

# Collatz Conjecture Proved Ingeniously & Very Simply

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## Abstract

Collatz conjecture states that beginning with a positive integer, if one repeatedly performs the following operations to form a sequence of integers, the sequence will eventually reach the integer one; the operations being that if the integer is even, divide it by 2, but if the integer is odd, multiply it by 3 and add one; and also, use the result of each step as the input for the next step. To prove Collatz conjecture, one would apply a systematic observation of the sequences produced by the Collatz process, the  $(3n + 1)\frac{1}{2}$  process, and note especially, the patterns of the sequence terms as the process reaches the equivalent powers,  $2^{2k}$  ( $k = 2, 3, \dots$ ) and the sequences reach the integer 1, by repeated division by 2, i.e.,  $2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ . Two main cases are covered. In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ). In this case, the sequence will reach the integer 1 by repeated division by 2, i.e.,  $2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ . In Case 2, the integer cannot be written as a power of 2, but the sequence terms of the integer reach the equivalent power,  $2^{2k}$  ( $k = 2, 3, \dots$ ), and by repeated division by 2, the sequence will reach the integer 1, i.e.,  $2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ . Thus, if an integer is of the form,  $2^k$  ( $k = 1, 2, 3, \dots$ ) or the sequence reaches  $2^{2k}$  ( $k = 2, 3, \dots$ ), the sequence will definitely continue to 1. In Case 2, when the sequence terms reach some particular integers such as 5, 21 and 85, the application of  $3n + 1$  to these integers will result in the powers,  $2^{2k}$  ( $k = 2, 3, \dots$ ). One would call these integers, the  $2k$ -power converters. There are infinitely many power converters as there are  $2^{2k}$  powers. If the sequence reaches a  $2^{2k}$  ( $k = 2, 3, \dots$ ) power, the terms will continue as  $2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, \dots, 2^{2k-2k}$ . A term of the sequence must be converted to an integer equivalent to  $2^{2k}$  ( $k = 2, 3, \dots$ ). There are infinitely many paths for converting integers to  $2^{2k}$  ( $k = 2, 3, \dots$ ) powers. Of these conversion paths, the integer 5-path, is the nearest  $2^{2k}$  ( $k = 2$ ) converter path to the integer 1 on the  $2^{2k}$ -route. For the 5-path, when a sequence terms reach the integer, 5, the next term would be  $3(5) + 1 = 16$  or  $2^4$ . Other integers can follow the integer 5-path to the  $2^4$  power as follows: Let  $n$  be an integer whose sequence terms would reach 16 or  $2^4$  in the  $(3n + 1)\frac{1}{2}$  process, and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$ , but the negative sign if  $n > 5$ . One will call the following, the 5-path  $2k$ -converter formula:  $\boxed{3(n \pm r) + 1 = 16 \text{ or } 2^4}$ . By the substitution axiom, using this formula, the sequence of every positive integer that cannot be written as a power of 2, would reach the integer, 16, as in Case 2; and once the sequence reaches 16, by repeated division by 2, the sequence would reach the integer 1. (16,8,4,2,1). In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ); and the sequence will reach the integer 1, by repeated division by 2, i.e.,  $2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ . Therefore, the sequence of every positive integer would reach the integer 1. It is worth mentioning that the integers, 1 quadrillion, 1 trillion, 1 billion, 1 million, and the integer, 123,456,789,012,345, all, use the 5-path to reach the  $2k$ -power forms.

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# Option 1

## Preliminaries

### Tables of Sequences of Positive Integers

1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	1	10	2	16	3	22	4	28	5	34	6	40	7	46	8	52	9	58	10
3	2		5	1	8	10	11	2	14	16	17	3	20	22	23	4	26	28	29	5
4	1		16		4	5	34	1	7	8	52	10	10	11	70	2	13	14	88	16
5			8		2	16	17		22	4	26	5	5	34	35	1	40	7	44	8
6			4		1	8	52		11	2	13	16	16	17	106		20	22	22	4
7			2			4	26		34	1	40	8	8	52	53		10	11	11	2
8			1			2	13		17		20	4	4	26	160		5	34	34	1
9						1	40		52		10	2	2	13	80		16	17	17	
10							20		26		5	1	1	40	40		8	52	52	
11							10		13		16			20	20		4	26	26	
12							5		40		8			10	10		2	13	13	
13							16		20		4			5	5		1	40	40	
14							8		10		2			16	16			20	20	
15							4		5		1			8	8			10	10	
16							2		16					4	4			5	5	
17							1		8					2	2			16	16	
18									4					1	1			8	8	
19									2									4	4	
20									1									2	2	
21																		1	1	
22																				
23																				

