wave equation and Newton's wave equation are special cases of the GOR wave equation: Einstein's wave equation is that of the optical agent OA(c); Newton's wave equation is that of the idealized agent OA_∞. The GOR wave equation is the wave equation of the general observation agent OA(η), including the optical agent OA(c) and the idealized agent OA_∞.

In this way, the GOR wave equation has generalized and unified Einstein's wave equation and Newton's wave equation.

However, this does not mean that the GOR wave equation supports Einstein's wave equation, let alone Einstein's prediction of gravitational waves. Quite the reverse: the GOR wave equation indicates that Einstein's prediction of gravitational waves is a mistake!

19.3.3 Einstein's Prediction of Gravitational Waves is a Mistake!

The retarded integral formula (Eq. (19.14)) of the GOR field equation and the GOR wave equation (Eq. (19.15)) indicates that:

Einstein's prediction of gravitational waves is wrong!

After the establishment of general relativity, based on the wave equation (19.7) and the retarded solution (Eq. (19.6)) of Einstein field equation, Einstein made his famous prediction of gravitational waves.

As stated in Sec. 19.1, in Einstein's theory of general relativity, the metric-perturbation tensor $h^-_{\mu\nu}=h^-_{\mu\nu}(\chi,c)$ is connected with the Newtonian gravitational potential χ and the speed c of light. Accordingly, Einstein believed that his retarded integral formula (Eq. (19.6)) represented gravitational radiation, and his wave equation (19.7) represented the wave of gravitational radiation, i.e., the gravitational wave that propagated at the speed c of light.

Thus, Einstein's specious prediction of gravitational waves was born.

As Zhao said [166]: "In essence, $h^-_{\mu\nu}$ is $h_{\mu\nu}$." In Sec. 15.5 of Chapter 15, the theory of GOR has clarified that the curved metric $h_{\mu\nu}=h_{\mu\nu}(\chi,\eta)$ of gravitational spacetime does not really represent gravitational radiation, let alone gravitational wave. Actually, as stated in Sec. 12.1.1 **The Gravitational Locality** of Chapter 12, Newton's theory of universal gravitation and Einstein's theory of general relativity, and even the theory of GOR, have on prior information about gravitational radiation or gravitational waves, which imply the important idealized hypothesis: gravity or gravitational interaction is action at a distance. Therefore, logically, it is impossible for Einstein to predict gravitational waves based on his theory of general relativity, let alone to calculate the speed of gravitational radiation or gravitational waves.

As stated in Sec. 12.1.2 **The Observational Locality** of Chapter 12, the locality in Einstein's theory of general relativity is the observational locality ($c<\infty$) of the optical observation agent OA(c), rather than the gravitational locality. Einstein's theory of relativity, including the special and the general, is the theory of optical observation, in which the observation agent OA(η) is the optical agent OA(c), and the transmission speed η of observed information is the speed c of light in vacuum. Therefore, the so-called retard or delay in Einstein's retarded integral formula (Eq. (19.6)): $|x^i-x'^i|/c$, is the retarded or delayed of observed information, not the retard or delay of gravitational interaction. In particular, it should be pointed out that: the wave speed c in Einstein wave equation (19.7) is the information-wave speed c of the optical agent OA(c), not the gravitational-wave speed κ.

The theory of GOR does not doubt the existence of gravitational waves.

However, as one of the objective interactions between matter and matter, gravitational waves or the waves of gravitational radiation do not rely on observation or the observation agent OA(η), do not rely on the speed η of observed information transmitted by the observation media of OA(η).

The metric-perturbation tensor $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(\chi,\eta)$ in the GOR retarded integral formula (Eq. (19.14)) and the GOR wave equation (19.15) does not represent gravitational waves or the waves radiated by the gravitational source, but the information wave of the observation agent OA(η) transmitting observed information, which is naturally carrying the information about gravitational radiation (χ). As the information wave of OA(η), the speed at which $h^{-}_{\mu\nu}(\chi,\eta)$ transmits observed information is naturally the information-wave speed η of OA(η), conforming to the GOR retarded integral formula (Eq. (19.14)) and the GOR wave equation (19.15). Specifically, as OA(η) is the optical agent OA(c), the speed at which $h^{-}_{\mu\nu}(\chi,c)$ as the information wave of OA(c) transmits observed information is naturally the speed c of light, conforming to Einstein's retarded integral formula (Eq. (19.6)) and Einstein's wave equation (Eq. (19.7)).

As stated in Sec. 15.5.3 of Chapter 15, for the solution (Eqs. (15.17) and (15.32)) of the GOR field equation, regardless of the Cartesian coordinates or the spherical coordinates, regardless of the approximate solution or the exact solution, the nonzero elements of the curved metric $h_{\mu\nu}(\chi,\eta)$ all contains an important dimensionless factor: $C_W=|\chi|/\eta^2$, i.e., the ratio of the Newtonian gravitational potential $|\chi|$ to the square of the information-wave speed η of OA(η). As $\eta\to\infty$, $C_W\to 0$ and $h_{\mu\nu}\to 0$, which suggests that, both the curved metric $h_{\mu\nu}(\chi,\eta)$ and the metric-perturbation tensor $h^{-}_{\mu\nu}(\chi,\eta)$ do not represent gravitational radiation or gravitational wave.

The theory of GOR refers to $C_W=|\chi|/\eta^2$ as **the Factor of Carrier Wave**. In the theory of GOR, the wave function $h^{-}_{\mu\nu}(\chi,\eta)$ in the GOR wave equation (19.15) is actually the information wave of the observation agent OA(η), the so-called **Carrier Wave**, loaded with the observed information of gravitational interaction (χ), which is the information wave of OA(η) modulated by the gravitational-radiation signal, and naturally, propagates at the information-wave speed η.

The GOR wave equation (19.15) means that there is a wave in the observational spacetime $X^{4d}(\eta)$ of the observation agent OA(η): $h^{-}_{\mu\nu}(\chi,\eta)$, that is, the information wave of OA(η), whose speed is the speed of observation medium of OA(η) transmitting observed information. Naturally, different observation agents have different information-wave speeds, which may not necessarily be the speed c of light. Einstein's wave equation (19.7) is only a special case of the GOR wave equation (19.15), where OA(η) is the optical agent OA(c).

The GOR wave equation, actually, is the GOR information-wave equation.

It is thus clear that the wave function $h^{-}_{\mu\nu}(\chi,\eta)$ in the GOR wave equation (19.15) is not the gravitational wave radiated by matter systems, but the information wave of the observation agent OA(η).

This means that, as a special case of the GOR wave equation (19.15), the so-called gravitational wave predicted by in Einstein's wave equation (19.7) is not the gravitational wave but the information wave of the optical observation agent OA(c): light wave, which transmits observed information at the speed c of light.

So, Einstein's wave equation is not the gravitational-wave equation, but the information-wave equation, that is, the electromagnetic-wave equation or the light-wave equation of the optical observational agent OA(c).

This is the essence of Einstein's prediction of gravitational waves.

19.3.4 Information waves are physical reality

The significance of the GOR retarded integral formula (Eq. (19.14)) and the GOR wave equation (19.15) lies not only in having proved that Einstein's prediction of gravitational waves is a mistake, but also in having proved that the information waves as observation media are physical reality, or in other words, the objectively physical existence.

As stated in the first chapter of OR theory: "Human being's understanding of the objective world depends on and is restricted by observation." However, we have not yet truly recognized and understood the role and special statue of observation and observation media in physics and its theoretical system.

By starting from the definition of time (Def. 2.2, the most basic logical premise), the theory of OR, has derived the general Lorentz transformation, which has generalized and unified the Galilean transformation and the Lorentz transformation [26,27]. Thereby, the theory of OR has discovered that both Newton's classical mechanics and Einstein's relativity theory are the partial theories of physics that depend on and are restricted by their respective observation system and observation condition: Newton's classical mechanics is the theory under idealized observation, depending on and being restricted by the idealized observation agent OA_∞; Einstein's relativity theory is the theory of optical observation, depending on and being restricted by the optical observation agent OA(c). This means that, in the theoretical systems or mathematical models of physics, in addition to the observer and the observed object, there must also be another important and indispensable role: **Observation Agent**, which takes advantage of certain observation media for transmitting the spacetime information of the observed object to the observer.

So, the theory of OR has defined the concept of **Information Wave**. The so-called information waves refer to the matter waves, such as water waves, sound waves, light waves, electromagnetic waves, and even gravitational waves, employed by observation agents for transmitting the observed information.

Different theoretical systems have different observation agents, employ different observation media, and therefore, have different information-wave speeds: the speed of the information wave of the idealized agent OA_∞ is infinite, the speed of the information wave of the optical agent OA(c) is the speed c of light in vacuum, and the speed of the information wave of the bat-agent OA(v_S) is the speed v_S of ultrasonic wave in the atmosphere, and so on.

So, does a theoretical system or mathematical model of physics really need a certain observation agent? In other words, does a theoretical system or mathematical model of physics really need a certain observation medium for transmitting the information of observed objects to observers?

For this matter, the value and significance of the GOR retarded integral formula (Eq. (19.14)) and the GOR information-wave equation (19.15) lies in having proved that observation media, or information waves, or observation agents paly the indispensable role in the theoretical systems or mathematical models of physics.

The retarded solution of GOR field equation has proved that:

In a theoretical system of physics, there must exist a specific observation agent OA(η), which employs a specific observation medium to transmit the information of observed objects to observers at a specific speed η.

The GOR wave equation has proved that:

In a theoretical system of physics, there must exist a specific matter wave in the observational spacetime $X^{4d}(\eta)$ of the observation agent OA(η), that is, the information wave of OA(η) transmitting observed information.

Observation Agent, **Information Wave** and **Informon**, were originally coined by the theory of OR. Now, **Observation Agent** and **Information Wave** have been proven to be the objectively physical existence by the GOR retarded integral formula (Eq. (19.14)) and the GOR information-wave equation (19.15). Thus, as the matter particles that make up the information wave and as the mesons that transmit observed information, the existence of **informons** is natural and rational.

19.4 GOR Interpreting: What Has LIGO Discovered?

According to the GOR information-wave equation, the theory of GOR has proved that Einstein's prediction of gravitational waves is a mistake.

So, what are the gravitational waves detected by LIGO Observatory? Has LIGO really detected gravitational waves? Do gravitational waves really exist?

The theory of GOR does not doubt the existence of gravity or gravitational radiation. Actually, the so-called gravitational waves are namely gravity or gravitational radiation: the waves of gravitational radiation.

LIGO claimed that it had detected gravitational waves. Also, in 2017, LIGO won the Nobel Prize in Physics for detecting gravitational waves. However, LIGO's gravitational-wave detection is not strictly empirical observation or experiment. As stated in Sec. 19.2 and depicted in Fig. 19.3, LIGO's gravitational-wave detection is half real and half virtual: the first half is listening; the second half is guessing. Perhaps, the only thing we could affirm is that LIGO heard the chirping sound from a certain matter system. As for the merging or coalescing of binary-blackhole systems, it was just an imagination or a speculation of LIGO, the product of the technology of computer simulation or the technology of **Virtual Reality**, which might not necessarily exist in reality.

Since Einstein's prediction of gravitational waves, which is based on Einstein's theory of general relativity, is a mistake, could LIGO's computer simulation of gravitational waves, which is also based on Einstein's theory of general relativity, still hold true?

19.4.1 GOR Interpreting LIGO Principle

For the principle or scheme of LIGO system detecting gravitational radiation or gravitational waves, the understanding of the theory of GOR is naturally different from the view of Einstein's theory of general relativity.

As stated in Sec. 19.2, the detection principle of LIGO originated from Pirani's scheme. Based on Einstein's theory of general relativity, Pirani believed that [168]: gravitational radiation or a gravitational wave could lead to the expansion and contraction of space; and therefore, testing the variation of the spatial distance between two matter bodies could detect the gravitational radiation or gravitational wave sweeping past the two test bodies. Based on Pirani's view, Weber further imagined that [169]: spatial expansion or contraction would shrink or stretch matter objects; and therefore, by measuring the shrinking and stretching of an matter object, one could determine the expansion and contraction of space. In Weber's scheme, the expansion and contraction of space is equivalent to the shrinking and stretching of matter objects.

LIGO follows Pirani's ideas and scheme. As stated in Sec. 19.2.3 and depicted in Fig. 19.2(a), the architecture of LIGO detector is designed based on the Michelson interferometer, in which the core is the two mutually perpendicular laser arms (the arm-X and the arm-Y) for measuring the spatial expansion and contraction between the test objects by taking use of the laser interference effect.

It should be pointed out that, originally, what Einstein's advocated in his theory of general relativity was the curvature of 4d spacetime, rather than the expansion or contraction of 3d space.

According to the GOR factor (Eq. (12.36)) of spacetime transformation, the theory of GOR has clarified that, regardless of whether there are gravitational fields or not, the objective and real spacetime will not be curved, and naturally, will not expand or contract. In Chapter 13, the theorem of Cartesian spacetime has also proved this conclusion: the observational spacetime $X^{4d}{}_{\infty}$ of the idealized agent OA_{∞} must be flat, regardless of whether there is matter in spacetime.

In the theory of GOR, the distribution of matter or the existence of gravitational fields would also lead to the optical-path difference between the two laser beams in the arm-X and the arm-Y of the LIGO laser interferometer, and then, produce interference effect. However, that is not due to the curvature of spacetime or the expansion and contraction of space, but due to the variation or perturbation of the speed of laser in gravitational spacetime.

In Sec. 15.4 of Chapter 15, based on the GOR logical way of idealized convergence, the theory of GOR has obtained the solution and line-element formula (Eq. (15.32)) of the static spherically-symmetric gravitational spacetime:

$$\mathrm{d}s^2 = \left(1+2\chi/\eta^2\right)\eta^2\mathrm{d}t^2 - \left(1+2\chi/\eta^2\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$$

where η is the information-wave speed of the observation agent OA(η); $\chi=-GM/\eta^2$ is the Newtonian gravitational potential, and M is the gravitational mass (also representing the center of gravity).

It is worth noting that the LIGO detector is actually an optical observation system, the optical agent OA(c), which employs the laser interferometer to measure the optical-path difference between the two perpendicular laser beams, thereby to sense the variation of the laser speeds. Therefore, like the Michelson interferometer in the Michelson-Morley experiment, the observation agent OA(η) of LIGO detector is the optical observation agent OA(c).

Suppose that the observed object P is a photon m. According to the theory of GOR, as OA(η) is the optical agent OA(c), the line-element ds of the photon m is zero: ds=0 (this is consistent with Einstein's theory of general relativity). Given the isotropy of spherically-symmetric gravitational spacetime, it follows that: $\mathrm{d}\theta=0$ and $\mathrm{d}\varphi=0$. Thus, the line-element formula (Eq. (15.32)) reduces to:

$$0 = \left(1+2\chi/c^2\right)c^2\mathrm{d}t^2 - \left(1+2\chi/c^2\right)^{-1}\mathrm{d}r^2$$

$$\text{that is}\quad v = \frac{\mathrm{d}r}{\mathrm{d}t} = \pm\left(1+\frac{2\chi}{c^2}\right)c \quad \left(\chi = -\frac{GM}{r}\right) \qquad (19.18)$$

where v is the speed of the photon m in the gravitational spacetime under OA(c).

Equation (19.18) suggests that: the speed of light or photons in gravitational spacetime is different from the speed c of light in vacuum; in addition, the speed of light or photons will be different for different gravitational potentials, and for different energies of gravitational radiation or gravitational wave. As gravitational radiation or gravitational wave invades the LIGO spacetime, in particular, as the gravitational radiation or gravitational wave around the arm-X and arm-Y of the LIGO laser interferometer is asymmetry, the speeds of laser in the arm-X and the arm-Y would be different. Then, the two laser beams in the arm-X and the arm-Y of the LIGO laser interferometer would exhibit optical-path difference and produce interference effects or interference fringes.

The Basic Principle of LIGO detector: According to the theory of GOR, the LIGO detector could detect the variation or perturbation of the speed of laser by making use of the LIGO laser interferometer; thus, LIGO could detect gravity or gravitational waves, detect gravitational radiation or gravitational fields; and furthermore, could explore the matter systems that act as gravitational sources sweeping the LIGO detector.

19.4.2 The Distance and Mass of LIGO's Gravitational-Wave Sources

In order to detect the gravitational waves coming from deep space or out space, LIGO requires the celestial bodies as gravitational-wave sources to possess huge mass and radiate the gravitational waves with huge energy.

As stated in Sec. 19.2.4, the sea of the earth is a natural gravitational-wave observatory, which could detect gravitational radiation or gravitational waves coming from celestial bodies: the sea presents tidal phenomena by sensing the gravitational radiation or gravitational waves from the moon and the sun. According to Newton's law of universal gravitation, the induced tidal force F_T of the matter or sea water with unit mass on the earth's surface is proportional to the mass M of gravitational source and inversely proportional to the cube of the distance d of gravitational source:

$$F_T = \Delta m a_T \quad \left(a_T = GM \frac{2R_E}{d^3} \right) \tag{19.19}$$

where, G is the gravitational constant, M is the mass of gravitational source or gravitational-wave source, R_E is the earth radius, Δm is the unit mass of the matter or sea water on the earth's surface, and a_T is the equivalent acceleration.

The LIGO detector itself is a matter system of the earth's surface.

Equation (19.19) has important enlightenment significance: a detector of gravitational radiation or gravitational waves on the earth's surface, including the sea and LIGO, are restricted by the law of inverse cubic of the gravitational-source distance d. Equation (19.19) means that whether the gravitational-wave observatory on the earth's surface, whether the sea or LIGO, could detect gravitational radiation or gravitational waves depends largely on the distance d between the gravitational source M and the gravitational-wave observatory on the earth.

The solar mass is 27.112 million times the lunar; while the distance between the sun and the earth is 390.6 times that between the moon and the earth. According to Eq. (19.19): the acceleration of the solar tidal force is a_T=5.05 $\times 10^{-7}$ m·s^{-2}; the acceleration of the lunar tidal force is a_T=1.10$\times 10^{-6}$ m·s^{-2}, 2.18 times that of the sun. This means that at least one more sun needs to be added in order for the tidal force of the sun to reach the level comparable to that of the moon.

The nearest fixed star to the solar system, Proxima Centauri, has the mass of about 0.12 suns and is about 4.22 light-years away from the earth. According to Eq. (19.19), the mass M of it must reach that of 1.88×10^{16} suns in order for its tidal force on the sea or LIGO detector on the earth's surface to reach the level comparable to that of the sun:

$$\frac{M}{M_{Sun}}=\frac{d^3}{d_{Sun}^3}=\left(\frac{4.22\times365\times24\times60\times60\times c}{1.5\times10^{11}}\right)^3\approx1.88\times10^{16} \tag{19.20}$$

Proxima Centauri, even if its mass M reaches 1.88×10^{16} suns, might also not trigger the LIGO detector. As stated in Sec. 19.2.4, as far as the LIGO detector and its surrounding spacetime are concerned, like the situations of the moon and the sun, the gravitational field of Proxima Centauri is a relatively static gravitational field, which is difficult to make the two laser beams in the arm-X and arm-Y of the LIGO laser interferometer form enough optical-path difference to produce the interference effects or interference fringes.

The first gravitational-wave signal detected by LIGO: GW150914, was imagined by LIGO as a gravitational wave erupted from a binary-blackhole system during merging or coalescing: the binary-blackhole system has the total mass of about 67 suns and the distance of about 1.3 billion light-years away from the earth. How many suns do such a distant celestial body need to accumulate together to generate the tidal effects equivalent to the sun on the earth's sea or LIGO detector? Even if considering the merging and coalescing of the two black holes, no one could imagine how LIGO detector could detect a gravitational wave from so far away.

What makes us curious is that, since GW150914 could trigger LIGO detector, why GW150914 could not trigger the earth's sea and set up huge tides?

19.4.3 LIGO's Gravitational Waves and the Coalescing of Black Holes

During the three phases of detection activities, including O1, O2 and O3a+b, LIGO detected the total of 91 gravitational-radiation signals, each had matched the coalescing model of a binary-star system, of which the vast majority were binary-blackhole systems: 84 cases consisting of two black holes, 5 cases of one black hole and one neutron star, and 2 cases of two neutron stars.

Actually, as stated in Sec. 19.2.4, LIGO took binary-blackhole systems as the detection target of gravitational waves from the very beginning, and looked forward to their merging or coalescing. However, whether neutron stars or black holes, whether binary neutron-star systems or binary-blackhole systems, whether merging or coalescing, all stay in the hypothetical mathematical models, and all stay in computer simulation and virtual reality.

Do the universe really have binary-blackhole celestial bodies? Do black holes really merge or coalesce so frequently and so regularly? Does the universe need the coalescing of black holes or does LIGO need the coalescing of black holes?

LIGO's Need for the Coalescing of Black Holes

In 1967, a radio telescope detected a periodic electromagnetic-pulse signal [172-174]. Thus, it was speculated that there were pulsars in the universe. Furthermore, it was believed that pulsars were neutron stars (a sort of fictional celestial bodies that had almost been abandoned by astronomers). The envisaged neutron stars have huge mass, however, as stated in Sec. 19.4.1, they are not massive enough to produce high-energy gravitational waves for LIGO's need.

In 1974, a radio telescope detected an electromagnetic-pulse signal, whose pulse period presented periodic variation [175,176]. Thus, it was speculated that there were binary-pulsar systems in the universe. Furthermore, it was believed that [177-179], the semi-major axis and orbital period of a binary pulsar would gradually shorten, the pulsar and its companion star would eventually merge or coalesce. The merging or coalescing of a binary-pulsar system must violently explode and fiercely erupt high-energy gravitational waves. However, the mass of a neutron star is limited. Even if a binary-pulsar system really merges or coalesces, the gravitational-wave energy erupted by it might still not meet LIGO's need.

So, physicists came up with the concept of black holes and conceived of binary-blackhole systems: what if a binary-blackhole system merges or coalesces [183,184]? The mass of a black hole could be freely imagined, and in this way, the merging or coalescing of a binary-blackhole systems could erupt the gravitational waves with arbitrarily high energy.

From neutron stars to pulsars, from pulsars to binary-pulsar systems, from binary-pulsar systems to binary-blackhole systems, and then to the merging or coalescing of binary-blackhole systems, it seems that LIGO's everything is based on conjecture and illusion. So far, except for computer simulation or virtual reality, there is no sufficient empirical basis supporting for the merging or coalescing of binary-blackhole celestial bodies.

As a matter of fact, those chirping sounds heard by LIGO detector do not really come from the merging or coalescing events of binary-blackhole systems.

LIGO's Ring Toss Game

As stated in Sec. 19.2.5: the universe still has too many unknowns for human beings; each gravitational radiation signal detected by LIGO detector implies infinite possibilities. However, from the very beginning, LIGO aimed its detection target at binary-blackhole systems.

LIGO's identification of the gravitational-radiation signals heard by the LIGO detector is somewhat like one of the street games: **Ring Toss Game**.

Through the computer simulation of virtual reality, a **chirp** could always match one of the mathematical models of binary-stars systems in LIGO's computer database, just as the ring falls over a doll. As shown in Tab. 19.1, the first gravitational-radiation signal detected by LIGO: GW150914, matched the merging or coalescing event of a binary-blackhole model: the primary star has the mass of 36.2 suns and the secondary star has the mass of 29.1 suns, the merged black hole has the mass of 62.3 suns, $13_{-5.5}^{+6.0}$ billion light-years away from the earth, erupting about the matter or energy of about 3.0 suns. If GW150914 was located in a sky region 2 billion light-years away from the earth (instead of 1.3 billion light-years), then LIGO would only need to make slight adjustments to the masses of the primary and secondary black holes in the binary-blackhole model, so that GW150914 could still match one binary-blackhole model, in other words, GW150914 could still fall over a doll. The probability of a gravitational-radiation signal detected by the LIGO detector matching with one of the binary-blackhole coalescing models is much higher than the probability of the ring falling over a doll in playing ring toss.

Of the 91 cases of the gravitational-radiation signals detected during LIGO's three phases of detection activities, including O1, O2 and O3a+b, only two cases matched the binary neutron-star coalescing models: GW170817 and GW190425. Compared with binary-blackhole models, both GW170817 and GW190425 are closer to the earth: GW170817 is only 130 million light-years away from the earth and GW190425 is only 520 million light-years away from the earth. Perhaps, just because they are closer to the earth, the matching mass could not be too large to reach the magnitude of black holes. Whether a gravitational-radiation signal detected by LIGO detector matches a binary-blackhole model or a binary neutron-star model depends on their distance from the earth, or vice versa.

It is worth noting that: LIGO's binary-blackhole systems could not be too close to the earth – if they are too close to the earth, LIGO's binary-star systems would have to be binary neutron-star systems; moreover, the farther away from the earth, the blacker LIGO's binary-blackhole systems, and the greater the total mass.

LIGO's binary-blackhole systems depend on the distances between the binary-blackhole systems and the earth.

What exactly does such a phenomenon in LIGO's detection data mean?

The Misconception of LIGO's Binary-Blackhole Models

Take the GW150914 in Fig. 19.3 and Tab. 19.1 as an example: on the one hand, based on the computer simulation of binary-blackhole systems, LIGO claimed that the GW150914 detected by LIGO detector was a gravitational wave predicted by Einstein, which verified the existence of gravitational waves; on the other hand, based on the gravitational-radiation signal GW150914, LIGO claimed that the GW150914 detected by LIGO detector came from a binary-blackhole system during its merging or coalescing, which verified the existence of binary-blackhole celestial bodies and the merging or coalescing of binary-blackhole systems.

Such a circular reasoning is logically questionable.

According to LIGO's computer simulation, the GW150914's binary-blackhole system has lost the mass of about 3.0 suns after merging or coalescing; according to Einstein formula $E=mc^2$, the lost mass has been transformed into the energy erupted or radiated outward in the form of gravitational radiation or gravitational waves as the two black holes merged or coalesced.

However, there are many doubts in the interpretation of LIGO

Firstly, LIGO's binary-blackhole models are the product of numerical relativity, which is based on Einstein's relativity theory, including the special and the general. However, LIGO has not realized that Einstein's theory was only a partial theory, belonging to optical observation, being true or valid only under the optical agent OA(c). The theory of OR has already clarified that the physical quantities of Einstein's relativity theory are all that of optical observation, containing the observational effects of OA(c). We are not sure whether LIGO limits its theoretical model based on Einstein's relativity theory to the optical observation conditions. And, we are not sure what makes LIGO conclude that, during merging or coalescing, the GW150914's binary-blackhole system has transformed the mass of about 3 suns into gravitational radiation or gravitational waves.

Secondly, LIGO mistakenly employed Einsteins formula $E=mc^2$ to calculate the energy of the gravitational waves that were erupted by binary-blackhole systems during merging and coalescing. The theory of OR has already clarified [26,27] that: Einstein formula $E=mc^2$ is only a special case of the OR mass-energy relation $E=m\eta^2$, which holds true only if the observation agent OA(η) is the optical agent OA(c); $E=mc^2$ only represents the observational kinetic energy of the observed object m moving at the speed of light under OA(c), and does not mean that the mass of the observed object m could be transformed into energy, just as the energy of nuclear explosion is the release of energy confined within the nucleus, rather than the transformation of mass. Therefore, it is theoretically wrong that LIGO calculates the energy of gravitational waves erupted by binary-blackhole systems during merging and coalescing based on Einstein formula $E=mc^2$.

