CMB, Hawking, Planck and Hubble scale relations consistent wth recent Quantization of General Relativity Theory

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Abstract

We are demonstrating new relationships between the Hawking temperature, the CMB temperature, and the Planck scale. When comprehended at a deep level, this is in line with recent developments in the quantization of cosmology and its connection to the Planck scale. This is also entirely consistent with a recently published approach to quantizing Einstein's general theory of relativity.

1 Background on the Hawking Temperature and the New CMB Temperature Formula

Hawking introduced the Hawking temperature [1, 2] in 1974, which is given by:

$$T_{Hw} = \frac{\hbar g}{k_b 2\pi c}. \tag{1}$$

Where k_b is the Boltzman constant, and $\hbar$ is the reduced Planck constant also known as the Dirac constant ($\hbar = \frac{h}{2\pi}$). Further, g represents the gravitational acceleration at the horizon of a Schwarzschild [3] black hole and is defined as:

$$g = \frac{GM}{r_s^2} = \frac{GM}{\frac{(2GM)^2}{c^4}} = \frac{c^4}{4GM}. \tag{2}$$

By substituting this expression back into the original Hawking formula, we arrive at another well know way to express the Hawking temperature:

$$T_{Hw} = \frac{\hbar c^3}{k_b 8\pi GM}. \tag{3}$$

For the Hubble sphere, the critical Friedmann [4] mass is defined as :

$$M_c = \frac{c^2 R_H}{2G}. \tag{4}$$

Here, $R_H = \frac{c}{H_0}$ represents the Hubble radius. Solving this equation for the Hubble radius, we obtain $R_H = \frac{2GM_c}{c^2}$.

It's noteworthy that the Hubble radius is mathematically identical to the Schwarzschild radius $r_s = \frac{2GM}{c^2}$ when considering a critical universe. This similarity has led several researchers to speculate that we could be inside a gigantic black hole, as discussed by Patheria [5] and Stuckey [6]. This question continues to be a topic of discussion in recent papers [7, 8]. In this discussion, we will not argue for or against the universe being a black hole, but will follow the mathematics of a Hubble sphere with mass (equivalent energy) equal to the critical Friedmann mass. It's important to note that the equivalence between the Schwarzschild radius and the Hubble radius holds true only in a critical universe and not after the expansion of space. However, for the sake of our current discussion, we can replace M with M_c in the Hawking radiation formula and hypothetically treat the Hubble sphere as a black hole, resulting in the following Hawking temperature:

$$T_{Hw} = \frac{\hbar c^3}{k_b 8\pi G M_c} \approx 1.32 \times 10^{-30} k. \tag{5}$$

Next we will do a trivial re-writing of the Hawking temperature, but despite its triviality it will help s later understand some important relation between the cosmic scale, the Hawking temperature and the CMB temperature:

$$\begin{aligned} T_{Hw} &= \frac{\hbar c^3}{k_b 8\pi G \frac{M_c}{m_p} m_p} \\ T_{Hw} &= \frac{\hbar c^3}{k_b 8\pi G m_p} \frac{m_p}{M_c} \\ T_{Hw} &= \frac{\frac{\hbar c}{G} c^2}{k_b 8\pi m_p} \frac{m_p}{M_c} \end{aligned} \tag{6}$$

where m_p is the Planck [9, 10] mass, be aware $\frac{\hbar c}{G} = m_p^2$ so we get

$$T_{Hw} = \frac{m_p c^2}{k_b 8\pi} \frac{m_p}{M_c} \tag{7}$$

and the Planck temperature [9, 11] is given by $T_p = \sqrt{\frac{\hbar c^5}{G k_b^2}} = \frac{m_p c^2}{k_b}$ so we can re-write the equation above as

$$T_{Hw} = \frac{T_p}{8\pi} \frac{m_p}{M_c} \tag{8}$$

or as

$$\begin{aligned} T_{Hw} &= \frac{T_p}{8\pi} \frac{m_p}{\frac{c^2 R_H}{2G}} \\ T_{Hw} &= \frac{T_p}{8\pi} \frac{\frac{2Gm_p}{c^2}}{R_H} \\ T_{Hw} &= \frac{T_p}{8\pi} \frac{2l_p}{R_H}. \end{aligned} \tag{9}$$