1	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
2	64	11	70	12	76	13	82	14	88	15	94	16	100	17	106	18	112	19	118	20
3	32	34	35	6	38	40	41	7	44	46	47	8	50	52	53	9	56	58	59	10
4	16	17	106	3	19	20	124	22	22	23	142	4	25	26	160	28	28	29	178	5
5	8	52	53	10	58	10	62	11	11	70	71	2	76	13	80	14	14	88	89	16
6	4	26	160	5	29	5	31	34	34	35	↓	1	38	40	40	7	7	44	268	8
7	2	13	80	16	88	16	94	17	17	106			19	20	20	22	22	22	134	4
8	1	40	40	8	44	8	47	52	52	53			58	10	10	11	11	11	67	2
9		20	20	4	22	4	142	26	26	160			29	5	5	34	34	34	202	1
10		10	10	2	11	2	71	13	13	80			88	16	16	17	17	17	101	
11		5	5	1	34	1	214	40	40	40			44	8	8	52	52	52	304	
12		16	16		17		107	20	20	20			22	4	4	26	26	26	152	
13		8	8		52		322	10	10	10			11	2	2	13	13	13	76	
14		4	4		26		161	5	5	5			34	1	1	40	40	40	38	
15		2	2		13		484	16	16	16			17			20	20	20	19	
16		1	1		40		242	8	8	8			52			10	10	10	58	
17					20		121	4	4	4			26			5	5	5	29	
18					10		364	2	2	2			13			16	16	16	88	
19					5		182	1	1	1			40			8	8	8	44	
20					16		91						20			4	4	4	22	
21					8		274						10			2	2	2	11	
22					4		137						5			1	1	1	34	
23					2		↓						↓						↓	
24					1															

1	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
2		21		22	136	23		24	148		154	26	160			28		29		30
3		64		11		70		12	74		77	13	80			14		88		15
4		32		34		35		6	37		232	40	40			7		44		46
5		16		17		106		3	112		116	20	20			22		22		23
6		8		52		53		10	56		58	10	10			11		11		70
7		4		26		160		5	28		29	5	5			34		34		35
8		2		13		80		16	14		88	16	16			17		17		106
9		1		40		40		8	7		44	8	8			52		52		53
10				20		20		4	22		22	4	4			26		26		160
11				10		10		2	11		11	2	2			13		13		80
12				5		5		1	34		34	1	1			40		40		40
13				16		16			17		17					20		20		20
14				8		8			52		52					10		10		10
15				4		4			26		26					5		5		5
16				2		2			13		13					16		16		16
17				1		1			40		40					8		8		8
18									20		20					4		4		4
19									10		10					2		2		2
20									5		5					1		1		1
21									16		16									
22									8		8									
23									4		4									
24									2		2									
25									1		1									
26																				

## Collatz sequence for the integer 27

Note: Sequence for 27 below, has 111 steps

27	47	484	137	233	395	668	1132	319	3238	911	9232	433	122	35	10
82	142	242	412	700	1186	334	566	958	1619	2734	4616	1300	61	106	5
41	71	121	206	350	593	167	283	479	4858	1367	2308	650	184	53	16
124	214	364	103	175	1780	502	850	1438	2429	4102	1154	325	92	160	8
62	107	182	310	526	890	251	425	719	7288	2051	577	976	46	80	4
31	322	91	155	263	445	754	1276	2158	3644	6154	1732	488	23	40	2
94	161	274	466	790	1336	377	638	1079	1822	3077	866	244	70	20	1

## Net Change

(Sum of both the pluses and minuses as in Example 2, below)

Let  $n$  be an integer whose sequence would reach 16 or  $2^4$  in the  $(3n + 1)\frac{1}{2}$  process and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$  but the negative sign if  $n > 5$ .

