# Some remarks on the generalization of atlases

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**Abstract**

We generalize atlases for flat stacks over smooth bundles by constructing local-global bijections between modules of differing order. We demonstrate an adjunction between a special mixed module and a holonomy groupoid.

## 1 Notation and Conventions

Throughout, $Strat_M^{\{*\}} \subseteq Man$ will denote a point-for-point stratification on a manifold $M$.[1] We will let $Strat_M^{\Delta}$ denote the conical stratification, $Strat_{U_\alpha}^{u_i}$ will denote an open (portable) cover of a parameter space, which is dependent upon the character $i$ for topological realization.

Call every $u_i$ a *covering sieve*, and let every

$$U_\alpha = \bigcup_i \alpha_i \in s_i \times s_i$$

Let, for every path $x \longrightarrow -x$, there be a corresponding value $\Psi_\theta : \mathcal{X} \rightrightarrows \mathcal{Y}$. In other words, we define a *polar path* (of polarity 1), by the map

$$\Pi_x : \pi \xrightarrow{x^{-1}} -\pi$$

by *inducing* an isomorphism,

$$Id_x \simeq y \cdot i \in x_i$$

To explicitly define induction, as a first-principle operator, would be difficult, though not entirely intractable. Recall from the adjunction

$$Hom(x, y) \xrightarrow{\sim} x \otimes y$$

that there is a path

$$min \to \max(exp(\pm x \times y))$$

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[1]Compare this notation with [6], section 2.1

where, for every nth order operation,

$$x \otimes_n y$$

,

there is a rank n retract, consisting of 2n arrows $\ker(x) \to im(x) \sim \ker(y) \to im(y)$. Let $\mathcal{H}^2$ denote the upper half-plane, and

$$\mathcal{H}^1 \simeq \mathbb{A}^1$$

hold by isometry between the cross product of diagonals

$$\sqrt{\Delta^2}(x \times x) \cdot (y \times y) \simeq Pull_\delta(Hom(x, y))$$

**Definition 1.** *Let $\mathcal{C}_\infty$ be an infinite ordered chain. Let every morphism be injective, and surnjective, and therefore a bijection. We let, for every $\varepsilon \in (x \in X) \times (y \in Y)$, there is a corresponding fraction, $\frac{1}{n} \simeq \delta$.*

*Call every map $A \xrightarrow{\delta} B$ of generalized spaces a $\delta$-pushout, and call its inverse $\delta^{-1}$ a $\delta$-pullback.*

The $\hookrightarrow$ will denote a monomorphism, and $\twoheadrightarrow$ will denote an epimorphism.

# 2 Lucid sets and inner homs

Let $\mathscr{C} : \mathcal{C} \longrightarrow \mathcal{C}_{SET}$ be a perfect immersion. Call the image of a distinguished character $i \in \mathscr{C}$ a *lucid* map, if its pullback is an etale object in the $\mathcal{C}$-category, which we will later enrich.

Let there be a bijection between an index $\mathfrak{A}$, and a category $\mathcal{C}$, and another between $\mathcal{C}_{SET}$ and $\mathscr{C}$. Let the index generate a class of open submersions

$$U_{\alpha_{i,*}} : x \hookrightarrow y \cap *$$

**Proposition 1.** *There is a $\delta$-pushout for every element $x_i \in \mathscr{C}$.*

**Example 1.** *The $\mathcal{C}^1$ space has a mirror with* Holder continuity $\mathcal{C}^\infty \to |\mathcal{C}|_\pm$. *This allows the bi-crossed module of inner homs*

$$\amalg_i x \star y$$

*to have a one-to-one bijection with orbital elements, which, as have been previously shown, act as local isotropy groupoid realizations.*

**Example 2.** *The* counting operad $x \star_+ y$ *induces a transitive relationship,*

$$\mathcal{R}_{xy} : \sum_{i=0}^{\infty} x_i \to y_i$$

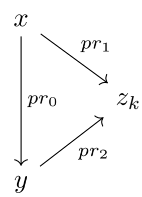
*on the set of open objects in the* collective isotropy group module,

$$\bigcup_i \mathcal{G}_{x_i}$$

**Example 3.** *As the fiber $x \rightrightarrows y$ splits, we obtain morphisms $pr_0$, $pr_1$, and $pr_2$, inducing a conical foliation on an abstract space. The critical point may be written $z_k$, where the harmonic function*

$$\sum_{n=\aleph_0}^{n=\aleph^{\omega^{\omega}}} \alpha_n \frac{1}{n} + \alpha_{n-1} \frac{1}{n+1} ... \alpha_{\emptyset} \frac{1}{n+\infty}$$

