

# Collatz Conjecture Proved Ingeniously & Very Simply

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## Abstract

Collatz conjecture states that beginning with a positive integer, if one repeatedly performs the following operations to form a sequence of integers, the sequence will eventually reach the integer one; the operations being that if the integer is even, divide it by 2, but if the integer is odd, multiply it by 3 and add one; and also, use the result of each step as the input for the next step.

To prove Collatz conjecture, one would apply a systematic observation of the sequences produced by the Collatz process, the  $(3n + 1)\frac{1}{2}$  process, Two main cases are covered. In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ), and in this case, the sequence will reach the integer 1 by repeated division by 2, i.e.,  $2^k, 2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ . In Case 2, the integer cannot be written as a power of 2, but the sequence terms of the integers reach integers equivalent to  $2^{2k}$  ( $k = 2, 3, \dots$ ), and by repeated division by 2, the sequences will reach the integer 1, i.e.,  $2^{2k}, 2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ . In Case 2, when the sequence terms reach some particular odd integers such as 5, 21 and 85, the application of  $3n + 1$  operation to these integers will result in the integers equivalent to the powers,  $2^{2k}$  ( $k = 2, 3, \dots$ ). One would call these integers, the  $2k$ -power converters. There are infinitely many power converters as there are  $2^{2k}$  powers. A term of the sequence must be converted to an integer equivalent to  $2^{2k}$  ( $k = 2, 3, \dots$ ). There are infinitely many paths for converting integers to  $2^{2k}$  ( $k = 2, 3, \dots$ ) powers on the  $2^{2k}$ -route, a route on which a  $2^{2k}$ -power can be divided repeatedly by 2 until the sequence reaches the integer 1. Of these conversion paths, the integer 5-path is the nearest  $2^{2k}$  ( $k = 2$ ) converter path to the integer 1 on the  $2^{2k}$ -route. Other paths to the  $2^{2k}$ -route include the 21-path, ( $k = 3$ ), and the 85-path, ( $k = 4$ ), For the 5-path, when a sequence terms reach the integer, 5, the next term would be  $3(5) + 1 = 16$ . Similarly, for the integers 21, and 85, the next terms, respectively, would be  $3(21) + 1 = 64$ ,  $3(85) + 1 = 256$ . Some non- $2k$  power converters can follow the integer 5-path to the  $2^4$  power as follows: Let  $n$  be an integer whose sequence terms would reach 16 or  $2^4$  in the  $(3n + 1)\frac{1}{2}$  process, and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$ , but the negative sign if  $n > 5$ . One will call the following, the 5-path  $2k$ -converter formula,  $3(n \pm r) + 1 = 16$ . The integers  $2^p(5)$  ( $p = 1, 2, 3, \dots$ ) will take the integer 5-path to convert to a  $2k$ -power. Similar definitions and formulas for the 21-path, and the 85-path, are as follows: For the 21-path:  $n \pm r = 21$ , The 21-path  $2k$ -converter formula is  $3(n \pm r) + 1 = 64$ . For the 85-path:  $n \pm r = 85$ . The 85-path  $2k$ -converter formula is  $3(n \pm r) + 1 = 256$ . There are similar definitions and formulas for the other infinite paths. The integers,  $2^p(21)$ ,  $2^p(85)$  with ( $p = 1, 2, 3, \dots$ ), will take, respectively, the integer 21-path, and the 85-path, to reach  $2k$ -powers. Just as there are infinitely many positive integers, there are infinitely many  $2^{2k}$ -power converters,  $2k$ -powers,  $2k$ -power converter paths, and descendants,  $2^p C$  ( $p = 1, 2, 3, \dots$ ) of  $C$ . With all the above functioning together, the sequence of every positive integer that cannot be written as a power of 2, would reach the equivalent integer,  $2^{2k}$  and by repeated division by 2, the sequence would reach the integer 1. Therefore, using the approaches in Cases 1 and 2, above, the sequence of every positive integer would eventually reach the integer 1.

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# Option 1

## Preliminaries

### Tables of Sequences of Positive Integers

1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	1	10	2	16	3	22	4	28	5	34	6	40	7	46	8	52	9	58	10
3	2		5	1	8	10	11	2	14	16	17	3	20	22	23	4	26	28	29	5
4	1		16		4	5	34	1	7	8	52	10	10	11	70	2	13	14	88	16
5			8		2	16	17		22	4	26	5	5	34	35	1	40	7	44	8
6			4		1	8	52		11	2	13	16	16	17	106		20	22	22	4
7			2			4	26		34	1	40	8	8	52	53		10	11	11	2
8			1			2	13		17		20	4	4	26	160		5	34	34	1
9						1	40		52		10	2	2	13	80		16	17	17	
10							20		26		5	1	1	40	40		8	52	52	
11							10		13		16			20	20		4	26	26	
12							5		40		8			10	10		2	13	13	
13							16		20		4			5	5		1	40	40	
14							8		10		2			16	16			20	20	
15							4		5		1			8	8			10	10	
16							2		16					4	4			5	5	
17							1		8					2	2			16	16	
18									4					1	1			8	8	
19									2									4	4	
20									1									2	2	
21																		1	1	
22																				
23																				

