Cooldown Time Estimation Methods for Stirling Cycle Crycoolers

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Abstract

Miniature crycoolers are small refrigerators that can reach cryogenic temperatures in the range of $60K$ to $150K$. They have the capability of accumulating a small temperature drop into a large overall temperature reduction. The cooldown time estimation is becoming more and more as a design parameter, certainly in hands-on applications. The various complicated physical processes involved in crycooler operation make it hardly possible to explicitly simulate the temperature time response. The numerical methods for solving a typical crycooler suffer from numerical instability, time step restrictions and high computational costs, among others. Since the operation of crycoolers involve processes in range of $15Hz-120Hz$, actually solving the crycooler transient response would require different software tools to support the design and analysis of physical processes such as heat transfer, fluid dynamic, electromagnetic and mechanical. These processes would also require an excessive amount of calculations, incurring time consuming and precision penalty. In this article we try to bridge the gap between the explicit impractical approach to steady state based approach. A framework developed in Python for calculating the cooldown time profile of any crycooler based on a steady state database, is introduced, while utilizing a semi-analytic approach under various operating conditions. The cooldown time performance can be explored at various target and ambient temperature conditions, and also the effects of an external load, material properties or thermal capacitance on the overall cooldown time response. Two case studies based on linear and rotary crycoolers developed at Ricor are used for verfication, with a good agreement between the simulated and measured values.

Keywords: Cooldown time estimation, rotary crycoolers, linear crycoolers.

1 Background

Miniature crycoolers are small refrigerators that can reach cryogenic temperatures in the range of 60 K to 150 K. It is relatively simple to reduce the temperature by about 25 K below the ambient, but it is much harder to reduce the temperature by about 250 K. A single phenomenon such as compressed gas

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expansion or thermo-electric effect may reduce the temperature by 25 K. However, there is no single physical effect that alone can take us 250 K below ambient. Crycoolers that perform cooling based on a Stirling cooling cycle do so. They have the capability of accumulating a small temperature drop into a large temperature reduction below ambient, and this feature is the essence of crycoolers [1].

Crycoolers performance and reliability are continually improving. They are frequently implemented for military, commercial and space applications. The most commonly used cooling cycles for various applications are Stirling, Brayton, Joule-Thomson, Gifford-McMahon, and pulse tube crycoolers. They are small devices (for certain applications) that generate low temperatures by compression and expansion of gas in a closed cycle manner. Stirling crycooler consist of compressor, a heat exchanger and an expander. With a cryogenic fluid as the working substance and moving parts, the fluid is moved around the thermodynamic cycle. The temporal development of this evolution is the cooldown process, which is the main issue of the present research [2, 3, 4].

There are some applications where the cooldown time to steady state is a crucial design parameter. These applications are usually carried by a man (except applications such as missile war head that requires fast cool down time and it is not man handled). Typically, the requirement involves cooling down a certain mass, which can be an IR detector, for example, from room temperature to cryogenic temperature within a stipulated amount of time. If one wants to calculate the transient response of the crycooler, it is possible, at least in theory. The main physical processes involved in crycoolers operation are heat transfer, fluid dynamic, acoustics, mechanical and electromagnetic. The physical equations describing these processes may be written as time dependent, discretized, and numerically solved over the time history. Since these processes are physically different, so does the numerical behavior, the discretization method (mechanical problems are discretized with finite element approach while fluid dynamic equations are discretized with finite volume approach) and the path to reach convergence. Consequently, different physical issues involve different commercial software, and there is no platform, at least commercially available, that can deal with electromagnetic and fluid dynamic, for example, in the same platform. In addition, crycoolers operate at frequency range of 15 Hz to 500 Hz, and in order to capture physical events occurring in this range the time step must be sufficiently small. From the Currant-Friedrichs-Lewy (CFL) (A condition that is related to the distance that any information travels within the mesh during a time step) and the tiny computational cells involved in a typical discretized crycooler model, the time step would have to be less than 0.5 ms. Taking into account that typical crycoolers cooldown time range is 3-5 minutes, it requires hundred thousands time steps to reach convergence for each variable (pressure, temperature, velocity field, etc.). Combining all together , number of unknown variables, nonlinear iterations per time step, time step restrictions, mesh size, parallel scalability issues, high computational costs and smooth interactions between the different codes, make this mission almost impossible, especially for industrial applications.

The present research aims to bridge the gap between the complicated explicit approach for cooldown time estimation and the steady state approach. Two methods are described for cooldown time estimation of any cryogenic system in various ambient conditions, external loads and power input. Both methods uses fast and cost-efficient design simulations to map crycooler performance in steady state conditions under various range of operating conditions. The steady state solver used is Sage, which is a commercially available crycooler performance software, from Gedeon Associates [5].The main advantage of Sage is in encapsulating gas flow, heat transfer, electromagnetic and other modeling details within a number of specialized model components which may be freely inter-connected form to complete models of complicated systems [5]. It is important to mention here that it is a one dimensional simplification model, and some of the models are empirical in nature and do not cover the whole range but use a rather weak correlation in some cases. It has an enormous strength but accurately modeling is a meister art.

The cooldown time estimation problem is essentially a system of first order nonlinear ordinary differential equations (ODE) comprising the material properties and the components intertwined with the heat transfer process (gas enthalpy, heat transfer between the solid and the gas, and mechanical work). The remainder of this paper presents two methods for cooldown time estimation. Then, two test cases based on crycoolers developed and manufactured at RICOR, were identified as potential candidates for the investigation of this approach. The chosen cases were those for which experimental results are readily available in various ambient conditions.

2 Cooldown time estimation

The cases presented here are used to cool down an IR detector and therefore need a vacuum isolated dewar that cover the cold finger - which in turn separate the hot part from the cooled part. Two approaches for cool down time estimation were analyzed: The first is a finger-based approach that focuses on the cold finger as the main heat transfer mechanism, which is typically divided to seven different substances. The heat capacity of each substance is integrated over time, from the initial (ambient) temperature to the target cold temperature. This approach is named "finger-based" since it focuses on the cold finger mechanism as the main source of the heat transfer process, including the detailed geometry and the substance properties.

The second approach is actually more general, as it is based on an energy balance on a control volume which transports energy in the form of heat, enthalpy and mechanical work across its boundaries. The energy flows contain information such as internal gas flows, oscillations, and the net result of many complicated process occurring within the crycooler. The latter approach is chosen as the methodology for the cool down time calculator, mainly since its generality and the fact that this method can be used to estimate the cooldown time of any cryogenic system subject to general power input, ambient conditions and heat loads. Our purpose here is to discuss the basic aspects of the cooldown time estimation process - we choose not to obscure the computational aspects by carrying along the extra complications and physics associated within the crycooler itself. For this reason, the detailed physical phenomena occurs within the crycooler are not modeled in the equations. With the above restrictions in mind, we first describe the control-volume approach.

