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EQUALITY OF THE VALUES OF THE AREA AND PERIMETER OF A NUMBER OF TWO–DIMENSIONAL FIGURES, VOLUME AND AREA - THREE-DIMENSIONAL

Annotation. Possible variants of the equality of the values of the area and perimeter of a number of two–dimensional figures (square, circle, rectangular, obtuse and equilateral triangles), volume and area - three-dimensional (Platonic bodies, cone, cylinder, pyramid and sphere) are considered.

Keywords: equality of values, two-dimensional shapes, three-dimensional shapes, parameters of geometric shapes, perimeter, volume, area.

Introduction. In one of his publications [1], possible cases of equality (numerical equality) of a number of two-dimensional and three-dimensional geometric figures were considered. As the research material accumulated, the number of such figures increased. And it can still increase by the forces of geometry lovers. In this regard, there is hope that by significantly replenishing the "register" of such geometric figures, a very beautiful theorem can be formulated.

The main part. Calculations of the parameters of a number of two-dimensional and three-dimensional figures were carried out using the online calculator "Geleot". Calculations requiring accuracy of more than three decimal places were performed independently on the basis of the corresponding formulas, using a calculator.

According to the results of calculations , the following numerical equalities of the area and perimeter of a number of two-dimensional figures were revealed:

– of a square when the side is equal to 4 (the area and the length of the perimeter, respectively, will be equal to the value 16), the radius of the inscribed circle is 2, and the circumscribed circle is equal to the value 8, the diagonal of the square is equal to 32;

– circle, when the equality of the area and the circumference is observed at a value of 12.566 ... or 4 π (the radius of the inscribed circle is 2);

– a right triangle with an irrational value of the area and perimeter, when the area and length of the perimeter is equal to the value 27,416324... ≡($\sqrt{5}$+3)2 (where the smaller cathet - 5,236... will be equal to $\sqrt{27,416324...}$, and the larger one - 10,472 ...). In this case, the hypotenuse in the triangle will be equal to twice the value of the radius of the circle described around the triangle (or its diameter). The radius of the circle inscribed in the triangle is 2;

– heron triangles with sides: (5, 12, 13 and 6, 8, 10 – right triangles). In this case, the hypotenuse in right triangles will be equal to twice the value of the radius of the circle described around the triangle (or its diameter); 6, 25, 29; 7, 15, 20; 9, 10, 17 ... (obtuse triangles), the radius of the circle inscribed in the named triangles is 2.

– an equilateral triangle, with an area value of 20.7846 ... (while the length of the side is 6.928...≡√48), the radius of the inscribed circle is 2, and the radius of the described one is its doubled value;

According to the results of calculations, the following numerical equalities of volume and area of a number of three-dimensional figures were revealed:

– a cube with a face equal to the value of √8, the volume and surface area of the cube is 216, the radius of the inscribed sphere is 3;

– spheres (equality of volume and surface area) equal to the value 113,097335526 ... or 36 π (in this case, the diameter of the sphere is 6, and its circumference is 18.85 ...≡6 π), the radius of the inscribed sphere is 3;

– tetrahedron (equality of area and volume) equal to the value 374,123 ... (while the edge length is 14,69693845669907...≡√216), the radius of the inscribed sphere is 3;

– octahedron (equality of area and volume) equal to the value 187.061 ... (while the edge length is equal to 7.344669228349534 ...≡√54), the radius of the inscribed sphere is 3;

– icosahedron (equality of area and volume) equal to the value of 136.4595 ... (while the edge length is 3.9695 ...), the radius of the inscribed sphere is 3;

– dodecahedron (equality of area and volume) equal to the value of 149.8578 ... (while the edge length is 2.694168 ...), the radius of the inscribed sphere is 3;

– cylinder (equality of area and volume) equal to the value of 54π ≈ 169.646 ... (in this case, the radius is 3, and the height is twice the value of the radius – 6). The area of the lateral surface is 113.097 ... (volume and area of the sphere inscribed in the figure) or 36π, and the area of one of the two bases is 9π, the radius of the inscribed the volume of the cylinder is equal to 3. The volume of the cylinder is greater than the volume of the sphere inscribed in it (where there is equality of the values of area and volume) by exactly 1.5 times;

– a cone (equality of area and volume) equal to the value 96π ≈ 301.593 ... (in this case, the radius of the base is 6, the generatrix is 10, and the height of the figure is 8). The area of the base (circle) is 113.097 ... (volume and area of the sphere inscribed in the figure) or 36π, the area of the lateral surface, respectively, – 60π, the radius of the inscribed sphere is 3;

– a three-sided pyramid (the equality of the area and volume of the tetrahedron) at a height of 12, the side of the base is 14,6969384567 ...≡√216 and is equal to the value of 374,123 ..., the radius of the inscribed sphere is 3;

– a tetrahedral pyramid (equality of area and volume) with a height of 12, the side of the base is 8,485281374 ...≡√72 and is equal to 288. The ratio of height to the side of the base is √2. At the same time, the area of the side surface of the pyramid is three times larger than the area of the base (216 and 72), the radius of the inscribed sphere is 3;

– a hexagonal pyramid with a height of 12, the side of the base is 4.898979485 ...≡√24 and is equal to 249.415..., the radius of the inscribed sphere is 3.

Based on the calculations carried out, the conclusion is formulated:

– in two-dimensional figures: square, circle, rectangular, obtuse and equilateral triangles, the radius of the inscribed circle with equal values of area and perimeter is 2;

– in three-dimensional figures tetrahedron, cube, octahedron, icosahedron, dodecahedron, cone, cylinder, 3-4-6-sided pyramid and sphere, the radius of the inscribed circle with equal values of area and volume is 3.

List of literature:

1. Ворон, А.В. Тождество значений площади и периметра ряда двумерных фигур, объема и площади – трехмерных // «Академия Тринитаризма», М., Эл № 77-6567, публ.25873, 14.11.2019.