1	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
2	64	11	70	12	76	13	82	14	88	15	94	16	100	17	106	18	112	19	118	20
3	32	34	35	6	38	40	41	7	44	46	47	8	50	52	53	9	56	58	59	10
4	16	17	106	3	19	20	124	22	22	23	142	4	25	26	160	28	28	29	178	5
5	8	52	53	10	58	10	62	11	11	70	71	2	76	13	80	14	14	88	89	16
6	4	26	160	5	29	5	31	34	34	35	↓	1	38	40	40	7	7	44	268	8
7	2	13	80	16	88	16	94	17	17	106			19	20	20	22	22	22	134	4
8	1	40	40	8	44	8	47	52	52	53			58	10	10	11	11	11	67	2
9		20	20	4	22	4	142	26	26	160			29	5	5	34	34	34	202	1
10		10	10	2	11	2	71	13	13	80			88	16	16	17	17	17	101	
11		5	5	1	34	1	214	40	40	40			44	8	8	52	52	52	304	
12		16	16		17		107	20	20	20			22	4	4	26	26	26	152	
13		8	8		52		322	10	10	10			11	2	2	13	13	13	76	
14		4	4		26		161	5	5	5			34	1	1	40	40	40	38	
15		2	2		13		484	16	16	16			17			20	20	20	19	
16		1	1		40		242	8	8	8			52			10	10	10	58	
17					20		121	4	4	4			26			5	5	5	29	
18					10		364	2	2	2			13			16	16	16	88	
19					5		182	1	1	1			40			8	8	8	44	
20					16		91						20			4	4	4	22	
21					8		274						10			2	2	2	11	
22					4		137						5			1	1	1	34	
23					2		↓						↓						↓	
24					1															

1	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
2		21		22	136	23		24	148		154	26	160			28		29		30
3		64		11		70		12	74		77	13	80			14		88		15
4		32		34		35		6	37		232	40	40			7		44		46
5		16		17		106		3	112		116	20	20			22		22		23
6		8		52		53		10	56		58	10	10			11		11		70
7		4		26		160		5	28		29	5	5			34		34		35
8		2		13		80		16	14		88	16	16			17		17		106
9		1		40		40		8	7		44	8	8			52		52		53
10				20		20		4	22		22	4	4			26		26		160
11				10		10		2	11		11	2	2			13		13		80
12				5		5		1	34		34	1	1			40		40		40
13				16		16			17		17					20		20		20
14				8		8			52		52					10		10		10
15				4		4			26		26					5		5		5
16				2		2			13		13					16		16		16
17				1		1			40		40					8		8		8
18									20		20					4		4		4
19									10		10					2		2		2
20									5		5					1		1		1
21									16		16									
22									8		8									
23									4		4									
24									2		2									
25									1		1									
26																				

## Collatz sequence for the integer 27

Note: Sequence for 27 below, has 111 steps

27	47	484	137	233	395	668	1132	319	3238	911	9232	433	122	35	10
82	142	242	412	700	1186	334	566	958	1619	2734	4616	1300	61	106	5
41	71	121	206	350	593	167	283	479	4858	1367	2308	650	184	53	16
124	214	364	103	175	1780	502	850	1438	2429	4102	1154	325	92	160	8
62	107	182	310	526	890	251	425	719	7288	2051	577	976	46	80	4
31	322	91	155	263	445	754	1276	2158	3644	6154	1732	488	23	40	2
94	161	274	466	790	1336	377	638	1079	1822	3077	866	244	70	20	1

## Net Change

(Sum of both the pluses and minuses as in Example 2, below)

Let  $n$  be an integer whose sequence would reach 16 or  $2^4$  in the  $(3n + 1)\frac{1}{2}$  process and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$  but the negative sign if  $n > 5$ .

**Example 1:** Let  $n = 12$  with the sequence terms 12,6,3,10,5. From 12 to 6, the change is -6; from 6 to 3, the change is -3; from 3 to 10, the change is +7; and from 10 to 5, the change is -5. The net change,  $r = -6 - 3 + 7 - 5 = -7$ ; and  $n \pm r = 12 - 7 = 5$ . Note: The negative sign confirms the rule of signs above. i.e.,  $n > 5$  ( $12 > 5$ ). Note: Since  $3(5 - 0) + 1 = 16$ ,  $3(12 - 7) + 1 = 16$ , Also,

$3(3 + 2) + 1 = 16$ , Suppose, one wants the sequence for the integer, 27 to reach 16. Then  $3(27 - 22) + 1 = 16$ , since  $27 - 22 = 5 - 0$ . One confirms the "-22" in  $3(27 - 22) + 1 = 16$ , below.

