A Classically Consistent Theory of Gravitation I:

The Assumption and its application in Astrophysics

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Abstract: Based on the assumption that mesons mediating the gravitation travel like photons in the space-time defined by gravitation and gravitational strength is proportional to the number density of the mesons, a non-Newtonian theory of gravitation is proposed. In part I of this paper, the application of this theory to the dynamics of the Milky Way galaxy and our solar system is presented. This new theory may lead to a new explanation of dark matter in spiral galaxies and the young star formation in the center of galactic center. By observing the Saturn motion when it is in the shadow of Jupiter, the validation of the assumption presented by the author can be tested. In part II of this paper, Dirac's big number principle is discussed. In Part III, a possible new field equation of gravitation is presented in the Kropinia geometry.

Introduction

Einstein's general relativity contains two parts: One part is the weak equivalence principle which embeds the gravitation into the curved space-time. Through weak equivalence principle, the particles motion in curved space time is described through local affine connections. The second part is the Einstein's field equations which is the relativity generalization of Newton's law of gravity and it prescribes how energy-momentum and boundary conditions determine the curvature of space time.

The weak equivalence principle and weak field limit of the Einstein's field equations are being well tested. But in the strong field limit, both Newton's law of gravity and Einstein's field equations has not been fully tested.

In modern field theory, the field is mediated by mesons including relativity gravitons for gravitation. Einstein's general relativity gives gravitons a unique path to travel which is different from photons. As a matter of fact that if gravitons travel like photons, there will be no gravitons outside of the black hole horizon. Obviously, this is not the case for Einstein's general relativity. In this paper, the term mesons mediating gravitation (MMG) is used to avoid confusion between the mesons defined in this paper and Einstein's gravitons.

Classically, our intuition is that the field strength shall be proportional to the field lines. The origin of the 1/r2 term in Newton's law of gravity and electro-magnetic field comes from the surface density of field lines. The current paper is to explore the semi-classic theory of gravitation based on the assumption that combines the theory of classic field strength being proportional to the surface density of field lines and the assumption that mesons mediating the gravitation travelling like photons in space time. This paper contains three parts: In Part I of this paper, the implication of this assumption in astrophysics is described. In Part II, the connection between strong interaction and gravitation is described. The Dirac large number principle is the direct conclusion of this assumption. In Part III, a possible new field equation of gravitation is written in Kropnia geometry and the bi term of kropinia geometry is related to the mass distribution of the system.

The Assumption and the analysis:

Einstein's weak equivalent principle is based on the equivalence of the mass of gravitation m_g to the mass of inertia m_i in Newton's law of gravitation. Thus,

$$\mathbf{F} = -\frac{GMm_g}{r^2} = m_i\mathbf{a}$$

can be rewritten as following in considering of m_i=m_g:

$$\mathbf{a} = -GM/r^2\,\hat{\boldsymbol{r}} \qquad (1)$$

where $\hat{r}$ is the unit vector in the direction of r.

The general theory of relativity

$$\frac{dy^n}{d\tau} + \Gamma^n_{ij}y^i y^j = 0$$

states that all particles, regardless of mass of particles, follow the same space time if the initial conditions are the same because Γ^n_{ij} is independent of mass and nature of particles without considering other interactions.

Another key variable of above Eq.(1) is $1/r^2$, where the 2 is exactly 2, not 1.9 or 2.1. Newton's Law of gravity indicates that the $1/r^2$ represents the surface density of the field lines in classic physics.

In classic physics domain, rewrite equation (1) as

$$\mathbf{a} = -GM\sigma_s\,\hat{\boldsymbol{r}} \qquad (2)$$

where σ_s is the surface number density of the mesons (or field) mediating the gravitational field (MMG).

Considering the speed of gravitation field (or virtual particles) is constant, Equation (2) can be rewritten as:

$$\mathbf{a} = -\chi\, M\rho\,\hat{\boldsymbol{r}} \qquad (3)$$

where ρ is the number density of the mesons (or field) mediating the gravitational field (MMG) and χ is the proportional constant.

If the mesons mediating gravitation behave like other particles in gravitational field, Newton's law of gravity needs to be modified in order to accommodate Equation (3). In this paper a non-Newtonian theory of gravity is proposed based on the following assumptions.

