

Gravitational Field Equations of the Theory of Self-Variation

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Abstract. In this article we present the gravitational field equations of the Self-Variation Theory. We formulate the differential equation for the gravitational interaction of two bodies and the orbits of the planets. The first calculations give consistency of the Theory at the distance scales that we have observational data. Theory predicts increased stellar velocities on the outskirts of galaxies. It also predicts increased velocities of galaxies on the outskirts of galaxy clusters. The properties of the interaction depend on the distance at which it acts and the distribution of the rest mass in space. The Theory of Self-Variation formulates gravity and electromagnetism with the same equations. We obtain an equation relating the rest mass or charge of the field source to the velocity of the source of the field relative to the observer and the propagation velocity of the field relative to the observer. In the frame of the Theory, this Equation applies to all interactions, not just gravity and electromagnetism. Further investigation of the equations will yield the full, precise prediction of the Theory.

1. Introduction

The Self-Variation principle postulates that the rest masses and charges of the fundamental particles ("Self-Varying Charge Q ") slowly increase (in absolute value) with time, while simultaneously radiating negative energy in the surrounding spacetime in order to balance the energy of their rest mass / charge increase. Through a series of mathematical calculations described in detail in [5], this axiom necessarily involves a modification of the electromagnetic potential. For comparison the classical electromagnetic Liénard–Wiechert potentials are

$$V_{LW} = \frac{q}{4\pi\epsilon_0 r \left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}\right)}$$

$$\mathbf{A}_{LW} = V \frac{\mathbf{u}}{c^2},$$

whereas the corresponding Self-Variation potentials are

$$V = \frac{\left(1 - \frac{u^2}{c^2}\right)q}{4\pi\epsilon_0 r \left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}\right)^2} + \frac{(\mathbf{v} \cdot \mathbf{a})q}{4\pi\epsilon_0 c^3 \left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}\right)^2},$$

$$\mathbf{A} = V \frac{\mathbf{v}}{c^2}.$$

The striking difference is that there is a second term in V , which is independent of the distance r , from the source charge and which has significant theoretical implications. Also the Equations for \mathbf{A} , are not identical. We shall examine below the implications in detail for the gravitational field. Notice that the field intensity resulting from either of the potentials (Liénard–Wiechert or Self-Variation’s) depends on the distance r . It is also crucial to realize that the electromagnetic potential of the Self - Variation Theory gives the correct field intensity, whether we consider the charge q , to be constant or varying in time in accordance with the principle of the Self-Variation. In the case of constant charges the field intensity of the Self-Variation potentials is exactly the same as the one implied by the Liénard–Wiechert potentials. However for charges which vary in time while satisfying the Self-Variation principle, the field intensity implied by the Liénard–Wiechert potential differs in an essential way.

The Liénard–Wiechert potentials are compatible with the Lorentz–Einstein transformations for constant charges but not for charges that vary in time. In contrast the Self-Variation potentials are compatible with the Lorentz–Einstein transformations both for constant charges and for charges that vary in time. In this sense the Self-Variation potentials present a much more strict formulation of the field potentials.

Gravitational potential

The Self-Variation potential for the gravitational interaction is derived from the above Equations of the electromagnetic Self-Variation potential by substituting the charge q , with the rest mass

M , of the source of the gravitational field hence, $\frac{q}{4\pi\epsilon_0} \rightarrow -GM$, where G , is the constant of

gravity and by substituting the acceleration \mathbf{a} of the particle in the electromagnetic field, with the intensity \mathbf{g} of the gravitational field, hence, $\mathbf{a} \rightarrow \mathbf{g}$. Also notice that now \mathbf{v} , represents the speed of propagation of the gravitational field, hence we must substitute the speed of light in vacuum c , with the speed of propagation of the gravitational field ν , hence, $c \rightarrow \nu$. These substitutions lead to the corresponding gravitational potentials of the Self-Variation,

$$V = -\frac{GM}{r} \frac{1 - \frac{u^2}{\nu^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{\nu^2}\right)^2} - \frac{GM}{\nu^3} \frac{\mathbf{v} \cdot \mathbf{g}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{\nu^2}\right)^2}, \quad (1)$$

$$\mathbf{A} = V \frac{\mathbf{v}}{\nu^2}.$$

where \mathbf{u} , is the velocity of the rest mass M relative to the observer, and r , is the distance from the rest mass M . Deriving the gravitational potentials in this way, implies that there is a gravitational analog to the magnetic field \mathbf{B} , and has units s^{-1} (see [5], Equations (3)–(6)). Notice that in the limit case where the speed of propagation of the gravitational field approaches infinity, $\nu \rightarrow \infty$, we get the limit potential

$$V = -\frac{GM}{r},$$

which is no other than the one of classical mechanics which assumed instant action of gravity at distance r .