2.1 Control volume based approach

First let us define a typical crycooler with all its complicated subsystems as a thermodynamic control volume and make an energy balance. The conservation of energy in a control volume states that the time rate of change of energy in a control volume at a certain time equals the net rate of energy transfer into the control volume at that time by three mechanisms: heat transfer, work and mass transfer. Such a control volume is sketched in Fig. 2.1. If the inflow and generation of thermal and mechanical energy exceed the outflow, then must be an increase in the amount of thermal and mechanical energy accumulated in the control volume. If the converse is true, there will be a decrease in thermal and mechanical energy stored. So the first law of thermodynamic can be written as a rate equation:

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \tag{2.1}$$

where $\dot{Q} = \lim_{dt \to 0}(\frac{\triangle Q}{dt})$, $\dot{W} = \lim_{dt \to 0}(\frac{\triangle W}{dt})$. The physical idea is that any rate of change of energy in the control volume must be caused by rates of energy flow into or out of the volume. In addition to the heat transfer and work that already included, the fluid that enters or leaves has an amount of energy per unit mass given by $e = u + c^2/2 + gz$ where u is the internal energy, and c is the fluid velocity relative to some coordinate system. Whenever fluid enters or leaves a control volume there is work term associated with the entry or exit. For example, flow exiting the volume push back the surrounding fluid, doing work on it which can be estimated as $dW_{flow} = pvdm_e$, where v is the volume. The rate of flow work is $\dot{W} = p_e v_e \dot{m}_e - p_i v_i \dot{m}_i$. Including all possible energy flows exist in crycoolers (heat, piston work, etc.) and neglecting others (shaft, shear work, etc.) the first law can then be written as:

$$\frac{d}{dt}\sum(E) = \sum\dot{Q} + \sum\dot{W}_{piston} + \dot{W}_{flow} + \dot{m}(u + \frac{c^2}{2} + gz) \tag{2.2}$$

By combining the specific internal energy u in e and the specific flow work pv, we get the enthalpy: $e + pv = u + \frac{c^2}{2} + gz + pv = h + \frac{c^2}{2} + gz$, and the first law can be written as:

$$\frac{d}{dt}\sum E = \dot{Q} + \dot{W}_{piston} + \dot{m}(h + \frac{c^2}{2} + gz) \tag{2.3}$$

It is important to mention here that if heat is added to the system then the sign is positive, and if work is extracted from system then the sign is negative. Since miniature crycoolers are small and closed systems with no entry or exit, then both potential energy and kinetic energy can be neglected. Finally, the first law can be written as:

$$\frac{dE}{dt} = \sum(\dot{Q} + \dot{H} + \dot{W}) \tag{2.4}$$

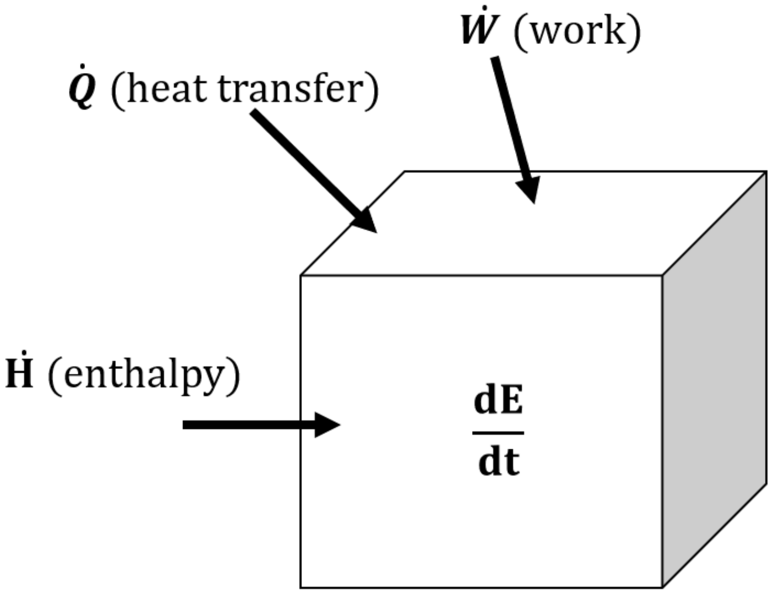

Figure 2.1: Energy balance in an arbitrary control volume.

Now, if we know the mass of the control volume and its specific heat, we can determine the amount of heat energy E entering or leaving the control volume by measuring the temperature change before and after the heat is gained or lost: $E = mC_p\triangle T$, whereC_p is specific heat at constant pressure, m is the mass and $\triangle T$ is the temperature change. The three main subsystems that compose a typical crycooler is compressor, expander and cold tip; each one gains or loose different heat, enthalpy and work values. Consequently, the heat, enthalpy and work flows are functions of the compressor, expander, and tip temperature and input voltage. Therefore it is reasonable to model our problem by three separated control volumes while the following set of equations is obtained:

$$\frac{d}{dt}[m_{cmp}C_{cmp}(T_{cmp})\triangle T_{cmp}] = \sum\dot{Q}_{cmp}(V, T_{cmp}, T_{exp}, T_{tip}) + \sum\dot{H}_{cmp}(V, T_{cmp}, T_{exp}, T_{tip}) + \sum\dot{W}_{cmp}(V, T_{cmp}, T_{exp}, T_{tip}) \tag{2.5}$$

$$\frac{d}{dt}[m_{exp}C_{exp}(T_{exp})\triangle T_{exp}] = \sum \dot{Q}_{exp}(V,T_{cmp},T_{exp},T_{tip}) + \sum \dot{H}(V,T_{cmp},T_{exp},T_{tip}) + \sum \dot{W}_{exp}(V,T_{cmp},T_{exp},T_{tip}) \tag{2.6}$$

$$\frac{d}{dt}[m_{tip}C_{tip}(T_{tip})\triangle T_{tip}] = \sum \dot{Q}_{tip}(V,T_{cmp},T_{exp},T_{tip}) + \sum \dot{H}(V,T_{cmp},T_{exp},T_{tip}) + \sum \dot{W}_{tip}(V,T_{cmp},T_{exp},T_{tip}) \tag{2.7}$$

where T_{cmp},T_{exp},T_{tip} are the compressor, expander and cold tip temperature. In addition, V is the input voltage and m_{cmp},m_{exp},m_{tip} are the mass values of the compressor, expander and tip. In this formulation we get three nonlinear ordinary differential equations that must be solved simultaneously for three unknowns T_{cmp}, T_{exp} and T_{tip}. Integrating by time and finding T_{cmp}, T_{exp} and T_{tip}, requires an information about the energy flows $\dot{Q},\dot{H}$ and $\dot{W}$ at each time step. For that purpose the calculation is done by using one dimensional steady-state solver [5]to calculate the energy flows in various ambient conditions during the crycooler operation. The steady state database gathered by Sage is sufficiently large and comprehensive in order contain all the physical situations and temperature relations that could be obtained by the compressor, expander and cold tip during the crycooler cooldown process. It is interesting to mention here that the control volume energy flow values that obtained at each steady state calculation are the outcome and final result of multiple intricate processes occur within a typical crycooler during its operation.

In order to solve the system of ODE as efficiently as possible, there are several existing solvers to choose from. In the present research the SciPy solvers [6] are used, which is a large suite of scientific software in Python, including tools for linear algebra, optimization, integration and other common tasks of scientific computing. For solving initial value problems, the tool of choice is the solve_ivp function for integrate module [7]. It is an adaptive solver which chooses the time step automatically to satisfy a given error tolerance. The numerical method used for the integration process is RK45 (Runge-Kutta method), while the nonlinear stage derivatives are solved by using Newton method. The steps of the implicit Runge-Kutta method are computational expensive per time step, but they are also more stable than explicit methods. The error is controlled assuming accuracy of the fourth-order method, but steps are taken as using the fifth-order accurate formula.

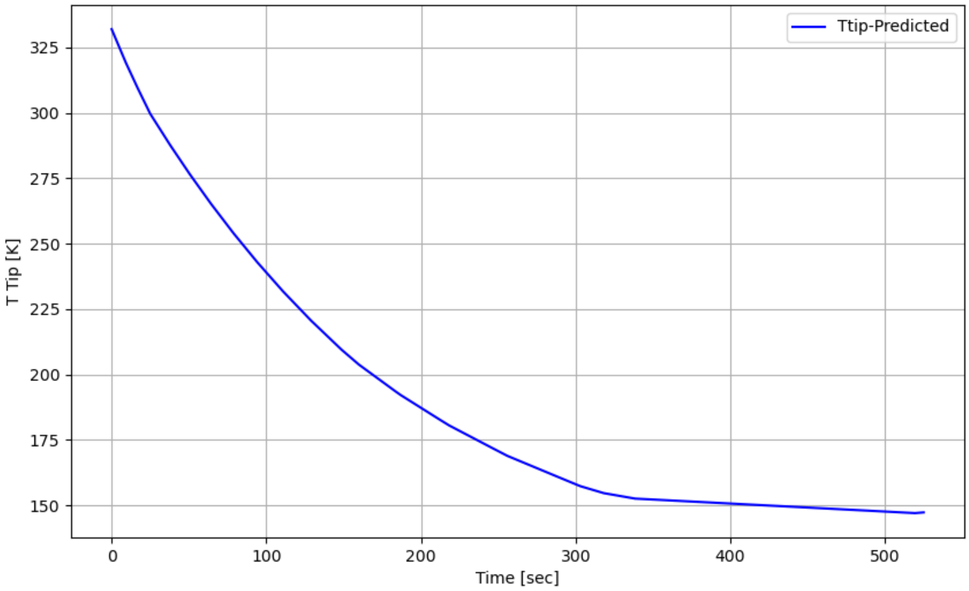

Figure 2.2: K580 computed cooldown time profile for 23°C ambient temperature.