At the same time, LIGO mistakenly employed Planck equation $E=hf$ to calculate the energy of gravitational waves. As far as the frequency f is concerned: the frequency detected by the LIGO detector is not the frequency of gravitational waves, but the frequency of relative perturbation of the speeds of laser in the arm-X and the arm-Y beams, and depends on the laser frequency. As far as the Planck constant h is concerned, the theory of OR has already clarified [26,27] that h is the Planck constant of the optical agent OA(c). According to the general Planck equation (6.16), different observation agents OA(η) have different Planck constant h_η (Eq. (6.29)). The quantum theory based on Planck equation $E=hf$, like Einstein's relativity theory, is the theory of optical observation, which is true or valid only if the observation agent OA(η) is the optical agent OA(c). Therefore, it is theoretically wrong that LIGO calculates the energy of gravitational waves erupted by binary-blackhole systems during merging and coalescing based on Planck equation $E=hf$.

In particular, the merging or coalescing of two black holes must violently explode and fiercely erupt matter and energy outwards. According to the theory of OR, all the erupted energy must be carried by the erupted matter, but not be transformed from the erupted matter. As stated in Sec. 19.2.6, if a binary-blackhole system merges or coalesces, then it will not only erupt gravitational radiation or gravitational waves, but also erupt electromagnetic matter or even any other form of matter outwards. For example, just 0.4s after LIGO had detected the first gravitational-radiation signal GW150914, Fermi GBM detected a gamma-ray burst from the same sky region [201,202]. If GW150914 really came from the merging or coalescing event of a binary-blackhole system, then this gamma-ray burst could be regarded as the electromagnetic matter erupted from the binary-blackhole system. Actually, even if there are really binary-blackhole systems in the universe, even if binary-blackhole systems really merge or coalesce, the erupted energy of gravitational waves may only account for a small part of all the erupted energy, which cannot trigger the LIGO detector.

It is thus clear that LIGO's gravitational-wave dynamic models for computer simulation or virtual reality is theoretically wrong. LIGO's so-called gravitational waves from binary-blackhole celestial bodies lack both sufficient empirical basis and sufficient theoretical basis.

19.4.4 The Problem of LIGO's Gravitational-Wave Speed

As stated earlier, the existence of gravity and gravitational waves need not be proved and verified. Actually, the sea has already proved and verified it to us

In a sense, as far as Einstein's prediction of gravitational waves is concerned, the important matter is not whether gravitational waves really exist, but whether the speed κ of gravitational radiation is really the speed c of light.

The theory of GOR has proven that the so-called gravitational waves predicted by Einstein is not the gravity or gravitational waves radiated by matter systems, but the information wave of the optical agent OA(c), that is, light wave, which transmits the information of observed objects for observers. The information wave of the optical agent OA(c) is light, and the speed of it is naturally the speed of light. However, that is not the speed of gravitational radiation or gravitational waves.

So, what about the speed of the gravitational radiation or gravitational waves?

There is no doubt that, according to the principle of physical observability or the principle of locality, gravity or gravitational interaction must not be action at a distance. Therefore, the speed of gravitational radiation must be finite or limited: it takes time for gravitational waves to cross space.

However, the speed of gravitational radiation or gravitational waves may not necessarily the speed of light.

Actually, observations and experiments seem more inclined to support the views and conclusions of Laplace [43] and Flandern [127] as well as others [203,204]: the speed of gravity or gravitational radiation is far greater than the speed of light – if the speed of gravitational waves were the speed of light, then the universe would lose the existing stable structure [127]. This kind of view has not been accepted by the mainstream school of physics only because it is contrary to Einstein's theory of general relativity and Einstein's prediction of gravitational waves.

An obvious fact is that: the reason why black holes are black is because light or photons is/are not fast enough to escape from black holes; while, black holes could not bind gravitational waves and gravitons, otherwise, black holes would lose their gravitational interaction with the external matter systems, and the binary-blackhole systems envisioned by LIGO would not exist: two black holes could not attract and orbit each other, and naturally, there would be no merging or coalescing event of binary-blackhole systems occurring in the universe.

It is thus clear that the speed κ of gravitational radiation, including gravitational waves and gravitons, may be greater than or far greater than the speed c of light as predicted by Laplace [43] and Flanderen [127].

LIGO claims that it has detected gravitational waves. Moreover, as stated in Sec. 19.2.6, based on the trilateration method and the joint observation data of multi-messenger astronomy, LIGO claims that it has proved the speed of gravitational waves is exactly the speed of light [199]. Actually, LIGO has not really measured or determined the speed κ of gravitational waves. Even by taking advantage of multiple base stations and multiple messengers, LIGO still could not accurately locate the source of gravitational waves, and hence, could not determine the speed of gravitational waves by means of the trilateration method. The LIGO Livingston and the LIGO Hanford are 3002 kilometers away, about 10ms optical path. It took the gravitational-radiation signal GW150914 about 7ms to sweep past the LIGO's two base stations. In this regard, LIGO could only draw a very vague conclusion: the speed of GW150914 conforms to Einstein's prediction of gravitational waves: gravitational waves propagate at the speed of light.

GW170817 is the first case of LIGO's gravitational-radiation signal that was matched with a binary neutron-star model. It is accompanied before and after it by numerous clumps of electromagnetic matter, i.e., the so-called electromagnetic counterparts, including the gamma-ray burst GRB170817A., LIGO believed that GW170817 and GRB170817A departed from the same binary neutron-stars system at the same time and almost simultaneously reached the earth, with a difference of only about 1.7s. Finally, on this basis, LIGO indirectly demonstrated that the speed κ of gravitational waves was equal to the speed c of light (Eq. (19.8)) [199].

So, is the speed κ of gravitational waves is really the speed c of light?

The problem is that: Do the gravitational-radiation signal GW170817 and the gamma-ray burst GRB170817A really originated from the merging or coalescing event of the same binary neutron-star system?

There is no doubt that: the gamma-ray burst GRB170817A came from a certain celestial body, perhaps, it is exactly the binary neutron-star system matched by LIGO's computer simulation of virtual reality: 85 million light-years away from the earth, with the total mass of three suns; moreover, merged or coalesced 85 million years ago, radiated or erupted GRB170817A which came to the earth after traveling 85 million years at the speed of light.

However, the LIGO's gravitational-radiation signal GW170817 was not originated from the binary neutron-star system of GRB170817A, even nor from any binary-star systems, including binary-blackhole systems.

As a matter of fact: LIGO is not sure whether there are really binary-star systems in the universe; even if there are, LIGO is not sure whether they will merge or coalesce; even if they will, LIGO is not sure how much energy they will radiate or erupt; even if LIGO theoretically can calculate the erupted energy, LIGO is not sure how much of the erupted energy will belong to gravitational radiation or gravitational waves. Actually, regardless of whether there are binary-blackhole systems in the universe, regardless of whether binary-blackhole systems will merge or coalesce, all the gravitational-radiation signals detected by LIGO detector are not directly related to binary-star systems, including binary neutron stars and binary black holes. What LIGO actually heard were not the chirp sounds originated from the merging or coalescing of binary-star systems, but the chirps of the so-called electromagnetic counterparts near by the LIGO detector, such as GRB170817A. Actually, both the gravitational-radiation signal GW170817 and the gamma-ray burst GRB170817A are the clumps of electromagnetic matter that swept past the earth. In particular, the GW170817 (as an clumps of electromagnetic matter) invaded the spacetime around the LIGO detector at close quarters, and therefore, the gravitational field of GW170817 own triggered the LIGO detector.

Thus, LIGO mistakenly takes the speed of GW170817 being electromagnetic radiation, i.e., the speed c of light, as the speed κ of gravitational radiation.

Ironically, it is just the LIGO's mistake [199] that will reveal the essence of LIGO's gravitational waves.

19.4.5 What Exactly did LIGO Detected?

LIGO claims that it has detected gravitational waves, and verified Einstein's prediction of gravitational waves: gravitational waves not only exist objectively, but also, just as predicted by Einstein, the speed κ of gravitational waves is exactly the speed c of light. For this, LIGO members, Weiss, Barish and Thorne, won the Nobel Prize in Physics in 2017.

However, the theory of GOR has proven that Einstein's prediction of gravitational waves is a mistake: the wave in Einstein's wave equation (19.7) is not the gravitational wave radiated by a certain gravitational source, but the information wave of the optical agent OA(c). As a matter of fact, whatever LIGO has detected, it does not mean that Einstein correctly made the prediction about gravitational waves and the speed of gravitational waves.

So, has LIGO really detected gravitational radiation or gravitational waves? Is the speed of gravitational radiation or gravitational waves really the speed of light?

The Trigger Conditions of LIGO Detector

According to Newton's law of universal gravitation, all things attract each other.

In this sense, the LIGO system for detecting gravitational waves is a detector for detecting all things or all matter systems with both mass and energy: a matter system must carry the gravitational field of its own, radiate gravity or gravitational waves, and as stated in Sec. 19.2.4, trigger the LIGO detector under certain conditions.

Electromagnetic matter must carry the electromagnetic field of its own and radiate electromagnetic force and electromagnetic waves, and therefore, radio telescopes can sense electromagnetic radiation or electromagnetic waves. So, a radio telescope may be called **the detector of electromagnetic matter**. Any matter system has its own mass, carry gravitational fields, radiate gravity and gravitational waves, and therefore, LIGO detector can sense gravitational radiation or gravitational waves. So, the LIGO system for detecting gravitational waves may be called **the detector of everything** for detecting all matter systems in the universe.

As long as there is a matter system invading or sweeping past the spacetime around the LIGO detector, the gravitational field carried by it might trigger the LIGO detector. However, two conditions must be met for the gravitational field of the matter system to trigger the LIGO detector:

(i) The matter system has the enough strength $g=GM/d^2$ of gravitational field or the enough tidal-generating force $F_T=2\Delta mGMR_E/d^3$ (Eq. (19.19)): (a) the mass M of it is large enough; (b) the distance d of it is small enough, that is, the matter system is close enough to the LIGO detector.

(ii) Relative to the LIGO detector's two laser arms of X and Y, the gravitational field of the matter system is asymmetric.

LIGO did detect gravitational waves.

Gravitational waves are the waves of gravitational radiation. In this sense, LIGO did detect gravitational waves.

Based on the basic principle of LIGO detector interpreted by the theory of GOR, the gravitational-radiation signals that triggers the LIGO detector, from the first GW150914 to the 91th GW200322, do mean that there were numerous matter systems with the gravitational fields of their own invading and sweeping past the spacetime around the LIGO detector, leading to the perturbation of the speed of laser in the Fabry-Pérot cavity. The two beams of laser in the arm-X and the arm-Y presented the optical-path difference. Thus, the LIGO laser interferometer produced interference effects and formed interference fringes, triggering the LIGO detector.

It could be affirmed that LIGO has indeed discovered or detected the invasion of numerous matter systems carrying the gravitational fields of their own. In other words, indeed, the gravity or gravitational waves radiated by those matter systems ever triggered the LIGO detector.

However, this does not seem to imply that the gravitational-radiation signals triggering the LIGO detector were originated from the merging or coalescing event of binary-star or binary-blackhole systems.

LIGO's gravitational waves
did not come from binary-star coalescing.

It could be affirmed that the gravitational-radiation signals or gravitational-wave signals detected by LIGO did not come from distant binary-star systems, neither from binary neutron-star systems nor from binary-blackhole systems; moreover, the speed of gravitational radiation or gravitational waves is not the speed of light.

According to Sec. 19.4.1 and Sec. 19.4.2, binary-star or binary-blackhole celestial bodies, even if they exist and coalesce, could barely meet the triggering conditions of the LIGO detector.

First of all, as stated in Sec. 19.4.1, although the huge mass of a black hole could be freely imagined by LIGO, it is too far away from the earth: the strength of its gravitational field decays with distance according to the law of inverse square; its tidal-generating force decays with distance according the law of inverse cubic. Thus, when it sweeps past the earth, its gravitational radiation or gravitational wave would be no longer strong enough to trigger the LIGO detector.

Secondly, as stated in Sec. 19.4.2, we know that: (i) According to the theory of GOR, the energy erupted by a binary-star coalescing even must be carried by the matter erupted by the binary-star coalescing even, rather than be transformed from matter or mass – the total energy erupted is far less than that of the theoretical calculation made by LIGO's computer simulation of virtual reality, and moreover, the energy of gravitational radiation or gravitational wave only accounts for a small part of it; (ii) The gravitational radiation erupted by the binary-star coalescing even is approximate to a spherical wave, and the energy density of it must also decay with distance according to the law of inverse square, and while sweeping past the earth, its spherical wave is approximately uniform and symmetrical and equipotential with respect to the laser arms X and Y of LIGO detector.

So, it is difficult even for binary-blackhole coalescing to trigger LIGO detector.

Why did LIGO's gravitational waves seemingly move at the speed of light?

LIGO claims that the gravitational waves detected by it propagate at the exactly the speed of light just as Einstein's prediction.

At first, based on the trilateration method depicted in Fig. 19.2(b), LIGO dimly reached the conclusion that the speed of gravitational waves seemed to be the speed of light: for example, sweeping past the LIGO Livingston and the LIGO Hanford, GW150914 took 7ms, GW170814 took 6ms, and GW170817 took 3ms; the distance between Livingston and Hanford is 3002km, with the optical path of about 10ms. Later, according to that the GW170817 and the GRB170817A arrived at the earth almost simultaneously [199], with Eq. (19.8), LIGO reached the conclusion that the speed of gravitational waves was exactly the speed of light.

So, why did the gravitational waves detected by LIGO seemingly move at the speed of light? Alternatively, what does it mean that the speed of LIGO's gravitational waves is exactly the speed of light?

As a matter of fact, neither the speed measured based on LIGO's trilateration method nor the speed measured based on LIGO's electromagnetic counterparts is the speed of gravitational radiation or gravitational waves.

The gravitational-radiation or gravitational-wave signals detected and recorded by LIGO detector only suggest that:

(i) Some matter systems carrying gravitational fields invaded and disturbed the spacetime around LIGO detector at close quarters.

(ii) It is not that the gravitational waves detected by LIGO propagated at the speed of light, but that the matter systems as the sources of gravitational fields moved at the speed of light.

It could be imagined that, as depicted in Fig. 19.4, for the gravitational-radiation or gravitational-wave signals recorded by LIGO detector, from the GW150914 to the GW200322, the hosts or gravitational sources of them are actually not binary-star systems and binary-blackhole systems, but some certain clumps of electromagnetic matter sweeping at the speed of light past the LIGO detector.

Why were LIGO's gravitational waves accompanied by electromagnetic matter?

As depicted in Fig. 19.4(a), in LIGO's joint activity of detecting gravitational waves, whenever LIGO detected a gravitational-radiation or gravitational-wave signal, it was always found that there were some clumps of electromagnetic matter before and after it, including gamma-ray bursts, X-rays, and the electromagnetic matter in other frequency bands, which are known as the so-called electromagnetic counterparts, i.e., the electromagnetic matter corresponding to or accompanying LIGO's gravitational waves.

The events of electromagnetic matter sweeping past the earth are extremely common; but, the events of binary-star or binary-blackhole coalescing are previously unknown, and no way to verify them.

As a matter of fact, it is not that LIGO's gravitational-radiation signals were accompanied by electromagnetic counterparts, but that some clumps of electromagnetic matter, as matter systems carrying their own gravitational fields and radiating gravity or gravitational waves, swept past the spacetime around the LIGO detector at close quarters to trigger the LIGO detector.

According to the theory of OR, all matter or matter systems, including electromagnetic matter, have their own mass and carry their own gravitational fields, radiating gravity and gravitational waves.

Naturally, the gravitational field of a matter system must move together with the matter system of it.

Taking the earth as an example, it moves in the orbit around the sun at a speed of $3\times10^4\,\mathrm{ms}^{-1}$, and naturally, the gravitational field of the earth moves together with the earth at the speed of $3\times10^4\,\mathrm{ms}^{-1}$. However, this does not mean that the speed of gravitational radiation or gravitational waves is $3\times10^4\,\mathrm{ms}^{-1}$.

Electromagnetic matter moves at the speed of light. Naturally, the gravitational field of electromagnetic matter moves together with electromagnetic matter at the speed of light. However, this does not mean that the speed of gravitational radiation or gravitational waves is the speed of light.

Electromagnetic matter, or electromagnetic counterparts of LIGO's gravitational waves, as depicted in Fig. 19.4(a), are composed of electromagnetic particles (photons) and move at the speed of light, which may be called **Electromagnetic Particle Clump** (EPC). The theory of OR has already clarified that photons have their own rest mass, which is the intrinsic mass of photons, i.e., the objective and real mass. Therefore, as a matter system, an EPC must carry the gravitational field of its own, radiate gravity or gravitational waves. In particular, the gravitational field of an EPC must move together with the EPC at the speed of light.

As depicted in Fig. 19.4(b), the conclusion of the trilateral measurement shows that the speed of LIGO's gravitational waves sweeping past the LIGO's two base stations seems to be consistent with the speed of light. However, it is worth noting that, as we stressed repeatedly, this does not mean that the speed of gravitational waves is the speed of light, but only means that, for the gravitational-radiation or gravitational-wave signals detected by LIGO detector, their hosts or their gravitational sources move at the speed of light.

So, who are the hosts of LIGO's gravitational-radiation signals?

At this point, the author believes that you must have your own answer.

The Hosts of LIGO's Gravitational Waves: EPCs

Everything points to electromagnetic matter, or to **Electromagnetic-Particle Clumps** (EPCs), which moves at the speed of light.

Indeed, the LIGO detector has recorded numerous gravitational-radiation signals, which may be referred to as **Gravitational Waves**: the waves of gravitational radiation. However, the gravitational-radiation or gravitational-wave signals detected by LIGO detector were not originated from the merging or coalescing events of binary stars or binary black holes.

For the gravitational-radiation or gravitational-wave signals recorded by LIGO detector are concerned, the hosts or gravitational sources of them are EPCs. It is naturally that the LIGO's gravitational-radiation or gravitational-wave signals were accompanied by electromagnetic counterparts or EPCs. Actually, it is not that the LIGO's gravitational-radiation or gravitational-wave signals were accompanied by electromagnetic counterparts or EPCs, but that the LIGO's gravitational-radiation or gravitational-wave signals were emitted by electromagnetic counterparts or EPCs at extremely close quarters.

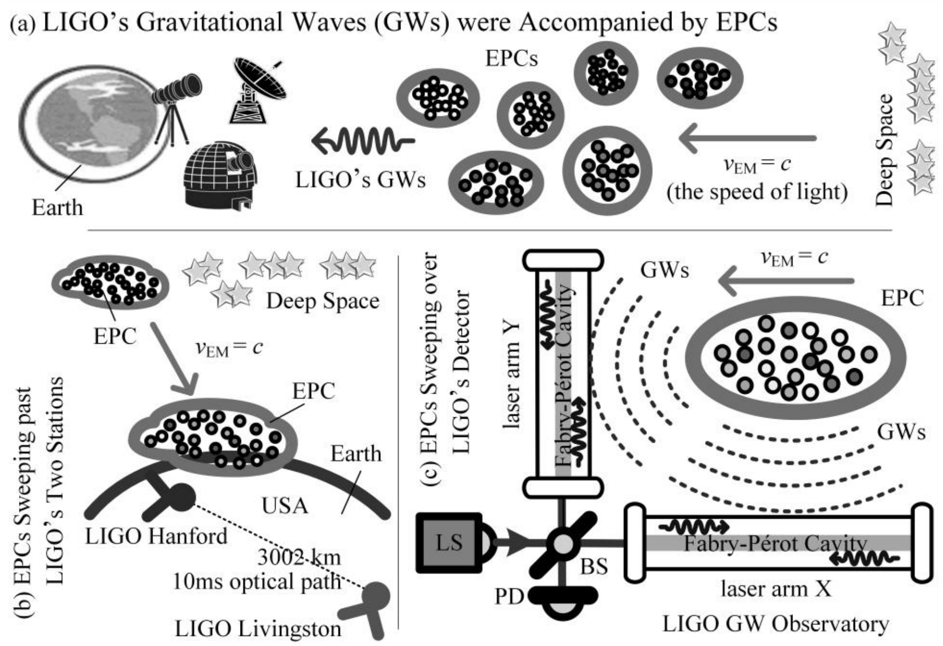

Figure 19.4 Why were LIGO's Gravitational Waves Always Accompanied by EPCs? (a) LIGO's Gravitational Waves were Accompanied by Electromagnetic Matter: Whenever LIGO detected a gravitational-radiation signal, it would always be found that there were some electromagnetic-particle clumps (EPCs) before and after it, which are known as electromagnetic counterparts, such as gamma-ray bursts and X-rays; naturally, EPCs move at the speed of light. (b) The Gravitational Fields of EPCs Sweeping at the Speed of Light past the LIGO Livingston and the LIGO Hanford: As matter systems, EPCs have the gravitational fields of their own moving together with EPCs; it is not that LIGO's gravitational-radiation signals or gravitational waves moved at the speed of light, but that EPCs moved at the speed of light. (c) The Gravitational Fields of EPCs Sweeping over the LIGO Detector: If an EPC swept the LIGO detector at close quarters and its gravitational field asymmetrically invaded and disturbed the spacetime around the laser arms X and Y of LIGO detector, the EPC might trigger the LIGO detector; what LIGO detected were not the gravitational wave originated from the distant black hole in deep space, but the gravitational radiation of the EPC in a short distance.

As depicted in Fig. 19.4(c), although the mass of an EPC is far less than the mass of a black hole, it could sweep past LIGO Gravitational-Wave Observatory at extremely close quarters. As an EPC sweeps very closely past the LIGO detector, it may meet the triggering conditions of LIGO detector and the gravitational field of it may trigger the LIGO detector.

So, according to the theory of GOR, we may reasonably make the following basic judgments:

LIGO's Detection or Discovery: There is no doubt that LIGO has detected gravitational-radiation signals and discovered gravitational waves, but LIGO has not determined the speed of gravitational waves. the gravitational waves detected by LIGO were not originated from the coalescing events of binary-blackhole celestial bodies but from EPCs sweeping past the LIGO detector at close quarters. The speed of gravitation waves is not the speed of light.

Now, we can explain that:

(i) Why were the gravitational waves detected by LIGO always accompanied by electromagnetic counterparts or EPCs?

(ii) Why did the gravitational waves detected by LIGO seemingly move at the speed of light?

The hosts or sources of LIGO's gravitational waves are not binary-star systems or binary-blackhole systems, but EPCs. For LIGO's gravitational-radiation signals, the so-called electromagnetic counterparts of them or EPCs are actually the hosts or gravitational sources of LIGO's gravitational waves.

20 The Unity of Newton and Einstein in GOR

One physical world, one logical system.

As stated in Chapter 8 **The Unity of Newton and Einstein in IOR** of **the 1st volume of OR: Inertially Observational Relativity** (IOR): "Both Newton's classical mechanics and Einstein's relativity theory are only the partial theories of physics in Hawking's words [31], and as a matter of fact, both are the partial theories of OR theory. The unity of the partial theories of physics is not the mechanical or formal reproduction or repetition of old theories, but the progress and even leap in human being's cognition of the objective world, which is a major step in tracing the logical origin of the theoretical systems of physics."

It is undoubtedly of far-reaching significance to unify Newton's theory of universal gravitation and Einstein's theory of general relativity, the two great gravitational theories of physics, into the same theoretical system under the same axiom system.

The theory of GOR, like the theory of IOR, is based on the most basic axiom system and starts from the most basic logical premises. Therefore, it has the high degree of generality and universality. The theory of GOR has generalized and unified Newton's theory of universal gravitation and Einstein's theory of general relativity, providing new insight into the theoretical system of universal gravitation or gravitational interaction.

Tab. 20.1 is a list for the analogy of the theory of GOR and Einstein's theory of general relativity as well as Newton's theory of universal gravitation, demonstrating the unification of Newton and Einstein in the theory of GOR.

Now, the theory of Gravitationally Observational Relativity (GOR), or the theory of GOR for short, has been established on the basis of the OR axiom system. Newton's theory of universal gravitation and Einstein's theory of general relativity have been generalized and unified into **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR).

Perhaps, as Hawking said [31]: "Then we should know the mind of God."

20.1 The unity of the Coordinate Systems of Gravitational Spacetime

The theory of GOR is the theoretical system of general observation agency OA(η). Newton's theory of universal gravitation is the theory of the idealized observation agent OA_∞; Einstein's theory of general relativity is the theory of the optical observation agent OA(c). As the GOR basic formulae shown in Tab. 20.1, if OA(η) is the optical agent OA(c), then the theory of GOR would strictly converge to Einstein's theory of general relativity, while if OA(η) is the idealized agent OA_∞, then the theory of GOR would strictly converge to Newton's theory of universal gravitation.

Tab. 20.1 takes the basic formulae of GOR theory as examples to demonstrate the generality and unity of GOR theory.

According to the theory of GOR, for a theoretical system in physics, the spacetime, no matter the inertial or the gravitational, is the observational spacetime of a specific observation agent OA(η), which is different from the objectively real spacetime. Under the principle of general correspondence (GC), following the logic of Einstein's theory of general relativity, **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR) extends the coordinate framework of Minkowski 4d spacetime from inertial spacetime to gravitational spacetime and from the optical observation agent OA(c) to the general observation agent OA(η)

This section analyzes the generality and unity of the GOR observation agent OA(η) and the coordinate framework of GOR spacetime $X^{4d}(\eta)$.

20.1.1 Cartesian Spacetime vs Minkowski Spacetime

Cartesian Coordinate System

Cartesian spacetime, as the observational spacetime $X^{4d}{}_{\infty}$ of the idealized observation agent OA_∞, can describe not only the inertial spacetime of classical mechanics, but also the gravitational spacetime of classical mechanics.

As stated in Sec. 8.1.1 of Chapter 8, the Cartesian coordinate system, or **the coordinate framework of Cartesian spacetime**, reflects the absolutist view of space and time held by Galileo and Newton: space and time are independent of each other; space is just space and time is just time. As stated in Sec. 1.4.3 **The Idealized Observation Agent** of Chapter 1, the Cartesian coordinate system represents the idealized observation system, i.e., the idealized observation agent OA_∞, in which the information-wave speed is infinite: it takes no time for the information of observed objects to cross space.

No matter inertial spacetime or gravitational spacetime, as shown in Eq. (1.4), the coordinate framework of Cartesian spacetime can be formalized as:

$$\mathrm{OA}_{\infty} \triangleq \left\{ \begin{array}{l} X^{4\mathrm{d}}{}_{\infty} : \left\{ x^0 = \eta t \;\; (\eta \to \infty);\, x^1 = x,\, x^2 = y,\, x^3 = z \right\} \\ \mathrm{d}t = \mathrm{d}\tau \;\; \left(\mathrm{d}\tau = \mathrm{d}s/\eta \right) \\ \mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \end{array} \right\}$$

where OA_∞ is the idealized observation agent, $X^{4d}{}_{\infty}$ is the idealized spacetime of OA_∞; dt is the idealized time-element, dτ is the objectively real time-element (**proper time**), i.e., the **mathematical time** in Newton's words; dl is the line-element of Cartesian 3d space (x,y,z), the time axis x^0 has no physical significance as $\eta \to \infty$.