2. Potential, propagation speed and intensity of the gravitational field caused by a single rest mass M

Considering that the vectors \mathbf{v} and \mathbf{g} have opposite directions, $\mathbf{v} = v \frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g \frac{\mathbf{r}}{r}$,

$\mathbf{v} \cdot \mathbf{g} = -vg$, where $v = \|\mathbf{v}\|$ and $g = \|\mathbf{g}\|$. Then

$$\frac{d\mathbf{v}}{dt} = \frac{dv}{dt} \frac{\mathbf{r}}{r} + v \frac{d}{dt} \left(\frac{\mathbf{r}}{r} \right) = \mathbf{g} = -g \frac{\mathbf{r}}{r}. \quad (2)$$

Then from Equation (1) we have

$$V = -\frac{GM}{r} \frac{1 - \frac{u^2}{v^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} + \frac{GM}{v^2} \frac{g}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2}. \quad (3)$$

The gravitational field intensity $\mathbf{g}(r)$ is given by

$$\mathbf{g}(r) = -\nabla V(r) = -\frac{dV}{dr} \frac{\mathbf{r}}{r}. \quad (4)$$

From Equation (2) in [5], we have that, $dr = -cdw$. However using the symbols of the current article this Equation is written as, $dr = -vdt$. From Equation (2) we get $\frac{dv}{dt} = -g$. Combining these Equations we get

$$v \frac{dv}{dr} = g = \frac{dV}{dr}, \quad (5)$$

where with t , we have denoted the time of the observer. From Equation (5) we have, $v^2 = 2V + a$ where a is a constant. In the limiting case of zero potential, $V \rightarrow 0$ the speed of propagation of the gravitational field tends to the speed of light in vacuum, that is, $v \rightarrow c$, hence we conclude that, $a = c^2$. Then

$$v^2 = c^2 + 2V. \quad (6)$$

From Equations (3) and (5) we get,

$$V = -\frac{GM}{r} \frac{1 - \frac{u^2}{v^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} + \frac{1}{v^2} \frac{GM}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} \frac{dV}{dr}. \quad (7)$$

Then, from Equations (6) and (7) we obtain the differential equation for the potential,

$$V = -\frac{GM}{r} \frac{1 - \frac{u^2}{v^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} + \frac{1}{c^2 + 2V} \frac{GM}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} \frac{dV}{dr} \quad (8)$$

and the differential equation for the speed of propagation of the gravitational field,

$$v^2 - c^2 = -\frac{2GM}{r} \frac{1 - \frac{u^2}{v^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} + \frac{1}{v^2} \frac{GM}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} \frac{dv^2}{dr}. \quad (9)$$

Through the substitutions $\frac{q}{4\pi\epsilon_0} \leftrightarrow -GM$, $\mathbf{a} \leftrightarrow \mathbf{g}$, $c \leftrightarrow v$ Equation (8) is common to gravity and electromagnetism.

Equation (9) relates three physics quantities. The rest mass M of the field source, the velocity \mathbf{u} of the field source relative to the observer, and the speed of propagation v of the field relative to the observer. The fact that this equation relates only these three physics quantities makes it fundamental to the gravitational interaction. For the observer, the properties of spacetime depend on the rest mass M .

In equation (8), the velocity \mathbf{u} of the mass M relative to the observer has implications for the Newtonian term

$$-\frac{GM}{r} \frac{1 - \frac{u^2}{v^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2}$$

of the potential and the Self-Variation term

$$\frac{1}{c^2 + 2V} \frac{GM}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} \frac{dV}{dr}.$$

The Newtonian term can be either greater than the potential

$$-\frac{GM}{r}$$

or less, depending on whether it is

$$1 - \frac{u^2}{v^2} < \left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2$$

or

$$1 - \frac{u^2}{v^2} > \left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2.$$

For the Self-Variation term we have

$$\frac{1}{c^2 + 2V} \frac{GM}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} \frac{dV}{dr} > \frac{GM}{c^2 + 2V} \frac{dV}{dr}.$$