What patterns do the time wise variations of the tip temperature take? Some feeling for the answer is provided by Fig. (2.2), which shows the variation of T_{tip} plotted versus the number of time steps. Examining the tip temperature, we obtained a natural behavior of a time-marching solution - the tip temperature changes from one time step to the next. However, in the approach toward the steady-state solution, after a large number of time steps, the changes in T_{tip} from one time step to the next become smaller and approach zero in the limit of large time. At this stage, the steady state has been achieved, and the calculation can be stopped. This termination of the calculation can be down automatically by the user itself by having a test in the program to sense when the changes in T_{tip} become smaller than some prescribed value. Another option, and that preferred when measured data is involved, is to simply stop the calculation after a prescribed number of time steps, or after the target cold tip temperature is reached.

2.2 A finger based approach

In this approach, the heat transfer model is based on the cold finger mechanism only, while totally neglecting the physical processes occur outside the cold finger, and their contribution to the overall heat transfer process.This concept was initially introduced by Sergay Riabzev[1], and even implemented and tested in Visual Basic for Applications (VBA), a programming language of Excel and other Office programs, for various crycooler models. Sergay contributed significantly to understanding the issue of cooldown time response, and his contribution is gratefully acknowledged.

From the crycooler point of view, a typical Stirling cold finger is composed of five different components: displacer, regenerator, finger tube, cold tip, and added thermal mass (detector). To illustrate its physical background, a schematic model of a typical cold finger is presented in Fig. (2.3). The main idea behind this approach is to calculate the thermal mass of each component in the finger, by integrating temperature, starting in ambient temperature and ending with the target cold tip temperature. The finger's components are categorized to two groups, one that is characterized with a long

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geometry, such as the cold finger tube and the regenerator, in which a temperature gradient is developed during the cooldown process. The second group includes small parts that are characterized with homogeneous distribution temperature, namely, the temperature gradient is neglected (i.e. cold tip, added thermal mass). Attaching a control volume to each component, one can say that the rate of change of the internal energy of the control volume is a function the heat flow into the control volume. The energy balance yields a first order nonlinear ordinary differential equation:

$$\frac{dE}{dt} = \sum \dot{Q} \tag{2.8}$$

where the heat flows$\dot{Q}$ is obtained from steady state calculations using Sage. In Eq. (2.8), the left hand side represents the rate thermal energy removal from a substance while the right hand side represents the net thermal energy flow through this substance at a given time. Sage is also considering interactions of shuttle losses between the tube and the displacer, which is neglected in the external model.

The enthalpy and work flows are not included in this approach, since their contribution are obtained indirectly, within the steady state calculations. For small parts with homogeneous temperature distribution, the time it takes to a specific substance to cooldown in a given ambient temperature is calculated as follows:

$$dt = \frac{dE}{\dot{Q}} = \frac{mC_p(T)dT}{\dot{Q}(T)} \tag{2.9}$$

As already mentioned, for small parts the temperature gradient is neglected, then, the time it takes to reach the target cold temperature is estimated as follows:

$$t_{final} = \sum_{T_h}^{T_c} \frac{mC_p(T)dT}{\dot{Q}(T)} \tag{2.10}$$

The summation process is done by choosing a small temperature interval dT and taking into account the specific heat capacity temperature dependence. Considering the temperature interval involved, the summation on T starts at $T = T_{high}$ and ends at $T = T_{cold}$.

For long parts characterized with a temperature gradient, the substance is divided into multiple sections lengthwise, while each section is characterized with a mass dm and a pre-defined temperature value. For each section an integration process is made, exactly as is given in Eq. (2.10), for its temperature boundaries in order to get its specific contribution to the overall temperature gradient and cooldown time. The summation process of the cooldown time of each slice, from the hot side to the cold tip, finally results in the cooldown time of the volume itself. The cool down time obtained in each slice are summed up until reaching the cold tip target temperature. This integration process can be written here as follows:

$$t_{final} = \sum_{T_c}^{T_h} \sum_{T_h}^{T_c} \frac{dmC_p(T)dT}{\dot{Q}(T)} \tag{2.11}$$

The first summation is for the specific heat and the second summation extends over all the mass sections. Summing up the cooldown time values of each substance results in the crycooler cooldown time estimation. The main benefit of this approach is in it's simplicity and relatively high accuracy (in terms of final cooldown time) that can be achieved due to the detailed geometry properties used. Another benefit is in the straightforward "dirty" engineering approach, which can be implemented and monitored using basic engineering tools. This fact can significantly reduce errors and increase accuracy and precision.

The drawback of this approach is that the user must have an accessibility to the finger's assembly cad model in order to extract each component properties. Another disadvantage is that the cooldown time calculation is performed in a series manner, for each component in the cold finger mechanism, and the final cooldown time is simply a summation of all the cooldown time values obtained for each component. This results in an overestimated cooldown time values, a fact that definitely reduce the accuracy of the cooldown time profile. This straightforward approach is presented here due to it's simplicity and relatively easy way to get an evaluation of the cooldown time, simply for clarity. After all, the cons outweigh the pros, and we decided to move forward with the first approach as the main methodology for the cooldown time calculator.

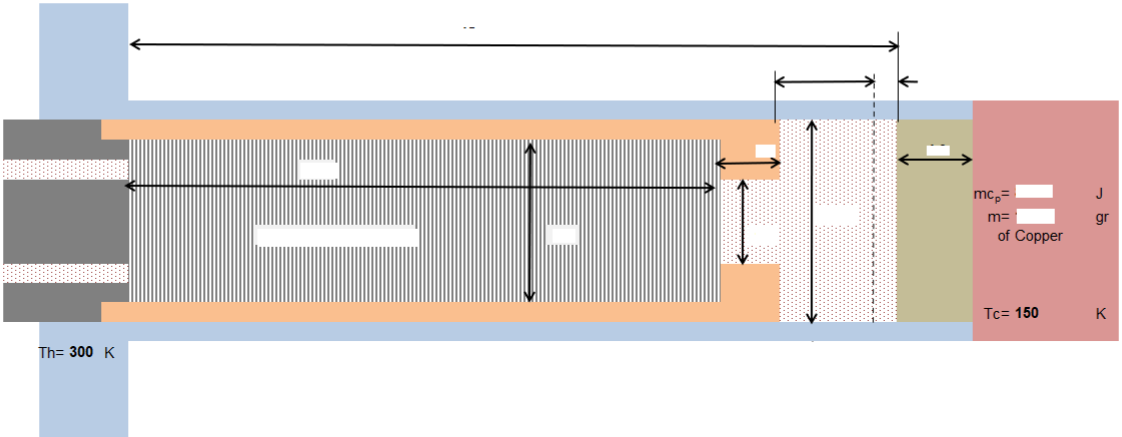

Figure 2.3: Cold finger schematic model

2.3 Steady state database construction

The energy flow $\dot{Q}, \dot{H}$ and $\dot{W}$ are obtained in steady state condition for any combination of compressor temperature, expander temperature, tip temperature and input voltage required. These energy flows were obtained with a steady-state solver, Sage, which operated on a various range of operating conditions $(T_{cmp}, T_{exp}, T_{tip}, V)$. Then $\dot{Q}, \dot{H}$ and $\dot{W}$ were integrated and recorded to four-dimensional array. The steady state database includes compressor and expander temperature range of 230 K to 350 K, tip temperature range of 70 K to 350 K, and input voltage ranges from 0 to 18 Volt. The interval $\triangle T = 20K$ is chosen for the compressor, expander and tip temperature, and an input voltage interval is $\triangle V = 1V$. Combining all together, the 4D array size is$7X7X15X18$, or $13,230$ elements.