It is worth noting that, as the information-wave speed η of OA_∞: $\eta \to \infty$, the time axis x^0 splits with the space axes (x^1,x^2,x^2). Therefore, time and space are independent of each other in the coordinate framework of Cartesian spacetime.

The Coordinate Framework of Minkowski Spacetime

Minkowski spacetime is a coordinate framework (Eq. (1.1) in Chapter 1) of 4d spacetime made by Minkowski [50,51] specially for Einstein's theory of special relativity. As clarified by the theory of OR, Minkowski spacetime represents the optical observation system, i.e., the optical observation agent OA(c), in which the observation medium is light, and the speed of transmitting observed information is the speed of light in vacuum, and is limited ($c<\infty$). Therefore, the optical observation agent OA(c) has the observational locality, and it takes time for the information of observed objects to cross space.

At the beginning, Einstein did not realize the value and significance of Minkowski spacetime. However, when he set about studying the theory of general relativity, Einstein found that his general relativity seemed to have to be built on the coordinate framework of 4d spacetime created by Minkowski.

So, Einstein expanded Minkowski spacetime (Eq. (10.1) in Chapter 10) from the inertial spacetime of special relativity to the gravitational spacetime of general relativity, which can be formalized as the GOR gravitational spacetime:

$$\mathrm{OA}(c) \triangleq \left\{ \begin{array}{l} X^{4\mathrm{d}}(c): \left\{ x^0 = ct;\ x^1 = x, x^2 = y, x^3 = z \right\} \\ \mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^{\mu} \mathrm{d}x^{\nu} \ \left(g_{\mu\nu} = g_{\mu\nu}\left(x^{\alpha}, c\right) \right) \end{array} \right\}$$

where OA(c) is the optical observation agent, $X^{4d}(c)$ is the optical observational spacetime of OA(c); dt is the optical observational time-element, ds is the line-element of Minkowski 4d spacetime (x^0,x^1,x^2,x^3), the space coordinates (x^1,x^2,x^3) may be the Cartesian coordinates (x,y,z).

20.1.2 GOR Gravitational Spacetime

The theory of OR has already clarified [26-30]: all the theories of physics depend on observation and are restricted by observation; in theory, all the forms of matter motion can be employed as observation media to transmit the information of observed objects to observers.

The 2nd volume of OR: Gravitationally Observational Relativity (GOR) extended the concept of the general observation agent OA(η) from inertial spacetime (Def. 1.1 in Chapter 1) to gravitational spacetime (Def. 10.1 in Chapter 10). In this way, the theory of GOR builds the coordinate framework of 4d gravitational spacetime (Eq. (10.2) in Chapter 10) for the gravitational spacetime $X^{4d}(\eta)$ the general observation agent OA(η), that is, **the GOR gravitational spacetime**:

$$\mathrm{OA}(\eta) \triangleq \left\{ \begin{array}{l} X^{4\mathrm{d}}(\eta): \left\{ x^0 = \eta t;\ x^1 = x, x^2 = y, x^3 = z \right\} \\ \mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^{\mu} \mathrm{d}x^{\nu} \ \left(g_{\mu\nu} = g_{\mu\nu}\left(x^{\alpha}, \eta\right) \right) \end{array} \right\}$$

where OA(η) is the general observation agent, $X^{4d}(\eta)$ is the observational spacetime of OA(η); dt is the observational time-element of OA(η), ds is the line-element of 4d observational spacetime $X^{4d}(\eta)=(x^0,x^1,x^2,x^3)$, the space coordinates (x^1,x^2,x^3) may be Cartesian coordinates (x,y,z).

Thus, the general observation agent OA(η) becomes the formalized coordinate framework of 4d spacetime in the theory of GOR.

It should be pointed out that no matter Def. 10.1 of the general observation agent

OA(η) or Eq. (10.2) is not the logical premise presupposed by the theory of OR, but the logical consequence derived by the theory of GOR following and analogizing Minkowski's logic [50,51] and Einstein's logic [8]

20.1.3 The Unity of Descartes and Minkowski

The Cartesian coordinate system is the coordinate framework of 3d space that can be employed to serve Newton's classical mechanics (including Newton's theory of universal gravitation); while the extended Minkowski spacetime (Eq. (10.1) in Chapter 10) is the coordinate framework of 4d spacetime that can be employed to serve Einstein's relativity theory (including the special and the general).

Actually, like in inertial spacetime, in gravitational spacetime, no matter Cartesian spacetime or Minkowski spacetime is just a special case of the coordinate framework of 4d spacetime of the general observation agent OA(η): Cartesian spacetime is the so-called idealized observation agent OA_∞ ($\eta\to\infty$); Minkowski spacetime is the so-called optical observation agent OA(c) ($\eta\to c$).

As shown in Tab. 20.1, in gravitational spacetime, the general observation agent OA(η) of GOR theory has generalized and unified the idealized observation agent OA_∞ and the optical observation agent OA(c); in other words, the coordinate framework of 4d spacetime of GOR theory has generalized and unified the Cartesian coordinate system and the coordinate framework of Minkowski 4d spacetime.

Naturally, if $\eta\to c$, then GOR agent OA(η) would strictly converge to the optical agent OA(c), i.e., the coordinate framework of Minkowski 4d spacetime:

$$\lim_{\eta\to c}\text{OA}(\eta)=\lim_{\eta\to c}\left\{\begin{array}{l} X^{4\text{d}}(\eta):\left\{\begin{array}{l} x^0=\eta t\\ x^1=x\\ x^2=y\\ x^3=z \end{array}\right\}\\ \text{d}s^2=g_{\mu\nu}\text{d}x^\mu\text{d}x^\nu\\ \left(g_{\mu\nu}=g_{\mu\nu}\left(x^\alpha,\eta\right)\right) \end{array}\right\}=\left\{\begin{array}{l} X^{4\text{d}}(c):\left\{\begin{array}{l} x^0=ct\\ x^1=x\\ x^2=y\\ x^3=z \end{array}\right\}\\ \text{d}s^2=g_{\mu\nu}\text{d}x^\mu\text{d}x^\nu\\ \left(g_{\mu\nu}=g_{\mu\nu}\left(x^\alpha,c\right)\right) \end{array}\right\}=\text{OA}(c) \quad (20.1)$$

In particular, as shown in Eq. (1.3), if $\eta\to\infty$, the 4d spacetime line-element ds in the observational spacetime $X^{4\text{d}}(\eta)$ of the GOR agent OA(η) would be split into independent time-element dt of 1d time (x^0) and independent line-element dl of 3d space (x^1,x^2,x^3):

$$\text{d}\tau^2=\frac{\text{d}s^2}{\eta^2}=\text{d}t^2-\frac{\text{d}l^2}{\eta^2} \quad \left\{\begin{array}{l} \text{d}\tau^2=\lim\limits_{\eta\to\infty}\dfrac{\text{d}s^2}{\eta^2}=\lim\limits_{\eta\to\infty}\left(\text{d}t^2-\dfrac{\text{d}l^2}{\eta^2}\right)=\text{d}t^2\\ \text{d}l^2=\text{d}x^2+\text{d}y^2+\text{d}z^2 \end{array}\right. \quad (20.2)$$

where dτ is the objectively real time (proper time).

Thus, the GOR agent OA(η) would strictly converge to the idealized agent OA_∞, i.e., the Cartesian coordinate system of Cartesian spacetime:

$$\lim_{\eta\to\infty}\mathrm{OA}(\eta)=\lim_{\eta\to\infty}\left\{\begin{array}{l} X^{4\mathrm{d}}(\eta):\left\{\begin{array}{l} x^0=\eta t\\ x^1=x\\ x^2=y\\ x^3=z \end{array}\right\} \\ \mathrm{d}s^2=\eta^2\mathrm{d}t^2 \\ \qquad -\mathrm{d}x^2-\mathrm{d}y^2-\mathrm{d}z^2 \end{array}\right\}=\left\{\begin{array}{l} X^{4\mathrm{d}}{}_{\infty}:\left\{\begin{array}{l} x^0=\infty t\\ x^1=x\\ x^2=y\\ x^3=z \end{array}\right\} \\ \mathrm{d}t=\mathrm{d}\tau \\ \mathrm{d}l^2=\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2 \end{array}\right\}=\mathrm{OA}_{\infty} \tag{8.3}$$

So, as shown in the row 20.1-1 of Tab. 20.1, like in inertial spacetime, in gravitational spacetime, the general observation agent OA(η) of the theory of OR or GOR has generalized and unified the idealized agent OA_∞ and the optical agent OA(c); in other words, the coordinate framework of 4d spacetime of GOR theory has generalized and unified the Cartesian coordinate system of Cartesian spacetime and the coordinate framework of Minkowski spacetime.

20.2 The Unity of the Factors of Spacetime Transformation

In Chapter 12, under the principle of GC, by analogizing and following the logic of Einstein's theory of general relativity, the theory of GOR has derived the most general spacetime-transformation factor Γ, that is:

The GOR Factor (Eq. (12.36)): $$\Gamma(\eta)=\frac{\mathrm{d}t}{\mathrm{d}\tau}=\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{\eta^2}}-\gamma_i\frac{v^i}{\eta}\right)^2-\frac{v^2}{\eta^2}}}$$

The theory of OR has already clarified that the GOR factor, i.e., the transformation factor of the gravitational spacetime $X^{4\mathrm{d}}(\eta)$ of the general observation agent OA(η), is the important representation of the relativistic effects of gravitational spacetime.

Naturally, if $\eta\to c$, then the GOR factor $\Gamma(\eta)$ would strictly converge to the spacetime-transformation factor $\gamma=\Gamma(c)$ of Einstein's theory of general relativity, that is, the factor of the spacetime transformation of the optical agent OA(c):

$$\mathrm{OA}(c):\ \gamma=\lim_{\eta\to c}\Gamma(\eta)=\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{c^2}}-\gamma_i\frac{v^i}{c}\right)^2-\frac{v^2}{c^2}}}=\Gamma(c) \tag{20.4}$$

In particular, if $\eta\to\infty$, then the GOR factor $\Gamma(\eta)$ would strictly converge to the spacetime-transformation factor $\Gamma_\infty=\Gamma(\infty)$ of Newton's theory of universal gravitation, that is, the factor of the spacetime transformation of the idealized agent OA_∞, or the Galilean factor Γ_∞ ($\equiv$1):

$$\mathrm{OA}_{\infty}:\ \Gamma_{\infty}=\lim_{\eta\to\infty}\Gamma(\eta)=\lim_{\eta\to\infty}\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{\eta^2}}-\gamma_i\frac{v^i}{\eta}\right)^2-\frac{v^2}{\eta^2}}}=1 \qquad (20.5)$$

So, as shown in the row 20.1-2 in Tab. 20.1, the GOR factor $\Gamma(\eta)$ of spacetime transformation has generalized and unified the spacetime-transformation factor $\gamma=\Gamma(c)$ of Einstein's theory of general relativity and the spacetime-transformation factor $\Gamma_\infty=\Gamma(\infty)$ of Newton's theory of universal gravitation. Both Einstein's factor $\gamma=\Gamma(c)$ of gravitational spacetime transformation and Newton's factor $\Gamma_\infty=\Gamma(\infty)$ of gravitational spacetime transformation are the special cases of the GOR factor $\Gamma(\eta)$: Einstein's factor $\gamma=\Gamma(c)$ belongs to the optical agent OA(c); while Newton's factor $\Gamma_\infty=\Gamma(\infty)$ belongs to the idealized agent OA_∞. Now, they have been generalized and unified into the GOR factor $\Gamma(\eta)$ of gravitational spacetime transformation.

Actually, as stated in Sec. 12.4 of Chapter 12, the GOR factor, not only has generalized and unified the spacetime-transformation factor $\gamma=\Gamma(c)$ of Einstein's theory of general relativity and the spacetime-transformation factor $\Gamma_\infty=\Gamma(\infty)$ of Newton's theory of universal gravitation, but also has generalized and unified the spacetime-transformation factors of static, scalar, and vector gravitational fields, and has generalized and unified the spacetime-transformation factors of inertial spacetime and gravitational spacetime.

20.3 The Unity of Gravitational-Field Equations and the Unity of Gravitational-Motion Equations

Einstein once imagined that his theory of general relativity should contain two basic formulae: one was gravitational field equation, describing how spacetime is curved in gravitational fields; the other was gravitational motion equation (the so-called geodesic), describing how an object move in the curved spacetime.

Later, Einstein et al [137] and Fock [138] independently proved that Einstein's field equation and motion equation were equivalent.

Actually, Newton's theory of universal gravitation also has two basic formulae:

Newton's Field Equation (14.1a): $\nabla^2\chi=4\pi G\rho \quad \left(\nabla^2=\sum_{i=1}^{3}\frac{\partial}{\partial x^i}\frac{\partial}{\partial x^i}\right)$

Newton's Geodesic Equation (14.1b): $F=\frac{GMm}{r^2}=-\frac{m}{r}\chi \quad \left(\chi=-\frac{GM}{r}\right)$

where Newton's field equation is the Poisson-equation form of Newton's law of universal gravitation; Newton's geodesic equation, i.e., Newton's motion equation, is the second-law form of Newton's law of universal gravitation.

So, Newton's field equation and Newton's motion equation are also equivalent.

Now, the GOR field equation has generalized and unified Newton's field equation and Einstein's field equation; the GOR motion equation has generalized and unified Newton's motion equation and Einstein's motion equation. From this, we can understand why Einstein's field equation and Einstein's motion equation are a pair of equivalent equations.

20.3.1 The Unification of Einstein's Field Equation and Newton's Field Equation

In Chapter 14, under the principle of GC, by analogizing and following the logic of Einstein's theory of general relativity, the theory of GOR has derived the gravitational-field equation under the general observation agent OA(η), that is:

The GOR Field Equation (14.63):
$$R_{\mu\nu}(\eta)-\frac{1}{2}g_{\mu\nu}(\eta)R(\eta)=-\kappa(\eta)T_{\mu\nu}(\eta)$$
$$\left(\kappa(\eta)=\kappa_{\mathrm{GOR}}=8\pi G/\eta^4\right)$$

The theory of OR has clarified that the GOR field equation is the theoretical model of the general observation agent OA(η). In theory, the observation medium of OA(η) could be any form of matter motion.

Naturally, if $\eta \to c$, then the GOR field equation would strictly converge to Einstein's field equation, that is, the gravitational-field equation under the optical observation agent OA(c):

$$\text{As } \eta \to c$$
$$\mathrm{OA}(\eta) \to \mathrm{OA}(c):\quad \begin{cases} R_{\mu\nu}(c)-\frac{1}{2}g_{\mu\nu}(c)R(c)=-\kappa(c)T_{\mu\nu}(c) \\ \left(\kappa(c)=\kappa_E=8\pi G/c^4\right) \end{cases} \tag{20.6}$$

In particular, as stated in Sec. 14.7 of Chapter 14, if $\eta \to \infty$, then the GOR field equation would strictly converge to Newton's field equation, the Poisson-equation form of Newton's law of universal gravitation, that is, the gravitational-field equation of the idealized agent OA_∞:

$$\begin{aligned} &\text{As } \eta \to \infty \\ &\mathrm{OA}(\eta) \to \mathrm{OA}_\infty:\quad \nabla^2\chi = 4\pi G\rho \end{aligned} \tag{20.7}$$

So, as shown in the row 20.1-3 of Tab. 20.1, the GOR gravitational-field equation has generalized and unified Einstein's field equation and Newton's field equation. Einstein's field equation belongs to the optical agent OA(c); Newton's field equation belongs to the idealized agent OA_∞. Now, both of them have been generalized and unified into the GOR gravitational-field equation under the general observation agent OA(η), becoming the special cases of GOR field equation.

More importantly, this means the unification of Einstein's field equation and Newton's law of universal gravitation.

20.3.2 The Unification of Einstein's Motion Equation and Newton's Motion Equation

In Chapter 14, under the principle of GC, by analogizing and following the logic of Einstein's theory of general relativity, the theory of GOR has derived the motion equation (the geodesic) under the general observation agent OA(η), that is:

$$\text{The GOR Motion Equation (14.37):}\quad \begin{aligned} &\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2}+\Gamma^\mu_{\alpha\beta}(\eta)\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}=0 \quad (\mu=0,1,2,3)\\ &\Gamma^\mu_{\alpha\beta}(\eta)=\frac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta}+g_{\nu\beta,\alpha}-g_{\beta\alpha,\nu}\right)\end{aligned}$$

The theory of OR has clarified that the GOR motion equation is the theoretical model of the general observation agent OA(η). In theory, the observation medium of OA(η) could be any form of matter motion.

Naturally, if $\eta\to c$, then the GOR motion equation would strictly converge to Einstein's motion equation, that is, the motion equation under the optical observation agent OA(c):

$$\text{As }\ \eta\to c$$

$$\mathrm{OA}(\eta)\to\mathrm{OA}(c):\ \begin{cases}\dfrac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2}+\Gamma^\mu_{\alpha\beta}(c)\dfrac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\dfrac{\mathrm{d}x^\beta}{\mathrm{d}\tau}=0 \quad (\mu=0,1,2,3)\\ \Gamma^\mu_{\alpha\beta}(c)=\dfrac{1}{2}g^{\mu\nu}\left(g_{\alpha\nu,\beta}+g_{\nu\beta,\alpha}-g_{\beta\alpha,\nu}\right)\end{cases}\tag{20.8}$$

In particular, as stated in Sec. 14.4 of Chapter 14, if $\eta\to\infty$, then the GOR motion equation (14.37), i.e., the so-called GOR geodesic, would be split into two independent equations (14.44a) and (14.44b):

$$\text{As }\ \eta\to\infty$$

$$\mathrm{OA}(\eta)\to\mathrm{OA}_\infty:\ \begin{cases}\dfrac{\mathrm{d}^2 t}{\mathrm{d}\tau^2}=0\\ \dfrac{\mathrm{d}^2 x^i}{\mathrm{d}\tau^2}=-\dfrac{1}{2}h_{00,i}\left(\dfrac{\mathrm{d}x^0}{\mathrm{d}\tau}\right)^2 \quad (i=1,2,3)\end{cases}\tag{20.9}$$

The time equation (14.44a) of Eq. (20.9) suggests: $\mathrm{d}t=\mathrm{d}\tau$; indicating that in the Cartesian spacetime $X^{4\mathrm{d}}{}_\infty$, or under the idealized agent OA_∞, the observational time dt is the objective and true time (proper time) dτ, and is independent of space. This is consistent with the theorem of Cartesian spacetime in the theory of GOR. The space equation (14.44b) of Eq. (20.9) suggests: the moving object m or the observed object P is subject to gravitational force F^i (i=1,2,3), and the GOR motion equation would strictly converge to the second-law form of Newton's law of universal gravitation:

$$\text{As }\ \eta\to\infty$$

$$\mathrm{OA}(\eta)\to\mathrm{OA}_\infty:\ \begin{cases}F=\dfrac{GMm}{r^2}=-\dfrac{m}{r}\chi \quad \left(\chi=-\dfrac{GM}{r}\right)\\ \text{or}\quad F^i=-m\dfrac{\partial\chi}{\partial x^i} \quad (i=1,2,3)\end{cases}\tag{20.10}$$

So, as shown in the row 20.1-4 of Tab. 20.1, the GOR motion equation has generalized and unified Einstein's motion equation and Newton's motion equation. Einstein's motion equation belongs to the optical agent OA(c); Newton's motion equation belongs to the idealized agent OA_∞. Now, both of them have been generalized and unified into the GOR motion equation under the general observation agent OA(η), becoming the special cases of GOR motion equation.

Likewise, this means the unification of Einstein's motion equation and Newton's law of universal gravitation!

20.4 The Unity of the Motion Equations of Celestial Two-Body System

All celestial bodies move by following Newton's law of universal gravitation in the universe.

Einstein's three major predictions: (i) the gravitational deflection of light, (ii) the gravitational redshift, and (iii) the orbital precession of planets, could be formalized as the problem of the celestial two-body system (M,m): How does the small celestial-body m (a moving object or an observed object P) moves in the gravitational spacetime around the large celestial-body M?

After the birth of Newton's law of universal gravitation [81], people built the classical motion model of the celestial two-body system (M,m) based on Newton's theory of universal gravitation to describe celestial motion. After the birth of Einstein's theory of general relativity [8], people built the relativistic motion model of the celestial two-body system (M,m) based on Einstein's theory of general relativity to describe celestial motion. More than one hundred years have passed, and human being's physics is still in struggle:

Who is right, Newton or Einstein?

Now, the theory of GOR has built the GOR motion model of the celestial two-body system (M,m) under the general observation agent OA(η), which has generalized and unified Newton's celestial-body motion model and Einstein's celestial-body motion model.

Now, physics may no longer be in struggle.

20.4.1 The Unification of the Metric Equations of Gravitational Spacetime

In the problem of the celestial two-body system (M,m), gravitational spacetime is often idealized as the static spherically-symmetric gravitational field. Based on his logic of weak-field approximation, Einstein obtained the approximate solution of Einstein field equation, i.e., the approximate metric of static spherically-symmetric gravitational spacetime [8]; Schwarzschild obtained the first exact solution of Einstein field equation, i.e, the exact metric of static spherically-symmetric gravitational spacetime [80].

Both Einstein's approximate metric and Schwarzschild's exace metric are the metrics of the observational spacetime $X^{4d}(c)$ of the optical observation agent OA(c). Newton's gravitational spacetime is Cartesian spacetime, whose spacetime metric is the metric of the idealized spacetime $X^{4d}{}_{\infty}$ of the idealized observation agent OA_{∞}, which is equivalent to the Minkowski metric $\eta_{\mu\nu}$.

In Chapter 15, under the principle of GC, by analogizing the logic of Schwarzschild exact solution [80], the theory of GOR has obtained the exact solution of GOR field equation, that is, the exact GOR metric of static spherically-symmetric gravitational spacetime:

The GOR spacetime metric (Eq. (15.32): $\begin{cases} g_{00}(\eta)=1+2\chi/\eta^2 \quad (\chi=-GM/r) \\ g_{11}(\eta)=-\left(1+2\chi/\eta^2\right)^{-1} \\ g_{22}(\eta)=-r^2 \\ g_{33}(\eta)=-r^2\sin^2\theta \\ g_{\mu\nu}(\eta)=0 \quad (\mu\neq\nu) \end{cases}$

Naturally, if $\eta\to c$, then the GOR spacetime metric $g_{\mu\nu}(\eta)$ would strictly converge to the exact solution of Einstein's field equation, that is, the Schwarzschild metric $g_{\mu\nu}(c)$ (Eq. (15.7)), the spacetime metric under the optical agent OA(c):

$$\lim_{\eta\to c}\begin{cases} g_{00}(\eta)=1+2\chi/\eta^2 \\ g_{11}(\eta)=-\left(1+2\chi/\eta^2\right)^{-1} \\ g_{22}(\eta)=-r^2 \\ g_{33}(\eta)=-r^2\sin^2\theta \\ g_{\mu\nu}(\eta)=0 \quad (\mu\neq\nu) \end{cases} = \begin{cases} g_{00}(c)=1+2\chi/c^2 \\ g_{11}(c)=-\left(1+2\chi/c^2\right)^{-1} \\ g_{22}(c)=-r^2 \\ g_{33}(c)=-r^2\sin^2\theta \\ g_{\mu\nu}(c)=0 \quad (\mu\neq\nu) \end{cases} \tag{20.11}$$

In particular, if $\eta\to\infty$, then the GOR spacetime metric $g_{\mu\nu}(\eta)$ would strictly converge to the Minkowski metric $\eta_{\mu\nu}$, that is, the Newtonian gravitational metric or Cartesian spacetime metric, the spacetime metric under the idealize agent OA_{∞}:

$$\lim_{\eta\to\infty}\begin{cases} g_{00}(\eta)=1+2\chi/\eta^2 \\ g_{11}(\eta)=-\left(1+2\chi/\eta^2\right)^{-1} \\ g_{22}(\eta)=-r^2 \\ g_{33}(\eta)=-r^2\sin^2\theta \\ g_{\mu\nu}(\eta)=0 \quad (\mu\neq\nu) \end{cases} = \begin{cases} \eta_{00}=1 \\ \eta_{11}=-1 \\ \eta_{22}=-r^2 \\ \eta_{33}=-r^2\sin^2\theta \\ \eta_{\mu\nu}=0 \quad (\mu\neq\nu) \end{cases} \tag{20.12}$$

where $\eta_{\mu\nu}=\mathrm{diag}(+1,-1,-r^2,-r^2\sin^2\theta)$ is the spherical-coordinate form of the Minkowski metric.

Equation (20.12) is consistent with the theorem of Cartesian spacetime.

So, as shown in the row 20.1-5 of Tab. 20.1, the GOR spacetime metric $g_{\mu\nu}(\eta)$ has generalized and unified Einstein's metric $g_{\mu\nu}(c)$ of gravitational spacetime and Newton's metric $g_{\mu\nu}(\infty)$ of gravitational spacetime. Einstein's spacetime metric

$g_{\mu\nu}(c)$ belongs to the optical agent OA(c); Newton's spacetime metric $g_{\mu\nu}(\infty)$ belongs to the idealized agent OA_∞. Now, both of them have been generalized and unified into the GOR gravitational-spacetime metric $g_{\mu\nu}(\eta)$ under the general observation agent OA(η), becoming the special cases of GOR spacetime metric.

20.4.2 The Unification of the Energy Equations of Gravitational Spacetime

According to classical mechanics, in gravitational spacetime (taking the celestial two-body system (M,m) as an example), the moving object m has both kinetic energy K and potential energy V. The total energy H of it is the sum of the kinetic energy K and the potential energy V: $H=K+V$.