In the electromagnetic interaction the Self-Variation Theory predicts two independent pairs of potentials. One gives the electromagnetic field of the moving electric charge and depends on the velocity \mathbf{u} of the charge (see [5], Equations (16) and (17)),

$$V_u = \frac{\left(1 - \frac{u^2}{c^2}\right) q}{4\pi\epsilon_0 r \left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}\right)^2}$$

$$\mathbf{A}_u = V_u \frac{\mathbf{v}}{c^2}.$$

The other gives the electromagnetic radiation emitted by the electric charge and depends on the acceleration \mathbf{a} of the charge (see [5], Equations (18) and (19)),

$$V_a = \frac{(\mathbf{v} \cdot \mathbf{a}) q}{4\pi\epsilon_0 c^3 \left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}\right)^2}$$

$$\mathbf{A}_a = V_a \frac{\mathbf{v}}{c^2}.$$

As a consequence of the substitution $\mathbf{a} \rightarrow \mathbf{g}$, this separation cannot be made in the gravitational interaction. The intensity of the gravitational field, the acceleration of gravity \mathbf{g} is related to both the

$$-\frac{GM}{r} \frac{1 - \frac{u^2}{v^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2}$$

term and the

$$-\frac{GM}{v^3} \frac{\mathbf{v} \cdot \mathbf{g}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2}$$

term in the second part of Equation (1). In addition to Equation (4), the potential and the gravitational field strength are also related to each other through Equation (1).

We formulated the Gravitational Field Equations for a point rest mass. If the rest mass M is distributed in space, we introduce in the Equations the density of the rest mass, depending on the application we are making.

3. Gravitational interaction of two bodies

We study the case where a body of rest mass m moves in the gravitational field of a stationary body of rest mass $M \gg m$.

In polar coordinates (r, θ) , the orbits $r = r(\theta)$ of the body of rest mass m is given by the solution of the system of equations,

$$L = mr^2 \frac{d\theta}{dt} = \text{constant} \quad (10)$$

$$\ddot{r} - r(\dot{\theta})^2 = -g(r) \quad (11)$$

and

$$mV(r) + K = E = \text{constant} . \quad (12)$$

In these Equations t is the time of the observer, L and K the angular momentum and the kinetic energy of the body of rest mass m , $\ddot{r} = \frac{d^2r}{dt^2}$, $\dot{\theta} = \frac{d\theta}{dt}$ and E the mechanical energy of the system of the two bodies.

From the solution of the differential Equation (8) we get the potential $V(r)$. Then, from Equation (5) we get the intensity $g(r)$ of the field. Alternatively, from the solution of the differential equation (9) we obtain the speed of propagation of the field $v(r)$. Then, from equation (6) we get the potential $V(r)$.

In the solutions of the differential equations (8) or (9), an integration constant k is introduced. Knowing the value of this constant we have the exact prediction of the Theory of Self-Variation for the gravitational field. In a first approach, the value of k can be estimated from the already known observational data (see, [1] – [3] and [6] – [15]). However, the orbits of the planets have been studied in detail. From the comparison of the theoretical prediction of Equations (8) or (9) with the observational data for the orbits of the planets the value of the constant k can be accurately measured.

4. Circular orbits

In the system of bodies in the previous section, if the body of rest mass m moves with velocity \mathbf{u} relative to the body of rest mass M , then the body of rest mass M moves with velocity $-\mathbf{u}$ relative to the body of rest mass m .

The inner product $\mathbf{u} \cdot \mathbf{v}$ in the differential Equations (8) and (9) has the consequence that complex calculations are required for their solution. These calculations are simplified in the case of circular orbits. The case of circular orbits is a first approach to the conclusions drawn from the Equations we present in this article.

For the body of rest mass m we have $\frac{d\mathbf{v}}{dt} = \frac{d\mathbf{u}}{dt} = \mathbf{g}$. In the case of circular orbit it is

$$\mathbf{u} \cdot \mathbf{g} = 0 \quad (13)$$

and taking into consideration that the vectors \mathbf{v} and \mathbf{g} have opposite directions we have

$$\mathbf{u} \cdot \mathbf{v} = 0. \quad (14)$$

Now we have

$$\frac{du^2}{dt} = \frac{d\mathbf{u}^2}{dt} = 2\mathbf{u} \frac{d\mathbf{u}}{dt} = 2\mathbf{u} \cdot \mathbf{g}$$

and with Equation (13) we get

$$\frac{du^2}{dt} = 0, \quad (15)$$

where $u = \|\mathbf{u}\|$. Therefore the velocity u is constant in the circular orbit.