Assumption: *The mesons mediating the gravitational field travel in space time like all other particles in gravitational field. And the classic gravitational force of any particle in gravitational field is proportional to the local number density of mesons mediating the gravitation.*

The traditional relativity theory is used to do analysis of the MMG behavior in the space time with gravitational field as the first order of approximation to this new theory before the full theory is developed. This above Assumption leads to the difference between this theory and classic Newton's Law of Gravity in the following three cases: 1). non-symmetric mass distribution systems because MMG can be focused by mass along it way and , σ_s is different in different direction even the radial distance is the same. 2). In strong field limit, MMG can be slowed down or even trapped by strong field and the local density increases as the field strength increases. 3). The source is rotating where the MMG radial speed is slower due to inertial frame rotation.

In the weak field limit and spherical system, σ_s in our theory shall not be so much different from the classic Newton's law of gravity. In the solar system, all the tests related to Newton's law of gravity shall be true except if the planet is in the shadow of another planet.

When the field strong, like the "black hole" of general relativity, no particles escape from the "black hole" and the photon is infinitely red shifted and trapped inside the balck hole, no MMG escape from the "balck hole", and **a** = 0 outside of a "black hole" based on equation (3).

Inside the “black hole”, for the ingoing MMG is infinitely violet shifted, **a** goes to infinite. The Schwarzschild “black hole” is not black, but does not interact with surrounding matters based on the Assumption.

Rewrite the number density in equation (3) as following based on the definition of number density:

$$\rho = \frac{\sigma_s}{y^s R} \tag{4}$$

where y^s is the classic speed of MMG in the direction σ_s and R is the bounce back (or trapped) radius of the MMG.

Putting Equation (4) back into (3),

$$\mathbf{a} = -\chi\, M \frac{\sigma_s}{y^s R}\,\hat{\boldsymbol{r}} \tag{5}$$

In the case of Einstein’s gravitons and the electromagnetic fields, y^s is close to 1 and it is the speed of MMG in the direction of σ_s. Comparing Equation (5) with (2), the Newtonian gravitational constant $G = \chi/R$. The gravitational constant is inversely proportional to the bound radius of the system which is the same as Dirac’s large number principle. This will be further analyzed in Part II of this paper.

The Implication of the Assumption in Astrophysics:

One of the difference between this classically consistent theory of gravitation and Newton’s law of gravity is that the gravitation field can be bent by the mass of an eccentric system, causing the focusing or de-focusing of the gravitational field. The effects of σ_s variation due to the focusing can be significant. An analysis of the field bending in galactic center, in the disk of the galaxies as well as the motion of Saturn under the influence of Jupiter is analyzed in the next three examples.

The gravitational field from mass M’ will be bent by mass M at distance d. The angle β expanded by the mass M at radius r by definition is

$$\beta = arc\ tg\ \frac{r}{d} \qquad (6)$$

The deflection angle α of MMG by mass M to the first order of approximation is

$$\alpha = \frac{4GM}{r} \qquad (7)$$

If β is smaller than α, then geodesic 1 and geodesic 2 of MMG in Fig. 1 will intersect, σ_s is infinite, **a** in equation (5) goes to infinite at some point. There may be some interesting new physics near the area of infinite σ_s with the final field equations of gravitation which incorporates this classically consistent theory of gravitation.

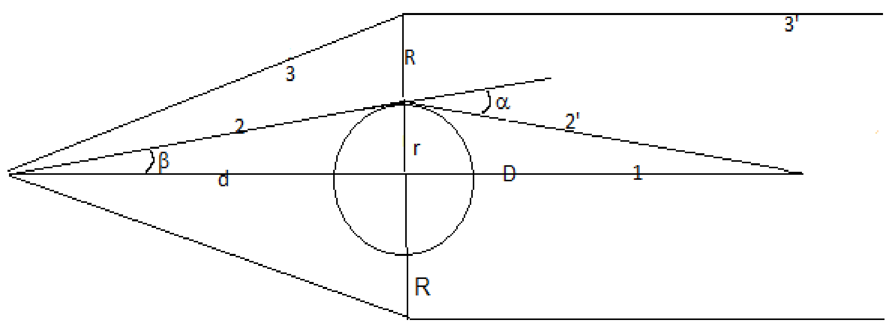

Figure 1. The deflection of MMG from the source at distance d by mass M of radius r. If the deflection angle α is greater than the expansion angle (β) of r over d, the MMG shall intersect at D on the axis of source and bending mass M. If $\alpha = \beta$, the MMG is travelling parallel to the axis.