**Example 2: Confirmation of the net change for Collatz sequence for the integer 27**

Read from top to bottom in the first column, and continue from top of the next column down and repeat the process for the other columns. Numbers with "+" signs are for the positive changes, and numbers with "-" signs are for the negative changes..

27	142	121	103	526	445	377	319	1619	1367	1154	976	23	20
+55	-71	+243	+207	-263	+891	+755	+639	+3239	+2735	-577	-488	+47	-10
82	71	364	310	263	1336	1132	958	4858	4102	577	488	70	10
-41	+143	-182	-155	+527	-668	-566	-479	-2429	-2051	+1155	-244	-35	-5
41	214	182	155	790	668	566	479	2429	2051	1732	244	35	5
+83	-107	-91	+311	-395	-334	-283	+959	+4859	+4103	-866	-122	+71	+11
124	107	91	466	395	334	283	1438	7288	6154	866	122	106	16
-62	+215	+183	-233	+791	-167	+567	-719	-3644	-3077	-433	-61	-53	
62	322	274	233	1186	167	850	719	3644	3077	433	61	53	
-31	-161	-137	+467	-593	+335	-425	+1439	-1822	+6155	+867	+123	+107	
31	161	137	700	593	502	425	2158	1822	9232	1300	184	160	
+63	+323	+275	-350	+1187	-251	+851	-1079	-911	-4616	-650	-92	-80	
94	584	412	350	1780	251	1276	1079	911	4616	650	92	80	
-47	-242	-206	-175	-890	+503	-638	+2159	+1823	-2308	-325	-46	-40	
47	242	206	175	890	754	638	3238	2734	2308	325	46	40	
+95	-121	-103	+351	-445	-377	-319	-1619	-1367	-1154	+651	-23	-20	

**Sum of the pluses, "+" = 40,552.; Sum of the minuses, "-" = -40,574.**

**Sum of the pluses and the minuses = 40,552 - 40574 = -22**

The net change,  $r = -22$ , and  $n - r = 27 - 22 = 5$

**Example 2** confirms that one can write the net change without the tedious addition of pluses and minuses in the above table. For example, for the integer, 33,  $n - r = 33 - 28 = 5$ .

For the integer, 45,  $n - r = 45 - 40 = 5$  One will call the following, the 5-path 2k-converter

formula:  $3(n \pm r) + 1 = 16$ . Using this formula, the sequence of some positive integers which cannot be written as a power of 2, can reach the integer, 16. Once the sequence reaches 16, applying repeated division by 2, the sequence will reach the integer 1. (16,8,4,2,1).

## Option 2 Introduction

To prove Collatz conjecture, one would be guided by a systematic observation of the sequences produced by the Collatz process,  $(3n + 1)\frac{1}{2}$  process, and note the patterns of the sequence terms as the process reaches integers equivalent to  $2^{2k}$  ( $k = 2, 3, \dots$ ) and continue straightforwardly as  $2^{2k}, 2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ , reaching the integer 1. For example,  $2^4$  continues as  $2^4, 2^3, 2^2, 2^1$ , to  $2^0$  (16, 8, 4, 2, to 1). If an integer is equivalent to the form  $2^k$  ( $k = 1, 2, 3, \dots$ ), by repeated division by 2, the sequence would reach the integer 1. In the numerical sequences of positive integers, in the preliminaries (Option 1), the following observations were made:

**Case 1:** If the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ) the sequence would reach the integer one by repeated division by 2, i.e.,  $2^k, 2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ .

Example 1:  $8 = 2^3$ , and  $2^3, 2^2, 2^1, 2^0$  or 8,4,2,1.

Example 2:  $16 = 2^4$ , and  $2^4, 2^3, 2^2, 2^1, 2^0$  or 16,8,4,2,1.