According to Sec. 18.4 **GOR and Gravitational Redshift** of Chapter 18, in the gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η), the kinetic energy $K(\eta)$ and the potential energy $V(\eta)$, as well as the total energy $H(\eta)$ of the moving object m, can be defined as:

$$H = H(\eta) = K(\eta) + V(\eta) = \left(\Gamma(\eta)\big|_{\chi=0} - \Gamma(\eta)\big|_{v=0} \right) m_o \eta^2 \tag{20.13}$$

$$\begin{cases} K = K(\eta) = \left(\Gamma(\eta)\big|_{\chi=0} - 1 \right) m_o \eta^2 \\ V = V(\eta) = \left(1 - \Gamma(\eta)\big|_{v=0} \right) m_o \eta^2 \end{cases} \quad \left(\Gamma(\eta) = \frac{1}{\sqrt{1 + 2\chi/\eta^2 - v^2/\eta^2}} \right)$$

Naturally, if $\eta \to c$, then in the GOR observational spacetime, the GOR kinetic energy $K(\eta)$ and potential energy $V(\eta)$ as well as total energy $H(\eta)$ of the moving object m would strictly converge to the kinetic energy $K(c)$ and potential energy $V(c)$ as well as total energy $H(c)$ of m in Einstein's theory of general relativity:

$$H = H(c) = K(c) + V(c) = \left(\Gamma(c)\big|_{\chi=0} - \Gamma(c)\big|_{v=0} \right) m_o c^2 \tag{20.14}$$

$$\begin{cases} K = K(c) = \left(\Gamma(c)\big|_{\chi=0} - 1 \right) m_o c^2 \\ V = V(c) = \left(1 - \Gamma(c)\big|_{v=0} \right) m_o c^2 \end{cases} \quad \left(\Gamma(c) = \frac{1}{\sqrt{1 + 2\chi/c^2 - v^2/c^2}} \right)$$

In particular, if $\eta \to \infty$, then in the GOR observational spacetime, the GOR kinetic energy $K(\eta)$ and potential energy $V(\eta)$ as well as total energy $H(\eta)$ of the moving object m would strictly converge to the kinetic energy K_∞ and potential energy V_∞ as well as total energy H_∞ of m in Newton's theory of universal gravitation:

$$H_\infty = \lim_{\eta \to \infty} H(\eta) = K(\eta) + V(\eta) = \frac{1}{2} m_\infty v^2 - \frac{GMm_\infty}{r} \quad \left(\chi = -\frac{GM}{r} \right) \tag{20.15}$$

$$\begin{cases} K_\infty = \lim_{\eta \to \infty} K(\eta) = \left(\Gamma(\eta)\big|_{\chi=0} - 1 \right) m_o \eta^2 = \frac{1}{2} m_o v^2 = \frac{1}{2} m_\infty v^2 \\ V_\infty = \lim_{\eta \to \infty} V(\eta) = \left(1 - \Gamma(\eta)\big|_{v=0} \right) m_o \eta^2 = \chi m_o = -\frac{GMm_\infty}{r} \end{cases}$$

So, as shown in the row 20.1-6 of Tab. 20.1, the GOR observational kinetic energy $K(\eta)$ of the moving object m in the GOR spacetime has generalized and unified the relativistic kinetic energy $K(c)$ in Einstein's theory of general relativity and the classical kinetic energy K_∞ in Newton's theory of universal gravitation; the GOR observational potential energy $V(\eta)$ of the moving object m in the GOR spacetime has generalized and unified the relativistic potential energy $V(c)$ in Einstein's theory of general relativity and the classical potential energy V_∞ in Newton's theory of universal gravitation. Thus, the GOR observational total energy $H(\eta)$ of the moving object m in the GOR spacetime has generalized and unified the relativistic total energy $H(c)$ in Einstein's theory of general relativity and the classical total energy H_∞ in Newton's theory of universal gravitation

The relativistic kinetic-energy formula $K(c)$ and relativistic potential-energy formula $V(c)$ as well as relativistic total-energy formula $H(c)$ in Einstein's theory of general relativity are the energy formulae in the observational spacetime $X^{4d}(c)$ of the optical observational agent OA(c); the classical kinetic-energy formula K_∞ and classical potential-energy formula V_∞ as well as classical total-energy formula H_∞ in Newton's theory of universal gravitation are the energy formulae in the idealized spacetime $X^{4d}{}_\infty$ of the idealized observational agent OA_∞. Now, they have been generalized and unified into the GOR energy formulae in the GOR observational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η), becoming the special cases of GOR energy formulae in the theory of GOR.

20.4.3 The Unification of the Motion Equations of the Celestial Two-Body System (*M*,*m*)

Before Einstein had established the theory of general relativity, astrophysicists built the classical motion equation of the celestial two-body system (M,m) based on Newton's theory of universal gravitation, that is:

Newton's celestial motion equation (16.5): $\dfrac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\dfrac{GM}{h_K^2}$

where the trajectory of the moving object m is a standard conic curve, the celestial body M as the gravitational source is located at a focus of the conic curve; in terms of a star M and a planet m, the trajectory of m is a closed ellipse, and therefore, the planet m has no orbital precession or perihelion precession.

Later, Einstein built the relativistic motion equation of the celestial two-body system (M,m) based on his theory of general relativity, that is:

Einstein's celestial motion equation (16.33): $\dfrac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\dfrac{GM}{h_K^2}\left(1+\dfrac{3h_K^2}{c^2}u^2\right)$

where, compared with Newton's celestial motion equation (16.5), Einstein's celestial motion equation (16.33) has one more term on the right end: $3GM/c^2r^2$ ($r=1/u$), and therefore, the trajectory of m is no longer a standard conic curve.

As far as the star M and the planet m are concerned, in Einstein's celestial motion equation (16.33), the trajectory of the planet m is no longer a closed ellipse, where $3GM/c^2r^2$ can be referred to as the orbital-precession term of planets, and could be employed to predict Mercury's orbital precession or perihelion precession of about 43″ every 100 years.

So, people believe in Einstein's theory of general relativity.

However, as clarified in Chapter 16, Einstein's celestial motion equation (16.33) cannot predict the objective and real precession of planet orbits. The so-called Mercury's orbital precession or perihelion precession of about 43″ every 100 years predicted by Einstein's celestial motion equation is actually the observational effect and apparent phenomenon of the optical observation agent OA(c), rooted from the observational locality ($c<\infty$) of OA(c).

In Chapter 16, under the principle of GC, by analogizing logic of Einstein's celestial motion equation (16.33), the theory of GOR has derived the GOR celestial motion equation (16.64) in the gravitational spacetime $X^{4d}(\eta)$ of the general observation agent OA(η), that is:

The GOR celestial motion equation (16.64): $\dfrac{d^2u}{d\varphi^2}+u=\dfrac{GM}{h_K^2}\left(1+\dfrac{3h_K^2}{\eta^2}u^2\right)$

where, like Einstein's celestial motion equation (16.33), the GOR celestial motion equation (16.64) also contains the orbital-precession term of planets: $3GM/\eta^2r^2$ ($r=1/u$). Thus, the GOR celestial motion equation (16.64) could also be employed to predict the orbital precession of planets.

However, under different observation agents, the same planet m, would present different degrees of orbital precession. This fact suggests that the orbital-precession term$3GM/\eta^2r^2$ of planets in Eq. (16.64) does not represent the objective and real precession of planet orbits, but the observational effect and apparent phenomenon of the observation agent OA(η), caused by the observational locality ($\eta<\infty$) of OA(η).

Naturally, if $\eta\to c$, then the GOR celestial motion equation (16.64) would strictly converge to Einstein's celestial motion equation (16.33):

$$\text{OA}(c):\quad \lim_{\eta\to c}\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{\eta^2}u^2\right)=\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{c^2}u^2\right) \tag{20.16}$$

In particular, if $\eta\to\infty$, then the GOR celestial motion equation (16.64) would strictly converge to Newton's celestial motion equation (16.5):

$$\text{OA}_\infty:\quad \lim_{\eta\to\infty}\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{\eta^2}u^2\right)=\frac{GM}{h_K^2} \tag{20.17}$$

So, as shown in the row 20.1-7 of Tab. 20.1, the GOR celestial motion equation has generalized and unified Einstein's celestial motion equation and Newton's celestial motion equation. Einstein's celestial motion equation belongs to the optical agent OA(c); Newton's celestial motion equation belongs to the idealized agent OA_∞. Now, both of them have been generalized and unified into the GOR celestial motion equation under the general observation agent OA(η), becoming the special cases of the GOR celestial motion equation in the theory of GOR.

20.4.4 The Unification of the Precession Equations of Planet Orbits

According to Newton's celestial motion equation (16.5), the orbit of a planet is a standard and closed ellipse, and therefore, as far as the celestial two-body system (M,m)=(Star,Planet) is concerned, the planet m has no the orbital precession or perihelion precession: $\Delta\varphi_N=0$ (see Sec. 16.2.3 of Chapter 16).

According to Einstein's celestial motion equation (16.33), the orbit of a planet is not a standard and closed ellipses, and therefore, the planet m has the orbital precession and perihelion precession: $\Delta\varphi_E=6\pi G^2M^2/c^2h_K^2$, as shown in Eq. (16.38). According to the calculation of Eq. (16.38), Mercury's orbital or perihelion would precesses about 43 arc second every 100 years.

After all, according to the actual observations, Mercury moving around the sun always exhibits the orbital precession or perihelion precession. So, people believe that Einstein's formula for the orbital precession of planets is better than Newton's.

However, according to the theory of GOR, under different observation agents, the same planet m would exhibit different degrees of orbital precession. The difference between Newton and Einstein's formula for the orbital precession of planets is the observational difference between different observation agents, that is, the observational difference between the idealized agent OA_∞ and the optical agent OA(c), which does not mean that Einstein is right, nor that Newton is wrong.

In Chapter 16, under the principle of GC, by analogizing the logic of Einstein's formula for the orbital precession of planets, the theory the GOR has derived the GOR formula (Eq. (16.68)) for the orbital precession of planets under the general observation agent OA(η), in which the precession rate of the planet m depends on the information-wave speed η of OA(η): $\Delta\varphi(\eta)=6\pi G^2M^2/\eta^2h_K^2$.

Like the other formulae in the theory of GOR, if $\eta\to c$, then the GOR precession equation (16.68) of planet orbits would strictly converges to Einstein's precession equation (16.38) of planet orbits; if $\eta\to\infty$, then the GOR precession equation (16.68) of planet orbits would strictly converges to Newton's precession equation (see Sec. 16.2.3) of planet orbits.

Just as it is expressed in the following equation (20.18):

$$\text{OA}(\eta):\quad \Delta\varphi(\eta)=\frac{6\pi G^2M^2}{\eta^2h_K^2}\text{rad}$$
$$\begin{cases}\text{OA}(c):\quad \lim\limits_{\eta\to c}\Delta\varphi(\eta)=\lim\limits_{\eta\to c}\dfrac{6\pi G^2M^2}{\eta^2h_K^2}=\dfrac{6\pi G^2M^2}{c^2h_K^2} \\ \text{OA}_\infty:\quad \lim\limits_{\eta\to\infty}\Delta\varphi(\eta)=\lim\limits_{\eta\to\infty}\dfrac{6\pi G^2M^2}{\eta^2h_K^2}=0\end{cases}\tag{20.18}$$

So, as shown in the row 20.1-8 of Tab. 20.1, the GOR precession equation of planet orbits has generalized and unified Einstein's precession equation of planet orbits and Newton's precession equation of planet orbits. Einstein's precession equation of planet orbits belongs to the optical agent OA(c); Newton's precession equation of planet orbits belongs to the idealized agent OA_∞. Now, both of them have been generalized and unified into the GOR precession equation of planet orbits under the general observation agent OA(η), becoming the special cases of the GOR precession equation of planet orbits in the theory of GOR.

20.4.5 The Unification of the Equations of Gravitational Deflection

Based on Newton's celestial motion equation (16.5), one could derive Newton's motion equation (17.7) of photons, and calculate Newton's gravitational-deflection angle δ_N of light: $\delta_N=2GM/R_Sc^2$. Based on Einstein's celestial motion equation (16.33), one could derive Einstein's motion equation (17.22) of photons, and calculate Einstein's gravitational-deflection angle δ_E of light: $\delta_E=4GM/R_Sc^2$.

Einstein's gravitational-deflection angle δ_E of light is twice Newton's δ_N, which seems to be more in line with **observation**. Therefore, people believe that Einstein's gravitational-deflection equation of light is better than Newton's. (It is worth noting that the term **observation** here refers to optical observation, which is the observation under the optical agent OA(c).)

However, according to the theory of GOR, under different observation agents, light would exhibit different degrees of gravitational deflection. The difference between Newton's gravitational-deflection equation (17.7)) and Einstein's gravitational-deflection equation (17.22), or the difference between Newton's gravitational-deflection angle $\delta_N=2GM/R_Sc^2$ of light and Einstein's gravitational-deflection angle $\delta_E=4GM/R_Sc^2$ of light, is the observational difference between different observation agents, that is, the observational difference between the idealized observation agent OA_∞ and the optical observation agent OA(c), which does not mean that Einstein is right, nor that Newton is wrong.

In Chapter 17, under the principle of GC, by analogizing the logic of Einstein's gravitational-deflection calculation, the theory of GOR has derived the GOR gravitational-deflection equation of light under the general observation agent OA(η), and then, as stated in Sec. 17.4.4 of Chapter 17, has made the calculation of the GOR gravitational-deflection angle $\delta_{\text{GOR}}=\delta_{\text{OA}(\eta)}$ of light:

(i) The case of the optical agent OA(η) ($\eta=c$): by solving the GOR motion equation (17.30) of photons as $\eta=c$, one could have the gravitational deflection angle of light sweeping over the sun M:

$$\delta_{\text{GOR}}=\delta_{\text{OA}(\eta)}=\frac{4GM}{R_S c^2} \quad (\eta\to c) \tag{20.19}$$

where the photon m is both the observed object P and the informon of OA(c); the speed c of light is both the motion speed of m as the observed object P and the information-wave speed of the optical agent OA(c).

(ii) the case of the superluminal agent OA(η) ($\eta>>c$): by solving the GOR motion equation (17.31) of photons as $\eta>>c$, one could have the gravitational deflection angle of light sweeping over the sun M:

$$\delta_{\mathrm{GOR}} = \delta_{\mathrm{OA}(\eta)} = \frac{2GM}{R_S c^2}\left(1+\frac{c^2}{3c^2+2\eta^2}\right) \quad (\eta \gg c) \tag{20.20}$$

where the photon *m* is the observed object *P*, but not the informon of OA(η); the speed *c* of light is the motion speed of *m* as the observed object *P*, but not the information-wave speed of the observation agent OA(η).

Naturally, if $\eta \to c$, then the GOR gravitational-deflection angle $\delta_{\mathrm{GOR}}=\delta_{\mathrm{OA}(\eta)}$ (Eq. (20.19)) of light would strictly converge to Einstein's gravitational-deflection angle δ_E (Eq. (17.25)) of light:

$$\mathrm{OA}(c): \quad \delta_E = \delta_{\mathrm{OA}(c)} = \lim_{\eta\to c}\delta_{\mathrm{OA}(\eta)} = \frac{4GM}{R_S c^2} \tag{20.21}$$

In particular, if $\eta \to \infty$, then the GOR gravitational-deflection angle $\delta_{\mathrm{GOR}}=\delta_{\mathrm{OA}(\eta)}$ (Eq. (20.20)) of light would strictly converge to Newton's gravitational-deflection angle δ_N (Eq. (17.12)) of light:

$$\begin{aligned} \mathrm{OA}_\infty: \quad \delta_N &= \delta_{\mathrm{OA}_\infty} = \lim_{\eta\to\infty}\delta_{\mathrm{OA}(\eta)} \\ &= \lim_{\eta\to\infty}\frac{2GM}{R_S c^2}\left(1+\frac{c^2}{3c^2+2\eta^2}\right) = \frac{2GM}{R_S c^2} \end{aligned} \tag{20.22}$$

So, as shown in the row 20.1-9 of Tab. 20.1, the GOR gravitational-deflection equation of light has generalized and unified Einstein's gravitational-deflection equation of light and Newton's gravitational-deflection equation of light. Einstein's gravitational-deflection equation of light belongs to the optical agent OA(c); Newton's gravitational-deflection equation of light belongs to the idealized agent OA_∞. Now, both of them have been generalized and unified into the GOR gravitational-deflection equation of light under the general observation agent OA(η), becoming the special cases of the GOR gravitational-deflection equation of light in the theory of GOR.

20.4.6 The Unification of the Equations of Gravitational Redshift

As stated in Chapter 18, based on his theory of general relativity, Einstein derived the gravitational-redshift equation (18.10) of light, that is, Einstein's gravitational-redshift equation of light:

Einstein's redshift (Eq. (18.10)): $$Z_E = 1-\frac{\sqrt{g_{00}(r_{\mathrm{B}})}}{\sqrt{g_{00}(r_{\mathrm{A}})}} \quad \begin{pmatrix} g_{00} = 1+2\chi/c^2 \\ \chi = -GM/r \end{pmatrix}$$

Taking the observers on the earth observing the solar spectrum as an example, according to Einstein's gravitational-redshift equation (18.10) of light, Einstein's redshift Z_E of the solar spectrum from the sun to the earth is as shown in Eq. (18.11): $Z_E \approx -GM_S/R_S c^2 \approx -2.12\times10^{-6}$, which is consistent with observation.

Based on Newton's theory of universal gravitation, one could also build the gravitational-redshift equation of light. As stated in Chapter 18, the current

Newtonian formula for the gravitational redshift of light is:

Pseudo Newtonian redshift (Eq. (18.20)): $Z_{PN} = \dfrac{GM}{c^2}\left(\dfrac{1}{r_{\mathrm{B}}} - \dfrac{1}{r_{\mathrm{A}}}\right)$

The pseudo Newtonian gravitational-redshift equation (Eq. (18.20)) of light is approximate to Einstein's gravitational-redshift equation (18.10) of light, as far as the earth's observers observing the solar spectrum are concerned, the Z_E and the Z_{PN} are not distinguishable in observation.

As stated in Chapter 18, Eq. (18.20) is not the true Newtonian theory of gravitational redshift, but a hybrid of Newton's classical mechanics, Einstein's relativity theory, and even quantum theory.

Under the principle of conservation of energy, Chapter 18 has established the gravitational-redshift theory of light, which is purely based on Newton's classical mechanics, and can be formally referred to as Newton's gravitational-redshift equation of light as described in Eq. (18.34):

Newton's redshift (Eq. (18.34)): $Z_N = \dfrac{2GMr_{\mathrm{B}}}{r_{\mathrm{B}}c^2 + 2GM}\left(\dfrac{1}{r_{\mathrm{B}}} - \dfrac{1}{r_{\mathrm{A}}}\right)$

As far as the earth's observers observing the solar spectrum are concerned, Newton's gravitational redshift $Z_N \approx -2GM_S/R_Sc^2$ of light is not equal or approximate to Einstein's gravitational redshift $Z_E \approx -GM_S/R_Sc^2$ of light. This suggests that, similar to the case in the gravitational-deflection problem of light, the gravitational redshifts presented by the idealized observation agent OA_∞ and the optical observation agent OA(c) are also different.

In Chapter 18, under the principle of conservation of energy, the theory of GOR has built the GOR gravitational-redshift theory (see Sec. 18.4 of Chapter 18), and has derived the GOR gravitational-redshift equation of light:

$$Z_{\mathrm{GOR}} = Z_{\mathrm{OA}(\eta)} = \frac{1\big/\sqrt{g_{00}(r_{\mathrm{B}})} - 1\big/\sqrt{g_{00}(r_{\mathrm{A}})}}{K_{F\eta}\big/m_o\eta^2 - \left(1 - 1\big/\sqrt{g_{00}(r_{\mathrm{B}})}\right)}$$
$$\left(K_{F\eta} = \left(\varGamma(\eta)\big|_{\chi=0} - 1\right)m_o\eta^2;\ g_{00}(r) = 1 + \frac{2\chi}{\eta^2};\ \chi(r) = -\frac{GM}{r}\right) \tag{20.23}$$

Naturally, as stated in Sec. 18.4.3 of Chapter 18, if $\eta \to c$, then the GOR gravitational-redshift equation (20.23) of light would strictly converge to Einstein's gravitational-redshift equation (18.10) of light:

$$\begin{aligned}\lim_{\eta\to c} K_{F\eta} &= \lim_{\eta\to c}\left(1 - \varGamma^{-1}\big|_{\chi=0}\right)m_\eta\eta^2 \\ &= \lim_{\eta\to c}\left(1 - \sqrt{1 - c^2/\eta^2}\right)m_\eta\eta^2 = mc^2 = m_oc^2\end{aligned} \tag{20.24a}$$

$$\text{OA}(c):\quad Z_E = Z_{\text{OA}(c)} = \lim_{\eta\to c} Z_{\text{OA}(\eta)}$$
$$= \lim_{\eta\to c}\frac{m_o\eta^2\big/\sqrt{g_{00}(r_\text{B})} - m_o\eta^2\big/\sqrt{g_{00}(r_\text{A})}}{K_{F\eta} - \left(1-1\big/\sqrt{g_{00}(r_\text{B})}\right)m_o\eta^2}$$
$$= \frac{m_oc^2\big/\sqrt{g_{00}(r_\text{B})} - m_oc^2\big/\sqrt{g_{00}(r_\text{A})}}{m_oc^2 - \left(1-1\big/\sqrt{g_{00}(r_\text{B})}\right)m_oc^2} = 1-\frac{\sqrt{g_{00}(r_\text{B})}}{\sqrt{g_{00}(r_\text{A})}} \tag{20.24b}$$

In particular, as stated in Sec. 18.4.3 of Chapter 18, if $\eta\to\infty$, then the GOR gravitational-redshift equation (20.23) of light would strictly converge to Newton's gravitational-redshift equation (18.34) of light:

$$\lim_{\eta\to\infty} K_{F\eta} = \lim_{\eta\to\infty}\left(\Gamma|_{\chi=0} - 1\right)m_o\eta^2$$
$$= \lim_{\eta\to\infty}\left(\frac{1}{\sqrt{1-c^2/\eta^2}}-1\right)m_o\eta^2 = \frac{1}{2}m_oc^2 \tag{20.25a}$$

$$\text{OA}_\infty:\quad Z_N = Z_{\text{OA}_\infty} = \lim_{\eta\to\infty} Z_{\text{OA}(\eta)}$$
$$= \lim_{\eta\to\infty}\frac{m_o\eta^2\big/\sqrt{g_{00}(r_\text{B})} - m_o\eta^2\big/\sqrt{g_{00}(r_\text{A})}}{K_{F\eta} - \left(1-1\big/\sqrt{g_{00}(r_\text{B})}\right)m_o\eta^2}$$
$$= \lim_{\eta\to\infty}\frac{\left(1+GM/r_\text{B}\eta^2\right)-\left(1+GM/r_\text{A}\eta^2\right)}{c^2/2\eta^2 + GM/r_\text{B}\eta^2}$$
$$= \frac{2GMr_\text{B}}{r_\text{B}c^2+2GM}\left(\frac{1}{r_\text{B}}-\frac{1}{r_\text{A}}\right) \tag{20.25b}$$

So, as shown in the row 20.1-10 of Tab. 20.1, the GOR gravitational-redshift theory has generalized and unified Einstein's gravitational-redshift theory and Newton's gravitational-redshift theory. Einstein's gravitational-redshift theory belongs to the optical agent OA(c); Newton's gravitational-redshift theory belongs to the idealized agent OA_∞. Now, both of them have been generalized and unified into the GOR gravitational-redshift theory under the general observation agent OA(η), becoming the special cases of the GOR gravitational-redshift theory in the theory of GOR.

20.5 The Unity of the Information-Wave Equations

As stated in Chapter 19, based on his theory of general relativity, Einstein derived an important wave equation:

$$\begin{cases} \nabla^2 h^-_{\mu\nu}(c) - \dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}h^-_{\mu\nu}(c) = 0 \\ h^-_{\mu\nu}(c) \equiv h_{\mu\nu}(c) - \dfrac{1}{2}\eta_{\mu\nu}h(c) \quad \left(h\equiv h^\mu_\mu = \eta^{\mu\nu}h_{\mu\nu}\right) \end{cases} \tag{20.26}$$

In Eq. (20.26), the wave function $h^{-}_{\mu\nu}(c)$ is referred to as the metric-perturbation tensor and is defined by the curved metric $h_{\mu\nu}$, which is related with the Newtonian gravitational potential χ: $h_{\mu\nu}\sim\chi$. Accordingly, Einstein believed that the wave function $h^{-}_{\mu\nu}(c)$ in Eq. (20.26) represented gravitational radiation, and called it **Gravitational Wave**. Meanwhile, according to Eq. (20.26), the wave speed of $h^{-}_{\mu\nu}(c)$ is the speed c of light in vacuum. From this, Einstein made the famous prediction: there are gravitational waves in gravitational spacetime, which propagate at the speed c of light in vacuum.

In this way, Einstein's specious prediction of gravitational waves was born.

However, as the theory of GOR has already clarified: Einstein's prediction of gravitational waves was a historic mistake!

Actually, the wave function $h^{-}_{\mu\nu}(c)$ in Einstein's wave equation (20.26) is not gravitational radiation, let alone a gravitational wave, but the information wave of the optical observation agent OA(c) that transmits the information of observed objects for observers, that is, light, naturally travelling at the speed c of light.

In Chapter 19, under the principle of GC, by analogizing the logic of Einstein's wave equation, the theory of GOR has derived the wave equation (19.15) in the gravitational spacetime $X^{4\text{d}}(\eta)$ of the general observation agent OA(η):

$$\begin{cases} \nabla^2 h^{-}_{\mu\nu}(\eta) - \dfrac{1}{\eta^2}\dfrac{\partial^2}{\partial t^2} h^{-}_{\mu\nu}(\eta) = 0 \\ h^{-}_{\mu\nu}(\eta) \equiv h_{\mu\nu}(\eta) - \dfrac{1}{2}\eta_{\mu\nu} h(\eta) \quad \left(h \equiv h^{\mu}_{\mu} = \eta^{\mu\nu} h_{\mu\nu}\right) \end{cases} \tag{20.27}$$

The GOR wave equation (20.27), exactly the GOR information-wave equation, is isomorphically consistent with Einstein's wave equation (20.26).

It is worth noting that: the wave function $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(\eta)$ in the GOR wave equation (20.27) depends on observation, depends on the observation agent OA(η), and depends on the information-wave speed η of OA(η); moreover, the speed of the wave in the wave function $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(\eta)$ is exactly the information-wave speed η of the observation agent OA(η).

The theory of GOR does not doubt the existence of gravitational waves.

However, the objectively gravitational radiation or gravitational waves must not rely on observation, and the intrinsic speed of gravitational radiation or gravitational waves must be unique and definite. If the wave function $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(\eta)$ in the GOR wave equation (20.27) was interpreted as **Gravitational Wave** following Einstein's logic, then, under different observation agents, the same gravitational wave $h^{-}_{\mu\nu}$ would represent different gravitational waves or would have different speeds.

Actually, with respect to the GOR wave equation (including Einstein's wave equation), the connection between the wave function $h^{-}_{\mu\nu}$ and the Newtonian gravitational potential χ does not means that $h^{-}_{\mu\nu}$ represents gravitational wave, but only means that $h^{-}_{\mu\nu}$ as the information wave of the observation agent OA(η) is carrying the information (χ) about gravitational radiation or gravitational interaction.

So, the GOR wave function $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(\eta)$ (including Einstein's wave function $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(c)$) does not represents gravitational radiation or gravitational wave, but the information wave of the observation agent OA(η). The GOR wave equation (including Einstein wave equation) is not the so-called gravitational-wave equation, but the information-wave equation of OA(η).

In Sec. 19.3.2 of Chapter 19, the theory of GOR has already clarified that: if $\eta\to c$, then the GOR wave equation (20.27) would strictly converge to Einstein's wave equation (20.26):

$$\mathrm{OA}(c):\quad \lim_{\eta\to\infty}\left(\nabla^2 h^{-}_{\mu\nu}(\eta)-\frac{1}{\eta^2}\frac{\partial^2}{\partial t^2}h^{-}_{\mu\nu}(\eta)\right)=\nabla^2 h^{-}_{\mu\nu}(c)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}h^{-}_{\mu\nu}(c) \quad (20.28)$$

There should be also the information-wave equation in Newton's gravitational spacetime, where the information-wave speed is infinite.