If the expansion angle β is equal to deflection angle α, geodesic 1 and geodesic 3 of MMG will be travelling parallel to each other after the deflection.

The analysis below is based on a first order approximation similar to the geometric optics with mass M as convex lens of MMG.

Example 1: The gravitation focusing by the galactic center. The distance D of infinite σ_s causing by the mass M (M' is the source of MMG at distance d from M) is given by:

$$arc\ tg\,\frac{r}{D} = \alpha - \beta \qquad (8)$$

Thus,

$$D= \frac{r}{tg(4GM/r-arctg(\frac{d}{r}))} \qquad (8')$$

The galactic center Sagittarius A* may contain a "black hole" of M which is 4.1 million solar mass in a radius of r of 44 million km [1]. This gives the deflection angle of α,

$$\alpha = \frac{4GM}{r} = 0.55$$

The expansion angle b is The deflection angle α by the mass at radius r seeing far away from the deflection mass is about 31.5^0. There are many MMG geodesics will intersect in space (see line 2 and 2' of Figure 1) near the galactic disk and causing the local infinite gravitational force near Sagittarius A* according to Eq.(5).

A gravitational source of mass M' at 4.4 billion kilometer (d equal to 100 times of radius r) away from Sagittarius A*, then the expansion angle is given by equation (6)

$$\beta = arc\ tg\,\frac{r}{d} = 0.01$$

Putting the value of α and β into equation (8), it gives that D= 73.3 million kilometers which means MMG from source M' can be focused to infinite gravitation force by Sagittarius A* at 73.3 million kilometers (1.66 r) away from the center of Sagittarius A*. The gravitational force pattern defined by this classically consistent theory of gravitation will change dramatically due to MMG focusing by Sagittarius A*. There should be many new stars are formed continuously due to the gravitational field focusing. The Youth Paradox, that the new stars forming near the galactic center may be the evidence of this theory which cannot be explained by Newton's Law of gravity, based on which the tidal forces from the central Sagittarius A* to prevent their formation.

The galactic nucleus is much more active than what Newton's Law gravity can explain.

Example 2: Focusing by the Disk of galaxies. A geodesic line of MMG travelling through the disk of a spiral galaxy will be bent many times by the mass it travels nearby. The total deflection can be projected towards to the disk's Z coordinate. It can be seen from previous example, that the "black hole" will focus MMG to intersect. After leaving the center of the galaxy, the MMG will continue to be bent to the disk because the mass of the disk.

The total deflection angle

$$\alpha_t = \sum \theta_i f_i$$

Where θ_i is the individual deflection angle and f_i is the projection factor towards the disk axis Z. The total deflection angel needs to be calculate through a model of detail statistics. A simple analysis will be presented here.

For a bending stars like the Sun in the disk, the deflection angle is α and its expansion angle to a source M' is β. If $\alpha=\beta$, the MMG 3 in Fig. 1 will be travelling parallel to MMG 1. Any source beyond $d=r/\alpha$, will be either parallel to 1 or intersect with 1. For the Sun, α is 0.848×10^{-5} and the radius of the Sun is 6.9×10^{5} kilometer. Any source beyond $d= 6.9 \times 10^{5}/ 0.848 \times 10^{-5} = 8.13 \times 10^{10} = 1.16 \times 10^{-2}$ light year will be bent to travel parallel to the axis of the source mass and the bending mass. For those MMG travelling parallel to the disk surface, the MMG is trapped in 2 dimension of the disk, σ_s will be inversely proportional to r, instead r^2.

By doing the same analysis, the gravitational field from one side is first being bent by the mass on its way to the center, then bent by the center of galaxy, and finally by the mass on the other side of galaxy. Part of the gravitational field is going to be parallel to the galactic disk due to the focusing by the center and the mass in the disk. All field lines parallel to the disk will be trapped in a 2 dimension surface. For the field being trapped in the 2 dimension surface, σ_s is inverse proportional to r, putting this result to Equation (5)

$$\chi \prod_i M_i \frac{\sigma_{s_i}}{y^s R} = \frac{v^2}{r} \qquad (9)$$

Where σ_{si} can be written as b_i/r (b_i are the constants) and taking y_s =1 and G=χ/R Equation (9) can be written as:

$$\mathrm{G} \prod_i M_i \frac{b_i}{r} = \frac{v^2}{r}$$

Thus, the stars far away from the galactic center are affected by the force generated by the field trapped in the galactic disk gives a constant rotational curve v=$\sqrt{\mathrm{G} \prod_i M_i b_i}$. The rotation curve of the globular clusters (no disk) will not have this kind of property due to its location away from the disk.