From Equations (9) and (14) we have

$$v^2 - c^2 = -\frac{2GM}{r} \left(1 - \frac{u^2}{v^2}\right) + \frac{GM}{v^2} \frac{dv^2}{dr}. \quad (16)$$

Let

$$x = \frac{c^2}{GM} r \quad (17)$$

and let

$$f(x) = \frac{v^2(x)}{c^2}. \quad (18)$$

From Equations (16), (17) and (18) we get the following differential equation,

$$\frac{df}{dx} - f^2 + f - \frac{2}{x} \left(f - \frac{u^2}{c^2} \right) = 0. \quad (19)$$

In Equation (19) it is

$$0 \leq \frac{u^2}{c^2} < 1. \quad (20)$$

The solution we get from the differential equation (19) depends on the value of the quotient $\frac{u^2}{c^2}$. In the solution of the differential equation, the integration constant k is introduced. Then solving the differential Equation (19), the velocity $v = v(x)$ is given by Equation (18).

From equations (6) and (18) we get the field potential as a function of x ,

$$V(x) = -\frac{c^2}{2} (1 - f(x)). \quad (21)$$

From equations (21) and Transformation (17) we get the field potential as a function of r .

From Equation (4) we have

$$\mathbf{g}(r) = -\frac{dV}{dr} \frac{\mathbf{r}}{r} = -\frac{dV}{dx} \frac{dx}{dr} \frac{\mathbf{r}}{r}$$

and with the Transformation (17) we have

$$\mathbf{g}(x) = -\frac{c^2}{GM} \frac{dV}{dx} \frac{\mathbf{r}}{r}$$

and with Equation (6) we have

$$\mathbf{g}(x) = -\frac{c^2}{2GM} \frac{dv^2}{dx} \frac{\mathbf{r}}{r}$$

and with Equation (18) we have

$$\mathbf{g}(x) = -\frac{c^4}{2GM} \frac{df}{dx} \frac{\mathbf{r}}{r}$$

and with Equation (19) we obtain the intensity \mathbf{g} of the field as a function of x ,

$$\mathbf{g}(x) = -\frac{c^4}{2GM} \left(f^2(x) - f(x) + \frac{2}{x} \left(f(x) - \frac{u^2}{c^2} \right) \right) \frac{\mathbf{r}}{r}. \quad (22)$$

The functions $v = v(r)$, $V = V(r)$ and $\mathbf{g} = \mathbf{g}(r)$ are obtained from Equations (18), (21) and (22) through the Transformation (17).

As a consequence of Equation (19) the velocity u and the gravitational field are mutually dependent. This is a clear conclusion of the Theory, which has its origin in Equation (1). The solution of the differential Equation (19) gives pairs (f, u) . Pairs of these solutions gives increased velocities u of stars on the outskirts of galaxies and increased velocities u of galaxies on the outskirts of galaxy clusters.

5. Potential, propagation speed, and intensity of the field induced by a stationary rest mass relative to an observer

If the observer and the source of the field M are stationary between them, $\mathbf{u} = \mathbf{0}$, then from Equations (8) and (9) we have

$$V = -\frac{GM}{r} + \frac{GM}{c^2 + 2V} \frac{dV}{dr} \quad (23)$$

and

$$v^2 - c^2 = -\frac{2GM}{r} + \frac{GM}{v^2} \frac{dv^2}{dr}. \quad (24)$$

From Equation (24), again applying Transformations (17) and (18) we get,

$$\frac{df}{dx} - f^2 + f - \frac{2}{x} f = 0. \quad (25)$$

Solving (25) for f , we have

$$f(x) = \frac{x^2}{ke^x + x^2 + 2x + 2} \quad (26)$$

where k , is the integration constant. Then from (18), (26) we have

$$v^2(x) = c^2 \frac{x^2}{ke^x + x^2 + 2x + 2}. \quad (27)$$

Finally applying the Transformation (17) to (27) we get the speed of propagation of the gravitational field, as derived from the Self-Variation gravitational potential, with respect to r ,

$$v^2(r) = c^2 \frac{a^2 r^2}{ke^{ar} + a^2 r^2 + 2ar + 2}, \quad (28)$$

$$\text{where } a = \frac{c^2}{GM}. \quad (29)$$

From Equations (6) and (27) we obtain the gravitational potential with respect to x ,

$$V(x) = -c^2 \frac{ke^x + 2x + 2}{2(ke^x + x^2 + 2x + 2)}. \quad (30)$$

Then from Equations (30) and Transformation (17) we obtain the gravitational potential with respect to r ,

$$V(r) = -c^2 \frac{ke^{ar} + 2ar + 2}{2(ke^{ar} + a^2 r^2 + 2ar + 2)}. \quad (31)$$