**Case 2** The integer cannot be written as a power of 2, but the sequence terms of this integer reach the equivalent power,  $2^{2k}$  ( $k = 2, 3, \dots$ ) which will continue as  $2^{2k}, 2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ . In particular, if the sequence reaches the integer, 16, the previous term would be the integer 5. This observation is from the sequences of the first 100 positive integers, except for about five integers and the equivalent  $2^k$  ( $k = 1, 2, 3, \dots$ )-power forms such as 8, 16, and 64. In Case 2, when the sequence terms reach some particular odd integers such as 5, 21 and 85, the application of  $3n + 1$  operation to these integers will result in the integers equivalent to the powers,  $2^{2k}$  ( $k = 2, 3, \dots$ ). One would call these integers, the  $2k$ -power converters. There are infinitely many  $2k$ -power converters as there are  $2^{2k}$  powers. A term of the sequence must be converted to an integer equivalent to  $2^{2k}$  ( $k = 2, 3, \dots$ ). There are infinitely many paths for converting integers to  $2^{2k}$  ( $k = 2, 3, \dots$ ) powers on the  $2^{2k}$ -route, a route on which a  $2^{2k}$ -power can be divided repeatedly by 2 until the sequence reaches the integer 1. Of these conversion paths, the integer 5-path is the nearest  $2^{2k}$  ( $k = 2$ ) converter path to the integer 1 on the  $2^{2k}$ -route. Other paths to the  $2^{2k}$ -route include the 21-path, ( $k = 3$ ), the 85-path, ( $k = 4$ ), and the 341-path ( $k = 5$ ). For the 5-path, when a sequence terms reach the integer, 5, the next term would be  $3(5) + 1 = 16$ . Similarly, for the integers 21, 85, and 341, the next terms, respectively, would be  $3(21) + 1 = 64$ ,  $3(85) + 1 = 256$  and  $3(341) + 1 = 1024$ . Non- $2k$  power converters can follow the integer 5-path to the  $2^4$  power as follows: Let  $n$  be an integer whose sequence terms would reach 16 or  $2^4$  in the  $(3n + 1)\frac{1}{2}$  process, and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$ , but the negative sign if  $n > 5$ . One will call the following, the 5-path  $2k$ -converter formula,  $3(n \pm r) + 1 = 16$ . The integers with factors  $2^p(5)$  ( $p = 1, 2, 3, \dots$ ) would take the integer 5-path to convert to a  $2k$ -powers. Similar definitions and formulas for the 21-path, the 85-path and the 341-path are as follows: For the 21-path:  $n \pm r = 21$ , The 21-path  $2k$ -converter formula is  $3(n \pm r) + 1 = 64$ . For the 85-path:  $n \pm r = 85$ , The 85-path  $2k$ -converter formula is  $3(n \pm r) + 1 = 256$ . For the 341-path:  $n \pm r = 341$ , The 341-path  $2k$ -converter formula is  $3(n \pm r) + 1 = 1024$ . There are similar definitions and formulas for the other infinite number of paths.

## Option 3

### Collatz Conjecture Proved Ingeniously & Very Simply

**Given:** 1. A positive integer,  $n$

$$2. \text{ The function } f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ 3n + 1 & \text{if } n \text{ is odd} \end{cases}$$

**Required:** Begin with the positive integer,  $n$ , and form a sequence by applying the above operation repeatedly, using the result of each step as input for the next step and prove or convince the reader that eventually, the sequence reaches the integer 1.

**Plan:** Write the positive integer as a power of 2 or change a term of the sequence to an integer which can be written as a power of 2.