**Example 1:** Let  $n = 12$  with the sequence terms 12,6,3,10,5. From 12 to 6, the change is -6; from 6 to 3, the change is -3; from 3 to 10, the change is +7; and from 10 to 5, the change is -5. The net change,  $r = -6 - 3 + 7 - 5 = -7$ ; and  $n \pm r = 12 - 7 = 5$ . Note: The negative sign confirms the rule of signs above. i.e.,  $n > 5$  ( $12 > 5$ ). Note: Since  $3(5 - 0) + 1 = 16$ ,  $3(12 - 7) + 1 = 16$ , Also,

$3(3 + 2) + 1 = 16$ , Suppose, one wants the sequence for the integer, 27 to reach 16. Then  $3(27 - 22) + 1 = 16$ , since  $27 - 22 = 5 - 0$ . One confirms the "-22" in  $3(27 - 22) + 1 = 16$ , below.

**Example 2: Confirmation of the net change for Collatz sequence for the integer 27**

Read from top to bottom in the first column, and continue from top of the next column down and repeat the process for the other columns. Numbers with "+" signs are for the positive changes, and numbers with "-" signs are for the negative changes..

27	142	121	103	526	445	377	319	1619	1367	1154	976	23	20
+55	-71	+243	+207	-263	+891	+755	+639	+3239	+2735	-577	-488	+47	-10
82	71	364	310	263	1336	1132	958	4858	4102	577	488	70	10
-41	+143	-182	-155	+527	-668	-566	-479	-2429	-2051	+1155	-244	-35	-5
41	214	182	155	790	668	566	479	2429	2051	1732	244	35	5
+83	-107	-91	+311	-395	-334	-283	+959	+4859	+4103	-866	-122	+71	+11
124	107	91	466	395	334	283	1438	7288	6154	866	122	106	16
-62	+215	+183	-233	+791	-167	+567	-719	-3644	-3077	-433	-61	-53	
62	322	274	233	1186	167	850	719	3644	3077	433	61	53	
-31	-161	-137	+467	-593	+335	-425	+1439	-1822	+6155	+867	+123	+107	
31	161	137	700	593	502	425	2158	1822	9232	1300	184	160	
+63	+323	+275	-350	+1187	-251	+851	-1079	-911	-4616	-650	-92	-80	
94	584	412	350	1780	251	1276	1079	911	4616	650	92	80	
-47	-242	-206	-175	-890	+503	-638	+2159	+1823	-2308	-325	-46	-40	
47	242	206	175	890	754	638	3238	2734	2308	325	46	40	
+95	-121	-103	+351	-445	-377	-319	-1619	-1367	-1154	+651	-23	-20	

**Sum of the pluses, "+" = 40,552.; Sum of the minuses, "-" = -40,574.**

**Sum of the pluses and the minuses = 40,552 - 40574 = -22**

The net change,  $r = -22$ , and  $n - r = 27 - 22 = 5$

**Example 2** confirms that one can write the net change without the tedious addition of pluses and minuses in the above table. For example, for the integer, 33,  $n - r = 33 - 28 = 5$ .

For the integer, 45,  $n - r = 45 - 40 = 5$  One will call the following, the 5-path 2k-converter

formula:  $3(n \pm r) + 1 = 16$ . Using this formula, the sequence of every positive integer which cannot be written as a power of 2, can reach the integer, 16. Once the sequence reaches 16, applying repeated division by 2, the sequence will reach the integer 1. (16,8,4,2,1).