One way to reduce the array size is to obtain the voltage values by implementing a "soft start" process that reduces dramatically the maximum peak power, and it is essential to prevent unrestrained displacement in the free piston elements due to low viscosity and changes in the cold head acoustic impedance. In this approach the voltage value becomes a part of the Sage model and it is estimated as a polynomial function of the tip temperature (by polynomial curve fitting), which is fed into the Sage software as a parameter. In this way a continuous voltage value is obtained through all the conditions occur during the cooldown process. In linear crycooler simulation, the soft start process is a critical parameter for ensuring a certain level of accuracy between the simulated and measured results.There are two advantages using this approach. First, it definitely imitates the controller action by using the real voltage value as a function of the tip temperature during steady state calculations. Second, instead of using a large 4D array, the steady state database is reduced to 3D array with $7X7X15$ (735) elements. For further increasing the tip temperature resolution, a 2D interpolation is implemented for each combination of T_{cmp}, T_{exp}reducing the tip temperature interval to 1K only. Once the steady state values of the energy flows are obtained for any combination of T_{cmp}, T_{exp} and T_{tip} it then can be fed into the system of equations to calculate the transient response.

The steady state database which involves a detailed and comprehensive Sage model, is by far the most difficult to set up as it requires a full representation of the actual components, and modeling losses and interactions as accurate as possible. For this reason, the vast majority of cooldown time estimation of a given crycooler is invested on the validation process of the Sage model that supposed to imitate the crycooler in action. The issue of validation of the Sage model for a given crycooler is an extremely complex process as it is hardly possible to generate experimental data at the same level of details as provided by the Sage simulation. In addition, both experimental and numerical data are subjected to many error and uncertainty sources, that the absolute validation of simulation results remains an unreachable objective. It requires, at the end, to take into account the uncertainties and to call upon good judgment and experience of both, physical and numerical properties. Usage of a large quantity of an empirical data and implementing validation techniques can make the model more accurate. In general, error of 20% is considered reasonable. In the cases presented in the present article we have reached an accuracy of less than 10% in the input-output tested parameters.

2.4 Specific heat capacities of common substances

Solving the ODE systems requires a significant thermodynamic coefficient database that defines the specific heat at constant pressure (Cp) as a function of temperature, from the cryogenic to ambient and above. The specific heat is the amount of heat energy per unit mass required to cause a unit increase in the temperature of a material, the ratio in the change of energy to the change in temperature. Specific heats are strong function of temperature, especially bellow 200 K [**?**]. A large database for the specific heat at constant pressure for various cryogenic materials have been constructed, based on NIST (National Institute Of standards and Technology) [**?**]. Materials that we have compiled includes Copper, 6061-T6 Aluminum, Titanium alloy, 304 stainless steel, Invar, Peek etc.. These were chosen as some of the most common materials used in cryogenic systems. The general form of the equation for specific heat, C_p, is

$$\begin{aligned} log_{10}(C_p) = A + Blog_{10}T + Clog_{10}T^2 + Dlog_{10}T^3 + Elog_{10}T^4 \\ + Flog_{10}T^5 + Glog_{10}T^6 + Hlog_{10}T^7 + Ilog_{10}T^8 \end{aligned} \tag{2.12}$$

where $a, b, c, d, e, f, g, h, i$ are the fitted coefficients and T is the temperature. Fig. (2.4) graphically shows the specific heat values for common substances used in crycooler. The measured results (the discrete data points) are compared with the numerical curve fitting results (the solid lines) for various substances, as a function of temperature. The agreement between the numerical results and the tabulated data is excellent, as clearly seen in Fig. (2.4). This fact ensures stability of the algorithm during the convergence process.

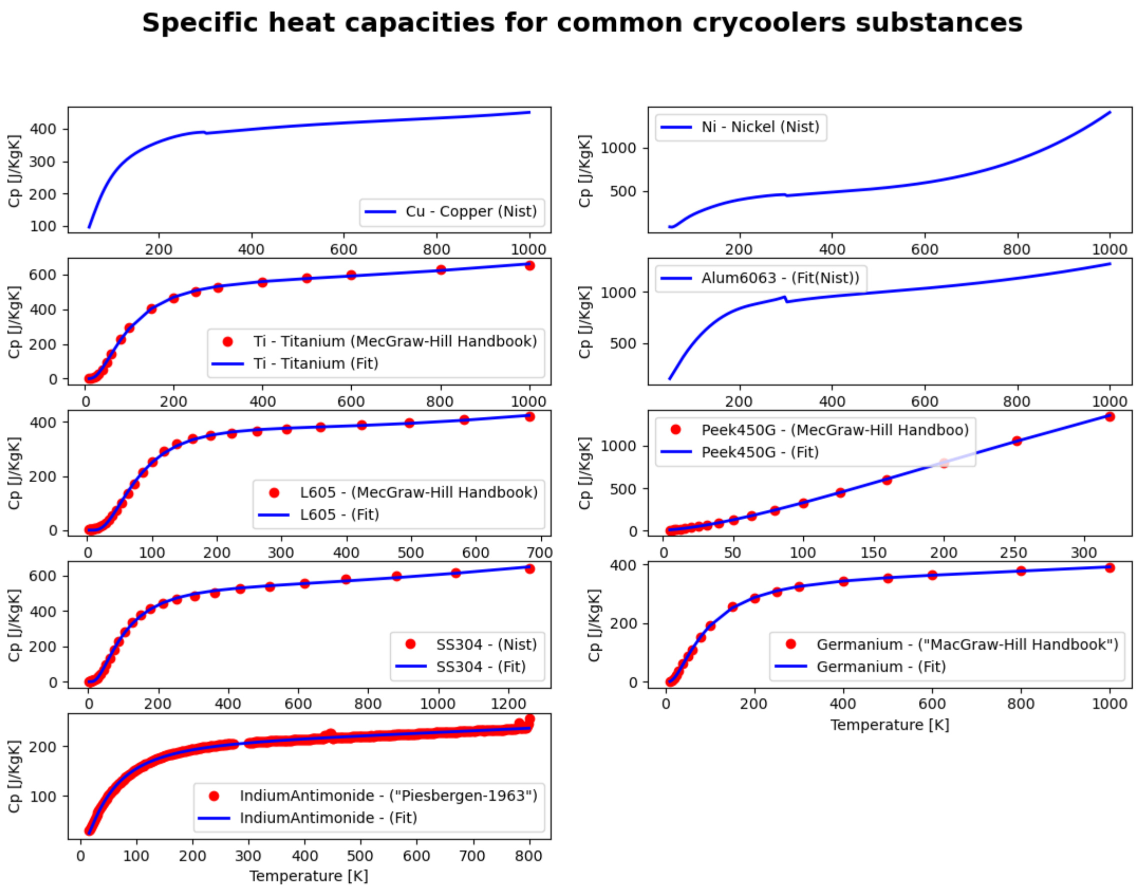

Figure 2.4: Specific heats capacities for common substances [8]

3 Results

The cooldown calculator validation process is done by utilizing two different crycoolers technology developed at RICOR: the K580 Integral rotary crycooler and the k590 split linear crycooler. An experimental cooldown database is used for validation, and the thermodynamic aspects were designed using Sage according to the maximum efficiency approach alongside validation of the SAGE model with real life data with error of 3-10%.