In the bar of the galactic center, the MMG will be trapped in the bar (1 dimension) and gravitational strength shall be even stronger. This is maybe the reasoning of the star formation in the Milky Way arms.

Example 3: The abnormal motion of Saturn in the shadow of Jupiter: In above two analyses, the Youth Paradox and the flatting of rotation curve cannot be tested easily to confirm the Assumption of this paper. Applying this theory to the solar system can be very interesting. In the case of solar system, the Sun is the source of gravitation, while take the Jupiter as the bending mass, where d $\gg r$, the solar gravitational field deflected by the Jupiter. The force of the Sun after Jupiter and far away from Jupiter, is amplified by a factor in the σ_s as Saturn travelling in the ϕ coordinate and located a Z distance above the rotational surface (see Figure 2).

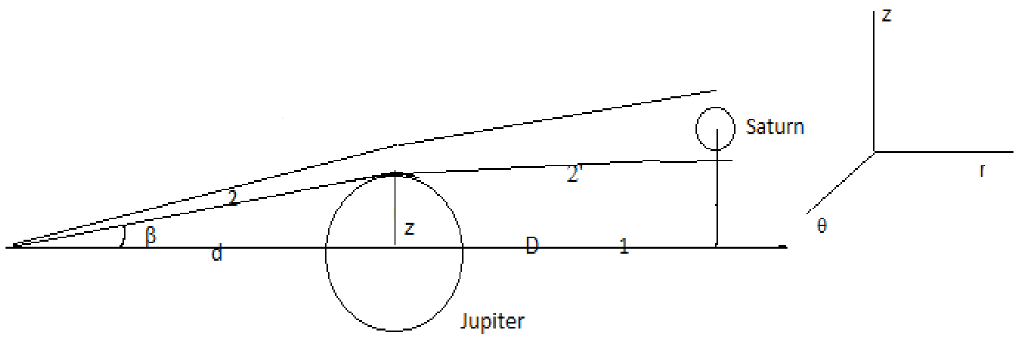

Figure 2. The gravitational field from the Sun is being deflected by Jupiter causing the solar force on Saturn is changed by a factor depening on the location of Saturn in reference to Jupiter.

The original expansion angle β_s of Saturn is

$$\beta_s = \frac{r_s}{d+D}$$

Where r_s is the radius of Saturn d is the Jupiter distance to the Sun and D is the distance between Jupiter and Saturn.

The angle β_s times d is the impact parameter difference, thus the two MMG has different bending angle and the amplification factor A of the angle comparing with the original expansion angle in the Z direction is:

$$A_z = \frac{\beta_s - \left(\frac{4GM}{z} - \frac{4GM}{z+\beta_s d}\right)}{\beta_s} = \left(1 \pm \frac{4GM}{z\,(z+\beta_s d)}\, d\right)$$

Where '+' is for the Saturn is below the rim of the Jupiter shadow and "-" is for Saturn is above the rim of the shadow. If Saturn's center is right at the rim, then the total amplification factor is zero, but the Saturn will experience the tidal force (upper half has less solar force and lower half has stronger solar force. If Z is smaller than the radius of Jupiter, M = $4\pi\rho z^3/3$ and the amplification factor A_z takes the positive sign.

$$A_z = (1 + \frac{16\pi G\rho z^2}{3(z+\beta_s d)} d)$$

The amplification factor in the θ direction is given by:

$$A_\theta = \frac{\beta_s - (\frac{4GM}{\theta d} - \frac{4GM}{\theta d + \beta_s d})}{\beta_s} = (1 \pm \frac{4GM}{\theta\,(\theta+\beta_s)d})$$

Where '+' is for the Saturn is away the rim of the Jupiter shadow and "-" is for Saturn is inside the rim of the shadow. If Saturn's center is right at the rim, then the total amplification factor is zero, but the Saturn will experience the tidal force (outside half has less solar force and inside half has stronger solar force). If θd is smaller than the radius of Jupiter, M = $4\pi\rho(\theta d)^3/3$ and the amplification factor A_θ takes the positive sign.