## Option 2 Introduction

To prove Collatz conjecture, one would be guided by a systematic observation of the sequences produced by the Collatz process,  $(3n + 1)\frac{1}{2}$  process, and note the pattern of the sequence terms as the process reaches the integers,  $2^{2k}$  ( $k = 2, 3, \dots$ ) and continue straightforwardly as  $2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ , reaching the integer 1. For example,  $2^4$  continues as  $2^3, 2^2, 2^1$ , to  $2^0$  (8, 4, 2, to 1). If an integer is equivalent to the form  $2^k$  ( $k = 1, 2, 3, \dots$ ), the sequence will definitely continue to 1. In the numerical sequences of positive integers, in the preliminaries (Option 1), the following observations were made:

**Case 1:** If the integer can be written as a power of 2,  $2^k$  ( $k = 1, 2, 3, \dots$ ). the sequence will reach the integer one by repeated division by 2, i.e.,  $2^k, 2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ .

Example 1:  $8 = 2^3$ , and  $2^3, 2^2, 2^1, 2^0$  or 8,4,2,1.

Example 2:  $16 = 2^4$ , and  $2^4, 2^3, 2^2, 2^1, 2^0$  or 16,8,4,2,1.

**Case 2** The integer cannot be written as a power of 2, but the sequence terms of this integer reach the equivalent power,  $2^{2k}$  ( $k = 2, 3, \dots$ ) which will continue as

$2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ . In particular, if the sequence reaches the integer, 16, the previous term would be the integer 5. This observation was from the sequences of the first 100 positive integers, except for about five integers and the equivalent  $2^k$  ( $k = 1, 2, 3, \dots$ )-power forms such as 8, 16, and 64. Thus the integer, 5, would be the guiding integer for converting other integers to  $2k$ -power forms, since  $3(5) + 1 = 16$ . Note that  $16 = 2^4$ , a  $2^{2k}$  form ( $k = 2$ )

In Case 2, two main requirements must be satisfied in order for a sequence of positive integers to reach the integer 1. The first requirement is that when the sequence terms reach some particular integers such as 5, 21 and 85, the application of  $3n + 1$  to these integers will result in the powers,  $2^{2k}$  ( $k = 2, 3, \dots$ ). One would call these integers, the  $2k$ -power converters. Thus an integer,  $n$ , would be called a  $2k$ -power converter if  $3n + 1 = 2^{2k}$  ( $k = 2, 3, \dots$ ). Examples of the converters are 5, 21, and 85 with the respective  $2^{2k}$ -powers,  $2^4, 2^6$ , and  $2^8$ . There are infinitely many power converters as there are  $2^{2k}$  powers. To find more power converters, let  $3n + 1 = 2^{2k}$ . If  $k = 5$ ,  $3n + 1 = 2^{10}$ ,  $3n = 2^{10} - 1$ ,  $3n = 1024 - 1$ , and  $n = 341$ . Satisfying the first requirement would be straightforward since, if the sequence reaches a  $2^{2k}$  ( $k = 2, 3, \dots$ ) power, the terms will continue as  $2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ . For example:  $2^4$  ( $k = 2$ ) will continue as 8, 4, 2, to 1.

The second requirement is that a term of the sequence of every positive integer which cannot be written as a power of 2 must be converted to  $2^{2k}$  ( $k = 2, 3, \dots$ ). Satisfaction of the second requirement involves the integer 5-path, and the other infinitely, many  $2^{2k}$  ( $k = 3, 4, \dots$ )-paths. For the integer 5-path, when a sequence terms reach the integer, 5, the next term would be  $3(5) + 1 = 16 = 2^4$ . Similarly, for the integer, 21, the next term would be  $3(21) + 1 = 64 = 2^6$ . Any integer which is  $(21)2^p$  ( $p = 1, 2, 3, \dots$ ) can reach integer, 21, By the substitution axiom, the sequence of every positive integer that cannot be written as a power of 2 can follow the integer 5-path to the  $2^4$  power as follows: Let  $n$  be an integer whose sequence would reach

$16 = 2^4$  in the  $(3n + 1)\frac{1}{2}$  process, and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$ , but the negative sign if  $n > 5$ . One will call the following, the 5-path 2k-converter formula:  $3(n \pm r) + 1 = 16$  or  $2^4$ . Using this formula, as in Case 2, the sequence of every positive integer which cannot be written as a power of 2, can reach the integer, 16. Once the sequence reaches 16, using repeated division by 2, the sequence will reach the integer 1. (16,8,4,2,1). In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ); and the sequence will reach the integer 1, by repeated division by 2, i.e.,  $2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ .

### About the 5-path formula

Some possible meanings of the formula  $3(n \pm r) + 1 = 16$  or  $2^4$

$n - r = 5$  can mean when the net change,  $r$  is removed,  $n$  would behave as the integer 5.

$n + r$  can mean when the net change,  $r$  is added,  $n$  would behave as the integer 5.

**Note:** For the integer, 5:  $3(5) + 1 = 16$ ;  $3(5 - 0) + 1 = 16$ ,  $n = 5$ ,  $r = 0$  (see also Preliminaries)

For the integer **27**:  $3(27 - 22) + 1 = 16$ , where  $n = 27$ , and  $r = 22$



## Option 3

### Collatz Conjecture Proved Ingeniously & Very Simply

**Given:** 1. A positive integer,  $n$

2. The function  $f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ 3n + 1 & \text{if } n \text{ is odd} \end{cases}$

**Required:** Begin with the positive integer,  $n$ , and form a sequence by applying the above operation repeatedly, using the result of each step as input for the next step and prove that eventually, the sequence reaches the integer 1.

#### Proof

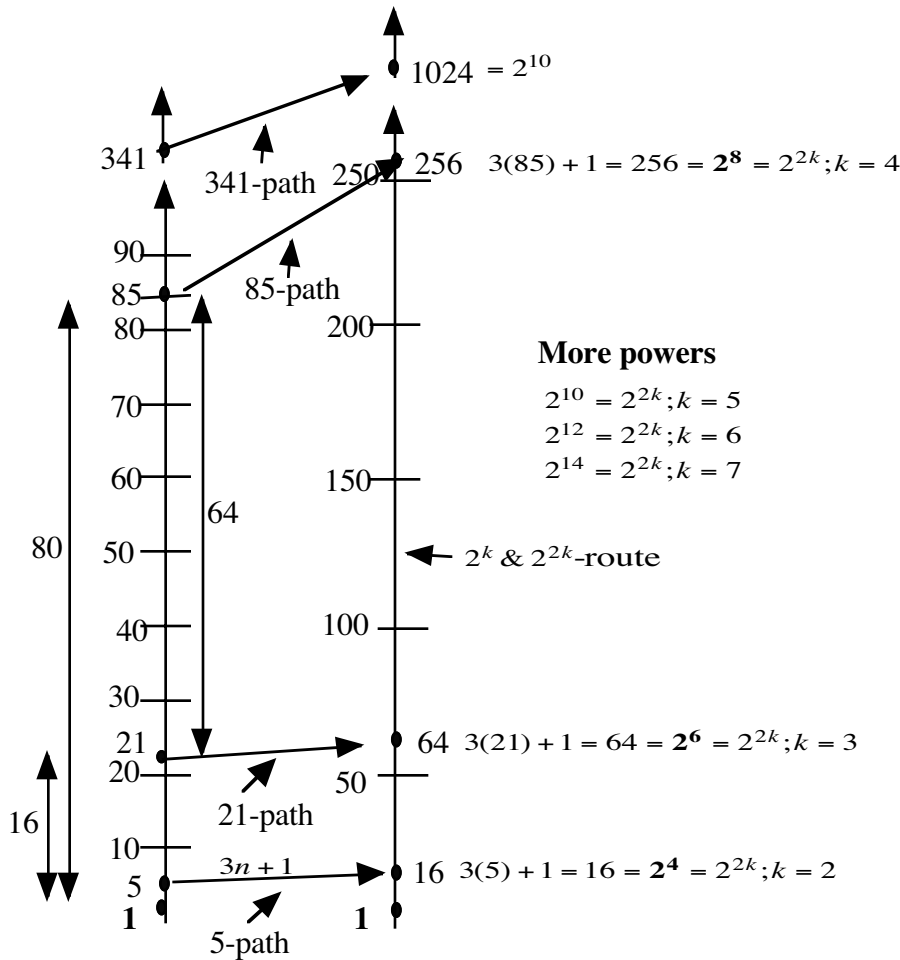
**Case 1:** The integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ )

