Black Hole Cosmology : Entropy , Holography and Dark Energy.

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Abstract.

This is a short article in which we present some curious results based on the concepts of black hole entropy, the event horizon of the observable universe and the cosmic microwave background.By means of an arithmetic *quantifier parameter* two cosmological parameters are deduced: the fraction of what is known as dark energy Ω_Λ and the fraction of what corresponds to baryonic matter Ω_b. A variation is observed over time in the ratio between Ω_Λ and Ω_b ,which is in accordance with what the cosmological standard model proposes, namely, that dark energy varies throughout the life of the universe from its initial moments (when it was very low) to the current moment in which it predominates and constitutes something more than $2/3$ of the total of what the universe is made of.

Keywords. *Black hole entropy.Cosmological parameters.Dark energy.Cosmic microwave background.Event horizon.*

Introduction.

In theory, when the gravitational force acts on the matter of a star whose mass is greater than a certain amount, there is a process of successive *contraction* of said stellar body in such a way that at a certain point what is called a black hole is produced.
A *singularity* occurs where time ceases and physical laws lose their meaning as such.
Bekenstein [1] hypothesized that the area of the event horizon of a black hole is directly proportional to the entropy.
The *boundary* of a black hole is known as the event horizon. Something like an *imaginary* surface on which the speed of light and the escape velocity have the same magnitude.
In Cosmology the event horizon of the observable universe [2] is the largest comoving distance from which light emitted at the present cosmological time can ever reach an observer in the future . Light of any event beyond that distance , has not had time to reach *our place* in the universe.
We cannot confuse the horizon of a black hole and the horizon of the observable universe. We , the observers, live in a region *inside* the event horizon of the universe.

Method and results.

We start with the Bekenstein-Hawking formula [3] that describes the entropy of a black hole

$$S_{BH} = \frac{1}{4}\frac{C^3 k}{G\hbar} A \quad (1)$$

Items involved in the equation:

Speed of light in vacuum , $c = 299792458\, m\, s^{-1}$

Boltzmann constant , $k = 1.38065\, x\, 10^{-23} JK^{-1}$

Newtonian constant of gravitation:

$G = 6.674\, x\, 10^{-11}\, m^3 kg^{-1} s^{-2}$

Reduced Planck constant , $\hbar = 1.05457\, x\, 10^{-34} Js^{-1}$

The parameter A represents the area of the *event horizon* of a black hole.An event horizon can be defined as a certain surface that *separates*, so to speak, two regions. *Inside*, the gravitational field is so intense that not even light can escape and be captured by an *observer* located in the region *outside* the event horizon.

We can try to calculate the area of the event horizon of the observable universe by multiplying the speed of light by the *current age* of the universe, t_0 , and squaring this product

$$A_{OU} = c^2 t_0^2 \quad (2)$$

assuming $t_0 = 4.3\, x\, 10^{17} s$ [4]   then

$$A_{OU} = 1.66\, x\, 10^{52} m^2 \tag{3}$$

So, for our purpose, we use the Bekenstein-Hawking formula to calculate the entropy of the observable universe

$$S_{OU} = \frac{1}{4}\frac{C^3 k}{G\hbar} A_{OU} = 2.2\, x\, 10^{98} JK^{-1} \tag{4}$$

In what physical units is entropy expressed?Let's write the physical units involved in the equation (4) one by one, knowing that the area is expressed in square meters,then S_{OU} is expressed in joules per degree kelvin:

$$\frac{m^3\, kg\, m^2\, kg\, s^2 s\;\; m^2}{s^3 s^2 m^3 kg\, m^2 K} = \frac{m^7 kg^2 s^3}{m^5 kg\, s^5 K} = kg\, m^2 s^{-2} K^{-1} = JK^{-1} \tag{5}$$

From among the set of cosmological parameters that the standard cosmological model [4] offers, we are going to select several:

1. $\upsilon_0 = 1.603\, x\, 10^{11} s^{-1}$: Frequency mode of vibration of the cosmic microwave background (CMB)
2. $T_0 = 2.73\, K$: Temperature associated to the CMB

3. $\Omega_{\Lambda} = 0.687$: The fraction of the total energy of the universe that is *dark energy.*
4. $\Omega_{b} = 0.049$: The fraction of the total energy of the universe that is baryonic matter.
Symbols with subscripts "0" like T_{0} , υ_{0} , represent values measured or *deduced* in the *current state* of the universe.
The values of both cosmological parameters, Ω_{Λ} and Ω_{b} , are *consensus values* of the standard model of cosmology.