In Sec. 19.3.2 of Chapter 19, the theory of GOR has already clarified that: if $\eta\to\infty$, then the GOR wave equation (20.27) would strictly converges to Newton's wave equation, that is, the vacuum form of the Poisson equation of Newton's law of universal gravitation, known as Laplace equation:

$$\begin{gathered}\mathrm{OA}_\infty:\quad \lim_{\eta\to\infty}\left(\nabla^2 h^{-}_{\mu\nu}-\frac{1}{\eta^2}\frac{\partial^2}{\partial t^2}h^{-}_{\mu\nu}\right)=\nabla^2 h^{-}_{\mu\nu}\\ \text{that is}\quad \nabla^2 h^{-}_{\mu\nu}=0\quad\text{or}\quad \nabla^2\chi=0\end{gathered} \quad (20.29)$$

So, as shown in the row 20.1-11 of Tab. 20.1, the GOR information-wave equation has generalized and unified Einstein's wave equation and Newton's wave equation. Einstein's information-wave equation belongs to the optical agent OA(c); Newton's information-wave equation belongs to the idealized agent OA_∞. Now, both of them have been generalized and unified into the GOR information-wave equation under the general observation agent OA(η), becoming the special cases of the GOR information-wave equation in the theory of GOR.

Table 20.1. The Generality and Unity of Newton and Einstein in the theory of GOR

	The Theory of GOR (the general observation agent OA(η))	Einstein's General Relativity (the optical agent OA(c): $\eta \to c$)	Newton's Gravitational Theory (the idealized agent OA$_\infty$: $\eta \to \infty$)
20.1-1	OA(η) and GOR spacetime $X^{4d}(\eta)$: $$\mathrm{OA}(\eta)=\left\{\begin{array}{l} X^{4d}(\eta):\left\{\begin{array}{l} x^0=\eta t \\ x^1=x, x^2=y, x^3=z \end{array}\right\} \\ \mathrm{d}s^2=g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\ \left(g_{\mu\nu}=g_{\mu\nu}\left(x^\alpha,\eta\right)\right) \end{array}\right\}$$	OA(c) and Minkowski spacetime $X^{4d}(c)$: $$\mathrm{OA}(c)=\left\{\begin{array}{l} X^{4d}(c):\left\{\begin{array}{l} x^0=ct \\ x^1=x, x^2=y, x^3=z \end{array}\right\} \\ \mathrm{d}s^2=g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\ \left(g_{\mu\nu}=g_{\mu\nu}\left(x^\alpha,c\right)\right) \end{array}\right\}$$	OA$_\infty$ and Cartesian spacetime $X^{4d}{}_\infty$: $$\mathrm{OA}_\infty=\left\{\begin{array}{l} X^{4d}{}_\infty:\left\{\begin{array}{l} x^0=\infty t \\ x^1=x, x^2=y, x^3=z \end{array}\right\} \\ \mathrm{d}t=\mathrm{d}\tau \\ \mathrm{d}l=\sqrt{\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2} \end{array}\right\}$$
20.1-2	The GOR factor of spacetime transformation: $\Gamma=\Gamma(\eta)$ $$\Gamma=\Gamma(\eta)=\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{\eta^2}}-\gamma_i\frac{v^i}{\eta}\right)^2-\frac{v^2}{\eta^2}}}$$	The Einstein factor: $\gamma=\Gamma(c)$ $$\gamma=\Gamma(c)=\lim_{\eta\to c}\Gamma(\eta)$$ $$=\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{c}}-\gamma_i\frac{v^i}{c}\right)^2-\frac{v^2}{c^2}}}$$	The Newtonian factor: Γ_∞ $$\Gamma_\infty=\lim_{\eta\to\infty}\Gamma(\eta)$$ $$=\lim_{\eta\to\infty}\frac{1}{\sqrt{\left(\sqrt{1+\frac{2\chi}{\eta^2}}-\gamma_i\frac{v^i}{\eta}\right)^2-\frac{v^2}{\eta^2}}}=1$$

20.1-3	The GOR field equation: $$\Box\chi_{\mu\nu}(\eta) = -\frac{\eta^2}{2}\kappa_{\text{GOR}}T_{\mu\nu}(\eta)$$ $$\begin{cases}\Box\chi_{\mu\nu} \equiv \frac{\eta^2}{2}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) \\ \kappa_{\text{GOR}} = 8\pi G/\eta^4\end{cases}$$	Einstein's field equation: $$\Box\chi_{\mu\nu}(c) = -\frac{c^2}{2}\kappa_E T_{\mu\nu}(c)$$ $$\begin{cases}\Box\chi_{\mu\nu} \equiv \frac{c^2}{2}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) \\ \kappa_E = 8\pi G/c^4\end{cases}$$ As $\eta\to c$, the GOR field equation converges to Einstein's field equation.	Newton's field equation: $$\nabla^2\chi = 4\pi G\rho$$ $$\begin{cases}\Box\chi_{\mu\nu} = 0 \ \ (\mu\nu \neq 00) \\ \Box\chi_{00} = -4\pi G\rho \\ \Box\chi_{00} = -\nabla^2\chi\end{cases}$$ As $\eta\to\infty$, the GOR field equation converges to Newton's law of universal gravitation in the form of Poisson equation.
20.1-4	The GOR motion equation: (i.e., the GOR geodesic equation) $$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}s^2} + \Gamma^\mu_{\alpha\beta}(\eta)\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s} = 0$$ $$(\mu = 0,1,2,3)$$	Einstein's motion equation: $$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}s^2} + \Gamma^\mu_{\alpha\beta}(c)\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s} = 0$$ $$(\mu = 0,1,2,3)$$ As $\eta\to c$, the GOR motion equation converges to Einstein's motion equation.	Newton's motion equation: $$\frac{\mathrm{d}^2t}{\mathrm{d}\tau^2} = 0 \quad \text{and} \quad \frac{\mathrm{d}^2x^i}{\mathrm{d}\tau^2} = -\frac{\partial\chi}{\partial x^i} \quad (i = 1,2,3)$$ As $\eta\to\infty$, the GOR motion equation splits into two independent relations: the time-element dt; the space-element dr which converges or reduces to Newton's law of universal gravitation in the form of the second law: $$F^i = -m\frac{\partial\chi}{\partial x^i} \quad (i = 1,2,3)$$

20.1-5	The GOR metric and line-element: $$g_{\mu\nu}(\eta):\begin{cases} g_{00}(\eta)=1+2\chi/\eta^2 \\ g_{11}(\eta)=-\left(1+2\chi/\eta^2\right)^{-1} \\ g_{22}(\eta)=-r^2 \\ g_{33}(\eta)=-r^2\sin^2\theta \\ g_{\mu\nu}(\eta)=0 \quad (\mu\neq\nu) \end{cases}$$ $$\mathrm{d}s^2=\left(1+2\chi/\eta^2\right)\eta^2\mathrm{d}t^2-\left(1+2\chi/\eta^2\right)^{-1}\mathrm{d}r^2 -r^2\mathrm{d}\theta^2-r^2\sin^2\theta\mathrm{d}\varphi^2$$	Einstein's metric and line-element: $$g_{\mu\nu}(c):\begin{cases} g_{00}(c)=1+2\chi/c^2 \\ g_{11}(c)=-\left(1+2\chi/c^2\right)^{-1} \\ g_{22}(c)=-r^2 \\ g_{33}(c)=-r^2\sin^2\theta \\ g_{\mu\nu}(c)=0 \quad (\mu\neq\nu) \end{cases}$$ $$\mathrm{d}s^2=\left(1+2\chi/c^2\right)c^2\mathrm{d}t^2-\left(1+2\chi/c^2\right)^{-1}\mathrm{d}r^2 -r^2\mathrm{d}\theta^2-r^2\sin^2\theta\mathrm{d}\varphi^2$$	Newton's metric and line-element: $$g_{\mu\nu}=\lim_{\eta\to\infty}g_{\mu\nu}(\eta)=\eta_{\mu\nu} =\mathrm{diag}\left(+1,-1,-r^2,-r^2\sin^2\theta\right)$$ $$\begin{cases}\mathrm{d}t=\mathrm{d}\tau \\ \mathrm{d}r^2=\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\end{cases}$$ As $\eta\to\infty$, the GOR metric converges to the Minkowski metric, and the line-element ds splits into two independent relation: the time-element dt and the space-element dr.
20.1-6	The GOR energy of the observed object P: P's kinetic $K=K(\eta)$ and potential $V=V(\eta)$: $$\begin{cases} K=K(\eta)=\left(\Gamma(\eta)\big\|_{\chi=0}-1\right)m_o\eta^2 \\ V=V(\eta)=\left(1-\Gamma(\eta)\big\|_{v=0}\right)m_o\eta^2 \end{cases}$$ P's total energy $H=H(\eta)$: $$H=H(\eta)=K(\eta)+V(\eta) =\left(\Gamma(\eta)\big\|_{\chi=0}-\Gamma(\eta)\big\|_{v=0}\right)m_o\eta^2$$	Einstein's energy of the observed object P: P's kinetic $K=K(c)$ and potential $V=V(c)$: $$\begin{cases} K=K(c)=\left(\Gamma(c)\big\|_{\chi=0}-1\right)m_oc^2 \\ V=V(c)=\left(1-\Gamma(c)\big\|_{v=0}\right)m_oc^2 \end{cases}$$ P's total energy $H=H(c)$ $$H=\lim_{\eta\to c}H(\eta)=H(c)=K(c)+V(c) =\left(\Gamma(c)\big\|_{\chi=0}-\Gamma(c)\big\|_{v=0}\right)m_oc^2$$	Newton's energy of the observed object P: P's kinetic $K=K_\infty$ and potential $V=V_\infty$: $$K=K_\infty=\frac{1}{2}m_\infty v^2 \quad \text{and} \quad V=V_\infty=\chi m_\infty$$ As $\eta\to\infty$, the GOR kinetic energy $K(\eta)$ and potential energy $V(\eta)$ converge to classical kinetic energy K_∞ and classical potential energy V_∞., respectively: $$\lim_{\eta\to\infty}K(\eta)=K_\infty \quad \text{and} \quad \lim_{\eta\to\infty}V(\eta)=V_\infty$$ P's total energy $H=H_\infty$: $$H=\lim_{\eta\to\infty}H(\eta)=H_\infty=K_\infty+V_\infty$$

20.1-7	The GOR motion equation of celestial two-body system (M,m): $$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{\eta^2}u^2\right)$$	Einstein's motion equation of celestial two-body system (M,m): $$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{c^2}u^2\right)$$ As $\eta\to c$, the GOR motion equation of celestial bodies converges to Einstein's motion equation of celestial bodies.	Newton's motion equation of celestial two-body system (M,m): $$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}$$ As $\eta\to\infty$, the GOR motion equation of celestial bodies converges to Newton's motion equation of celestial bodies.
20.1-8	The GOR precession-angle equation of planet orbits: $\Delta\varphi_{\mathrm{GOR}}$ $$\Delta\varphi_{\mathrm{GOR}}=\Delta\varphi_{\mathrm{OA}(\eta)}=\frac{6\pi G^2M^2}{\eta^2h_K^2}$$	Einstein's precession-angle equation of planet orbits: $\Delta\varphi_E$ $$\Delta\varphi_E=\Delta\varphi_{\mathrm{OA}(c)}=\lim_{\eta\to c}\Delta\varphi_{\mathrm{OA}(\eta)}=\frac{6\pi G^2M^2}{c^2h_K^2}$$ As $\eta\to c$, the GOR precession angle $\Delta\varphi_{\mathrm{GOR}}$ of planet orbits converges to Einstein's precession angle $\Delta\varphi_E$ of planet orbits.	Newton's precession-angle equation of planet orbits: $\Delta\varphi_N$ $$\Delta\varphi_N=\Delta\varphi_{\mathrm{OA}_\infty}=\lim_{\eta\to\infty}\varphi_{\mathrm{OA}(\eta)}=0$$ As $\eta\to\infty$, the GOR precession angle $\Delta\varphi_{\mathrm{GOR}}$ of planet orbits converges to Newton's precession angle $\Delta\varphi_N$ of planet orbits.

20.1-9	The GOR gravitational-deflection angle of light sweeping over the sun: δ_{GOR} in the case of the optical agent OA(η): $\eta \to c$ $$\delta_{\mathrm{GOR}} = \delta_{\mathrm{OA}(\eta)} = \frac{4GM}{R_S c^2} \quad (\eta \to c)$$ where the photon m is both the observed object P and an informon of OA(c); the speed c of light is both the speed of P and the speed of the information wave of OA(c). in the case of the superluminal agent OA(η): $\eta >> c$ $$\delta_{\mathrm{GOR}} = \delta_{\mathrm{OA}(\eta)} \quad (\eta \gg c)$$ $$= \frac{2GM}{R_S c^2}\left(1 + \frac{c^2}{3c^2 + 2\eta^2}\right)$$ where the photon m is the observed object P, but not an informon of OA(η); the speed c of light is the speed of P, but not the speed of the information wave of OA(η).	Einstein's gravitational-deflection angle of light sweeping over the sun: δ_E in the case of the optical agent OA(η): $\eta \to c$ Einstein's gravitational-deflection equation is a special case of GOR equation: $$\delta_E = \delta_{\mathrm{OA}(c)} = \lim_{\eta \to c} \delta_{\mathrm{OA}(\eta)}$$ $$= \frac{4GM}{R_S c^2}$$	Newton's gravitational-deflection angle of light sweeping over the sun: δ_N in the case of the superluminal agent OA(η): $\eta >> c$ Newton's gravitational-deflection equation is a special case of GOR equation: $$\delta_N = \delta_{\mathrm{OA}_\infty} = \lim_{\eta \to \infty} \delta_{\mathrm{OA}(\eta)}$$ $$= \lim_{\eta \to \infty} \frac{2GM}{R_S c^2}\left(1 + \frac{c^2}{3c^2 + 2\eta^2}\right)$$ $$= \frac{2GM}{R_S c^2}$$

20.1-10	The GOR gravitational-redshift equation of light: Z_{GOR} $$Z_{\text{GOR}}=Z_{\text{OA}(\eta)}=\frac{1\big/\sqrt{g_{00}(r_{\text{B}})}-1\big/\sqrt{g_{00}(r_{\text{A}})}}{K_{F\eta}\big/m_o\eta^2-\left(1-1\big/\sqrt{g_{00}(r_{\text{B}})}\right)}$$ $$\begin{pmatrix} K_{F\eta}=\left(\Gamma\vert_{\chi=0}-1\right)m_o\eta^2 \\ g_{00}(r)=1+\frac{2\chi}{\eta^2};\ \chi(r)=-\frac{GM}{r}\end{pmatrix}$$ where η ($\geq c$) is the information-wave speed of the general observation agent OA(η); the speed c of light is the speed of the photon m as the observed object P.	Einstein's gravitational-redshift equation of light: Z_E $$Z_E=Z_{\text{OA}(c)}=\lim_{\eta\to c}Z_{\text{OA}(\eta)}=1-\frac{\sqrt{g_{00}(r_{\text{B}})}}{\sqrt{g_{00}(r_{\text{A}})}}$$ $$\left(g_{00}(r)=1+\frac{2\chi}{c^2};\ \chi(r)=-\frac{GM}{r}\right)$$ As $\eta\to c$, the GOR gravitational-redshift equation of light converges to Einstein's gravitational-redshift equation of light.	Newton's gravitational-redshift equation of light: Z_N $$Z_N=Z_{\text{OA}_\infty}=\lim_{\eta\to\infty}Z_{\text{OA}(\eta)}=\frac{2GMr_{\text{B}}}{r_{\text{B}}c^2+2GM}\left(\frac{1}{r_{\text{B}}}-\frac{1}{r_{\text{A}}}\right)$$ As $\eta\to\infty$, the GOR gravitational-redshift equation of light converges to Newton's gravitational-redshift equation of light.
20.1-11	The GOR information-wave equation: $$\nabla^2 h^-_{\mu\nu}(\eta)-\frac{1}{\eta^2}\frac{\partial^2}{\partial t^2}h^-_{\mu\nu}(\eta)=0$$ where the wave function $h^-_{\mu\nu}(\eta)$ is the metric-perturbation tensor under OA(η).	Einstein's information-wave equation: $$\nabla^2 h^-_{\mu\nu}(c)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}h^-_{\mu\nu}(c)=0$$ As $\eta\to c$, the GOR wave equation converges to Einstein's wave equation.	Newton's information-wave equation: $$\nabla^2 h^-_{\mu\nu}=0\quad\text{or}\quad\nabla^2\chi=0$$ As $\eta\to\infty$, the GOR wave equation converges to Newton's wave equation.

Notes: The theory of GOR has generalized and unified Einstein's theory of general relativity and Newton's theory of universal gravitation. All formulae or relationships in the theory of GOR, as $\eta\to c$, strictly converge to that of Einstein's theory of general relativity; $\eta\to\infty$, strictly converge to that of Newton's theory of universal gravitation. It is thus clear that the theory of GOR is logically consistent not only with Einstein's theory of general relativity, but also with Newton's theory of universal gravitation. Moreover, such strict corresponding relationship between different theoretical systems, from one aspect, confirms the logical self-consistency and theoretical validity of the theory of GOR.

21 GOR and the Big Puzzles in Physics

Table 9.1 in Chapter 9 of **the 1st volume of OR: Inertially Observational Relativity** (IOR) lists 15 big puzzles in modern physics, labelled with BP-01 to BP-15, in which BP-01 to BP-08 mainly involve Einstein's theory of special relativity and de Broglie's theory of matter waves or quantum mechanics, and have been interpreted by the theory of IOR and the OR theory of matter waves in **the 1st volume of OR: Inertially Observational Relativity** (IOR), providing new idea and insight into Einstein's theory of special relativity and de Broglie's theory of matter waves or even quantum mechanics. BP-09 to BP-15 mainly involve Einstein's theory of general relativity and the so-called **Modern General Relativity**. Now, they can be interpreted by the theory of GOR in **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR), providing new idea and insight into Einstein's theory of general relativity and even the modern general relativity.

The theory of IOR in the 1st volume of OR has revealed the essence of inertial relativistic phenomena, where the core problem is: Why is the speed of light invariant? Now, the theory of GOR in 2nd volume of OR has revealed the essence of gravitational relativistic phenomena, where the core problem is: Why is the spacetime of gravitation is curved?

In the big puzzle BP-03: **The Essence of Relativistic Effects**, the relativity effects it states include both the inertial and the gravitational. According to the statements in Sec. 9.4 of Chapter 9 in the 1st volume of OR and in Sec. 12.5 of Chapter 12 in the 2nd volume of OR, the conclusion that **All relativistic phenomena are apparent phenomena** is valid for both the theory of IOR in the 1st volume of OR and the theory of GOR in the 2nd volume of OR.

With respect to the big puzzle BP-02: **The Problem of Photon Mass**, according to the mass-speed relation (Eq. (5.5)) in the theory of IOR, as stated in Sec. 5.1.5 of Chapter 5 and Sec. BP-02.3 of Chapter 9 in the 1st volume of OR, all matter particles, including photons and gravitons, have their own rest masses. According to the theory of IOR, the so-called rest mass is actually the intrinsic mass of matter, which is the objective and real mass of matter. However, in the statement of the big puzzle BP-02 **The Problem of Photon Mass** in the 1st volume of OR, the theory of IOR fails to provide the theoretical prediction of photon mass.

In the 2nd volume of OR, based on the GOR theory of gravitational redshift, the theory of GOR has theoretically calculated the rest mass m_o of photons, and has made the theoretical prediction for photons' rest mass.

The GOR Prediction for Photon Weight (stated in Eq. (18.46)):

$$m_o = m = \frac{E}{c^2} = \frac{hf}{c^2} \quad \begin{cases} E = mc^2 \\ E = hf \end{cases} \tag{21.1}$$

where m is Einstein's relativistic mass (moving mass) of photons, and E is Einstein's photon energy (the relativistic kinetic energy of photons). According to Einstein formula: $E=mc^2$; according to Planck equation: $E=hf$.

It turns out that, the mass or rest mass m_o of a photon is actually Einstein's relativistic mass m of a photon.

As state in Sec. 18.5 of Chapter 18 in the 2nd volume of OR, the theoretical value of photon rest mass in Eq. (21.1) is confirmed by observation and experiment as well as the GOR theory of gravitational redshift.

Now, we have finally understood why experimental physicists tried their best but still could not find the rest mass of photons [37,82-85].

In **the 1st volume of OR: Gravitationally Observational Relativity** (GOR), the Sec. 18.5 **GOR Gravitational Redshift and Photon Mass** of Chapter 18 has obtained the theoretical value of photon rest mass, and can be regarded as the further interpretation for the big puzzle BP-02: **The Problem of Photon Mass**.

There are numerous big puzzles in modern physics, some of them could be interpreted by the theory of IOR in the 1st volume of OR, and some of them could be interpreted by the theory of GOR in the 2nd volume of OR.

The theory of IOR enables us to understand the inertial relativistic effects – why the speed of light is invariant. The theory of GOR enables us to understand the gravitational relativistic effects – why the spacetime of gravitation is curved.

From the perspective of the general observation agency OA(η), the theory of IOR in the 1st volume of OR has clarified and interpreted the big puzzles BP-01 to BP-08 listed in Tab. 19.1. Now, from the perspective of the general observation agent OA(η), the theory of GOR in 2nd volume of OR will clarified and interpret the big puzzles BP-09 to BP-15 listed in Tab. 19.1.

In the view of the theory of GOR, the universe might not have had a Big Bang.

BP-09 Why is Spacetime Curved?

BP-09.1 The Statement of the Problem

The Curvature of Spacetime (Einstein's original intention): The existence of matter or energy makes spacetime curved.

The concept of **spacetime curvature** was coined by Einstein.

In 1915, Einstein established the theory of general relativity [8]: in Einstein's view, matter or energy determines how spacetime is curved, and the curved spacetime determines how matter moves.

Perhaps we can understand or imagine the curvature of space; however, it is difficult for us to understand the curvature of spacetime, and in particular, it is difficult for us to imagine how time is curved. Einstein's theory of general relativity has been established for over one hundred years, but we still cannot understand why spacetime is curved and how spacetime is curved.

So, is spacetime curved or not, or why is spacetime curved?

This is the big puzzle marked as BP-09 in the theory of OR.

BP-09.2 The Mainstream View

Actually, the curvature of spacetime is the logical consequence resulting from the invariance of the light speed as the logical premise of Einstein's theory of general relativity. The mainstream school of physics could not explain why the speed of light is invariant, and naturally, also could not explain why spacetime is curved. It could only be attributed to the distribution of matter or energy.

But why does matter and energy make spacetime curved?

To this question, the mainstream school of physics will never have an answer.

BP-09.3 The View of GOR Theory

With regard to the problem of spacetime curvature, the theory of GOR has clarified in Sec. 12.5 of Chapter 12 that spacetime is not really curved.

The theory of GOR has an important theorem, that is, the theorem of Cartesian spacetime: as $\eta\to\infty$, $g_{\mu\nu}\to\eta_{\mu\nu}$ and $\Gamma(\eta)\to\Gamma_\infty$. The theorem of Cartesian spacetime indicates that, under the idealized observation agent OA(η), the GOR spacetime metric, no matter the inertial or the gravitational, would converge to the Minkowski metric $\eta_{\mu\nu}$=diag(+1,–1,–1,–1), and the GOR factor $\Gamma(\eta)$ of spacetime transformation would converge to the Galilean factor $\Gamma_\infty\equiv 1$.

The theorem of Cartesian spacetime means that, the objective and real spacetime is the spacetime described by Galileo and Newton for us, which is flat rather than curved, regardless of matter and energy.

Thus, the theory of GOR has realized that spacetime is not really curved.

According to the theory of GOR, the so-called spacetime curvature relies on observation: under different observation agents, the observational spacetime has different degrees of curvature; the so-called spacetime curvature is only an observational effect or an apparent phenomenon of the observation agent OA(η): the observation agent OA(η) ($\eta<\infty$) with the observation locality ($\eta<\infty$) is like a wide-angle lens, making gravitational spacetime appear somewhat curved or deformed. As a matter of fact, the so-called spacetime curvature in Einstein's theory of general relativity is the observational effect and apparent phenomenon caused by the observational locality $c<\infty$) of the optical observation agent OA(c), which is actually **the effect of wide-angle lens**.

According to the theory of GOR, according to the theorem of Cartesian spacetime in the theory of GOR, the objective and real spacetime, including space and time, is never curved.

The problem of gravitational-spacetime curvature, and even the problem of gravitational relativistic effects, has been discussed in detail in Chapter 12.

BP-10 The Orbital Precession of Mercury

BP-10.1 The Statement of the Problem

The Problem of Mercury's Orbital Precession: Suppose that the planet m moves around the star M. Under the idealized conditions, the orbit of the planet m is a standard and closed ellipse; but in reality, the orbit of the planet m is always insistently precessing. Taking Mercury as an example, the observational data from the optical astronomy shows that the precession rate of Mercury's orbit around the sun reaches 5600.73 arc seconds every 100 years. However, the point is not whether Mercury' orbit precesses, or whether Mercury's orbit precesses at the rate of 5600.73 arc seconds every 100 years. The point is that Newton's celestial two-body model (M,m) based on Newton's theory of universal gravitation has no the orbital precession of Mercury, while Einstein's celestial two-body model (M,m) based on Einstein's theory of general relativity has the orbital precession of Mercury: 43.03 arc seconds every 100 years.

So, who is right, Newton or Einstein?

This is the big puzzle marked as BP-10 in the theory of OR.

BP-10.2 The Mainstream View

As stated in Chapter 16, Newton's celestial two-body model (M,m) is idealized: (i) the gravitation of the star M is action at a distance, and the speed of gravitational radiation is infinite; (ii) the observation agent of observers is the idealized agent OA_∞, and the speed of the information wave of OA_∞ transmitting the information on the planet m is also infinite. So, the orbit of the planet m in Newton's celestial two-body model (M,m) is a standard and closed ellipse and could not predict the orbital precession of Mercury. Actually, in considering the non-idealized factors, for example, the precession of the equinoxes caused by the non-inertial geocentric coordinate system and the perturbation made by other celestial bodies to Mercury, Newton's theory of universal gravitation could also calculate that Mercury's orbit precesses at the rate of 5557.62 arc seconds every 100 years. Finally, out of the 5600.73 arc seconds recorded in astronomical observation data, only 43.11 arc seconds remained to be determined.

Coincidentally, Mercury's orbital-precession rate of 43.03 arc seconds every 100 years in Einstein's celestial two-body model (M,m) **accurately** fills this gap.

For this result, in a letter to his friend, Einstein said: "The equation gives the correct numbers for Mercury's perihelion. You could imagine how happy I am. I couldn't help but be happy for several days."

The mainstream school of physics generally believes that, as far as the prediction of Mercury's orbital precession is concerned, Einstein's theory of general relativity is better than Newton's theory of universal gravitation.

BP-10.3 The View of GOR Theory

There are many doubts about the views of the mainstream school of physics.

Einstein's prediction of the 43.03″ is less than 0.8% of the 5600.73″ recorded in astronomical observation, and could almost be included in the observational error. Inexplicably, why could not Einstein's theory of general relativity predict Mercury's orbital precession of the actual 5557.62″ every 100 years?

According to the theory of GOR, as clarified in Chapter 16, both Newton's classical two-body model and Einstein's relativistic two-body model are the idealized motion models of celestial bodies, where there is no prior knowledge and information (such as the influence of non-inertial geocentric coordinate system or the perturbation of other celestial bodies) available to predict the actual orbital precession of planets or Mercury.

The theory of GOR has built the GOR celestial two-body model (M,m) (Eq. (16.64)) for the general observation agent OA(η), and then, has derived the GOR planetary-precession equation (16.68).