In this case, the sequence will reach the integer 1 by repeated division by 2, i.e.,  $2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$

**Case 2:** The integer cannot be written as a power of 2, but the sequence terms of the integer reach the equivalent power,  $2^{2k}$  ( $k = 2, 3, \dots$ ) which will continue as  $2^{2k-1}, 2^{2k-2}, 2^{2k-3}, \dots, 2^{2k-2k}$ . Particularly, if the sequence reaches the integer, 16, the previous term would be the integer 5. One would call the integer, 5, a 2k-power converter. Thus, an integer,  $n$ , would be called a 2k-power converter if  $3n + 1 = 2^{2k}$ . In Case 2, the sequence of every positive integer which cannot be written as a power of 2 can reach the power,  $2^{2k}$  ( $k = 2, 3, \dots$ ). There are infinitely many paths for reaching  $2^{2k}$  ( $k = 2, 3, \dots$ ) powers. Of these paths, the integer 5-path, is the nearest  $2^{2k}$  ( $k = 2$ ) converter path to the integer 1 on the  $2^{2k}$ -route. For the 5-path, when a sequence terms reach the integer, 5, the next term would be  $3(5) + 1 = 16$ . Any sequence which reaches 5, will continue as 16, 8, 4, 2, to 1. Other integers can follow the integer 5-path to the  $2^4$  power as follows: Let  $n$  be an integer whose sequence terms are to reach 16 or  $2^4$  in the  $(3n + 1)^{\frac{1}{2}}$  process; and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$ , but the negative sign if  $n > 5$ . Since  $5 = 5 - 0$ , one can write  $3(5 - 0) + 1 = 16$ . One can also view  $5 - 0$  as coming from single term sequence whose first term is 5 and whose net change is zero in the above  $n \pm r = 5$  definition. If a sequence begins with for example, the positive integer 27, one can write  $3(27 - 22) + 1 = 16 = 2^4$ , by the substitution axiom, since  $27 - 22 = 5$ . Here,  $n = 27$  and  $r = 22$ . Thus,  $3(n \pm r) + 1 = 16$  is for the integer 5-path. When the sequence reaches the integer, 16, repeated division by 2 will proceed as 8, 4, 2, 1. One will call  $\boxed{3(n \pm r) + 1 = 16 \text{ or } 2^4}$  the 5-path 2k-converter formula. Using this formula, the sequence of every positive integer which cannot be written as a power of 2, can reach the integer, 16. Once the sequence reaches 16, applying repeated division by 2, the sequence will reach the integer 1. (16, 8, 4, 2, 1). In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ); and the sequence would reach the integer 1, using repeated division by 2, i.e.,  $2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ . Therefore, the sequence of every positive integer would reach the integer 1; and the proof is complete.

# Option 4 Discussion

In addition to the  $2k$ -power converter, 5, there are other  $2k$ -power converters such as 21, 85 and 341 which can convert respectively, as follows: 1.  $3(21) + 1 = 64 = 2^6$ , 2.  $3(85) + 1 = 256 = 2^8$ , and 3.  $3(341) + 1 = 1024 = 2^{10}$ . There are infinitely many  $2k$ -power converters as there are  $2^{2k}$  powers.



Additionally, for the integer, 21, One can go from 21 to  $2^4$  by applying the 5-path,  $n \pm r = 5$ , where  $n = 21$  and  $r = 16$ . Then, since  $21 - 16 = 5$   $3(21 - 16) + 1 = 16 = 2^4$  (see Figure). Let one apply the 21-path to the positive integer 27.. Then  $3(27 - 6) + 1 = 64$  or  $2^6$  since  $27 - 6 = 21$ . To generalize for the 21-path here,  $n \pm r = 21$ . with  $n = 27$ , and  $r = 6$