3.1 K580 - An integral rotary crycooler

The K580 model is an integral rotary type crycooler with reduced size, low weight and low power consumption, for high operating temperature IR detectors. The rotary crycooler is driven by rotation motor and transfers the rotational movement into linear movement by a crankshaft with an excenter in order to move the piston and the displacer. The position of the piston and the displacer form an angle of 90 degrees and thereby create the cooling effects. This crycooler models operates in temperatures in the range of 130 K - 180 K, and a typical cooldown time at $210J@150K@23°C$ is around 3 minuets. For more detailed specification and experimental results the reader is referred to [9]. The cooldown time algorithm was tested at ambient temperatures of -41$°C$, 23$°C$, and 71$°C$, at cold tip temperature of 150 K. The main parameters of the cooler are listed in the table (1).

Parameter	Design
Cooling capacity	$500mw@150K@71°C$
Steady state input power	$220mw@150K@23°C$
Input voltage	$4-16DC$
Ambient temperature range	$-40°Cto+71°C$
Cooldown time	$210J@150K@23°C$:3:30 min type

Table 1: K580 crycooler typical parameters

3.1.1 Numerical and experimental results

The cooldown calculator was tested and validated under a variety of ambient conditions. The final cooldown time compared to experimental results are summarized in table (2). The experiments were conducted by using a climate control chamber in order to imitate a specific ambient temperature. The cold tip temperature time was monitored by using a data loger that takes temperature samples at regular intervals, every 10 seconds approximately. The cooldown time and temperature values of the expander, compressor and cold tip as a function of time are presented in Figs. (3.1,3.2,3.3,3.4), and the final results are listed in table (2). In the upper left figure the cooldown time profile (continuous line) is compared to the measured data (red dotted line). The compressor average temperature is presented in the upper right figure, the bottom left figure presents the detector Joule mass during the cooldown process, and the expander average temperature is presented in the bottom right figure. The calculations are stopped when the target cold temperature (150 K) is reached, similar to the measured temperature that is driven by an external digital temperature controller with an accurate long-term stability of $\pm 0.1K$ [9]. The controller reduces the input power as the target tip temperature meets the requirement.

As can be shown, a very good agreement is obtained between the computed and measured cooldown time values, in terms of the final cooldown time and the actual profile estimation. In an ambient temperature of $23°C$ it takes 202 seconds to reach the target temperature of 150 K while the simulated value is under estimated by 9.7% approximately (183 sec). The case of $-40°C$ results in a gap of only 1.2% between the simulated and measured values. One of the advantages of using the control-volume approach is that during the cooldown process we gather a valuable information about the expander and compressor average temperature, a fact that can help us to make a verification and identify errors during the integration process. In the complicated case of $71°C$ the predicted cooldown time is 423 seconds which is slower in 33% approximately than the measured values. One of the reasons to this gap may be in the inaccuracy of the detector Joule mass parameter which is used as an external parameter that is given by the detector manufacturer, and both the substance specifications and the specific geometry data are limited since proprietary reasons. Another reason is the lower validation level of the SAGE model in this temperature. Frictional effects may increase in real life and so does thermal expansion that affects the gaps and dynamic seals. The assimilation of this phenomena in the SAGE model might not be well defined.

Joule mass, g	Heat load, mw	Ambient temp.	Cold tip temp.	Com. cooldown time, sec	Exp. cooldown time, sec	Error
3.4	0	$-41°C$ (230K)	150K	139.8	141.57	-1.2%
3.4	0	$23°C$ (296K)	150K	182.65	202.25	-9.7%
3.4	0	$71°C$ (330K)	150K	422.56	283.15	+33%
3.4	50	$23°C$ (296K)	150K	64.62	78.02	-17.2%

Table 2: Simulated and experimental results for the K580 crycooler

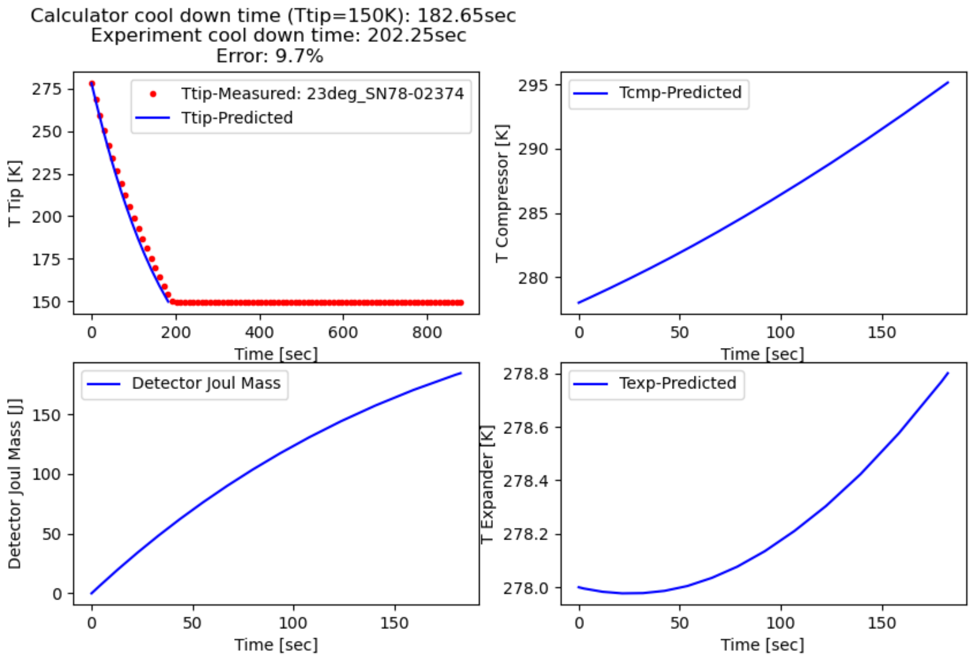

Figure 3.1: K580 computed cooldown time profile for $23°C$ ambient temperature.

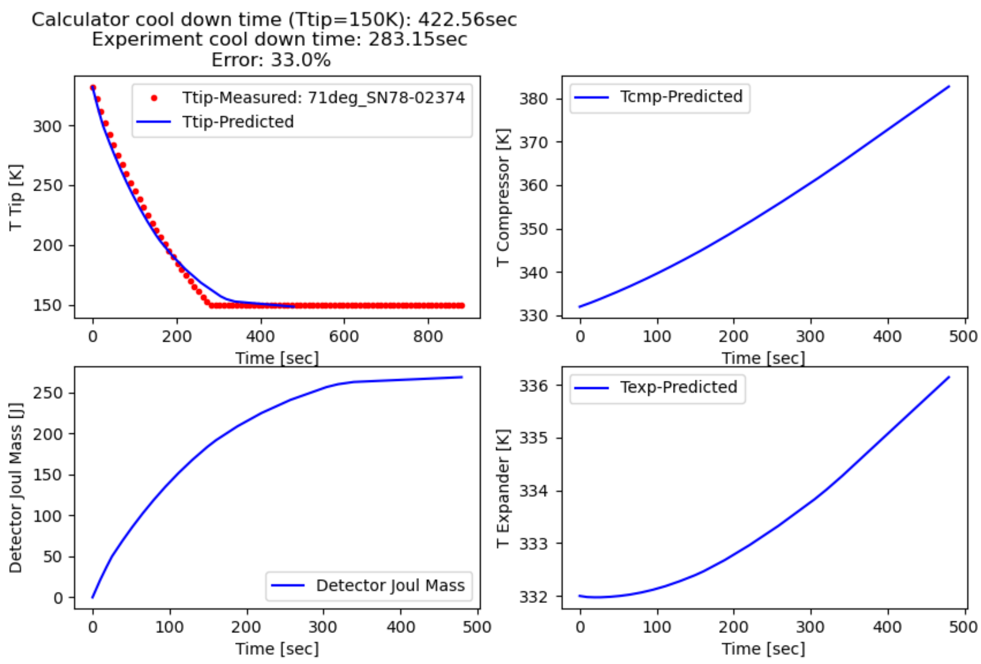

Figure 3.2: K580 computed cooldown time profile for $71°C$ ambient temperature.