#### Proof

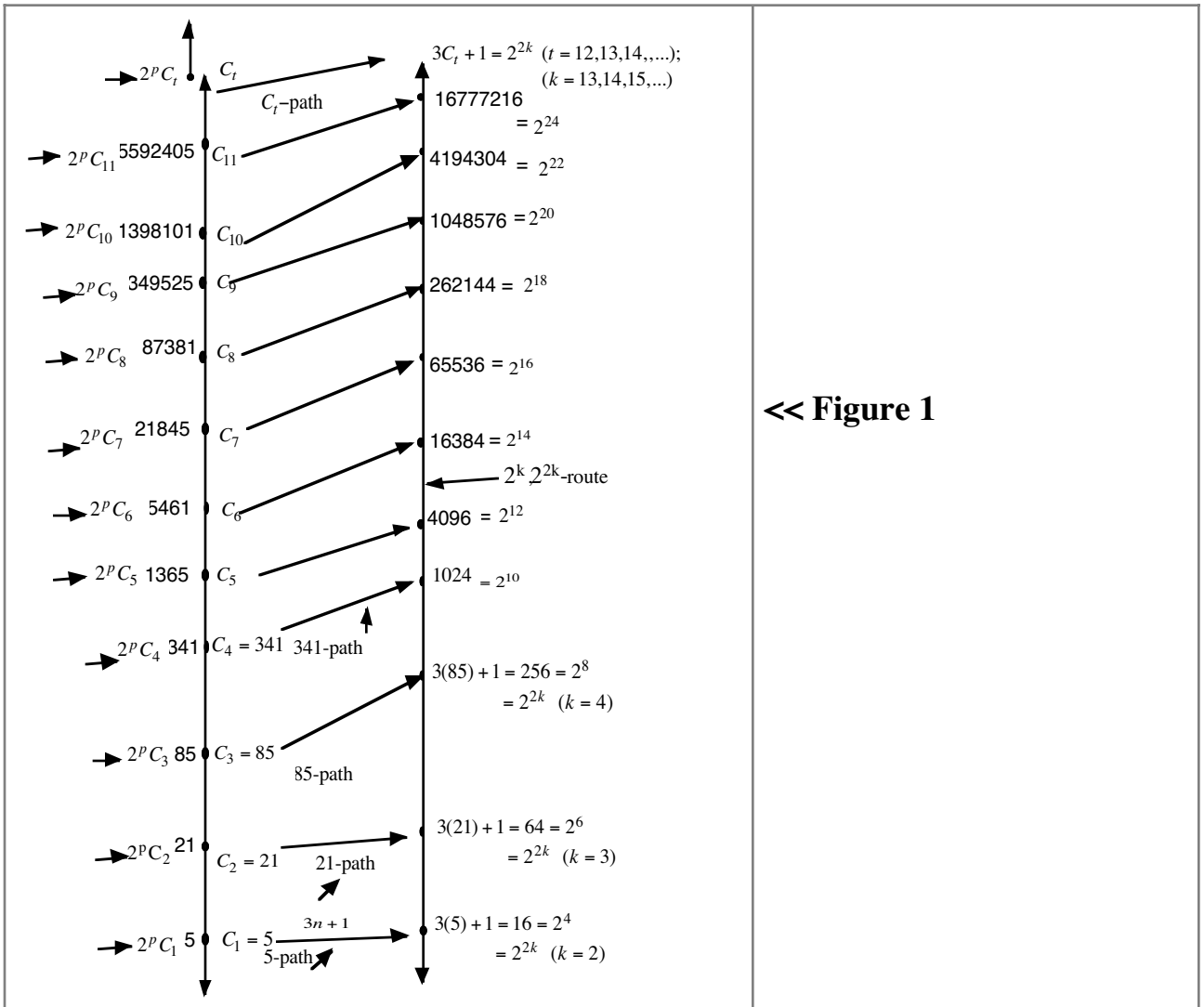
**Case 1:** If the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ) the sequence would reach the integer one by repeated division by 2, i.e.,  $2^k, 2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ .

**Case 2:** The integer cannot be written as a power of 2, but the sequence terms of this integer reach the equivalent power,  $2^{2k}$  ( $k = 2, 3, \dots$ ) which will continue as  $2^{2k}, 2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ .

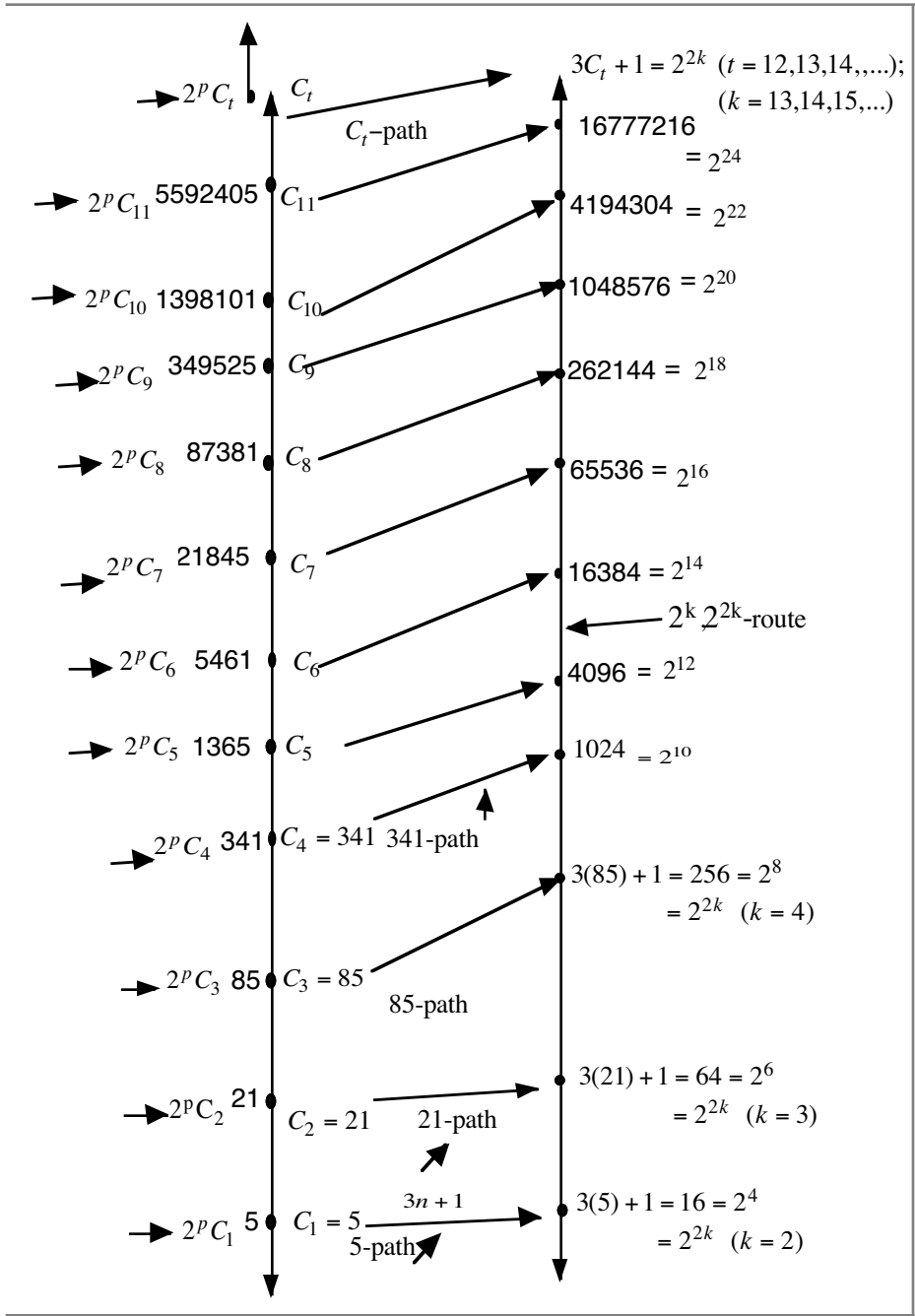
In Case 2, (see Fig.1 on p.9) when the sequence terms reach some particular odd integers such as 5, 21 and 85, the application of  $3n + 1$  operation to these integers will result in the integers equivalent to the powers,  $2^{2k}$  ( $k = 2, 3, \dots$ ). One would call these integers, the  $2k$ -power converters. There are infinitely many  $2k$ -power converters as there are  $2^{2k}$  powers. A term of the sequence must be converted to an integer equivalent to  $2^{2k}$  ( $k = 2, 3, \dots$ ). There are infinitely many paths for converting integers to  $2^{2k}$  ( $k = 2, 3, \dots$ ) powers on the  $2^{2k}$ -route, a route on which a  $2^{2k}$ -power can be divided repeatedly by 2 until the sequence reaches the integer 1. Of these conversion paths, the integer 5-path is the nearest  $2^{2k}$  ( $k = 2$ ) converter path to the integer 1 on the  $2^{2k}$ -route. Other paths to the  $2^{2k}$ -route include the 21-path, ( $k = 3$ ), the 85-path, ( $k = 4$ ), and the 341-path ( $k = 5$ ). For the 5-path, when a sequence terms reach the integer, 5, the next term would be  $3(5) + 1 = 16$ . Similarly, for the integers 21, 85, and 341, the next terms, respectively, would be  $3(21) + 1 = 64$ ,  $3(85) + 1 = 256$  and  $3(341) + 1 = 1024$ . Non- $2K$  power converters can follow the integer 5-path to the  $2^4$  power as follows: Let  $n$  be an integer whose sequence terms would reach 16 or  $2^4$  in the  $(3n + 1)\frac{1}{2}$  process, and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$ , but the negative sign if  $n > 5$ . One will call the following, the 5-path  $2k$ -converter formula,  $3(n \pm r) + 1 = 16$ . The integers with factors  $2^p(5)$  ( $p = 1, 2, 3, \dots$ ) would take the integer 5-path to convert to a  $2k$ -power. Similar definitions and formulas for the 21-path, the 85-path and the 341-path are as follows: For the 21-path:  $n \pm r = 21$ , where  $r$  is the net change before the integer 21; and one uses the positive sign if  $n < 21$ , but the negative sign if  $n > 21$ . The 21-path  $2k$ -converter formula is  $3(n \pm r) + 1 = 64$ . For the 85-path:  $n \pm r = 85$ , where  $r$  is the net change before the integer 85; and one uses the positive sign if  $n < 85$ , but the negative sign if  $n > 85$ . The 85-path  $2k$ -converter formula is  $3(n \pm r) + 1 = 256$ . For the 341-path:  $n \pm r = 341$ , where  $r$  is the net change before the integer 341;



