Generalization for specific type of continued fraction

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Abstract

I came across "The Ramanujan Machine" on the Internet and, using my intuition on those kind of stuff, I found some interesting results.

The link to the website mentioned above is https://www.ramanujanmachine.com/results/ and the results I found is listed below:

$$n+2-\cfrac{1}{n+3-\cfrac{2}{n+4-\cfrac{3}{n+5-\cfrac{4}{\dots}}}}=\frac{(-1)^n e}{d_n e-n!}$$

$$d_n = n!\sum_{k=0}^{n}\frac{(-1)^k}{k!}\approx\frac{n!}{e}$$

1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496, 1334961, 14684570, 176214841, 2290792932, 32071101049, 481066515734, 7697064251745, 130850092279664, 2355301661033953, …

https://oeis.org/A000166

or you can use this formula if you want:

$$n+2-\cfrac{1}{n+3-\cfrac{2}{n+4-\cfrac{3}{n+5-\cfrac{4}{\dots}}}}=\frac{\frac{(-1)^n}{n!}}{\sum_{k=0}^{n}\frac{(-1)^k}{k!}-\frac{1}{e}}$$

$$2-\cfrac{1}{3-\cfrac{2}{4-\cfrac{3}{5-\cfrac{4}{\cdots}}}}=+\frac{e}{1e-0!}=\frac{e}{e-1}$$

1.58197670686932642438…

$$3-\cfrac{1}{4-\cfrac{2}{5-\cfrac{3}{6-\cfrac{4}{\cdots}}}}=-\frac{e}{0e-1!}=e$$

(The Ramanujan Machine)

$$4-\cfrac{1}{5-\cfrac{2}{6-\cfrac{3}{7-\cfrac{4}{\cdots}}}}=+\frac{e}{1e-2!}=\frac{e}{e-2}$$

3.78442238235466562875...

$$5-\cfrac{1}{6-\cfrac{2}{7-\cfrac{3}{8-\cfrac{4}{\cdots}}}}=-\frac{e}{2e-3!}=-\frac{e}{2e-6}$$

4.82447016745576732333…

$$6-\cfrac{1}{7-\cfrac{2}{8-\cfrac{3}{9-\cfrac{4}{\cdots}}}}=+\frac{e}{9e-4!}=\frac{e}{9e-24}$$

5.85160064959487971142…

$$7-\cfrac{1}{8-\cfrac{2}{9-\cfrac{3}{10-\cfrac{4}{\cdots}}}}=-\frac{e}{44e-5!}=-\frac{e}{44e-120}$$

6.87129660173296177453...

$$8-\cfrac{1}{9-\cfrac{2}{10-\cfrac{3}{11-\cfrac{4}{\cdots}}}}=+\frac{e}{265e-6!}=\frac{e}{265e-720}$$

7.88628876557801878356...

$$9-\cfrac{1}{10-\cfrac{2}{11-\cfrac{3}{12-\cfrac{4}{\cdots}}}}=-\frac{e}{1854e-7!}=-\frac{e}{1854e-5040}$$

8.89810304707489785707...