What is the energy associated with a photon of the cosmic microwave background? We can calculate it by applying the Einstein-Planck formula [5], which consists of multiplying reduced Planck's constant by the frequency associated with one photon:

$$E_{0} = \hbar\, \upsilon_{0} = 1.69\, x\, 10^{-23} J \tag{6}$$

Let's introduce an *Arithmetic-quantifier* tool.It's a *large number's tool* , namely , one dimensionless parameter related to length dimension.We are interested in two physical constants of length: Bohr radius and Planck length.
The Bohr radius [6] in the hydrogen atom is the *longitudinal* measure of the radius of the hydrogen atom's *orbital* at its fundamental energy level. The Planck length [7] as a limit of magnitude length, below which Physics as such is *meaningless*.

What we are going to do now is divide Bohr radius by Planck length and the resulting amount is divided by the Euler's number:

$a_0 = 5.292 \, x \, 10^{-11} m$ is the Bohr radius

$l_P = 1.616 \, x \, 10^{-35} m$ is the Planck length

$e = 2.71828$, Euler's number

we carry out the relevant arithmetic operation:

$$(N...) = \frac{a_0}{e \, l_P} = 1.2045 \, x \, 10^{24} \tag{7}$$

Now that we know the defined value of $(N...)$ we can use it in the context of this article.

It is worth emphasizing that we do arithmetic operations in the *realm* of large numbers. Where *better* than *in the cosmos* are we going to find very *large numbers*?

Once we have defined the selected cosmological parameters, let's put them together in the following equation:

$$\frac{S_{OU} \, T_0 \, \Omega_b}{\hbar \, \nu_0 \, \Omega_\Lambda} = (Z...) \tag{8}$$

What is $(Z...)$? let's do the calculations carefully:

$$\frac{(2.2 \, x \, 10^{98} JK^{-1}) \, x \, 2.73 \, K \, x \, (0.049)}{(1.69 \, x \, 10^{-23} J) \, x \, (0.687)} = 2.5353272 \, x \, 10^{120} \tag{9}$$

One very large number indeed.Let us divide $(Z...)$ by $(N...)$ successively. The reader will easily verify that

$$(Z...) = (N...)^5 \tag{10}$$

What can we hypothesize about $(Z...)$? If we divide $(N...)$ by two the result is

$$\frac{1}{2}(N...) = 6.0225 \, x \, 10^{23} \tag{11}$$

What does that result remind us of? Is it not *equivalent* to the physical constant known as Avogadro's constant? Indeed, Avogadro's number , or Avogadro's constant [8],commonly denoted N_A or L (for Loschmidt), is the number of constituent particles of a substance (normally atoms or molecules) that can be found in the amount of one *mole* of the substance.So that

$$2N_A = (N...) \text{ and } 32N_A^5 = (Z...) \tag{12}$$

But here ,in this paper, we are talking about those kinds of things that have to do with cosmology, we do not study *moles*. If N_A counts particles per *mole* of chemical substance, what kinds of things do $(Z...)$ or $(N...)$ count?
At first sight, by means of both $(Z...)$ or $(N...)$, we are counting *particles of space* or *atoms of space.*By large, *bits of*

reality.Perhaps the *immense* number of the very small pieces of which the *reality's fabric* is made?
And this idea would be included in the hypothesis that states that space itself is not continuous but discrete or *granular*.

Discussion.