*gives us the value of $k$.*




Recall that, for a totally lossless projection of a perfect map, there exists a perfect inverse. In formal terms:

**Axiom 1.** $(Perf(x) \hookrightarrow Perf(y)) \longrightarrow Perf((x \cdot y)^{-1})$

In fact, by this we mean strictly

$$Perf(x, y) \implies Perf(x, y, \cdot, -)$$

So, for $\mathcal{C}$ a small category, whose objects are etale, the topological realization

$$c \in \mathcal{C} \longrightarrow \{*\}$$

, we are given (for free), a logical implication

$$c \implies \mathcal{C}_{SET}$$

be "remembering" the inner hom constructed in example 1.

Some topics of interest for this implication may include portability, which further implies holonomy if the underlying stratification element is a manifold. Thus, as a result, it may be worth considering orbifolds as well, leading to a more nuanced theory of orbispaces.

I am inclined to state that, at the macroscopic level, everything that is observed is portable; i.e., it exhibits actions which are derivatives with respect to time. This is to say, everything observed in the *practical* world, is a submodule of the enriched vector space overlying the stack $\mathscr{A}$ from which the topology is derived.

**Proposition 2.** *Let*

$$Hol_n \simeq (\mathscr{A} \twoheadrightarrow Strat^{\omega}_{\bullet})$$

*Then, there is an arithmetic mapping*

$$n \longrightarrow \bullet$$

*which is locally contractible if it is simply connected.*

**Remark 1.** *$Hol_n$, of course, denotes the holonomy groupoid of Ehresmann, with rank n isomorphisms. Recall that a rank n isomorphism is a cutting of a fiber q into n isotropic (of equal arity) segments.*

**Proposition 3.** *$\mathscr{A} \twoheadrightarrow Strat^{\omega}_{\bullet}$ may be extended to a logical implication*

$$\mathscr{A} \Longleftarrow Strat^{\omega}_{\bullet}$$

*Proof.* The inverse, $Id_n^{-1}$ may be composed with $Id_n$ to yield the null groupoid.[2] □

# 3 Classifying spaces, atlases, and diffeomorphisms

Let $\mathscr{C} \hookrightarrow \mathscr{C}_+$ be an additive, polar mapping. Let there be a manifold $\mathscr{M}_{\mathcal{C}} \to \mathcal{C}_+$ generated by taking the first derivative of a tangent fiber at any arbitrary point, with respect to time.

Let there be a Haefligger classifying space,

$$\Gamma^q \cdot BG$$

such that $\pi_q(\varepsilon) \implies \vec{\partial} U_{q_i \in \alpha}$.
Suppose that the implication splits as $\amalg_{\mathcal{R}} = x_y, y^x$
Then,

**Proposition 4.** *There is a canonical bijection*

$$((x_y \times y^x) \cdot (y^x \times x_y)) \leftrightarrow \pi_n(\Psi_\theta)$$

Recall from [4] the van Est theorem:

**Theorem 1.** *If G is (topologically) $p_0$-connected, the map induced by $VE$ in cohomology is an isomorphism for $p \leq p_0$ and injective for $p = p_0 + 1$, in the map:*

$$VE_{k-hom} : C^p_{k-hom}(\mathfrak{v}) \to C^p_{k-hom}(v)$$

That is to say, for two p-cochains obeying the (generalized) cocycle condition, there is a totally lossless projection[3]

$$\mathfrak{v}_p \hookrightarrow v_p$$

such that

$$\mathfrak{v}^n_p \cdot v^n_p = \sum_{i=0}^{n} fib_{pro}(v_i)$$

[2]Consult [1] for the relevant literature. It is a masterful work of art. [2] is highly recommended as well, but not, of course, necessary.