From Equation (30) we get,

$$V(0) = -\frac{c^2}{2} = \lim_{x \rightarrow \infty} V(x).$$

These limiting potential values are equal to each other, constant and independent of rest mass M .

The gravitational field intensity \mathbf{g} is calculated as follows. From Equation (4) and Transformation (17) we get

$$\mathbf{g}(x) = -\frac{c^2}{GM} \frac{dV(x)}{dx} \frac{\mathbf{r}}{r}$$

and with Equation (6) we get,

$$\mathbf{g}(x) = -\frac{c^2}{GM} \frac{dv^2(x)}{dx} \frac{\mathbf{r}}{r}$$

and with Equation (18) we get,

$$\mathbf{g}(x) = -\frac{c^4}{2GM} \frac{df(x)}{dx} \frac{\mathbf{r}}{r}$$

and with Equations (25) we get,

$$\mathbf{g}(x) = -\frac{c^4}{2GM} \left(f^2(x) - f(x) + \frac{2}{x} f(x) \right) \frac{\mathbf{r}}{r}$$

and with (26) we obtain,

$$\mathbf{g}(x) = -\frac{c^4}{2GM} \left(f^2(x) - f(x) + \frac{2}{x} f(x) \right) \frac{\mathbf{r}}{r} = -\frac{c^4}{2GM} \frac{2kxe^x - kx^2e^x + 2x^2 + 4x}{(ke^x + x^2 + 2x + 2)^2} \frac{\mathbf{r}}{r}. \quad (32)$$

The function $f = f(x)$ is given by Equation (26). From Transformation (17) and Equations (26) and (32) we get the field intensity $\mathbf{g} = \mathbf{g}(r)$ as a function of r .

6. The case $M = M(r)$

In this section we consider the case where the rest mass M is a function of the distance r , $M = M(r)$. We assume that

$$M = \rho r, \quad (33)$$

where ρ is a constant rest mass linear density.

From Equations (8) and (33) we get,

$$v^2 - c^2 = -2G\rho \frac{1 - \frac{u^2}{v^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} + \frac{1}{v^2} \frac{G\rho}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} \frac{rdv^2}{dr}. \quad (34)$$

Let

$$r = \mu x, \quad (35)$$

where μ is a constant with a unit of 1 meter (m) and $x \in \mathbb{R}$. Then we have

$$\frac{rdv^2}{dr} = \frac{bx dv^2}{bdx} = \frac{xdv^2}{dx}$$

and Equation (34) is written in the form,

$$v^2 - c^2 = -2G\rho \frac{1 - \frac{u^2}{v^2}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} + \frac{1}{v^2} \frac{G\rho}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{v^2}\right)^2} \frac{xdv^2}{dx}. \quad (36)$$

Now, consider again the function f ,

$$f(x) = \frac{v^2(x)}{c^2}, \quad (37)$$

where c is the speed of light in vacuum. From Equation (36) and Transformation (37) we get

$$\frac{1}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2 f}\right)^2} \frac{xdf}{dx} - \sigma f^2 + f \left(\sigma - 2 \frac{1 - \frac{u^2}{c^2 f}}{\left(1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2 f}\right)^2} \right) = 0, \quad (38)$$

where

$$\sigma = \frac{c^2}{G\rho}. \quad (39)$$

The gravitational field equations resulting from the solution of differential Equation (38) are analogous to those of sections 4 and 5.

Circular orbits

Taking into consideration Equations (13) and (14), from Equation (38) we get

$$\frac{xdf}{dx} - \sigma f^2 + (\sigma - 2)f + 2\frac{u^2}{v^2} = 0. \quad (40)$$

As in equation (19), the solution of differential Equation (40) depends on the quotient $\frac{u^2}{c^2}$.