From the arithmetic operations in equation (8), some lines of interest can be drawn.
We think of the parameter $(Z...)$ as a *physical-arithmetic* dimensionless parameter. It's useful to make a *partition* between the entropy of the observable universe, defined in equation (4), and $(Z...)$ which, as we have seen, is a very large number.
As a result, what we obtain is an enormous number of little *pieces of energy*. Each of them weighs about 10^{-23} joules.
What we have observed is that each of those little *bits* has the same energy value as a single photon from the cosmic microwave background.
The arithmetic operation that leads to that conclusion is that,besides that, we need to multiply $(Z...)$ by the ratio of two cosmological parameters, namely , $\frac{\Omega_{\Lambda}}{\Omega_{b}}$
At first glance, when we say that there is a physical-arithmetic relationship between S_{OU} and radiation energy in the universe, this can be interpreted as the *physical-arithmetic correlation* that exists between the entropy deduced from the

Bekenstein-Hawking formula for black holes, the energy *stored* in the cosmic microwave background and the relationship between the fraction of dark energy and the fraction of baryon matter.
We were able to do it this correlation by including the dimensionless parameter (Z...).
Thus, an interrelation is established between the *information* stored in the event horizon area and the information stored in the *four-dimensional* space of the universe in which we live.
The reader will wonder what does what was said above look like. It is not something similar or *analogous* to what the well-known *holographic paradigm* [9] postulates in its assertions?
For now we do not want to delve further into this matter of holography, which is far from our arithmetic *aspirations*.

Dark ages

We have already seen the numerical correlation between, on the one hand, the *encoded* entropy in the area of the event horizon at *the present moment* of the observable universe t_0 and, on the other hand, the cosmic microwave background, its associated temperature and the ratio between the fraction of dark energy with respect to the fraction of baryonic matter.

We are curious to calculate what the proportion $\frac{\Omega_\Lambda}{\Omega_b}$ would be in a primitive cosmological epoch.

According to the data published,in the so-called dark ages when the universe was around $1.3\ x\ 10^{6}$years old, equivalent to $\sim 10^{13} s$.
The temperature was about 4000K.
Applying Wien's displacement law [10] that consists in an inverse relationship between wavelength and temperature,

$$\lambda = \frac{b}{T}\ ,\ b = 0.0029\ mK \qquad (13)$$

b symbolizes Wien's constant.
When you know the value of wavelength you can know the value of the frequency mode of vibration of one photon of the cosmic microwave background, just divide the speed of light in vacuum by wavelength:

$$\nu = \frac{c}{\lambda} \qquad (14)$$

Thus, supposing some dark ages data (*subscript D*)

$$T_D \sim 4000\ K$$

$$\lambda_D = \frac{b}{4000K} \sim 10^{-7} m$$

$$\upsilon_D \sim 10^{14} s^{-1}$$

$$t_D \sim 10^{13} s$$

We can calculate an approximate value of the dark energy content with respect to the baryonic matter that the universe had at that distant time. First, the *dark ages entropy*,making the *analogy* with the event horizon of black hole:

$S_D = \frac{1}{4}\frac{C^3 k}{G\hbar} A_D \,; A_D = c^2 t_D^2 \sim 10^{42} m^2$ and

$S_D \, x \, 4000K \sim 10^{93} J$, therefore

$$\frac{10^{93} J}{(Z...)\, \hbar \, \nu_D} \Rightarrow \frac{\Omega_{\Lambda D}}{\Omega_{bD}} \sim 10^{-8} \qquad (15)$$

Perhaps such a *small* value of dark energy , the mysterious force that opposes gravitational attraction ,was important for the subsequent formation of galaxies *at the end* of the dark ages?

<u>Cosmological constant</u>

The *nature* of dark energy is unknown.There is a huge discrepancy in comparing the value predicted by theory and the observed value of the cosmological constant [11].
The value of the cosmological constant is usually expressed in m^{-2} , whose experimental value is around $\frac{1}{10^{52} m^2}$.

When it is expressed in Planck units, a *very small* dimensionless value is obtained , around 10^{-122}.

In the realm of very large numbers, to which it undoubtedly belongs 10^{-122}, it might be interesting to apply the parameter $(Z...)$.
Let's write an arithmetic version , *via* parameter $(Z...)$, where Λ_z symbolizes the dimensionless value of the cosmological constant:

$$\Lambda_z = \frac{1}{4\pi^2 (Z...)} \sim 10^{-122} \qquad (16)$$

Conclusion.

We have studied some concepts about the entropy of black holes. We have seen that when we perform some arithmetic operations the result we perceive is a small set of cosmological parameters. We have also seen a certain numerical relationship between what is known as the event horizon of black holes and the event horizon of the observable universe. We briefly describe the holographic issue and the analogy of the results with said concept.Finally we make a small foray back in time to see how there is a variation in the amount of dark energy with respect to the amount of baryonic matter.

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