The GOR Motion Equation (16.64) of Planets:

$$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2}+u=\frac{GM}{h_K^2}\left(1+\frac{3h_K^2}{\eta^2}u^2\right)\quad\left(u=\frac{1}{r}\right)$$

where G is the gravitational constant, M is the mass of the star, h_K is the velocity moment of the planet m orbiting the star M.

The GOR Orbital-Precession Equation (16.68) of Planets:

$$\Delta\varphi_{\mathrm{GOR}}\left(\eta\right)=\frac{6\pi G^2M^2}{\eta^2h_K^2}$$

where $\Delta\varphi_{\mathrm{GOR}}(\eta)$ is the orbital-precession angle of the planet m under the general observation agent OA(η).

The GOR orbital-precession equation (16.68) of planets shows that the orbital precession angle $\Delta\varphi_{\mathrm{GOR}}=\Delta\varphi_{\mathrm{GOR}}(\eta)$ depends on the information-wave speed η of the observation agent OA(η): under different observation agents, for the same two-body system (M,m) of celestial bodies, the planet m would exhibit different orbital precession angles.

It is worth noting that the GOR orbital-precession angle $\Delta\varphi_{\mathrm{GOR}}(\eta)$ (Eq. (16.68)) of planets has generalized Einstein’s precession angle $\Delta\varphi_E=\Delta\varphi_{\mathrm{GOR}}(c)$ and Newton’s precession angle $\Delta\varphi_N=\Delta\varphi_{\mathrm{GOR}}(\infty)$:

$$\begin{cases}\Delta\varphi_E=\Delta\varphi\left(c\right)=\lim\limits_{\eta\to c}\Delta\varphi_{\mathrm{GOR}}\left(\eta\right)=\dfrac{6\pi G^2M^2}{c^2h_K^2}\\ \Delta\varphi_N=\Delta\varphi_\infty=\lim\limits_{\eta\to\infty}\Delta\varphi_{\mathrm{GOR}}\left(\eta\right)=0\end{cases}\tag{21.2}$$

where $\Delta\varphi_E$ is Einstein’s planetary precession angle, i.e., the precession angle under $\Delta\varphi_{\mathrm{GOR}}(c)$ the optical agent OA(c); $\Delta\varphi_N$ is Newton’s planetary precession angle, i.e., the precession angle $\Delta\varphi_{\mathrm{GOR}}(\infty)$ under the idealized agent OA_∞.

It is thus clear that the so-called orbital precession presented in the GOR motion equation (16.64) of planets, including Einstein’s motion equation of planet, is not the objective and real precession of planet orbits, but in essence only an observation effect or an apparent phenomenon caused by the observational locality ($\eta<\infty$) of the observation agent OA(η).

Einstein’s prediction of Mercury’s orbital precession, based on Einstein’s theory of general theory, is actually only an observation effect caused by the observational locality ($c<\infty$) of the optical observation agent OA(c), rather than the actual orbital precession of Mercury. Our astronomical observation data all come from optical observation, are the observation data of the optical agent OA(c); Einstein's theory of general relativity is exactly the theory of optical agent OA(c). Therefore, Einstein's prediction for Mercury's orbital precession angle of the 43.03 arc seconds, based on Einstein's theory of general relativity, might indeed belong to Mercury's orbital precession angle of the 5600.73 arc seconds recorded by the optical astronomy. Alternatively, as far as the problem of Mercury's orbital precession is concerned, there are indeed the optical observational effects in the historical records of astronomical observation: Mercury's perihelion precesses at the rate of 5600.73 arc seconds every 100 years, of which 5557.62 arc seconds belong the actual perihelion precession of Mercury, while the rest 43.11 arc seconds belong the observation effects or apparent phenomena caused by the observational locality ($c<\infty$) of the optical agent OA(c).

In this way, the actual orbital precession of Mercury and the historical data of astronomical observation provide the strong support for the theory of GOR, verifying the GOR prediction for the orbital precession of planets: in the Mercury's orbital-precession data of the 5600.73 arc seconds every 100 years recorded by the optical observation agent OA(c), there is 43.11 arc seconds belonging to the observation effects or apparent phenomena of OA(c) and rooted from the observational locality ($c<\infty$) of OA(c).

The detailed discussion for the problem of Mercury's orbital precession has made and stated in Chapter 16.

BP-11 The Gravitational Deflection of Light

BP-11.1 The Statement of the Problem

The Problem of Gravitational Deflection of Light: Based on the principle of equivalence, Einstein predicted that light would be curved in gravitational spacetime, which is known as Einstein's prediction of the gravitational deflection of light. Actually, Newton's theory of universal gravitation could also draw the same conclusion. Before the formal establishment of general relativity, Einstein's calculation of the gravitational deflection angle of light was the same as the calculation based on Newton's theory of universal gravitation. However, after the formal establishment of general relativity, Einstein recalculated the gravitational deflection angle of light based on his general theory, which was twice Newton's.

So, who is right, Newton or Einstein?

This is the big puzzle marked as BP-11 in the theory of OR.

BP-11.2 The Mainstream View

Einstein proposed that, by taking advantage of total solar eclipses to observe the starlight sweeping over the sun, the deflection angle of starlight could be determined, thereby his prediction for gravitational deflection of light could be verified.

The theoretical values of Newton and Einstein are δ_N and δ_E, respectively:

$$\delta_N = \frac{2GM}{R_S c^2} \quad \text{and} \quad \delta_E = \frac{4GM}{R_S c^2} \tag{21.3}$$

where, G is the of universal gravitational constant, M is the mass of the sun, and R_S is the radius of the sun; $\delta_N = 0.875''$ is Newton's deflection angle of starlight, while $\delta_E = 1.75''$ is Einstein's deflection angle of starlight.

In 1919, the team led by British astronomer Eddington determined the starlight deflection angle through a total solar eclipse to be $\delta = 1.61'' \pm 0.40''$, which tended to support Einstein's prediction for the gravitational deflection of light.

So far, almost all observations of total solar eclipses have tended to support Einstein's prediction for the gravitational deflection of light. Therefore, as far as the gravitational deflection of light is concerned, the mainstream school of physics believe that Einstein's theory of general relativity is better and more accurate than Newton's theory of universal gravitation.

BP-11.3 The View of GOR Theory

It should be pointed out that:

(i) Einstein's observation agent is the optical agent OA(C), the speed c of light in Einstein's deflection angle δ_E is not only the moving speed v ($\approx c$) of the photon m as the observed object P, but also the information-wave speed η ($=c$) of the optical agent OA(c).

(ii) Newton's observation agent is the idealized agent OA_∞, the speed c of light in Newton's deflection angle δ_N is only the moving speed v ($\approx c$) of the photon m as the observed object P, but not the information-wave speed η ($=\infty$) of the idealized agent OA_∞.

Actually, it is natural for astronomical observations (including the observation of total solar eclipses) to support Einstein's theory of general relativity, for Einstein's theory of general relativity is the product of the optical observation agent OA(c), belonging to the theory of optical observation.

Human astronomy, including the optical and the radio, employs the optical observation agent OA(c) to observe celestial phenomena.

The support of observation or experiment for Einstein's prediction of the gravitational deflection of light does not mean that Einstein's theory of general relativity is better or more accurate than Newton's theory of universal gravitation, let alone that Einstein's theory of general relativity is more in line with the physical reality than Newton's theory of universal gravitation.

Newton's theory of universal gravitation is the product of the idealized observation agent OA_∞, belonging to the theory of idealized observation. If we could employ the idealized agent OA_∞ to observe the starlight sweeping over the sun, then we would find that the deflection angle of starlight is more in line with that calculated or predicted by Newton's theory of universal gravitation.

The idealized agent OA_∞ describes the objective and real physical world.

In this regard, Newton's theory of universal gravitation is more objective and in line with the objectively physical reality than Einstein's theory of general relativity.

As stated in Sec. 17.4 of Chapter 17, based on the GOR two-body model of the celestial system (M,m), i.e., the GOR motion equation of photons in gravitational spacetime, where M is the sun and m is a photon, the theory of GOR could make the theoretical calculation for the gravitational deflection angle of starlight sweeping over the sun under the general observation agent OA(η): $\delta_{\text{GOR}}=\delta_{\text{OA}(\eta)}$.

According to the theory of GOR: as $\eta\to c$, the GOR motion equation of photons would reduce to Einstein's motion equation of photons, and the GOR deflection angle $\delta_{\text{GOR}}=\delta_{\text{OA}(\eta)}$ of starlight would converge to Einstein's deflection $\delta_E=\delta_{\text{OA}(c)}$ of starlight; as $\eta\to\infty$, the GOR motion equation of photons would reduce to Newton's motion equation of photons, and the GOR deflection angle $\delta_{\text{GOR}}=\delta_{\text{OA}(\eta)}$ of starlight would converge to Newton's deflection $\delta_N=\delta_{\text{OA}(\infty)}$ of starlight. Due to the continuity and monotonicity of the solution to the GOR motion equation of photons, the GOR deflection angle δ_{GOR} of starlight should satisfy:

$$\delta_E \geq \delta_{\text{GOR}}(\eta) \geq \delta_N \tag{21.4}$$

where $\delta_{\text{GOR}}=\delta_{\text{GOR}}(\eta)$ depends on the observation agent OA(η). So, for the same gravitational scene, under different observation agents, the starlight sweeping over the sun must exhibit different degrees of gravitational deflection.

It is thus clear that different observation agents have different degrees of gravitational deflection of light: Newton's theory of universal gravitation describes the gravitational deflection of light under the idealized observation agent OA_∞, which is supported by objectively physical reality; Einstein's theory of general relativity describes the gravitational deflection of light under the optical observation agent OA(c), which is supported by optical observation.

Anyway, the objective and real gravitational deflection of starlight must be more in line with the predictions of Newton's theory of universal gravitation.

The detailed discussion for the problem of the gravitational deflection of light has made and stated in Chapter 17.

BP-12 The Gravitational Redshift of Light

BP-12.1 The Statement of the Problem

The Problem of Gravitational Redshift of Light: Based on the principle of equivalence, Einstein predicted that the frequency of light would decay in gravitational spacetime, which is known as Einstein's prediction of the gravitational redshift of light. Actually, Newton's theory of universal gravitation could also draw the same conclusion. As far as the problem of the gravitational redshift of light is concerned, Einstein's prediction and Newton's prediction seem to be the same or approximate, with only differences in second-order approximation, which has no the observational distinguishability.

People are accustomed to comparing Einstein's theory of general relativity with Newton's theory of universal gravitation. However, it is difficult for observation and experiment to identify or determine which one is better or more accurate on the second-order small scale.

So, why is this?

This is the big puzzle marked as BP-12 in the theory of OR.

BP-12.2 The Mainstream View

In any case, the theory of gravitational redshift has been verified and confirmed by observations and experiments, for example, the observation of the solar spectrum [152-154] and the gravitational-redshift experiment of Mössbauer Effect [157-159]. However, these observations and experiments seem to support not only Einstein's theory of gravitational redshift but also Newton's theory of gravitational redshift.

Whether for the observation of the solar spectrum or for the experiments of Mössbauer Effect, the observation agent is the optical observation agent OA(c). Therefore, it is understandable that the observational conclusions of OA(c) support Einstein's theory of general relativity and Einstein's prediction for the gravitational redshift of light under OA(c); but it is somewhat unreasonable that the observational conclusions of OA(c) supports Newton's theory of universal gravitation and Newton's prediction for the gravitational redshift of light under OA_∞.

The mainstream school of physics do not seem to fully understand the difference between different observation agents.

The mainstream school of physics appears to be confused about Newton's prediction under the idealized agent OA_∞ approximating Einstein's prediction under the optical agent OA(c). And so far, they have had no convincing answer.

BP-12.3 The View of GOR Theory

The theory of GOR has already clarified in Chapter 18 that the current so-called the Newtonian formula (Eq. (18.20)) for the gravitational redshift of light is the **pseudo** Newtonian gravitational-redshift equation, which is the mixture of Newton's theory of universal gravitation and Einstein's theory of general relativity, as well as quantum theory. This is why the pseudo Newtonian gravitational-redshift equation (18.20) approximates Einstein's gravitational-redshift.

Based on the viewpoint of the theory of GOR, the gravitational redshift of light is the decay of photon kinetic energy, being **the redshift of energy**. Essentially, it is the transformation of different forms of energy, following the principle of conservation of energy.

Under the principle of conservation of energy, the theory of GOR has redefined the concept of the gravitational redshift of light, which equivalently transforms the definition for the frequency redshift of $Z=\Delta f/f$ into the definition for energy redshift: $Z=\Delta K/K$.

According to the definition of energy redshift: $Z=\Delta K/K$, the theory of GOR has deduced the gravitational-redshift equation of light purely based on Newton's classical mechanics and Newton's law of universal gravitation, that is, the **real** Newtonian gravitational-redshift equation of light.

Newton's Gravitational-Redshift Equation (18.25) of Light:

$$Z_N = \frac{2GMr_{\mathrm{B}}}{r_{\mathrm{B}}c^2 + 2GM}\left(\frac{1}{r_{\mathrm{B}}} - \frac{1}{r_{\mathrm{A}}}\right)$$

where G is the gravitational constant, M is the gravitational source; as shown in Fig. 18.2, r_{A} and r_{B} are respectively the distances of the points A and B from the gravitational source M; Z_N is the redshift of light at the point A relative to the point B calculated purely by Newton's classical mechanics and Newton's law of universal gravitation, that is, the real Newtonian gravitational redshift.

Newton's gravitational-redshift equation (18.25) of light is different not only from the pseudo Newtonian gravitational-redshift equation (18.20), but also from Einstein's gravitational-redshift equation (18.14) of light.

Einstein's Gravitational-Redshift Equation (18.14) of Light:

$$Z_E = 1 - \sqrt{1 - \frac{2GM}{r_{\mathrm{B}}c^2}} \bigg/ \sqrt{1 - \frac{2GM}{r_{\mathrm{A}}c^2}}$$

where Z_E is Einstein's gravitational redshift of light.

The Pseudo Newtonian Gravitational-Redshift Equation (18.20) of Light:

$$Z_{PN} = \frac{GM}{c^2}\left(\frac{1}{r_{\mathrm{B}}} - \frac{1}{r_{\mathrm{A}}}\right)$$

where Z_E is the pseudo Newtonian gravitational redshift of light.

According to the statement in Chapter 18, as far as the problem of gravitational redshift for the solar spectrum:

$$Z_E \approx Z_{PN} \quad \text{and} \quad Z_N \approx 2Z_E \tag{21.5}$$

Thus, it is clear that the pseudo Newtonian gravitational redshift Z_{PN} is the same as or approximate to Einstein's gravitational redshift Z_E; the real Newtonian gravitational redshift Z_N is different from Einstein's gravitational redshift Z_E, but is twice Einstein's gravitational redshift: $Z_N=2Z_E$. This means that the gravitational redshift of light observed by the optical agent OA(c) is different from the gravitational redshift of light observed by the idealized agent OA_∞.

Based on the definition $Z=\Delta K/K$ of energy redshift, the theory of GOR has theoretically derived the gravitational-redshift equation (18.38) for the general observation agent OA(η):

$$Z_{\mathrm{GOR}} = \frac{\Delta K_\eta}{K_\eta} = \frac{m_o\eta^2 \Big/ \sqrt{g_{00}\left(r_{\mathrm{B}}\right)} - m_o\eta^2 \Big/ \sqrt{g_{00}\left(r_{\mathrm{A}}\right)}}{K_{F\eta} - \left(1 - 1\Big/\sqrt{g_{00}\left(r_{\mathrm{B}}\right)}\right)m_o\eta^2} \tag{21.6}$$

This is the GOR gravitational-redshift equation of light in theory of GOR.

As stated in Sec. 18.4.3 of Chapter 18, the GOR theory of gravitational redshift has generalized and unified Einstein's theory of gravitational redshift and Newton's theory of gravitational redshift: as $\eta \to c$, the GOR gravitational-redshift equation (18.38) or (21.6) of light would strictly converge to Einstein's gravitational-redshift equation (18.14) of light; as $\eta \to \infty$, the GOR gravitational-redshift equation (18.38) or (21.6) would strictly converges to Newton's gravitational-redshift equation (18.25) of light. This not only reflects the logical self-consistency of the GOR theory of gravitational redshift, but also confirms the theoretical validity of Newton gravitational-redshift equation (18.25)). Meanwhile, the GOR gravitational-redshift equation (18.38) or (21.6) of light indicates that, under different observation agents, light would exhibit different degrees of gravitational redshift.

Like the problem of the gravitational deflection of light, it does not mean who is right that Newton's gravitational redshift Z_N of light is different from Einstein's gravitational redshift Z_E of light. It only means that the gravitational redshift observed by different observation agents is different from each other. The difference between Einstein's gravitational redshift and Newton's gravitational redshift is in essence the difference between the optical agent OA(c) and the idealized agent OA_∞, which belongs to the differences in observation rather than the differences in the correctness of theories.

The observation of the solar spectrum [152-154] and the experiment of gravitational redshift based on Mössbauer Effect [157-159], employing the optical agent OA(c) as the observation agent, naturally support Einstein's prediction of the gravitational redshift of light. However, this does not mean that Einstein is right or Newton is wrong. If we could employ the idealized agent OA_∞ to observe the gravitational redshift of the solar spectrum, then we would find out that the objective and real gravitational redshift is more in line with Newton's theory of gravitational redshift.

The detailed discussion for the problem of the gravitational redshift of light has made and stated in Chapter 18.

BP-13 Gravitational Waves

BP-13.1 The Statement of the Problem

Einstein's Prediction of Gravitational Waves: In 1916, Einstein derived a wave equation from his field equation of general relativity, in which the wave in gravitational spacetime propagates at the speed c of light in vacuum. Einstein believed that the wave function in his wave equation represented gravitational radiation and might be called **Gravitational Wave** for it was related with the Newtonian gravitational radiation χ. Therefore, the corresponding wave equation is naturally referred to as the gravitational-wave equation.

This is Einstein's famous prediction of gravitational waves, which is based on Einstein's theory of general relativity [164,165].

In 2015, LIGO announced that it had detected gravitational waves [161], and moreover, the speed of gravitational radiation or gravitational waves it detected was exactly the speed c of light in vacuum as predicted by Einstein.

So far, the problem of gravitational waves seems to have been fully solved.

However, there are many doubts about Einstein's prediction of gravitational waves and about LIGO's discovery of gravitational waves. The point is not whether gravitational waves really exist, but the problems:

(i) Is the wave in Einstein's wave equation really gravitational radiation or a gravitational wave? Or, did Einstein correctly predict gravitational waves?

(ii) Did LIGO really detect gravitational radiation or gravitational waves?

(iii) Are gravitational waves really propagate at the speed *c* of light in vacuum as predicted by Einstein or as detected by LIGO?

This is the big puzzle marked as BP-13 in the theory of OR.

BP-13.2 The Mainstream View

Constrained by the perspective of optical observation or the optical observation agent OA(*c*), the mainstream school of physics believe that Einstein's prediction of gravitational waves, which is based on Einstein's theory of general relativity, is theoretically valid and correct with no doubt.

The theoretical supports for Einstein's prediction of gravitational waves lie not only in the wave equation derived from Einstein field equation, but also in the retarded integral formula or retarded solution of Einstein field equation.

Einstein's Retarded Integral Formula (19.6):

$$h^{-}_{\mu\nu}\left(c\right)=-\frac{\kappa_E}{2\pi}\int\frac{T_{\mu\nu}\left(t-\dfrac{|x^i-x'^i|}{c},x'^i,c\right)}{|x^i-x'^i|}\mathrm{d}^3x'^i$$

Einstein's Wave Equation (19.7):

$$\nabla^2 h^{-}_{\mu\nu}\left(c,\chi\right)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}h^{-}_{\mu\nu}\left(c,\chi\right)=0$$

According to the retarded solution (Eq. (19.6)) of Einstein field equation, Einstein concluded that gravitational interaction is not action at a distance: it takes time to cross space and propagate at the speed *c* of light in vacuum.

According to Einstein's wave equation (19.7), the wave function $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(c,\chi)$ is related with the Newtonian gravitational potential χ. Accordingly, Einstein believed that $h^{-}_{\mu\nu}(c,\chi)$ represented gravitational radiation and could be named as **Gravitational Wave**. Therefore, Einstein's wave equation (19.7) is known as the gravitational-wave equation, which suggests that Einstein's gravitational waves propagate exactly at the speed *c* of light in vacuum.

In order to test and verify Einstein's prediction of gravitational waves, the physics community has spent enormous manpower and material resources, as well as over 100 years of time, of which the most large-scale and representative operation is the LIGO project for detecting gravitational waves.

The mainstream school of physics as always firmly believe that Einstein theory of general relativity and his prediction of gravitational waves are valid and correct. All observations or experiments revolve around how to affirm or support Einstein's prediction of gravitational waves. Physicists strive to collect and even create the evidences that are beneficial for Einstein's prediction of gravitational waves.

It seems that no one has ever attempted to refute Einstein's theory of general relativity and his prediction of gravitational waves.

It seems that the mission of physicists is not to test Einstein's prediction of gravitational waves, but to make the first discovery of gravitational waves, and so as to win the Nobel Prize in Physics.

In 2015, LIGO announced the first discovery of gravitational waves [161], and moreover, the speed of the gravitational waves detected by LIGO was exactly the speed *c* of light in vacuum, affirming Einstein's prediction of gravitational waves.

Sure enough, the LIGO team was awarded the 2017 Nobel Prize in Physics.

BP-13.3 The View of GOR Theory

According to the theory of GOR, no matter Newton's theory of universal gravitation or Einstein's theory of general relativity, no matter Newton's law of universal gravitation or Einstein's field equation, has no prior knowledge or information about the speed of gravitational radiation. Therefore, it is impossible for both Newton's theory and Einstein's theory to deduce an equation of gravitational waves or calculate the speed of gravitational waves.

Of course, the theory of GOR does not suspect the existence of gravitational radiation or gravitational waves.

However, common sense tells us that the speed of gravitational radiation or gravitational waves must be higher than the speed of light. Otherwise, how could gravitational waves or gravitons escape from a black hole and interact with the external matter through gravitational interaction?

I. Einstein Mistakenly Predicted Gravitational Waves

As clarified in Sec. 19.3 of Chapter 19, Einstein's prediction of gravitational waves is a historic mistake.

In Chapter 19, under the principle of GC, by analogizing and following the logic of Einstein's deducing the retarded integral formula (Eq. (19.6) and wave equation (19.7) from Einstein field equation, the theory of GOR has derived the GOR retarded integral formula and GOR wave equation under the general observation agent OA(η).

The GOR Retarded Integral Formula (19.14):

$$h^{-}_{\mu\nu}(\eta) = -\frac{\kappa_{\text{GOR}}}{2\pi}\int \frac{T_{\mu\nu}\left(t - \frac{|x^i - x'^i|}{\eta}, x'^i, \eta\right)}{|x^i - x'^i|}\mathrm{d}^3 x'^i$$

The GOR Wave Equation (19.15):

$$\nabla^2 h^{-}_{\mu\nu}(\eta,\chi) - \frac{1}{\eta^2}\frac{\partial^2}{\partial t^2}h^{-}_{\mu\nu}(\eta,\chi) = 0$$

where η is the information-wave speed of the general observation agent OA(η).

The GOR retarded integral formula and GOR wave equation derived from the GOR field equation are isomorphically consistent with Einstein's retarded integral formula and wave equation, and has generalized Einstein's retarded integral formula and wave equation. Einstein's retarded formula and wave equation are only special cases of the GOR retarded integral formula and GOR wave equation, which could hold true only if the observation agent OA(η) is the optical agent OA(c).

It is worth noting that: for the GOR retarded integral formula and GOR wave equation, the wave function $h^{-}_{\mu\nu}=h^{-}_{\mu\nu}(\eta,\chi)$ depends on the observation agent OA(η); the wave speed is the information-wave η of OA(η).

This suggests that: the wave function $h^{-}_{\mu\nu}(\eta,\chi)$ of the GOR wave equation is not the so-called gravitational wave, but the information wave of the observation agent OA(η). Only if OA(η) is the optical agent OA(c), the wave speed η could be the speed c of light. The so-called **retard** or **delay** in the GOR retarded integral formula is not the delay of gravitational radiation caused by the locality of gravitational interaction, but the delay of observed information caused by the observational locality ($\eta<\infty$) of the observation agent OA(η).

Naturally, for the objective and real gravitational radiation or gravitational waves, the wave speed is determined, does not rely on observation or observation agents, which does not vary with the variation of observation agents.

Therefore, the so-called GOR wave equation is actually the information-wave equation" of the general observation agent OA(η), rather than the gravitational-wave equation. Einstein's theory of general relativity is the theory of the optical agent OA(c); Einstein's wave equation (19.7) is only a special case of the GOR wave equation (19.15), where the information wave is naturally the wave of light and the wave speed is naturally the speed of light.

So, the theory of GOR has discovered that Einstein's prediction of gravitational waves was actually a mistake: Einstein mistakenly regarded the information wave of the optical agent OA(c) in his wave equation as the wave of gravitational radiation.

Of course: it is not that there is no gravitational wave; but that the gravitational waves predicted by Einstein are not gravitational waves.

II. LIGO Mistakenly Discovered Gravitational Waves

Since Einstein's prediction of gravitational waves was a mistake, what should the gravitational waves discovered by LIGO be?

has LIGO really discovered gravitational waves?

The theory of GOR does not doubt the existence of gravitational waves. In the theory of GOR, the so-called gravitational wave refers to gravity or gravitational radiation. In this regard, LIGO did indeed detect the gravity or gravitational waves radiated by certain matter systems. However, they were not the gravitational waves came from distant binary-blackhole systems; the so-called merging or coalescing events of binary-blackhole systems are just the imagination of LIGO.

As clarified in Chapter 19, LIGO's gravitational-wave detection is not strictly empirical observation or experiment. LIGO's detection relies half on listening and half on guessing. The only thing we could affirm is that LIGO heard the chirping sound from a certain matter system; while, the so-called binary-blackhole system and the merging or coalescing events are only LIGO's computer simulation of virtual reality, which might not necessarily exist objectively.

The so-called discovery of LIGO is actually just a mistake!

As clarified in Sec. 19.4 of Chapter 19, LIGO did not detect the gravitational waves erupted from the coalescing events of binary-blackhole systems. Instead, LIGO only detected some clumps of electromagnetic matter that swept over the earth and disturbed the LIGO detector at close quarters: gamma-ray bursts, X-rays, or other electromagnetic-particle clumps (EPCs) in various frequency bands, which were the gravitational radiation signals emitted by such EPCs.

As depicted in Fig. 19.4: LIGO mistakenly regards the gravitational radiation of EPCs at close range as the gravitational waves came from distant binary-blackhole systems during merging or coalescing; LIGO mistakenly regards the moving speed of EPCs as the speed of gravitational radiation.

III. The Basic Judgement of GOR

In summary, based on the GOR information-wave equation (19.15), the theory of GOR has had the following basic judgments:

(i) Einstein's prediction of gravitational waves is a mistake

Einstein's theory of general relativity is the theory of the optical agent OA(*c*). The wave in Einstein's wave equation (19.7) is not a gravitational wave, but the information wave (light wave) of the optical agent OA(*c*), and therefore, the speed of it is the speed *c* of light in vacuum. Einstein mistakenly treated the information wave of OA(*c*) as the wave of gravitational radiation.