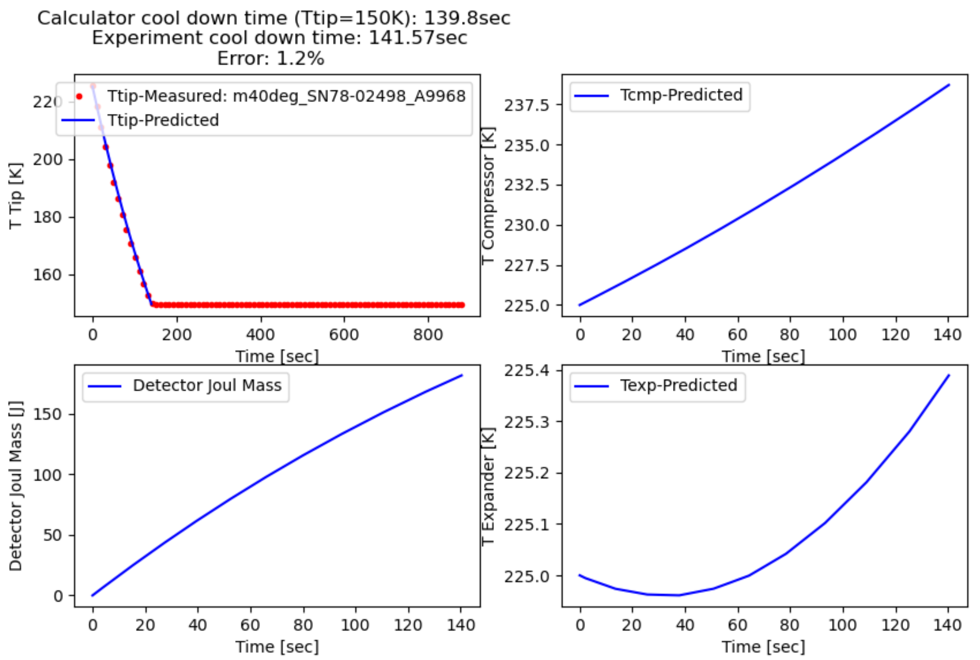

Figure 3.3: K580 computed cooldown time profile for $-40°C$ ambient temperature.

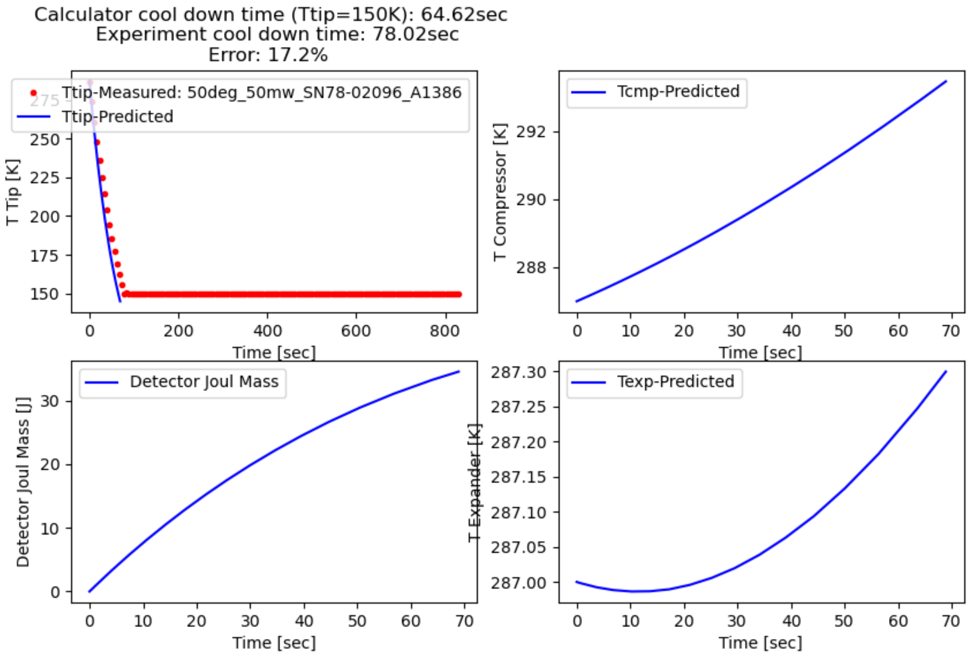

Figure 3.4: K580 computed cooldown time profile for $23°C$ ambient temperature and added heat load of $50mw$.

3.2 k590 - A split linear crycooler

The linear cooler is driven by a linear actuator that drives a piston for the purpose of gas compression. The displacer mechanism in the cold head of the split linear crycooler is pneumatically driven allowing connection to the compressor by a gas pipe. Regarding the coolers interface, the split linear crycooler advantage is in the flexibility of its assemblies due to separation of the compression part from the cold finger. The main parameters of the cooler are similar to the K580 listed in the table (1).

The linear crycooler is designed to be driven by a controller that drives the crycooler to the commanded set-point automatically, following software-programmable soft start power ramp, using the measured and indicated cold tip temperature to close the control loop. As already mentioned, the main purpose of the soft start process is to reduce the maximum peak power and prevent over stroking damage. In figures (3.5, 3.6, 3.7) the soft start profile is presented, including the actual digital temperature controller parameters and the curve-fit used for the crycooler optimization, utilized as an input in the Sage software. A 7-th order voltage-temperature polynomial curve fit were generated as a function of the ambient temperature:

$$T_{amb} = -40^o c, V(T_{\text{tip}}) = 120.1 - 4.16T_{tip} + 6.64e^{-2}T_{tip}^2 - 5.71e^{-4}T_{tip}^3 + 2.85e^{-6}T_{tip}^4 - 8.28e^{-9}T_{tip}^5 + 1.29e^{-11}T_{tip}^6 - 8.42e^{-15}T_{tip}^7 \quad (3.1)$$

$$T_{amb} = 23^o c, V(T_{\text{tip}}) = -265.49 + 11.87T_{tip} - 0.213T_{tip}^2 + 2.09e^{-3}T_{tip}^3 - 1.21e^{-5}T_{tip}^4 + 4.14e^{-8}T_{tip}^5 - 7.7e^{-11}T_{tip}^6 + 6.03e^{-14}T_{tip}^7 \quad (3.2)$$

$$T_{amb} = 71^o c, V(T_{\text{tip}}) = -16.22 - 23.57T_{tip} + 0.972T_{tip}^2 - 0.016T_{tip}^3 + 1.38e^{-4}T_{tip}^4 - 6.57e^{-7}T_{tip}^5 + 1.692e^{-9}T_{tip}^6 - 1.649e^{-12}T_{tip}^7 \quad (3.3)$$