and one uses the positive sign if  $n < 341$ , but the negative sign if  $n > 341$ . The 341-path 2k-converter formula is  $3(n \pm r) + 1 = 1024$ . There are similar definitions and formulas for the other infinite number of paths. The integers with the factors,  $2^p(21), 2^p(85), 2^p(341)$  with  $(p = 1, 2, 3, \dots)$ , will take, respectively, the integer 21-path, the 85-path and the integer 341-path to reach 2k-powers. To generalize the 2k-power conversions, let C be a 2k-power converter such that  $3C + 1 = 2^{2k}$  ( $k = 2, 3, 4, \dots$ ). Also, let  $2^p C$  ( $p = 1, 2, 3, \dots$ ) be descendants of C. Then, the integer  $2^p(5)$  ( $p = 1, 2, 3, \dots$ ) will take the integer 5-path to convert to a 2k-power. Similarly, the integers  $2^p(21), 2^p(85)$  and  $2^p(341)$  with  $(p = 1, 2, 3, \dots)$  would take, respectively, the integer 21-path, the integer 85-path, and the integer 341-path to convert to 2k-powers. With infinitely many positive integers, infinitely many  $2^{2k}$ -power converters, infinitely many 2k-powers, infinitely many 2k-power converter paths, and infinitely many descendants,  $2^p C$  ( $p = 1, 2, 3, \dots$ ) of C, a 2k-power converter, all with corresponding definitions, and formulas, the sequence of every positive integer that cannot be written as a power of 2, would reach the integer,  $2^{2k}$  ( $k = 2, 3, \dots$ ) and by repeated division by 2, the sequence would reach the integer 1. Therefore, using the approaches in Cases 1 and 2, the sequence of every positive integer would eventually reach the integer 1.



# Option 4 Discussion



**Example** for integer 84 using the 21-path to reach the power,  $2^6$   
 For the integer  $2^p C$ ,  
 If  $p = 2, C = 21$   
 $2^2(21) = 84$   
 For the sequence, 84, 42, 21,  
 $3(21) + 1 = 64 = 2^6$   
 That is, the sequence of the integer 84 reached  $2^6$  power using the 21-path.

It is worth noting that the integers, 1 quadrillion, 1 trillion, 1 billion, and 1 million, all use the integer 5-path to convert to the  $2^k$ -power forms