(ii) LIGO's discovery of gravitational waves is a mistake

The gravitational radiation signals or gravitational waves detected by LIGO are not gravitational waves erupted from the merging or coalescing events of binary-blackhole systems, but the gravity or gravitational waves radiated by the electromagnetic-particle clumps (EPCs) as matter systems as they swept over the earth and disturbed the LIGO detector at close range. The gravitational field of EPCs moves together with EPCs at the speed of light, but not that gravitational waves propagate at the speed of light.

The theory of GOR does not suspect the existence of gravitational radiation or gravitational waves. However, no matter Einstein's theory of general relativity or Newton's theory of universal gravitation, or even the theory of GOR, has no prior information about gravitational waves and the speed of gravitational radiation. Therefore, it is impossible for Einstein and Newton and even GOR to deduce their gravitational-wave equations, let alone calculate the speed of gravitational radiation.

One thing seems certain: the speed of gravity or gravitational radiation, or the speed of gravitational waves, is not the speed of light.

Black holes are black because light or photons could not escape from them. However, no matter how black a black hole is, the gravitational waves or gravitons of it are not bound by itself. Suppose there is a huge black hole: according to the theory of black hole, no matter how big it could be, it could still radiate the gravitational waves or gravitons to the external spacetime. This suggests that the gravitational waves or gravitons must have the speed that far higher than the speed of light as calculated by Laplace [43] or Flandern [127].

Anyway, the speed of gravitational radiation or gravitational waves requires to be measured or determined by observation and experiment.

Physicists ever made arduous efforts to determine the speed of light. Perhaps, the measurement or determination of the speed of gravitational radiation or gravitational waves requires physicists to make more arduous efforts than measuring or determining the speed of light.

The detailed discussion for Einstein's prediction of gravitational waves and LIGO's detection of gravitational waves has made and stated in Chapter 19.

BP-14 Black Holes

BP-14.1 The Statement of the Problem

Black Holes: A class of celestial bodies with huge mass and huge density, which are dark or black because light or photons could not escape from black holes, and therefore, the observers outside black holes could not see or observe the light emitted by black holes. According to the definition of the so-called **modern general relativity**, a **black hole** refers to a celestial body whose spacetime curvature reaches the point where light could not escape from its **Event Horizon**.

Einstein's theory of general relativity has been established for over 100 years since 1915. Now, Einstein's theory of general relativity is called the classical general relativity; while the so-called modern general relativity has newly been developed on the basis of Einstein's general relativity, which mainly involves the important application of Einstein's general relativity in cosmology and astrophysics, such as gravitational waves, black holes, binary-star systems, and the theory of quantum gravity. Nowadays, black holes play the important role in the modern general relativity.

In 1916, Schwarzschild obtained the first exact solution of Einstein field equation [80], in which there is **Schwarzschild's line-element equation (15.8)**:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{c^2 r}\right)c^2\mathrm{d}t^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$$

where there are two singularities: r=0 and $r=2GM/c^2$.

These two singularities are endowed with unique functions:

(i) The singularity r=0 of the big bang: leading to the Big Bang;

(ii) The singularity $r=2GM/c^2$ of a black hole: leading to the event horizon of the black hole.

This reminds us once again of Hawking's words in his **A Brief History of Time: From the Big Bang to Black Holes** [31]: "Mathematics cannot really handle infinite numbers. At singularity, the theory itself breaks down or fails."

The Schwarzschild line-element equation (15.8) and **the black-hole singularity** suggests that, as the radius r of a static spherically-symmetric celestial body is less than $2GM/c^2$, an **event horizon** in the surrounding spacetime would be formed: $r_s \equiv 2GM/c^2$ – entering this horizon means falling into a black hole, even light or photons could not escape from the black hole. Here, the horizon radius r_s is called the Schwarzschild radius.

A black hole is a theoretical massive celestial object. In encyclopedias and literature, **black holes** are often described as **incredible celestial bodies** or **the most mysterious natural phenomenon in the universe**. The term **black hole** itself is clothed upon with a strong mysterious hue, which meets people's psychology of seeking curiosity, and could be freely imagined.

Black holes are actually not mysterious, but they are much more massive than other celestial bodies so that light or photons could not escape from them.

Without Einstein's theory of general relativity and the Schwarzschild singularity, Newton's theory of universal gravitation could also produce the concept and theory of black holes. Based on Newton's law of universal gravitation, the larger the mass of a celestial body, the stronger the gravity or gravitation of it, and the harder it is for a moving object to escape from it. It could be imagined that, as a celestial body is massive enough, it would become a **black hole**, even if light or photons moving at an immense speed fall into its event horizon, it could also not escape.

Suppose there is a celestial body M, an object m moves at the speed v relative to M, the initial distance between M and m is R, and m tends to escape from M and fly towards the infinity. According to Newton classical mechanics:

$$\int_R^\infty F_G \mathrm{d}r = \int_R^\infty \frac{GMm}{r^2}\mathrm{d}r = \frac{1}{2}mv^2 \quad \text{so that} \quad R = \frac{2GM}{v^2} \tag{21.7}$$

where F_G is the gravitational force exerted by the celestial body M on the moving object m. According to Newton's formula of escape speed: whether the moving object m could escape from the celestial body M does not depend on the curvature of spacetime, but depends on the gravitational F_G between M and m.

If the moving object m is a photon, then $v=c$, R is the Schwarzschild radius.

It is thus clear that Newton's theory of universal gravitation could also produce the concept of black holes and deduce the theory of black holes.

Naturally, the theory of GOR could also deduce the theory of black holes.

Logically speaking, the existence of black holes seems to be reasonable, and the concept and theory of black holes could be theoretically deducible. However, whether the macroscopic matter motion or microscopic matter structure of black holes, there are too many unknowns about black holes for us. We could not even be completely certain whether black holes really exist.

At present, human astronomy, including the optical and the radio, is the astronomy of the optical observation agent OA(c): we must employ light or electromagnetic interaction as the observation medium to observe or measure celestial phenomena or celestial motion. Naturally, the optical agent OA(c) could not break through the optical horizon of black holes. With regard to black holes, we have to guess: or based on Einstein's theory of general of relativity, although it itself is only a partial theory; or based on purely mathematical deduction, although lacking clear and definite physical meaning. To this day, the black-hole theory of modern general relativity still lacks sufficient empirical evidence.

So, black holes are still just a mystery to us.

This is the big puzzle marked as BP-14 in the theory of OR.

BP-14.2 The Mainstream View

The black-hole theory of modern general relativity represents the viewpoint or understanding of black holes held by the mainstream school of physics. The mainstream school of physics have not recognized that Einstein's theory of general relativity is only a partial theory, and therefore, they do not think that, with regard to the concept and theory of black holes based on Einstein's theory of general relativity, including the event horizon, gravitational singularity, photon sphere, accretion, gravitational collapse, ergosphere, Hawking radiation or Hawking evaporation, spacetime-coordinate transformation, and even black-hole thermodynamics, the logical premises might have logical flaws from the very beginning, and as a result, the logical consequences might not be valid or correct.

Perhaps because of this, some scholars have questioned the authenticity or existence of black holes in recent years.

It is worth mentioning the research of American scholar Mersini-Houghton in 2014 [205,206], which involved Hawking's theory of radiation or evaporation. Like Hawking, Mersini-Houghton also believes that the death and collapse of a star would be accompanied by Hawking radiation or Hawking evaporation, which could lead to the mass loss of the star. However, the difference is that Mersini-Houghton has mathematically proved that the mass lost with Hawking radiation is quite large so that the dead star could not form a black hole.

Thus, Mersini-Houghton concluded that black holes do not exist at all!

Actually, Mersini-Houghton's theory, like theory of black holes, is also based on Einstein's theory of general relativity and even on the modern general relativity. Therefore, Mersini-Houghton's questioning of the black-hole theory is irrelevant to whether the theory of black holes is valid and correct, which only means that the modern general relativity, including the theory of black holes, logically lacks consistency and self-consistency.

Nevertheless, the mainstream school of physics still believe that the black holes in the black-hole theory of modern general relativity are the objectively physical existence or the objectively celestial phenomena. Thus, the black-hole theory of modern general relativity has derived many myths of black holes.

The Big Bang is exactly the most splendid myth of black holes.

BP-14.3 The View of GOR Theory

The theory of GOR repeatedly emphasizes that Newton's theory of universal gravitation and Einstein's theory of general relativity are two partial theories of gravitational interaction in physics: Einstein's theory is that of the optical observation agent OA(c), which would present the observation effects or apparent phenomena of OA(c) and would be valid only as we observe the physical world with light or the optical agent OA(c); Newton's theory is that of the idealized observation agent OA_∞, which describes the objectively physical world.

Naturally, the black-hole theory of modern general relativity has been developed on the basis of Einstein's theory of general relativity (the so-called classical general relativity). So, why could not physics develop the black-hole theory on the basis of Newton's theory of universal gravitation?

The theory of GOR has generalized and unified the two great gravitational theories of Newton and Einstein, providing us with the new insight into the theory of black holes and into the reconstruction of the theory of black holes.

I. Why are Black Holes Black?

As for why black holes are black, Einstein and Newton share the same view: because light or photons could not escape from black holes.

So, why could not light or photons escape from black holes?

In this regard, the views of Einstein's theory of general relativity and Newton's theory of universal gravitation are different. Einstein believed that it was due to the curvature of spacetime: the spacetime in a black hole was too curved; Newton believed that it was due to the universal gravitation: the mass of a black hole was too great and hence had huge gravity.

We do not fully understand how the curved spacetime prevents light or photons from escaping black holes, but we can understand that, under the enormous gravity, all moving matter objects (including light or photons) might be bound by the huge gravity of massive celestial bodies (such as a black hole). Of course, gravitational waves or gravitons might or must be exceptions.

The theorem of Cartesian spacetime in the theory of GOR has proved that the objective and real spacetime would never be curved.

We must abandon Einstein's erroneous doctrine of spacetime curvature and return to Newton's right stand: the reason why the earth orbits the sun, the reason why black holes are black, and the reason why light or photons could not escape from black holes, is not due to spacetime curvature, but due to the effects of universal gravitation, i.e., the gravitational interaction between matter and matter.

II. The Event Horizons of Different Observation Agents

According to the black-hole theory of modern general relativity, light or photons could not pass through the event horizon of a black hole, so the observers outside the event horizon could not see the black hole. Therefore, according to the black-hole theory of modern general relativity, black holes could not be directly seen or observed because they are dark and black.

However, according to the theory of Observational Relativity (OR), whether black holes could be directly seen or observed does not depend on whether they are black or how black they are, but depends on observers' observation agents.

Einstein's theory of general relativity is the theory of the optical observation agent OA(c), under which observers see or observe the world through light. The information wave of the optical agent OA(c) is light, which could not pass through the event horizon of black holes, and therefore, could not transmit the information inside black holes to the observers outside black holes. However, in the future, with the progress and development of science and technology, human beings will master the technology of superluminal observation agents, for example, the gravitational observation agent OA(κ). As the theory of OR repeatedly emphasizes, the gravitational-wave speed κ must be much higher than the speed c of light, as calculated by Laplace [43] or Flandern [127]. At that time, by taking advantage of the gravitational observation agent OA(κ), human beings will clearly see or observe the scene inside the optical horizon of black holes.

Under the principle of GC, by analogizing or following the logic of Schwarzschild's solving Einstein field equation [80], the theory of GOR has obtained the exact solution (Eq. (15.32)) of the GOR field equation of static spherically-symmetric gravitational spacetime under the general observation agent OA(η), in which there is **the GOR line-element equation (15.32)**:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{\eta^2 r}\right)\eta^2 \mathrm{d}t^2 - \left(1 - \frac{2GM}{\eta^2 r}\right)^{-1} \mathrm{d}r^2 - r^2 \mathrm{d}\theta^2 - r^2 \sin^2\theta \mathrm{d}\varphi^2$$

where there are also two singularities: $r=0$ and $r=2GM/\eta^2$.

The singularity $r=2GM/\eta^2$ of the general observation agent OA(η) can be called the GOR black-hole singularity of the general observation agent OA(η), which means that, as the radius r of the static spherically-symmetric celestial body M is less than $2GM/\eta^2$, the surrounding spacetime would form the OA(η) event horizon: $r_s(\eta)=2GM/\eta^2$ is the radius of OA(η) horizon – outside the OA(η) horizon is the observable spacetime of OA(η); inside the OA(η) horizon is the unobservable black hole of OA(η). Entering the OA(η) horizon means falling into the black hole of OA(η), the information wave or informons of OA(η) could not pass through the OA(η) horizon and could not transmit the information inside the OA(η) horizon to the observers outside the OA(η) horizon.

In particular, suppose that OA(η) is a superluminal observation agent: $\eta>c$, then the OA(η) event horizon represents a **super blackhole**: $r_s(\eta)=2GM/\eta^2<r_s(c)$. Entering the OA($\eta$) event horizon means falling into the super blackhole, even the superluminal information wave or informons of OA(η) ($\eta>c$) could not pass through the OA(η) horizon and could not transmit the information inside the OA(η) horizon to the observers outside the OA(η) horizon.

It is obvious that $r_s(\eta)<r_s(c)=2GM/c^2$ ($\eta>c$). Accordingly, a celestial body that is black and unobservable for the optical agent OA(c) might not necessarily be black or unobservable for a superluminal observation agent OA(η) ($\eta>c$).

It is thus clear that the so-called **Event Horizon** of a black hole depends on the observation agents, and the radius $r_s=r_s(\eta)$ of OA(η) event horizon is decided by the information-wave speed η of the observation agent OA(η).

III. To Reshape Black-Hole Theory with GOR

The theory of black holes is the product of modern general relativity.

Astrophysicist have made the theory of black holes based on Einstein's theory of general relativity, and furthermore, have made the theory of Big Bang based on the theory of black holes. In this way, the so-called modern general relativity has been formed. The theory of black holes and the theory of Big Bang made by the modern general relativity are actually built on the basis of Einstein's theory of general relativity. However, the mainstream school of physics has not realized that Einstein's theory of general relativity is only a partial theory of gravitational interaction, which is valid only under the optical observation agent OA(c).

The theory of OR has already clarified that all relativistic effects, whether the inertial or the gravitational, are observational effects and apparent phenomena, not the objective and real natural phenomena or physical reality. The objectively physical world is what described by Galileo and Newton for us. What Einstein's theory of relativity (whether the special or the general) described for us is only an optical image of the objective world, containing the observational effects and apparent phenomena of the optical observation agent OA(c), which does not completely represent the objective and real physical world.

Einstein's theory of general relativity is actually the partial theory of GOR theory. Naturally, the theory of black holes based on Einstein's theory of general relativity could only be a partial theory under the optical observation agent OA(c). In particular, it must have the limitations and contain erroneous understandings rooted from Einstein's theory of general relativity.

If black holes really exist, then we need to reshape the theory of black holes.

The theory of GOR has laid the base for the reshape of the black-hole theory.

As stated earlier, the theory of GOR can also deduce the theory of black holes.

Under the principle of GC, by analogizing and following the logic of modern general relativity making the theory of black holes under the optical agent OA(c), one would be able to build the GOR theory of black holes, that is, the black-hole theory of the general observation agent OA(η).

It could be imagined that, as the black-hole theory of the general observation agent OA(η), the GOR theory of black holes must be able to generalize and unify the black-hole theory of the optical agent OA(c) and the black-hole theory of the idealized agent OA_∞. In other words, it must be able to generalize and unify the black-hole theory based on Einstein's theory of general relativity and the black-hole theory based on Newton's theory of universal gravitation. In this way, the GOR theory of black holes should be logically consistent with both Einstein's theory of black holes and Newton's theory of black holes, and vice versa. Thus, the logical rationality and theoretical validity of the black-hole theory of modern general relativity would have to be tested by the GOR theory of black holes.

The GOR theory of black holes theory will undoubtedly provide new ideas and new insights into the black-hole theory of modern general relativity.

Now, the concept of **Gravitational Wave Astronomy** has been formed [144]. But that is just a pseudo concept of gravitational wave astronomy, where the so-called gravitational waves are not the real gravitational waves: the speed of gravitational waves is limited to the speed of light. A pseudo gravitational wave or a pseudo graviton moving at the speed of light could not break through the optical horizon of black holes, and therefore could not become the superluminal observation agent.

In the future, only when they master the technology of the gravitational observation agent OA(κ) (the gravitational-wave speed $\kappa >> c$) will human beings truly usher in gravitational wave astronomy. The gravitational observation agent OA(κ) will transmit the information inside the event horizon of super blackholes (no matter how dark or how black they are) to the observers outside black holes.

At that time, we will know what the real black hole looks like.

BP-15 The Big Bang

BP-15.1 The Statement of the Problem

The Theory of Big Bang: At first, all matter in the universe curled up into one point, with extremely high mass-density and temperature, approaching or reaching the Big Bang singularity of Schwarzschild's spherically-symmetric spacetime metric: $r_s = 0$, where space and time were meaningless, and spacetime was in the state of nothingness or nihility; Suddenly, there were a loud bang (no one could hear) and a dazzling flash (no one could see), and all matter erupted violently outward. Then, a new universe was born: the space began to expand; the time began to flow – until it has been evolving into the universe of today.

The theory of the Big Bang is a myth, which is fascinating to talk about.

Let us briefly review the formation of the theory of Big Bang.

In 1927, Belgian cosmologist Lemaitre proposed a theory: a long long time ago, the universe was a **Primeval Atom** (according to the law of conservation of matter, it should have gathered all the matter including the mass and the energy in the universe today), and afterwards, the **Primeval Atom** exploded, forming the present-day universe. In 1929, American astronomer Hubble proposed Hubble's law: the spectrum of galaxies exhibits redshift phenomena, and the spectrum redshift of a galaxy was proportional to the distance between the galaxy and the earth. Thereby, Hubble inferred that all galaxies were moving away from the earth or from each other, just like the scene during a huge explosion. This is Hubble's doctrine of cosmic expansion, which provides an excellent annotation to Lemaitre's doctrine of the Big Bang. In 1946, American physicist Gamow officially put forward the theory of Big Bang. Gamow built the concept and theoretical model of **the Hot Big Bang**: the universe had been formed by a Big Bang about 14 billion years ago. Later, with a more refined calculation, the time of the birth of the universe was determined to be 13.8 billion years ago. In 1964, Penzias and Wilson discovered the cosmic microwave background (CMB) predicted by the theory of Big Bang. Hubble's law and CMB are regarded as the key evidences for the theory of Big Bang.

Naturally, in order to explain how the universe have been evolving from a **Primeval Atom** to the present-day universe, how the time began to flow from stillness, and how the space began to expand from zero, the theory of Big Bang could not be separated from Einstein's theory of general relativity. In the theory of Big Bang of modern general relativity, Lemaitre's **Primeval Atom** evolved into the singularity of Big Bang, also known as the **Cosmic Singularity**. Thus, the universe or spacetime has possessed flexibility: the time could be slowed or sped; the space could be shrunk or stretched. In this way, the time could begin to flow from stagnation, and the space could begin to expand from zero.

The mainstream school of physics firmly believes in the theory of the Big Bang. However, the voice of questioning the theory of Big Bang have never stopped.

The simplest questions are often the most difficult questions for the theory of Big Bang to answer.

Did the universe really need a starting point of space and time?

Could all matter in the universe really be squeezed into the **Primeval Atom**?

Was the **Primeval Atom** in motion or in motionless?

If the **Primeval Atom** was in motionless, then there was no the Big Bang in the universe. If the **Primeval Atom** was in motion, then the spacetime would not be one 4d point: the 1d time would not be stopped into stagnation; the 3d space would not be shrunk into the **Primeval Atom** or the **Cosmic Singularity** ($r_s=0$).

Before the Big Bang, was the universe always the **Primeval Atom**?

Some people, including Einstein himself, ever imagined that Einstein's field equation should be given a suitable cosmological constant, so that the universe could over and over explode to expand from the **Cosmic Singularity**, and then, shrink into the **Primeval Atom**.

But why a huge celestial body might explode? How much mass might a celestial body need to explode? Why did all matter have to shrink to a single **Primeval Atom** or **Cosmic Singularity** before the universe exploded?

In his book **The First Three Minutes: A Modern View of the Origin of the Universe**, Weinberg once observed [63]: "One possibility is that there never really was a state of infinite density. The Big Bang may have begun when the density of the universe had reached some very high but finite value."

So, what kind of high mass-density state would a celestial body have to reach before it could explode? Suppose that, after all the celestial bodies in the Milky Way exhaust their energy, the matter of the Milky Way collapses and shrinks into a high mass-density celestial body. Then, would it lead to a Big Bang? If so, does this mean the birth of a new universe? If so, does a supernova explosion mean the birth of a new universe? Although black holes are generally not as black as the **Primeval Atom**, according to the logic of modern general relativity, they might explode when they get black to a certain extent. So, does the explosion of a general black hole mean the birth of a new universe? Does the merging or coalescing of a binary-blackhole system means the birth of a new universe?

We could not continue to imagine it, it would become even more absurd.

The **Primeval Atom**, or the **Cosmic Singularity** before the Big Bang, was the greatest black hole. If Mersini Houghton's theory or conclusion about the existence of black holes holds true, then the theory of Big Bang can not hold true. Actually, as stated in BP-14, Mersini-Houghton's theory [205,206], like theory of black holes of modern general relativity, is based on Einstein's theory of general relativity and the theory of quantum gravity. Alternatively, Mersini Houghton's theory is also the product of modern general relativity. Therefore, at least, Mersini Houghton's theory means that the so-called modern general relativity, including its theory of black holes and its theory of Big Bang, logically lacks consistency and self-consistency.

Has the universe of ours had really experienced the so-called Big Bang?

This is the big puzzle marked as BP-15 in the theory of OR.

BP-15.2 The Mainstream View

Actually, the theory of Big Bang made by modern general relativity itself represents the views or understandings held by the mainstream school of physics about the evolution of the universe and the Big Bang.

The mainstream school of physics believe that, before the Big Bang, the spacetime and matter of the universe curled up into one 4d point, known as the **Cosmic Singularity** or the **Big-Bang Singularity**: spacetime being infinitely curved, 1d time being stagnant, 3d space being null, mass density being infinitely great, and matter temperature being infinitely high. At some time about 13.8 billion years ago, the universe exploded at the **Big-Bang Singularity**: the time began flowing, the space began expanding.

Eventually, it has been evolving into the universe of today.

This is the theory of Big Bang.

The theory of Big Bang is based on two the fundamental principles:

(i) The principle of universality: physical laws have universal applicability;

(ii) the principle of cosmology: on large scales, the universe is homogeneous and isotropic.

The theoretical basis of the Big-Bang theory involves:

(i) Einstein's theory of general relativity;

(ii) The theory of quantum gravity.

The main observational evidences supporting the Big-Bang theory include:

(i) The cosmological redshift: Hubble's law of cosmic expansion;

(ii) The CMB: the Cosmic Microwave Background radiation;

(iii) The abundance of light elements or primordial elements.

The Timeline of Big Bang:

Let us listen to the story of Big Bang about the universe.

(i) Quantum Gravity Era ($0–10^{-44}$s): The temperature of the universe reached infinity, and the virtual spacetime exploded.

(ii) Planck Era (1 Planck Time: 10^{-43}s): The universe spanned a region of 1 Planck Length (10^{-35}m), and the phase transition temperature was the Planck Temperature (10^{32}K).

(iii) Grand Unification Era ($10^{-43}–10^{-36}$s); At 10^{-43}s, the phase transition of grand unification began, and the phase transition temperature was 10^{28}K; during $10^{-43}–10^{-36}$s, the real spacetime or vacuum was formed, the earliest elementary particles began to be created, and the force of gravity was separated from other fundamental forces.

(iv) Inflationary Era ($10^{-36}–10^{-32}$s): The universe had undergone an extremely rapid exponential expansion, the so-called cosmic inflation, matter and antimatter were asymmetric, quarks and leptons were separated.

(v) Electroweak Era (10^{-32}–10^{-12}s): At 10^{-32}s, the phase transition of electroweak unification began, and the phase transition temperature was 10^{16}K.

(vi) Quark Era (10^{-12}–10^{-6}s): quarks, electrons and neutrinos formed.

(vii) Hadron Era (10^{-6}–1s): At 10^{-6}, hadrons formed, quarks were confined, the temperature of the universe was 10^{12}K.

(viii) Lepton Era (1s–3min): Leptons and their antiparticles dominated, and the temperature of the universe was 10^{11}K.

(ix) Nucleosynthesis Era (3–20min): The nuclei of the simple elements of hydrogen, helium and lithium was formed, and the temperature of the universe was 10^{10}K.

(x) Photon Era (3min–2×10^5years): The universe was filled with plasma, the energy of the universe was dominated by photons, and the temperature of the universe fell to 4000K.

(xi) Galaxy Era (2×10^5–10^9years): Primitive galaxies formed, and galaxies gradually evolved.

(xii) Star Era (10^9–5×10^{10}years): The stars formed, heavy elements formed, planets formed, and molecules formed.

(xiii) Today (13.8 billion years): God created human beings who have observed the cosmic expansion and the 3K cosmic background radiation and have built the cosmic model of Big Bang based on Einstein’s theory of general relativity.

Of course, there are many different versions of the story of Big Bang.

However, in the view of OR theory, these are just fictional stories: the universe has never experienced the so-called Big Bang.

BP-15.3 The View of GOR Theory

It is the basic characteristic of relativistic effects or relativistic phenomena that they are **apparently real but actually unreal**. From the special relativistic effects to the general relativistic effects, from the invariance of light speed to spacetime curvature, from the gravitational redshift of light and the gravitational deflection of light to the orbital precession of planes, from Einstein’s prediction of gravitational waves to LIGO’s detection of gravitational waves, from cosmological redshift to Hubble’s law of cosmic expansion, from the theory of black holes to the theory of Big Bang, all these have reflected this basic characteristic. Now, from the classical general relativity to the modern general relativity, the characteristic of this sort has been continued and strengthened.

As clarified by the theory of OR, Einstein’s theory of relativity, including the special and the general, is the theory of the optical observation agent OA(c). The optical agent OA(c) has the observational locality ($c<\infty$), which presents to observers the optical image of the objective world, that is, the phenomena of optical observation rather than the essence of the physical world. This is the reason why relativistic effects or relativistic phenomena are apparently real but actually unreal.

So, the theory of Big Bang, which are based on Einstein's theory of general relativity are equally **apparently real but actually unreal**.

I. The Big Bang and Einstein's General Relativity

The theory of Big Bang, like the theory of black holes, is the product of modern general relativity. The so-called modern general relativity, including the theory of quantum gravity, is mainly based on Einstein's theory of general relativity.

However, the physicists whose made the theory of Big Bang have not realized that Einstein's theory of general relativity is the partial theory relying on the optical observation agent OA(*c*), and have not correctly understand Einstein's theory of relativity, including the special and the general. Alternatively, while making the theory of Big Bang, they did not realize that: all relativistic phenomena are in essence observational effects or apparent phenomena.

The relativistic effects in Einstein's relativity theory, such as the spacetime curvature, spacetime transformation, space expansion, time expansion, mass-energy transformation, and quantum-energy perturbation derived from the principle of uncertainty, have already become indispensable elements of the Big-Bang theory. The Big Bang of the universe began with the fictional virtual spacetime, where space and time were meaningless: time is stagnant, space is null, and then, there was a fictional quantum-energy perturbation triggering the Big Bang and leading to the extremely rapid exponential expansion of the universe.