The case $\mathbf{u} = \mathbf{0}$

If $\mathbf{u} = \mathbf{0}$, from Equation (38) we get,

$$\frac{xdf}{dx} - \sigma f^2 + (\sigma - 2)f = 0. \quad (41)$$

The Equations in this section were derived for a constant linear density ρ . In this case the value of the parameter $\sigma = \frac{c^2}{G\rho} > 0$ is critical for the solutions given by the differential Equation (41).

Their form depends on whether they are $0 < \sigma < 2$, $\sigma = 2$ or $\sigma > 2$. In the solution of the differential equation, the integration constant k is introduced.

Taking into consideration that the rest mass is distributed in space, we can introduce into Equations the rest mass density or particle density and the corresponding current density. The resulting differential equations depend on this distribution of rest mass in space. Equation (38) refers to the specific case of Equation (33) that we studied in this section. The study of this section is indicative of the consequences of the distribution of rest mass in spacetime.

7. The potentials V , \mathbf{A}

The complete investigation of the Theory of Gravitation is done through Equation (1) and the pair of potentials (V, \mathbf{A}) , where

$$\mathbf{A} = V \frac{\mathbf{v}}{v}.$$

These potentials relate the gravitational field to spacetime, through the equations,

$$\mathbf{g} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t},$$

$$\mathbf{B} = \nabla \times \mathbf{A},$$

where \mathbf{B} is a gravitational proportional to the magnetic field and has units s^{-1} (see [5], Equations (3) – (6)).

As a consequence of Self-Variation, at time t , the rest mass / electric charge located at point P acts on point A with the value it had at another point E (see [5], Fig. 1). The intensity of the gravitational field given by the potentials (V, \mathbf{A}) is the same whether we consider the rest mass M to be constant or consider it to vary according to the Principle of Self-Variation. This property of the potentials of the Self-Variation Theory allows us to solve the differential Equations of the gravitational field by considering the rest mass M as constant, as we did in the previous sections.

On the macroscopic scale it is clear that point E refers to a previous, compared to P , position that the rest mass / electrical charge had. However, given that at time t the rest mass / electric charge acts from point P to point A , there is a set of possible points E . Unlike the macrocosm, at the microscopic scale there is no certainty about the location of point E .

8. Discussion

We have presented the Equations for Gravity as predicted by the Theory of Self-Variation. The substitution $\alpha \rightarrow \mathbf{g}$, by which we get the Gravitational potential of Self-Variation from the corresponding Electromagnetic potential, is an idea belonging to Einstein. Without this substitution the Gravitational field of Self-Variation cannot arise. The Equations we have presented include Einstein's proposal for the equivalence of acceleration and gravity.

Through Equation (32) we can investigate the consequences of gravity at cosmological scale distances. It is a issue that needs further investigation, since the Self-Variation Theory justifies the cosmological data in a static universe (see, [4]). This investigation concerns the self-consistency of the Theory on the cosmological scale.

The potentials (V, \mathbf{A}) apply to all distance scales. Taking into account the principle of Self-Variation, then it follows that the Equations of this article apply whether we consider the masses to be fixed or to consider them to be variable. If we also take into account the principle of conservation of energy - momentum, then the Equations that justify the cosmological data arise.

In sections 4, 5 and 6 we made the simplest applications of the Equations of this article. In these applications we studied cases where $\mathbf{u} \cdot \mathbf{v} = 0$. In the general case, the role of the inner product $\mathbf{u} \cdot \mathbf{v}$ needs further investigation. The resulting solutions of the differential Equations depend on the inner product $\mathbf{u} \cdot \mathbf{v}$.

At the macroscopic scale, we made clear assumptions about the inner product $\mathbf{u} \cdot \mathbf{v}$ in the applications of Equations (8) and (9). At the microscopic scale it is not clear what form the inner product $\mathbf{u} \cdot \mathbf{v}$ might take. The uncertainty of this form is added to the uncertainty of the location of point E (see [5], Fig. 1). There are reasonable assumptions we can make on the macroscopic scale but cannot make on the microscopic scale.

Equation (8) is common to gravity and electromagnetism. If in this Equation we replace the rest mass with the "Self-Variating Charge Q " we get the "Unified Self-Variation Interaction". In gravitational interaction Self-Variating charge is the rest mass, $Q = M$. In electromagnetic interaction the Self-Variating charge is the electric charge, $Q = q$. An issue for investigation is the possible physics quantities Q .

References

[1] Corbelli, Edvige, and Paolo Salucci. "The extended rotation curve and the dark matter halo of M33." *Monthly Notices of the Royal Astronomical Society* 311.2 (2000): 441-447.