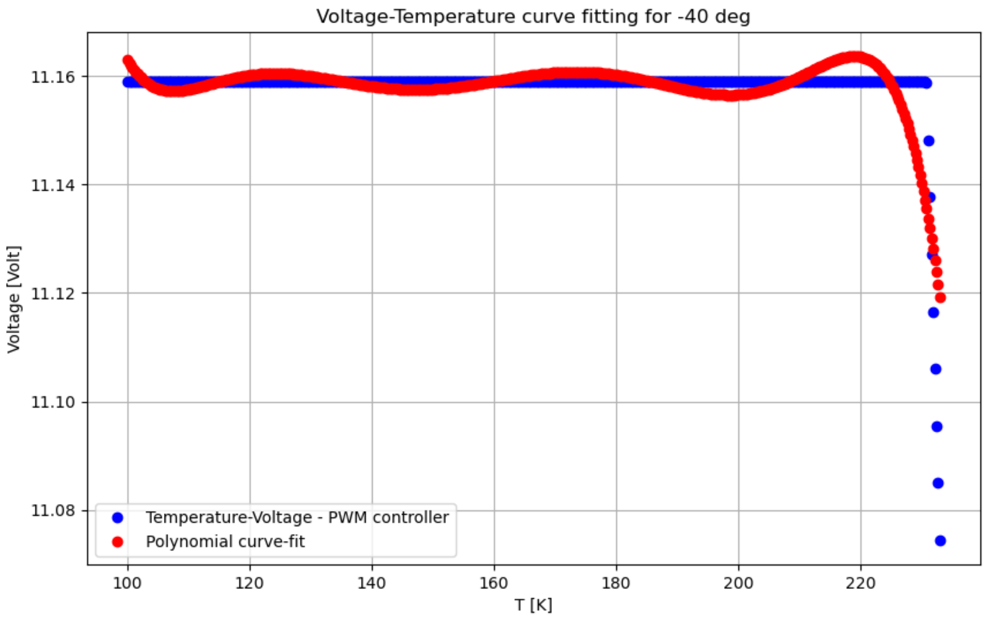

Figure 3.5: Soft start temperature-voltage relation for 100K@-40deg conditions

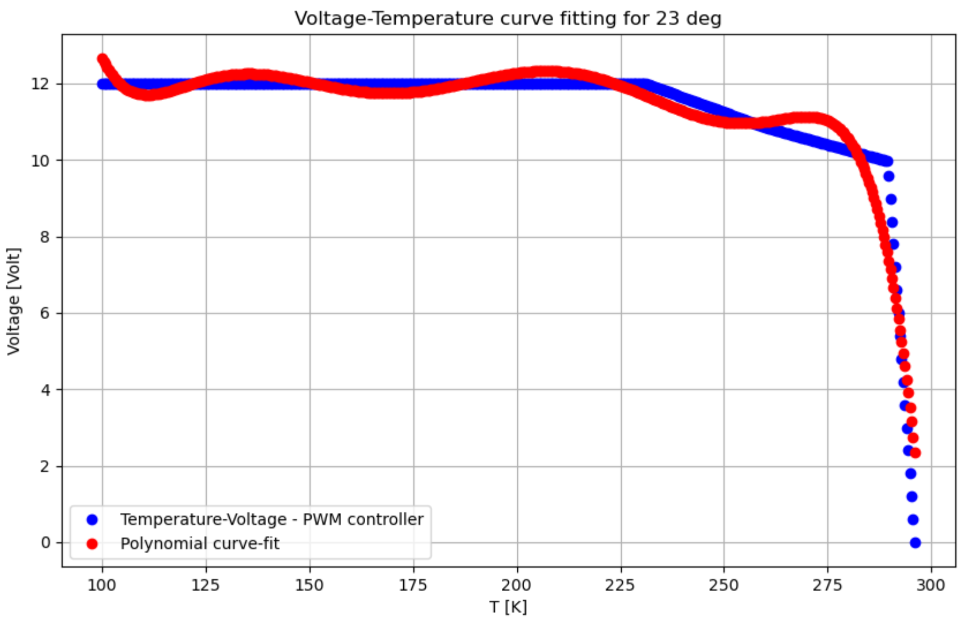

Figure 3.6: Soft start temperature-voltage relation for 100K@23deg conditions

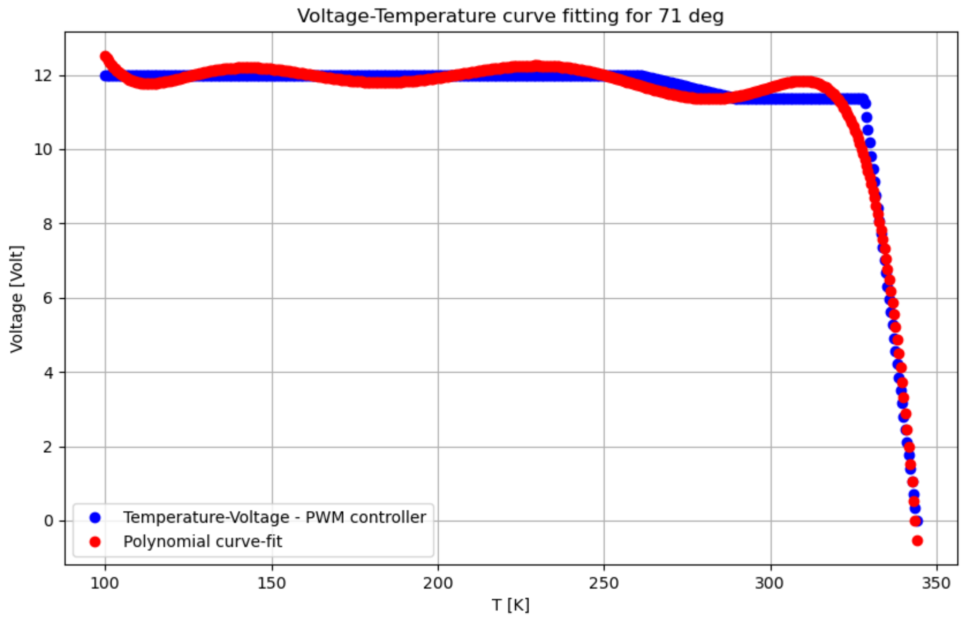

Figure 3.7: Soft start temperature-voltage relation for 100K@71deg conditions

3.2.1 Numerical and experimental results

In the linear crycooler we take a one step further in complication of the steady state model while adding the softstart process by varying the input voltage according to the cold tip temperature. The convergence histories of the cold tip temperature and the detector Joule mass are presented in figures (3.8, 3.9, 3.10) for ambient temperature values of $23°C$, $71°C$ and $-40°C$. The experimental measured results are given by the solid circle, while the computed instantaneous cold tip temperature is given by the solid line. The final cooldown time for different ambient conditions are summarized in table (3). In ambient temperature of $23°C$ it takes 160 seconds to reach the target temperature of 150 K while the simulated value is under estimated by 29.4% approximately (112 sec). As the ambient temperature increases the simulated accuracy degrades, albeit we find it still very efficient for this problem, in light of the complicated processes occurring within the crycooler. The case of $-71°C$ results in gap of 38% between the simulated and measured values. For the given problem we cannot establish a clear prediction of the measured values. In essence, from all the discussions above, we can point on four possible reasons for the gap between the simulated and measured values:

1. The cold finger mechanism is composed of 4-6 main parts, however, there are details that are not taken into consideration, such as: connectors, coatings, sealing and adhesives materials, which can significantly effect overall heat transfer process and the cooling time profile.

2. The Joul mass, which includes the detector mechanism, is given by the detector manufacturer as an external parameter, with a minimal information because of proprietary reasons. It is important to note here that the energy information obtained by the manufacturer is utilized to exact the actual detector mechanism's mass, while assuming it comprised from a single substance. Through this procedure we definitely damaged the actual information. Since the detector is strictly connected to the cold tip, it's effect on the overall heat transfer process is dramatic.

3. There is an uncertainty about the parameters of the soft start process which are actually used in the digital controller. There is an uncertainty associated to the different voltage stages used in the

experimental voltage profile, mainly at the starting stage (10-20 initial seconds). This process of varying the input voltage directly effects the piston stroke in linear crycooler, and hence, the overall cooling process.

4. Under the above circumstances, the cooldown time algorithm performs well at predicting the average cooldown time and profile estimation, with relatively reasonable dispersion. It is important to mention here that the control-volume approach assumes homogeneous temperature and uniform substance properties having a given heat capacity. But in reality the compressor is quite large and is compared of different substances characterized with various heat capacity values. Further improvement may be achieved by discretizing the compressor mass in order to take into account thermal gradients and material differences.