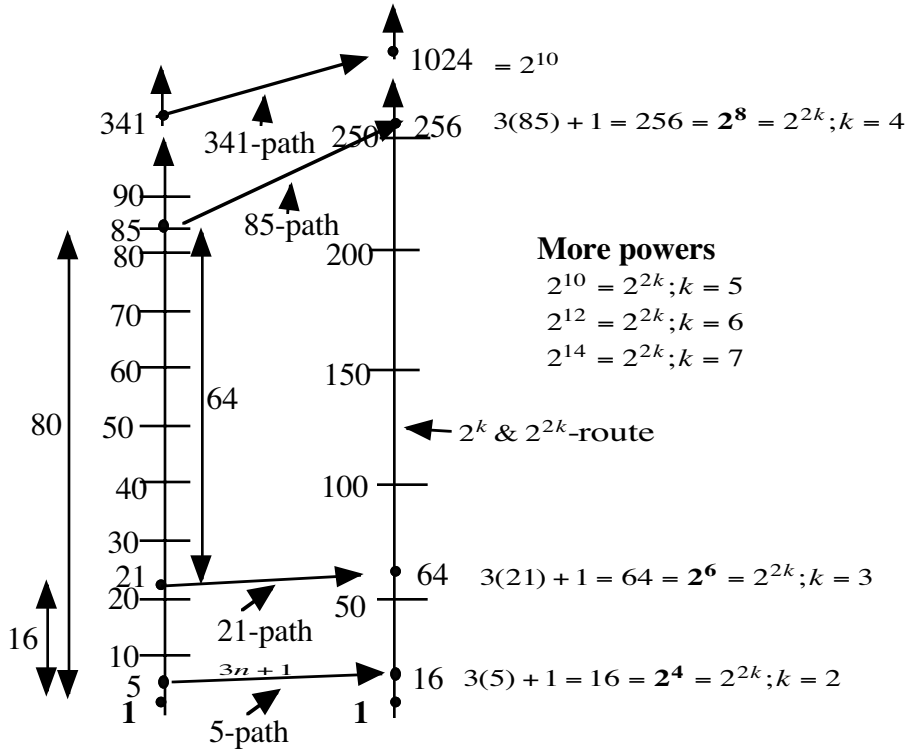
## Option 5 Conclusion

By applying a systematic observation of the sequences produced by the Collatz process, the  $(3n + 1)\frac{1}{2}$  process, the author has shown that Collatz conjecture is true. Particularly, one noted the patterns of the sequence terms as the process reaches the equivalent powers,  $2^{2^k}$  ( $k = 2, 3, \dots$ ) and continues straightforwardly as  $2^{2^k}, 2^{2^{k-1}}, 2^{2^{k-2}}, 2^{2^{k-3}}, \dots, 2^{2^{k-2^k}}$ . The approach used consists of two main cases, namely, Case 1 and Case 2. In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ), and the sequence would reach the integer one by repeated division by 2, i.e.,  $2^k, 2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ . In Case 2, the integer cannot be written as a power of 2, but the sequence terms of the integer reach the equivalent power,  $2^{2^k}$  ( $k = 2, 3, \dots$ ), and by repeated division by 2, the sequence will reach the integer 1, i.e.,  $2^{2^k}, 2^{2^{k-1}}, 2^{2^{k-2}}, 2^{2^{k-3}}, \dots, 2^{2^{k-2^k}}$ . Thus the sequence will definitely continue to 1. When the sequence terms reach some particular integers such as 5, 21 and 85, the application of  $3n + 1$  to these integers would result in integers equivalent to the powers,  $2^{2^k}$  ( $k = 2, 3, \dots$ ). One would call these integers, the  $2^k$ -power converters. Examples of the converters are 5, 21, and 85 with the respective  $2^{2^k}$ -powers,  $2^4, 2^6$ , and  $2^8$ . There are infinitely many power converters as there are infinitely many  $2^{2^k}$  powers. A term of the sequence of a positive integers must be converted to  $2^{2^k}$  ( $k = 2, 3, \dots$ ) power. There are infinitely many paths for converting integers to  $2^{2^k}$  ( $k = 2, 3, \dots$ ) powers. Of these conversion paths, the integer 5-path, is the nearest  $2^{2^k}$  ( $k = 2$ ) converter path to the integer 1 on the  $2^{2^k}$ -route. For the 5-path, when a sequence terms reach the integer, 5, the next term would be  $3(5) + 1 = 16$  or  $2^4$ . Similarly, for the  $2^k$ -power converter, 21, the next term would be  $3(21) + 1 = 64 = 2^6$ . Some other integers, can follow the integer 5-path to the  $2^4$  power as follows: Let  $n$  be a descendant of 5 and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$ , but the negative sign if  $n > 5$ . One will call the following, the 5-path  $2^k$ -converter formula:  $3(n \pm r) + 1 = 16$  or  $2^4$ . By the substitution axiom, using this formula, the sequence of some positive integers, that cannot be written as a power of 2, would reach the integer, 16, as in Case 2; and once the sequence reaches 16, by repeated division by 2, the sequence would reach the integer 1. The integers with factors  $2^p(5)$  ( $p = 1, 2, 3, \dots$ ) would take the integer 5-path to convert to a  $2^k$ -power. In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ); and the sequence will reach the integer 1, by repeated division by 2, i.e.,  $2^k, 2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ . Just as there are infinitely many positive integers, there are infinitely many  $2^{2^k}$ -power converters, infinitely many  $2^k$ -powers, infinitely many  $2^k$ -power converter paths, and infinitely many descendants,  $2^p C$  ( $p = 1, 2, 3, \dots$ ) of  $C$ , and with all the above functioning together, the sequence of every positive integer that cannot be written as a power of 2, would reach the equivalent power,  $2^{2^k}$ . By repeated division by 2, the sequence would reach the integer 1. Therefore, using the approaches in Cases 1 and 2, the sequence of every positive integer would eventually reach the integer 1. The approach used in this paper has applications in civil engineering, especially in road design and construction as well as town and country planning.

**References:** 1. <https://www.dcode.fr/collatz-conjecture>  
2. <https://www.goodcalculators.com/collatz-conjecture-calculator>

# Option 6

## Integer Humor



**Integer 5 speaks:** Integer 27, the integer 21-path is near you. Why did you not use the 21-path to cross to the  $2^{2^k}$ -route, but instead, you went through the jungle to use my path?

**Integer 27 answers:** I am not a descendant of integer 21, For me to use the 21-path my name should be  $(21)2^p$  ( $p = 1, 2, \dots$ ). Perhaps, when I reached the entrance to the 21-path, the Kamikaze typhoon blew me away to your path.

**Integer 84 speaks to Integer 21:** What should I write on the path formula if I want to use your path to go to the  $2^{2^k}$ -route?

**Integer 21 answers:** Write  $3(84 - 63) + 1 = 64 = 2^6$  on the formula.

**Integer 21 speaks to Integer 32:** Integer 32, When are you going to use my path to cross to the  $2^k, 2^{2^k}$ -route?

**Integer 32 answers:** I,  $2^5$ , do not need to use your path. I am already on the  $2^k, 2^{2^k}$ -route, and I am on my way to attend the UN meeting on 1st Avenue,

**Adonten**