[3]See [5]

Let $\mathcal{G}_0 = Id_{x \in X}$ for some $x \in \mathcal{G} \cap X$. This identity extends to a classifying space, $BG$, by way of a categorical stratification, $Strat^x_{\mathcal{G}}$, via the map

$$Strat^x_{\mathcal{G}} \cdot \Gamma^q$$

for $q \neq n \in \mathbb{N}$. Here, $x^n_i$ denotes the qth orbital of a point-like object in a topological stack. We may extend this to a map of displays:

$$\phi_{\mathfrak{X} \rightrightarrows \mathfrak{Y}} : (x^n_i)^q \longrightarrow \tilde{x}_j$$

,

where $\tilde{x}_j$ denotes the jth representative gneerator of a jet bundle $\mathcal{J}_x(\phi)$.

Denote by ${}_{\tilde{X}}Mix_{T_x J^n}$ the mixed module obtained by flattening a section of a topological space $X$ to a discrete foliation $\tilde{X} \simeq \mathcal{F}_{x_i}$ over the tangent space of a jet bundle of order n.

**Proposition 5.** *The diagram*

$$\begin{array}{ccc} {}_{\tilde{X}}Mix_{T_x \mathcal{J}^n} & \xrightarrow{|\sim|} & \mathfrak{X} \\ {\scriptstyle fib(x_i)}\downarrow & & \downarrow{\scriptstyle pr_0} \\ S^n & \xrightarrow[pr_1]{} & \mathfrak{Y} \times (x_i \circ Id_\delta) \end{array}$$

*is commutative, and the projections are totally lossless.*

*Proof.* Assume $\mathfrak{X}$ is perfect. Then, $\mathfrak{Y} \times (x_i \circ Id_\delta)$ must be perfect also. Since we have the quotient uniformity

$${}_{\tilde{X}}Mix_{T_x J^n} / \sim = |x_i| \in \mathfrak{X}$$

serving as the geometric realization for each tangent fiber

$$T_{x_i} \in fib(\mathfrak{X})$$

,

where $\mathfrak{X}$ is a topological space, we conclude our proof by noting that, for every section of $T_{x_i}$, there is a $\delta$-pullback $\phi^{-1} T_{x_i}$. □

At this point, we may construct a gerbe, $\mathscr{G}^{Hol}_{\tilde{X}}$ by inducing a levelwise, piecewise differentiable structure on the tangent bundle. This is given by the formula

$$\mathscr{G}^{Hol}_{\tilde{X}} = \{\mathfrak{X} \times_{x_i} \mathfrak{X} | x_i \in \sum_{i=0}^{n} X_i \{\partial^0 x + \partial x + \partial^2 x + ... + \partial^n x\}$$

Further yet, there is a diffeomorphism

$$\mathscr{G}^{Hol}_{\tilde{X}} \simeq \mathcal{J}^n(T_x x_i(fib(x)))$$

on connections parameterized by the nth order jet bundle over a typical fiber of x. This is because of the famous link between the structure sheaf, $\mathcal{O}_X$ of a stack of which $x$ is a germ, and the orbit group, $\mathscr{O}_x$ of a topological stack including $x$ as a *point-like* object.

**Definition 2.** *Call $A$ an* atlas *if it is obtained by a composition of transition maps*

$$A = \int_0^n (\phi(\partial^n(x_i)) \circ \phi^{-1}(\partial^n(x_i)) \circ ... \circ \phi(\partial^0(x_i)) \circ \phi^{-1}(\partial^0(x_i)))$$

**Example 4.** *Consider an atlas A where every natural transformation $\phi^{-1} \Rightarrow \phi$ is totally lossless. This is called the perfect atlas.*

**Example 5.** *Let A be an atlas, and let every $x_i$ belong to the category $Strat_M$ of stratified manifolds, with an unspecified stratification. Then, a* corner*, $\partial(\mu(x + y))$, is the orthogonal pseudo-orthogonal stratification of an atlas A, written $A(Strat_M^{\square})$.*

**Proposition 6.** *There is an adjunction*

$$\mathscr{G}_{\tilde{X}}^{Hol} \leftrightarrow_{\tilde{X}} Mix_{T_x J^n}$$

*Proof.* This follows from the famous "tensor-hom" adjunction. $\square$

# 4 References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