**For integer, 85**, since 85 is odd, by applying  $3n + 1$ , one obtains  $3(85) + 1 = 256$  or  $2^8$ . This path is straightforward. However, one can go from 85 to  $2^4$  by applying the 5-path,  $n \pm r = 5$ , where  $n = 85$  and  $r = 80$  Then,  $85 - 80 = 5$ , and  $3(85 - 80) + 1 = 16 = 2^4$ .(see Figure)

Let one apply the 85-path to the positive integer 27. Then  $3(27 + 58) + 1 = 256$  or  $2^8$  since  $27 + 58 = 85$ . To generalize for the 85-path here,  $n \pm r = 85$ . with  $n = 27$ , and  $r = 58$

**For integer, 341**, since 341 is odd, by applying  $3n + 1$ , one obtains  $3(341) + 1 = 1024$  or  $2^{10}$ . This path is straightforward. However, one can go from 341 to  $2^4$  by applying the 5-path,  $n \pm r = 5$ , where  $n = 341$  and  $r = 336$  Then,  $341 - 336 = 5$ , and  $3(341 - 336) + 1 = 16 = 2^4$ .(see Figure)

# Option 5

## Conclusion

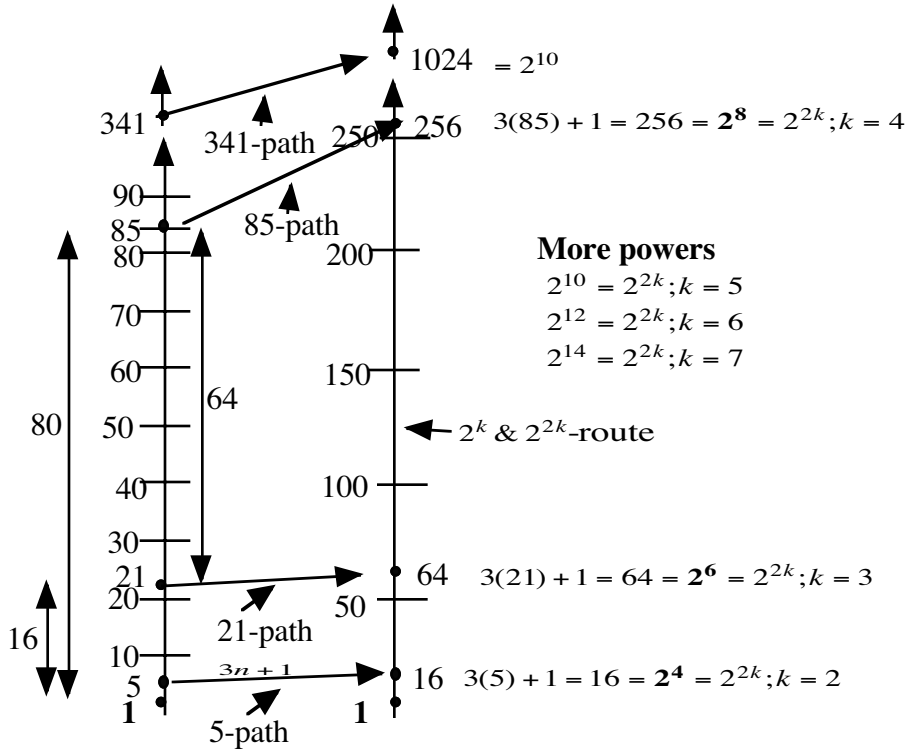
By applying a systematic observation of the sequences produced by the Collatz process, the  $(3n + 1)\frac{1}{2}$  process, the author has shown that Collatz conjecture is true. Particularly, one noted the patterns of the sequence terms as the process reaches the equivalent powers,  $2^{2^k}$  ( $k = 2, 3, \dots$ ) and continues straightforwardly as  $2^{2^{k-1}}, 2^{2^{k-2}}, 2^{2^{k-3}}, \dots, 2^{2^{k-2k}}$ . The approach used consists of two main cases, namely, Case 1 and Case 2. In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ), and the sequence would reach the integer one by repeated division by 2, i.e.,  $2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ . In Case 2, the integer cannot be written as a power of 2, but the sequence terms of the integer reach the equivalent power,  $2^{2^k}$  ( $k = 2, 3, \dots$ ), and by repeated division by 2, the sequence will reach the integer 1, i.e.,  $2^{2^{k-1}}, 2^{2^{k-2}}, 2^{2^{k-3}}, \dots, 2^{2^{k-2k}}$ . Thus the sequence will definitely continue to 1. In Case 2, two main requirements must be satisfied in order for a sequence of positive integers to reach the integer 1. The first requirement is that when the sequence terms reach some particular integers such as 5, 21 and 85, the application of  $3n + 1$  to these integers would result in the powers,  $2^{2^k}$  ( $k = 2, 3, \dots$ ). One would call these integers, the  $2k$ -power converters. Examples of the converters are 5, 21, and 85 with the respective  $2^{2^k}$ -powers,  $2^4, 2^6$ , and  $2^8$ . There are infinitely many power converters as there are  $2^{2^k}$  powers. The second requirement is that a term of the sequence of positive integers must be converted to  $2^{2^k}$  ( $k = 2, 3, \dots$ ). There are infinitely many paths for converting integers to  $2^{2^k}$  ( $k = 2, 3, \dots$ ) powers. Of these conversion paths, the integer 5-path, is the nearest  $2^{2^k}$  ( $k = 2$ ) converter path to the integer 1 on the  $2^{2^k}$ -route. For the 5-path, when a sequence terms reach the integer, 5, the next term would be  $3(5) + 1 = 16$  or  $2^4$ . Similarly, for the  $2k$ -power converter, 21, the next term would be  $3(21) + 1 = 64 = 2^6$ . Other integers can follow the integer 5-path to the  $2^4$  power as follows: Let  $n$  be an integer whose sequence terms would reach 16 or  $2^4$  in the  $(3n + 1)\frac{1}{2}$  process, and let  $n \pm r = 5$ , where  $r$  is the net change in the sequence terms before the integer 5; and one uses the positive sign if  $n < 5$ , but the negative sign if  $n > 5$ . One will call the following, the 5-path  $2k$ -converter formula:  $\boxed{3(n \pm r) + 1 = 16 \text{ or } 2^4}$ . By the substitution axiom, using this formula, the sequence of every positive integer that cannot be written as a power of 2, would reach the integer, 16, as in Case 2; and once the sequence reaches 16, by repeated division by 2, the sequence would reach the integer 1. (16,8,4,2,1). In Case 1, the integer can be written as a power of 2 as  $2^k$  ( $k = 1, 2, 3, \dots$ ); and the sequence will reach the integer 1, by repeated division by 2, i.e.,  $2^{k-1}, 2^{k-2}, 2^{k-3}, \dots, 2^{k-k}$ . Therefore, the sequence of every positive integer would reach the integer 1. It is worth noting that the integers, 1 quadrillion, 1 trillion, 1 billion, and 1 million, all, use the 5-path to convert to the  $2k$ -power forms. The approach used in this paper has applications in civil engineering, especially in road design, road construction. as well as town and country planning.

### References:

1. <https://www.dcode.fr/collatz-conjecture>
2. <https://www.goodcalculators.com/collatz-conjecture-calculator>

# Option 6

## Integer Humor



**Integer 5 speaks:** Integer 27, the integer 21-path is near you. Why did you not use the 21-path to cross to the  $2^{2k}$ -route, but instead, you went through the jungle to use my path?

**Integer 27 answers:** I am not a descendant of integer 21, For me to use the 21-path my name should be  $(21)2^p$  ( $p = 1, 2, \dots$ ). Perhaps, when I reached the entrance to the 21-path, the Kamikaze typhoon blew me away to your path.

**Integer 5 speaks to Integer 21:** What should I write on the path formula if I want to use your path to go to the  $2^{2k}$ -route?

**Integer 21 answers:** Write  $3(5 + 16) + 1 = 64 = 2^6$  on the formula.

**Integer 21 speaks to Integer 32:** Integer 32, When are you going to use my path to cross to the  $2^k, 2^{2k}$ -route?

**Integer 32 answers:** I,  $2^5$ , do not need to use your path. I am already on the  $2^k, 2^{2k}$ -route.

**Adonten**