$$A_\theta = (1 + \frac{16\pi G\rho\theta^2 d}{3(\theta+\beta_s)})$$

Based on Equation (5), to the first order of approximation, the acceleration of Saturn has the following form passing the shadow of Jupiter at Z:

$$\mathbf{a} = A_\theta\, A_z\, \mathbf{a_s} = (1 \pm \frac{4GM}{\theta\,(\theta+\beta_s)d} \pm \frac{4GM}{z\,(z+\beta_s d)} d)\, \mathbf{a_s}$$

where $\mathbf{a_s}$ is the acceleration without the gravitational field deflection. The amplification factor $\frac{d}{r_j}$ when Z is the radius of Jupiter, A_z is 1.113 x10^4. $\mathbf{a}$ = (1.0 - 0.95 x10^{-3}) $\mathbf{a}_s$ is about 0.95 x 10^{-3} smaller than normal if the Saturn is just off the rim of the Jupiter shadow (see Figure 2) or about 0.95 x 10^{-3} bigger if Saturn is just inside the shadow of Jupitor. This result shall be observable to confirm the Assumption of this paper. This disturbance of the path by Jupiter may be the reason of large amount of smaller objects near Jupiter.

Discussion: From the above analysis, it is clear that the Assumption introduced in this paper may be used to explain some of the problems encountered in studying the dynamics of the galactic systems. The equation of motion of a test particles as well as MMG in gravitational field can still be discribed by the potentials defined traditionally, except the field equations needs to be modified dramatically. To find the potential, the equation of motion for the MMG is found first, insert the speed as well as surface density of MMG to equation (5), the motion of MMG and other testing particles is found. Using the computer technology, the final equation of motion can be found through repeating the processes of putting MMG distribution into the equation of motion and re-calculating the the MMG distibution and speed.

From the Gauss Law of gravity aspect, the equation can to be modified as:

$$\oiint_{\partial V} \mathbf{g} \cdot d\boldsymbol{A} = -\, 4\pi\, G\, \varepsilon\, M$$

Where ε is the distribution factor of gravitation similar to that of electromagnetism polarization factor and it depends on the mass distribution.

From the Assumption, we know that for a infinite mass sheet, the MMG will be trapped in the surface, the force will be 1/r away from the center of mass which is different from that of Gauss Law of gravity.

In the general covariance, consider the acceleration of MMG obeys also the equition of motion in general relativity, use y^i as the speed of MMG,

$$y^n{}_{,m} y^m = -\, G\, M \frac{\sigma_s}{y^s} \hat{y}^n$$

Where $\hat{y}^n$ is the unit vector in the y direction and σ_s can be written in terms of focus of null geodesic.

$$\frac{1}{A}\frac{dA}{d\tau} = k^{\mu}_{;\mu}$$

Where A is the area of the null geodesics and σ_s = 1/A, as well as

$$y^s = \frac{dx^s}{d\tau} / \frac{dt}{d\tau}$$

thus

$$y^n{}_{,m} y^m = -\, G\, M \frac{e^{-\int k^{\mu}_{;\mu} d\tau}}{\frac{dx^s}{d\tau}} \frac{dt}{d\tau}\, \hat{y}^n$$

In comparing with Einstein's geodesic equation.

$$y^n{}_{,m} y^m + \Gamma^n_{ij} y^i y^j = 0$$

where $\Gamma^n_{ij} = \Gamma^n_{ij}\left(y^n{}_{,m}, y^i, \phi\right)$ and ϕ is the ratio of $\frac{y^i}{y^j}$. Obviously, the affine connection has the Finsler geometry signature. The y^m and ϕ have the role of A_i in the electromagnetic field. The difference between gravitational field and electromagnetic field is that electromagnetic field does not interact with itself, while gravitational field will interact with itself. Einstein's field equations need to be replaced by its counterpart of equations on Finsler spaces for the MMG. In order to introduce the self-interaction terms, the tensor term $D^n_{ij} = \Gamma^n_{ij} - \gamma^n_{ij}$ is used to replace Γ^n_{ij} [2]. Once the affine connection is determined, the space time and motion of test particles is just in normal Riemannian geometry. The principle of equivalence and the equation of motions will remain unchanged. This new field equation is like the macroscopic Maxwell equations, while Einstein's field equations are the microscopic Maxwell equations if the MMG density is not involved.

References:

[1]. Gillessen, S.; et al.: "Monitoring stellar orbits around the Massive Black Hole in the Galactic Center". *The Astrophys. J.* **692** (2) (2009): 1075–1109.

[2]. Shibata, C.: On Finsler Spaces with Kropina Metric., *Rep. on Math. Phys.* **13** (1978), 117-128.

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