<https://ui.adsabs.harvard.edu/abs/2000MNRAS.311..441C/abstract>

[2] Green Anne M., and Bradley J. Kavanagh. "Primordial Black Holes as a dark matter candidate." *Journal of Physics G: Nuclear and Particle Physics* 48.4 (2021): 043001.

<https://iopscience.iop.org/article/10.1088/1361-6471/abc534/meta>

[3] Hong, S. E., Jeong, D., Hwang, H. S., & Kim, J. "Revealing the local cosmic web from galaxies by deep learning." *The Astrophysical Journal* 913.1 (2021): 76.

<https://iopscience.iop.org/article/10.3847/1538-4357/abf040>

[4] Manousos, Emmanuil. "Awaiting the Measurements by the James Webb Space Telescope: 'A Hitherto Unrecognized Principle of Nature' Justifies the Cosmological Data." *European Journal of Applied Physics* 5.3 (2023): 31-35.

<https://www.ej-physics.org/index.php/ejphysics/article/view/222/176>

[5] Manousos, Emmanuil. "The replacement of the potentials as a consequence of the limitations set by the law of selfvariations on the physical laws." *IJEIT* 3.10 (2014): 181-185.

https://www.ijeit.com/Vol%203/Issue%2010/IJEIT1412201404_36.pdf

[6] Najita, Joan R., Glenn P. Tiede, and John S. Carr. "From stars to superplanets: The low-mass initial mass function in the young cluster IC 348." *The Astrophysical Journal* 541.2 (2000): 977.

<https://iopscience.iop.org/article/10.1086/309477/pdf>

[7] Natarajan, P., Chadayammuri, U., Jauzac, M., et al. "Mapping substructure in the HST Frontier Fields cluster lenses and in cosmological simulations." *Monthly Notices of the Royal Astronomical Society* 468.2 (2017): 1962-1980.

<https://academic.oup.com/mnras/article/468/2/1962/2970349>

[8] Orkney, M. D., Taylor, E., Read, J. I., et al. "EDGE: the shape of dark matter haloes in the faintest galaxies." *Monthly Notices of the Royal Astronomical Society* 525.3 (2023): 3516-3532.

<https://academic.oup.com/mnras/article/abstract/525/3/3516/7246895?redirectedFrom=fulltext>

[9] Peacock, J. A., Cole, S., Norberg, P., et al. "A measurement of the cosmological mass density from clustering in the 2dF Galaxy Redshift Survey." *Nature* 410.6825 (2001): 169-173.

<https://www.nature.com/articles/35065528>

[10] Roberts, Morton S., and Robert N. Whitehurst. "The rotation curve and geometry of M31 at large galactocentric distances." *Astrophysical Journal*, Vol. 201, p. 327-346 201 (1975): 327-346.

<https://adsabs.harvard.edu/full/1975ApJ...201..327R>

[11] Rubin, Vera C., and W. Kent Ford Jr. "Rotation of the Andromeda nebula from a spectroscopic survey of emission regions." *Astrophysical Journal*, vol. 159, p. 379 159 (1970): 379.

<https://adsabs.harvard.edu/full/1970ApJ...159..379R/0000383.000.html>

[12] Taylor, A. N., Dye, S., Broadhurst, T. J., et al. "Gravitational lens magnification and the mass of abell 1689." *The Astrophysical Journal* 501.2 (1998): 539.

<https://iopscience.iop.org/article/10.1086/305827/pdf>

[13] Weijmans, A. M., Cappellari, M., Bacon, R., De Zeeuw, et al. "Stellar velocity profiles and line strengths out to four effective radii in the early-type galaxies NGC 3379 and 821." *Monthly Notices of the Royal Astronomical Society* 398.2 (2009): 561-574.

<https://academic.oup.com/mnras/article/398/2/561/1184730>

[14] Yu, X., Shi, Y., Chen, Y., et al. "What drives the velocity dispersion of ionized gas in star-forming galaxies?." *Monthly Notices of the Royal Astronomical Society* 486.4 (2019): 4463-4472.

<https://academic.oup.com/mnras/article/486/4/4463/5480470>

[15] Zwicky, Fritz. "Die rotverschiebung von extragalaktischen nebeln." *Helvetica Physica Acta*, Vol. 6, p. 110-127 6 (1933): 110-127.

<https://adsabs.harvard.edu/full/1933AcHPh...6..110Z/0000119.000.html>