No Collatz Conjecture integer series have looping

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Abstract

If the series of Collatz Conjecture integer has looping in it, it is sure the members of the loop cannot reach to value 1. Here it is proven that the possibility of looping is zero except one.

1. Introduction

Procedure of Collatz Conjecture is recognized as following operations.

It starts with positive odd integer n.

It continues following calculation up to $n_i = 1$.

- Compute $n_w = 3 \times n_{i-1} + 1$. (1)
- n_w is divided by 2, m_i times until it becomes positive odd integer.

$$n_i = \frac{n_w}{2^{m_i}} \quad (2)$$

n_i becomes n_{i-1} for (1).

2. Looping

Collatz conjecture procedure is represented as follow.

$$(3((3((3 \times n + 1)/2^{m_1}) + 1)/2^{m_2}) + 1)/2^{m_3} \cdots$$

On this, relation between n_{i-1} and n_i is

$$n_i = (3n_{i-1} + 1)/2^{m_i}. \quad (n_0 = n) \quad (3)$$

Then

$$3n_{i-1} - 2^{m_i} n_i = -1 \quad (4)$$

Regarding to (3)

$$m_i = maximum\ integer\ to\ make\ n_i\ odd\ integer \quad (5)$$

Initial value is n_0. Then if

$$n_i = n_0 \quad (6)$$

This makes looping from n_0 to n_{i-1}.

Above (4)(5)(6) are relations of variables, and recognized as constrains for variables.

In general,

if (number of variables) = (number of unique and exact (not approximated) constrains), there is a unique solution if exists.

The reason of this is that all degree of freedoms (=number of variables) is constrained by same number of constrains.

About this system,

Number of variables n_i:　$i+1$

Number of variables m_i:　i

Number of constrains type (4); i

Number of constrains type (5); i

Number of constrains type (6); 1

This system has same number of variables as number of constrains. Therefore, there is a unique solution if exists. This is true even if it is impossible to get solution or how the solution could be gotten.

Trying around $i=1$ case, followings solution could be obtained.

$$m_1 = m_2 = \cdots = m_i = 2 \tag{7}$$

$$n_0 = n_1 = \cdots = n_i = 1 \tag{8}$$

All constrains are satisfied with these or this is a solution of this system for every i. Therefore, there is no other solution and it is not needed to find others.

This looping is only one member looping or self-looping when n_0=1. Therefore, n_0=1 can be terminal point of Collatz Conjecture operation.

3. Consideration

No looping proof in this report could be considered with *1 and *2 which investigate Collatz Conjecture Space. These show that the space expectation value of 2^{m_i} of (3) is $2^2 = 4$.

Also, this no looping report could be considered with *3 which investigates the series of Collatz Conjecture integer.

Therefore, these combinations show Collatz Conjecture is correct.

*1) viXra:2204.0151

*2) viXra:2304.0182

*3) viXra:2302.0015