Joule mass, g	**heat load, mw**	**Ambient temp.**	**Cold tip temp.**	**Com. cooldown time, sec**	**Exp. cooldown time, sec**	**Error**
3.4	0	$-40°C$ (230K)	150K	56.51	80	-29.4%
3.4	0	$23°C$ (296K)	150K	112.91	160	-29.4%
3.4	0	$71°C$ (330K)	150K	156.22	252.81	-38.2%

Table 3: Simulated and experimental results for the K590 crycooler

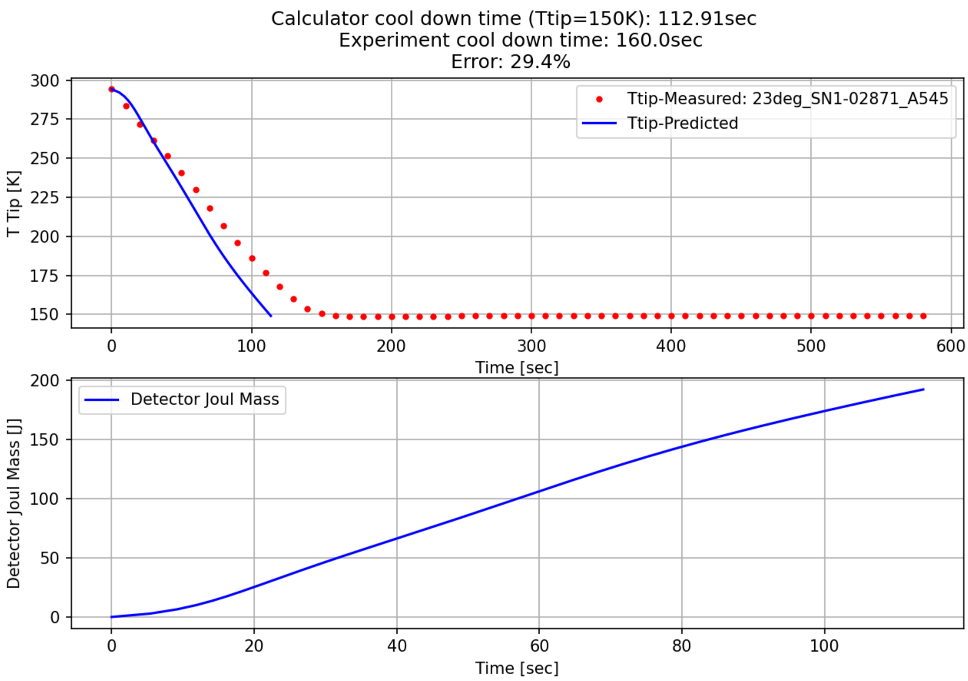

Figure 3.8: K590 computed cooldown time profile for $23°C$ ambient temperature.

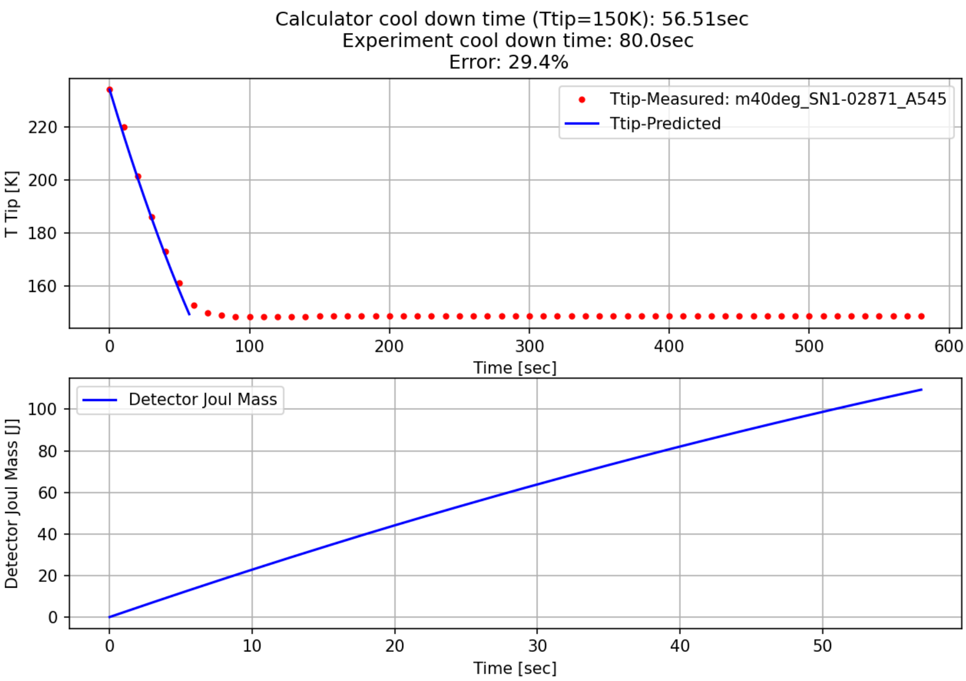

Figure 3.9: K590 computed cooldown time profile for $-40°C$ ambient temperature.

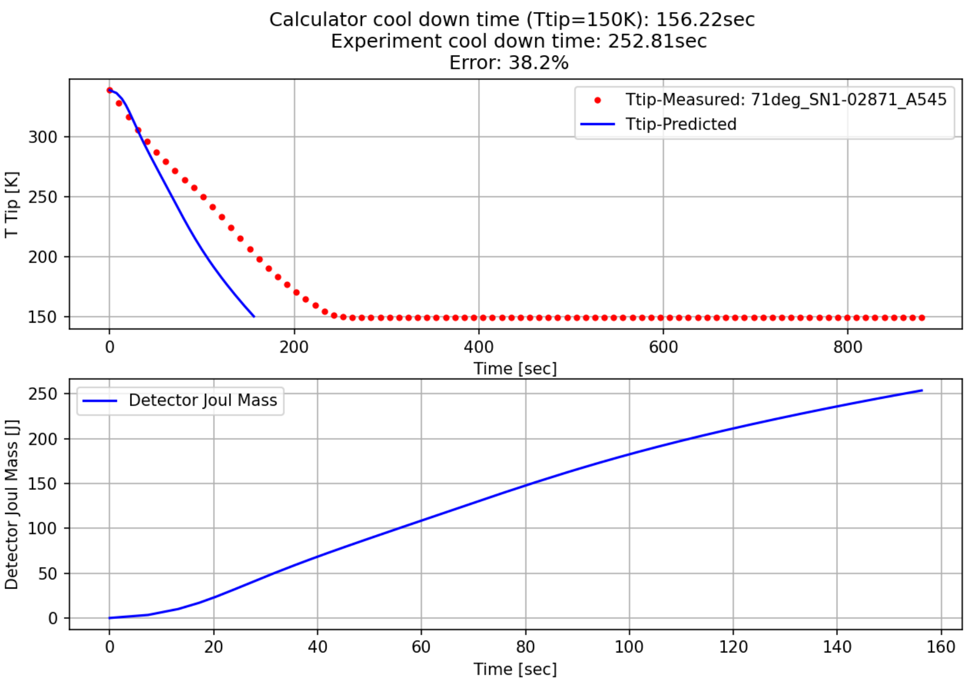

Figure 3.10: K590 computed cooldown time profile for $71°C$ ambient temperature.

4 Conclusions

The cooldown time estimation is rather complex one from a computational point of view. The present research aims to bridge the gap between the complicated explicit approach for cooldown time estimation and the steady state approach. The objective of this article is to provide a tool allowing researchers to predict the time response of a any crycooler either rotary, linear or pulse tube. One of the main difficulties is the construction a comprehensive steady state database which includes the energy flow values heat transfer, enthalpy and mechanical work for any various combination of compressor, expander, tip temperature and input voltage. These energy flows were obtained by using Sage, with various range of operating conditions. The cooldown time estimation problem is essentially a system of ordinary first-order nonlinear differential equations while applying the conservation of energy to various internal parts of the crycooler. The ODE system includes the material properties (temperature, specific heat capacity, mass) of the components that control the heat transfer channels: conduction, radiation and crycooler heat capacity. The formulation of the framework developed in Python for calculating the cooldown time profiles is introduced. Two case studies based on K580 and K590 crycoolers developed at RICOR were used for verfication, with a good agreement between simulated and measured values.

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