The theory of OR repeatedly emphasizes that the relativistic effects in Einstein's relativity theory is not the objective and real physical reality. In the theory of GOR, the theorem of Cartesian spacetime has proved that spacetime could never be curved. In the objective and real physical world, time and space are independent of each other: time could never dilate and space could never extend; mass and energy are independent of each other: energy must be conserved and mass must also be conserved. As stated in the big puzzle BP-06 in Chapter 9 of **the 1st volume of OR**, Heisenberg's uncertainty is in essence the observational uncertainty that cannot generate the so-called **quantum perturbation** or **quantum fluctuation**.

Since spacetime could never be curved, the universe would never be curled up into the so-called **Primeval Atom** or **Big-Bang singularity**: time would never be stagnant and space would never be shrunk into one point. Since there is no Heisenberg's **quantum perturbation** or **quantum fluctuation**, even if all matter in the universe was accumulated together, the Big Bang could not be triggered. Even if the all matter of the universe was accumulated together with extremely high density and temperature and led to the Big Bang, it would not mean the birth of a new universe or the new beginning of time. Even if the Big Bang really happened, it would only be an event at a certain time point in the evolution of the universe.

The theory of Big Bang employs Einstein's theory of general relativity as the main theoretical foundation of its own. However, according to the theory of GOR, Einstein's theory of general relativity is a partial theory, i.e., the theory of optical observation, which is valid only under the optical agent OA(*c*), and the general relativistic effects or gravitational relativistic effects are not the objective and real physical reality. Therefore, Einstein's theory of general relativity is logically difficult to serve as the theoretical foundation of the Big-Bang theory.

So, without Einstein's theory of general relativity as the theoretical foundation of modern general relativity could the theory of Big Bang still hold true?

II. Hubble's Law: Hubble's Interpretation for Cosmological Redshift

The so-called observational evidences supporting the theory of Big Bang, such as cosmological redshift, cosmic microwave background radiation, the abundance of light elements, primordial gas clouds, and galactic evolution and distribution, might have an infinite number of possible interpretations, and not the evidences or source materials specifically prepared for the theory of Big Bang.

The most important and direct observational evidence listed in the theory of Big Bang is naturally Hubble's cosmological redshift. In particular, when interpreted by Hubble's Law, the cosmological redshift became the crucial foundation and pillar of the theory of Big Bang made by the modern general relativity.

Astronomical observations indicate that the spectra of starlight observed by the observers of the earth exhibit the redshift phenomenon of frequencies, and moreover, the farther away from the earth a star is, the more significant the spectral redshift of it is. In other words, the spectral redshift Z of starlight is directly proportional to the co-moving distance D the star.

Under the optical agent OA(c), it follows that:

$$\begin{aligned} &|Z| = R_{ED} D \\ &Z = \frac{\Delta f}{f} = \frac{f_O - f_e}{f_O} = 1 - \frac{f_e}{f_O} = 1 - \frac{\lambda_O}{\lambda_e} \end{aligned} \tag{21.8}$$

where Z is the cosmological redshift of starlight, f_e and f_O are the emission frequency and observational frequency of light respectively, λ_e and λ_O are the emission wavelength and observational wavelength of light respectively, R_{ED} is the redshift coefficient, and D is the co-moving distance of the star.

Hubble speculated that the cosmological redshift, like the Doppler effect, was caused by the recession of stars or galaxies relative to the earth.

Let v_r be the recession velocity of a galaxy or a star, then according to the Fizeau-Doppler formula, it follows that:

$$\begin{cases} \dfrac{\lambda_O}{\lambda_e} = \dfrac{f_e}{f_O} = \sqrt{\dfrac{1+\beta}{1-\beta}} = \sqrt{\dfrac{1+v_r/c}{1-v_r/c}} \\ Z = 1 - \sqrt{\dfrac{1+v_r/c}{1-v_r/c}} \approx -\dfrac{v_r}{c} \end{cases} \tag{21.9}$$

where the minus sign "–" represents the spectral shift to red.

Thus, Hubble's law was born [112]:

$$v_r = H_0 D \quad \left(v_r = cZ;\ H_0 = cR_{ED}\right) \tag{21.10}$$

where v_r the recession velocity of the galaxy, H_0 is the Hubble constant.

It is generally believed that H_0, v_r, and D change with the expansion of the universe; while $H_0=70.4^{+1.3}_{-1.4}$ km/s/Mpc is only the Hubble constant of the present-day universe.

Hubble's Law: According to Eq. (21.10), all celestial bodies or galaxies in the universe are rapidly retreating relative to the earth, and the farther away from the earth a galaxy is, the higher the recession velocity of it is.

Hubble's Theory of Cosmic Expansion: According to Hubble's law, Hubble further inferred that the present-day universe is rapidly expanding, just like the scene presented after a Big Bang.

The Hubble Distance: According to Hubble's law, there must be a certain distance D_H from the earth, known as the Hubble distance, where the recession velocity v_r of celestial bodies or galaxies has reached the speed c of light, and therefore, the optical observers on the earth could not observe the celestial bodies or galaxies beyond the Hubble distance of D_H.

Naturally, the Hubble distance D_H could be calculated by Hubble's law:

$$D_H = \frac{c}{H_0} \approx 4.26^{+0.09}_{-0.08} \times 10^3 \text{ Mpc} \quad (v_r = c) \tag{21.11}$$

The mainstream school of physics believes that Hubble's Law of cosmic expansion is the significant discovery that has laid the foundation of modern cosmology and is the indispensable component of the theory of Big Bang.

However, the universe might not really follow Hubble's law.

Or rather, the universe might not really be expanding.

III. The Law of Light Traveling: the GOR Interpretation for Cosmological Redshift

With regard to the cosmological redshift of starlight, originally, there could have been an infinite number of possible interpretations. Perhaps, inspired by Lemaitre's explosion doctrine of **Primeval Atom**, which predicted the expansion of the universe based on Einstein's theory of general relativity, Hubble conceived Hubble's law of cosmic expansion.

Actually, based on the definition and interpretation of the gravitational redshift of light in Chapter 18 in the 2nd volume of OR, the redshift of spectral, no matter the gravitational redshift of light or the cosmological redshift of starlight, from the perspective of waves, it is the frequency decay of light or starlight, this is only a phenomenon; from the perspective of particles, it is the energy decay of light or photons, this is the essence.

Astronomers' observation of the gravitational redshift of starlight belongs to optical observation. Naturally, the observation agent OA(η) is the optical agent OA(c), where the kinetic energy K of photons is $K=hf$ (that is, Planck's energon equation: $E=hf$). Thus, the cosmological redshift $Z_K=\Delta K/K$ defined based on the kinetic-energy decay of light or photons is equivalent to the cosmological redshift $Z=\Delta f/f$ defined based on the frequency decay of light or photons:

$$Z_K = \frac{\Delta K}{K} = \frac{K_O - K_e}{K_O} = \frac{hf_O - hf_e}{hf_O} = \frac{f_O - f_e}{f_O} = \frac{\Delta f}{f} = Z \tag{21.12}$$

where K_e and K_O represent the emission kinetic energy of photons and observational kinetic energy of photons, respectively; the cosmological redshift Z_K of starlight represents the kinetic-energy decay of photons, while Z represents the frequency decay of starlight.

Thus, Hubble's Law should be reshaped into the law of light traveling:

$$\begin{cases} |Z_K| = R_{ED} D \\ R_{ED} = H_0 / c = 23.5^{+0.43}_{-0.47} \times 10^{-5} / \mathrm{Mpc} \end{cases} \tag{21.13}$$

where Z_K represents the kinetic-energy redshift (decay) of photons, and R_{ED} is the kinetic-energy redshift (decay) coefficient of photons, that is, the ratio of the Hubble constant H_0 to the speed c of light in vacuum; pc (≈3.2616 light-years) is the photometric distance, and M=10^6.

The GOR law of Light Traveling: According to Eq (21.13), while light or photons emitted by stars or galaxies is traveling in the universe, the kinetic-energy K of the light or photons gradually decays and the frequency gradually shifts to red. The redshift or decay of the kinetic energy $Z_K=\Delta K/K$ of a photon is proportional to the co-moving distance D of the star that emits the photon: $|Z_K|=R_{ED}D$. The larger the distance D between the star and the earth, the more significant the redshift or decay of photons' kinetic energy is.

The GOR law of light traveling is self-evident. the rationality of the GOR law of light traveling is obvious: taking a bullet leaving a rifle as an example, the kinetic energy of the bullet will inevitably decay gradually, and ultimately, it will stop moving due to the depletion of its kinetic energy; the motion of light or photons in the universe is in theory the same as that of the bullet. The universe is not a pure vacuum or the free spacetime: just like a bullet flying in the atmosphere, a photon traveling in the universe must consume energy.

Waves, such as light waves and sound waves, could always keep their specific speeds regardless of whether they are moving in vacuum, atmosphere, or water. Therefore, in the same medium, light waves of different frequencies or sound waves of different frequencies have the same speeds. It could be imagined that waves have a sort of mechanism to keep their specific speeds: as their kinetic energies decay, waves could keep their specific speeds by reducing their frequencies; as their kinetic energies grow, waves could keep their specific speeds by raising their frequencies.

In this way, a photon could fly in the universe at the specific speed c before its kinetic energy has been depleted. With the gradual decay of energy, the frequency of the photon would gradually red-shift until the kinetic energy of the photon has been depleted, and finally, the photon has to stop its moving in the universe.

Naturally, due to the limited energy of light or photons, there must be the limited time or the limited space as the upper limit for light or photons to travel in the universe. According to the GOR law of light traveling, the upper bound of the distance that light or photons could travel in the universe might be called **the GOR**

Distance of Light Traveling.

The GOR Distance of Light Traveling: According to the GOR law of light traveling, there must exist an upper limit D_{GOR} of the distance for light or photons to travel in the universe. For stars or galaxies that are beyond the distance of D_{GOR} away from the earth, the starlight emitted by those stars or galaxies could never reach the earth due to the limitation of energy, and therefore, the observers of the earth could not see or observe those stars and galaxies.

The relative redshift (decay) $Z_K=1$ of the kinetic energy of light or photons represents the depletion of the kinetic energy K_e of light or photons. At this point, the corresponding the co-moving distance D of stars or galaxies is the upper limit D_{GOR} of the distance for light or photons to travel in the universe.

According to the GOR law of light traveling (Eq. (21.13)):

$$\begin{aligned} D_{\text{GOR}} &= \left(R_{ED}\right)^{-1} = c/H_0 \approx 4.255\times10^{3}\ \text{Mpc} \\ &\approx 1.388\times10^{10}\ \text{ly} \quad \left(Z_K = 1\right) \end{aligned} \tag{21.14}$$

where, according to the data provided by the International Organization for Standardization (ISO), the distance D_{GOR} of light traveling in the universe is approximately 13.9 billion light-years.

One could make a comparison: a bullet can fly in the earth's atmosphere only within the flight distance of about 1000m and the flight time of about 2s; a photon can fly in the universe for up to the flight distance of 13.9 billion light-years and the flight time of 13.9 billion years.

So, what exactly lead to the cosmological redshift of starlight, the cosmic expansion after the Big Bang or the energy decay of light? What law exactly the cosmological redshift of starlight follows, Hubble's law of cosmic expansion or the GOR law of light traveling?

IV. The Universe Has No Hubble Distance and Hubble Expansion.

It is very interesting that the GOR distance D_{GOR} (Eq. (12.14)) of light traveling is exactly equal to the Hubble distance D_H (Eq. (21.11): $D_{\text{GOR}}=D_H$.

The GOR distance of light traveling and the Hubble distance share a common meaning: at this distance, the light or photons emitted by stars or galaxies are unobservable by the observers of the earth.

However, it is worth noting that the GOR distance and the Hubble distance are two completely different concepts: the GOR distance D_{GOR} is the upper bound of photon flight distance; while the Hubble distance means, at the distance of D_H, the stars or galaxies recede relative to the earth at the speed c of light.

So, according to Hubble's law, you must ask: if the distance between a star and the earth is greater that the Hubble distance, will the recession velocity of the star relative the earth be higher than the speed c of light? Is this not contrary to Einstein's principle of the invariance of light speed?

Actually, there is no the so-called Hubble distance in the universe: it is very difficult for us to imagine that there are celestial bodies or galaxies in the universe moving at light speed or superluminal speed relative the earth

However, the GOR distance of light traveling is necessary: no matter particle could travel endlessly in the universe without consuming energy.

The GOR law of light traveling and the GOR distance of light traveling tell us that: Hubble’s doctrine of cosmic expansion is not the objectively physical reality, but just a conjecture in pursuit of the myth of the Big Bang.

The phenomenon of cosmological redshift is not the effect of cosmic expansion, but the manifestation of the kinetic-energy redshift or decay of light or photons traveling in the universe, that is, the effect of the kinetic-energy redshift or decay of light or photons.

The kinetic-energy redshift or decay of light or photons traveling in the universe is inevitable and not a conjecture; while the recession of celestial bodies or galaxies or the expansion of the universe is purely an imagination or a conjecture.

V. The Universe Has Never Experienced the Big Bang.

As a matter of fact, from the beginning of the **Primeval Atom**, the theory of the Big Bang has only been a myth that caters to the psychology of curiosity.

Now, without Hubble’s Law, without the cosmic expansion of the universe, could the theory of Big Bang still hold true?

GOR Summary

In 1915, based on **Einstein's theory of special relativity**, under **the principle of equivalence** and **the principle of general covariance**, starting from **the hypothesis of the invariance of light speed**, Einstein established **the theory of general relativity** and expounded the relativistic phenomenon of gravitational spacetime and gravitational interaction. Einstein's theory of general relativity has been verified and supported by observation and experiment. Einstein believed that the gravitational relativistic effects or phenomena were the essential characteristics of gravitational interaction.

Einstein's theory of general relativity has been established for over 100 years.

Now, the mainstream school of physics still believe that gravitational relativistic phenomena are the essential characteristics of gravitational interaction.

In **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR), under **the principle of general correspondence** (GC), the theory of OR has established the GOR three principles as the axiom system of GOR theory: (i) the principle of GOR equivalence; (ii) the principle of GOR covariance; (iii) the principle of the invariance of information-wave speeds. Based on the three principles of GOR, by analogizing or following the logic of Einstein's general relativity, the theory of OR has derived the GOR gravitational-field equation, generalizing and unifying Newton's field equation and Einstein's field equation; and furthermore, has established the whole theoretical system of GOR, named as the theory of **Gravitationally Observational Relativity** (GOR), generalizing and unifying Newton's theory of universal gravitation and Einstein's theory of general relativity, i.e., the two great gravitational theories in physics, revealing the root and essence of the relativistic effects of gravitational interaction.

The theory of GOR is not only the challenge to Newton's theory of universal gravitation and Einstein's theory of general relativity, but also the development of Newton's gravitational theory and Einstein's gravitation theory.

The principle of the invariance of information-wave speeds was originally the logical consequence of the theory of IOR. Now, it has become the most fundamental logical premise of the theory of GOR, the so-called **Gravitationally Observational Relativity** or **General Observational Relativity** (GOR).

Like in the theory of inertia motion, **observation** plays an indispensable role in the theory of gravitational interaction.

The theory of GOR has clarified that Newton's law of universal gravitation is the gravitational model for idealized observation, and Einstein's field equation is the gravitational model for optical observation; Newton's theory of universal gravity is the gravitational theory for idealized observation, and Einstein's theory of general relativity is the gravitational theory for optical observation. Newton's theory of universal gravitation is the true portrayal of gravitational spacetime and gravitational interaction, while Einstein's theory of general relativity is only the optical image of gravitational spacetime and gravitational interaction.

This is the origin of the name of **Gravitationally Observational Relativity** or **General Observational Relativity** (GOR).

The new theory leads to the new discoveries.

The theory of GOR provides new insights into physics.

The theory of GOR has discovered that: Spacetime is not really curved.

According to the theorem of Cartesian spacetime in the theory of GOR, all gravitational relativistic effects are observational effects and apparent phenomena. The theory of GOR tells us that: the scientific predictions based on Einstein's theory of general relativity are not more accurate than the scientific predictions based on Newton's theory of universal gravitation; Einstein's prediction of gravitational waves is a mistake, and LIGO did not detect the gravitational waves came from deep space; Hubble's cosmological redshift does not mean the expansion of the universe, and the universe has never experienced the so-called Big Bang.

The elements of GOR theory, or the contents of **the 2nd volume of OR: Gravitationally Observational Relativity** (GOR), can be summarized as follows.

(i) The Essence of Gravitational Relativistic Effects

The theory of GOR has discovered that, like inertial relativistic effects, the gravitational relativistic effects or phenomena are in essence observational effects and apparent phenomena, but not the essential characteristics of gravitational spacetime and gravitational interaction.

(ii) The Root of Gravitational Relativistic Effects

The theory of GOR has discovered that, like inertial relativistic effects, the root of gravitational relativistic effects or phenomena lies in the observational locality – the speeds of observation media transmitting the information of observed objects to observers are all finite: it takes time for the observed information to cross space.

(iii) Spacetime is not Really Curved

According to the theory of GOR, under different observation agents, spacetime exhibits different degrees of curvature. This means that: spacetime is not really curved; the so-called spacetime curvature is actually an observational effect or an apparent phenomenon, depending on observational agents and rooted from the observational locality of observational agents. The optical agent OA(c) ($c<\infty$) is just like a wide-angle lens, making the gravitational spacetime in Einstein's theory of general relativity appear somewhat curved or deformed.

(iv) The Problem of Photon Rest Mass

In **the 1st volume of OR,** the theory of IOR has already clarified that: any particle of matter, including photons, must have the rest mass m_o of its own, that is, the intrinsic mass of it, which is the objectively real mass and has real gravitational effects. Now, in **the 2nd volume of OR**, based on the theory of GOR gravitational redshift, the theory of GOR has calculated and predicted that: the rest mass m_o of a photon is actually the relativistic mass m of it – $m_o=m=hf/c^2$ (see Eq. (18.46) in Chapter 18). This conclusion may have the enlightening meaning to the experimental physicists who are dedicated to detecting the rest mass of photons.

(v) The Perihelion Precession of Planetary Orbits

In the theory of GOR, the GOR model of the celestial two-body system (M,m) has generalized Einstein's celestial two-body model and can also calculate and predict the perihelion precession of planetary orbits. However, the theory of GOR has discovered that this sort or precession depends on observation and observation agents: under different observation agents, the perihelion of a planet exhibits different precession rates. This suggests that the precession rate predicted by the GOR motion model of planets is only an observational effect or an apparent phenomenon. Einstein's celestial two-body model is a special case of the GOR celestial two-body model; and therefore, Einstein's prediction of Mercury's perihelion-precession rate of the 43.03 arcseconds every 100 years is only the apparent phenomenon presented by the optical agent OA(c), but not the objectively real precession of Mercury's perihelion. As a matter of fact, whether the theory of GOR or Einstein's theory of general relativity, the idealized celestial two-body model cannot predict the actual precession of planetary orbits, including Mercury's actual precession of the 5557.62 arcseconds every 100 years.

(vi) The Gravitational Deflection of Light

In the theory of GOR, the GOR model of the gravitational deflection of light has generalized Einstein's model of the gravitational deflection of light and Newton's model of the gravitational deflection of light, and can also predict the gravitational deflection angle of light. However, the theory of GOR has discovered that this sort of gravitational deflection depends on observation and observation agents: under different observation agents, the light in gravitational spacetime exhibits different degrees of deflection. This suggests that the gravitational deflection of light predicted by the GOR model of the gravitational deflection of light contains the observational effects or apparent phenomena of observation agents.

As far as the deflection angle of starlight passing over the sun is concerned: Newton's prediction is δ_N=0.875″; while Einstein's prediction is δ_E=1.75″, being twice Newton's prediction.

Astronomical observation tends to support Einstein's prediction. However, this does not mean that Einstein's theory of general relativity is better than Newton's theory of universal gravitation. On the contrary, according to the theory of GOR, Newton's prediction is the observational value of the idealized observation agent OA_∞, representing the objective and real gravitational deflection of light; while Einstein's prediction is the observational value of the optical observation agent OA(c), which, although supported by OA(c), contains the optical observation effects or apparent phenomena of OA(c), is not completely objective and real. Actually, the gravitational deflection of starlight observed during total solar eclipses provides another enlightening empirical evidence for the theory of GOR.

(vii) The Gravitational Redshift of Light

In the theory of GOR, the GOR model of the gravitational redshift of light has generalized Einstein's model of the gravitational redshift of light and Newton's model of the gravitational redshift of light, and can also predict the gravitational redshift of light. However, the theory of GOR has discovered that this sort of gravitational redshift depends on observation and observation agents: under different observation agents, the light in gravitational spacetime exhibits different degrees of redshift. This suggests that the gravitational redshift of light predicted by the GOR model of the gravitational redshift of light contains the observational effects or apparent phenomena of observation agents.

As far as the gravitational redshift of the solar spectrum is concerned: Newton's prediction is $Z_N=-4.14\times10^{-6}$; while Einstein's prediction is $Z_E=-2.12\times10^{-6}$, being only half of Newton's prediction.

According to the theory of GOR, Newton's prediction is the observational value of the idealized agent OA_∞, representing the objective and real gravitational redshift of light; Einstein's prediction is the observational value of the optical agent OA(c), which, although supported by OA(c), contains optical observational effects and is not completely objective and real.

(viii) GOR's Information Waves and Einstein's Gravitational Waves

Based on the gravitational-field equation in his general relativity, Einstein derived a wave equation, in which the wave had the factor χ/c^2 with the Newtonian gravitational potential χ and the light speed c. Accordingly, Einstein believed that is a gravitational wave, and made the famous prediction: there were gravitational waves propagating at the speed c of light in gravitational spacetime.

From the GOR gravitational-field equation, the theory of GOR has also derived a wave equation that is isomorphically consistent with Einstein's wave equation, in which the wave has the factor χ/η^2 with the Newtonian gravitational potential χ and the information-wave speed η of the general observation agent OA(η). The GOR wave equation has generalized and unified Einstein's wave equation and Newton's wave equation (that is, Laplace equation). This suggests that the wave in the GOR wave equation is the information wave of the general observation agent OA(η), and the wave speed naturally depends on the information-wave speed η of the general observation agent OA(η).

It is thus clear that: the wave in Einstein's wave equation is not a gravitational wave, but rather the information wave of the optical observation agent OA(c) that transmits the information of observed objects at the speed c of light.

(ix) GOR and the Expansion of the Universe

Hubble's doctrine of cosmic expansion and Einstein's theory of general relativity are the important evidence and theoretical foundation for the theory of Big Bang. However, there seems to be contradiction between Hubble's doctrine and Einstein's theory: Einstein believed that the speed of light could not be exceeded; Hubble believed that galaxies at Hubble's distance regressed relative to earth at the speed of light, and naturally, the galaxies beyond Hubble's distance regressed at a speed faster than the speed of light. It is unimaginable that massive galaxies could move relative to earth at the speed of light or even faster than light. Of course, what the theory of GOR questions is not only the recession velocity of galaxies, but also the doctrine of cosmic expansion: the spectrum redshift of starlight, the so-called cosmological redshift, does not mean that the universe is expanding. In the view of

GOR theory, the redshift of light is the kinetic-energy redshift or decay of photons. The observers of the earth could not observe the starlight emitted by the stars or galaxies beyond the Hubble distance, not because the stars or galaxies are retreating at the speed faster than light, but because the energy of light or photons is limited and the long-distance travel of light or photons must consume energy. So, it is impossible for starlight to cross the Hubble distance (rightly **the GOR distance of light traveling**) to reach the earth.

Actually, the phenomenon of cosmological redshift does not mean the so-called cosmic expansion or the expansion of the universe. Cosmological redshift follows the GOR Law of Light traveling rather than Hubble's Law of cosmic expansion. As clarified in BP-15 of Chapter 21, the cosmological redshift is the manifestation of the kinetic-energy redshift or decay of starlight traveling over long distances in the universe. Naturally, the farther the distance from the earth, the more significant the energy redshift or decay is.

(x) GOR and the Big Bang

Einstein's theory of general relativity is the important theoretical foundation of the theory of Big Bang. However, those cosmologists, who are keen on the theory of Big Bang, have not realized that Einstein's theory of general relativity is only a partial theory and the physical model of optical observation, which is effective and valid only when the observation agent OA(η) is the optical agent OA(c). Since cosmologists could create the theory of Big Bang under the optical observation agent OA(c), they could also create the theory of Big Bang under the general observation agent OA(η). Naturally, based on Newton's theory of universal gravitation, cosmologists could also create the theory of Big Bang under the idealized observation agent OA_∞.

The Hubble doctrine of cosmic expansion and Einstein's theory of general relativity are the two pillars of the theory of Big Bang. In the view of the theory of GOR, the cosmological redshift of starlight does not mean the expansion of the universe. So, could the theory of Big Bang still hold true without Einstein's theory of general relativity and Hubble's doctrine of cosmic expansion?

(xi) The GOR Gravitational-Field Equation: Generalizing and Unifying Newton and Einstein's Field Equations

The Poisson-equation form of Newton's law of universal gravitation is the gravitational-field equation of the idealized observation agent OA_∞; while Einstein's field equation in Einstein's theory of general relativity is the gravitational-field equation of the optical observation agent OA(c). In the theory of GOR, the GOR field equation is the gravitational-field equation of the general observation agent OA(η). The GOR gravitational-field equation has generalized and unified Newton's field equation and Einstein's field equation: as $\eta \to c$, the GOR field equation would strictly (rather than approximately) converge to Einstein's field equation; as $\eta \to \infty$, the GOR field equation would strictly (rather than approximately) converge to Newton's field equation.

(xii) The Theory of GOR: Generalizing and Unifying Newton's Theory of Universal Gravitation and Einstein's Theory of General Relativity

In the theory of GOR, the GOR field equation generalizes and unifies Einstein's field equation and Newton's field equation; the GOR motion equation generalizes and unifies Einstein's motion equation and Newton's motion equation; The GOR formula for the GOR total-energy $H(\eta)$ (the GOR kinetic energy $K(\eta)$ and the GOR potential energy $V(\eta)$), or **the GOR Hamiltonian**, generalizes and unifies Einstein's formula for the relativistic total energy $H(c)$ and Newton's formula for the classical total energy H_∞; the GOR celestial motion model generalizes and unifies Einstein's celestial motion model and Newton's celestial motion model; and so on.

Finally, the theory of GOR has generalized and unified Einstein's theoretical system of general relativity and Newton's theoretical system of universal gravitation, the two greatest theoretical systems in the history of human physics.

The theory of GOR, so-called **Gravitationally Observation Relativity** or **General Observational Relativity**, has generalized and unified Einstein's gravitational theory and Newton's gravitational theory. This indicates that the theory of GOR is logically consistent with both Einstein's theory of general relativity and Newton's theory of universal gravitation. In particular, such logical consistency confirms the logical self-consistency of GOR theory, and from one aspect, confirms the logical rationality and theoretical validity of GOR theory.

The theory of GOR is both speculative and empirical.

It should be pointed out that the theory of GOR has empirical basis and is supported by observation and experiment.

As a matter of fact, the support of observation and experiment for Einstein's theory of general relativity is also the support for the theory of GOR; the support of observation and experiment for Newton's theory of universal gravitation is also the support for the theory of GOR.

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