Tatum et al. [12, 13] suggested that the temperature inside the Hubble sphere is given by a slightly modified Hawking temperature formula:

$$T_H = T_{CMB} = \frac{\hbar c^3}{k_b 8\pi G \sqrt{M_c m_p}} \approx 2.725 k. \tag{10}$$

Haug and Wojnow [14] have recently demonstrated that this temperature can indeed be derived from the Stefan-Boltzmann [15, 16] law. They also provide the formula:

$$T_{CMB} = T_H = \frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_H}} \approx 2.725k. \tag{11}$$

Equations 10 and 11 are identical from a deeper perspective. In a recent paper, Tatum, Haug, and Wojnow [17] have demonstrated that this new understanding of a deeper theoretical relationship between the CMB temperature and the Planck scale can be used in practice to significantly reduce the uncertainty in predictions of H_0, while fully taking into account uncertainty in input variables. We mention this not only because it has theoretical implications, but also because it leads to practical improvements, opening the door to a new area of high-precision cosmology, where R_H t_H and M_c can be predicted more accurately than every before. Part of the reason for this is that the precession in CMB temperature measurements and predictions have increased dramatically in recent years. For example, see [18–21]. Additionally, an exact mathematical relation between CMB temperature and the Hubble constant also plays an important role here.

However one should at the same time be aware that there are unsolved challenges like the Hubble tension [22, 23]. We will not try to resolve the Hubble tension in this paper, but we are mentioning this just to make us all humbly aware that there naturally could be changes to the foundation of cosmology that also potentially can affect the formulas that will be presented here.

In this paper, we will build on this foundation and introduce some very interesting relationships between Hawking temperature and the CMB temperature and the Planck scale and the large scale structures of cosmos.

2 Relationships between Hawking temperature, CMB, Planck scale, and the Hubble scale

Here, we will simply start by taking the square of T_{CMB} divided by the square of T_{Hw}. This gives:

$$\frac{T_{CMB}^2}{T_{Hw}^2} = \frac{\left(\frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_H}}\right)^2}{\left(\frac{T_p}{8\pi}\frac{2l_p}{R_H}\right)^2} = \frac{\frac{2l_p}{R_H}}{\frac{4l_p^2}{R_H^2}} = \frac{R_H}{2l_p}. \tag{12}$$

This means we must have:

$$R_H = 2l_p\frac{T_{CMB}^2}{T_{Hw}^2}. \tag{13}$$

Similarly, for the Hubble time, we get:

$$t_H = 2t_p\frac{T_{CMB}^2}{T_{Hw}^2} \tag{14}$$

where $t_p = \frac{l_p}{c}$ is the Planck time. Further for the critical mass, we get:

$$M_c = m_p\frac{T_{CMB}^2}{T_{Hw}^2} \tag{15}$$

and for the crtitical energy we get

$$M_c c^2 = E_p \frac{T_{CMB}^2}{T_{Hw}^2}. \tag{16}$$

And for the Hubble constant, we get:

$$H_0 = \frac{1}{2} f_p \frac{T_{Hw}^2}{T_{CMB}^2} \tag{17}$$

where $f_p = \frac{c}{l_p}$ is the Planck frequency. Further the entropy of the Hubble sphere with a critical mass is then

$$S_H = k_b 4\pi \frac{T_{CMB}^4}{T_{Hw}^4}. \tag{18}$$

That is we have established a meaningful relation between the Hawking temperature, the CMB temperature, the Planck scale, and the large scales of the cosmos (R_H, t_H, M_c). In all of these, we have the parameter $\frac{T_{CMB}^2}{T_{Hw}^2}$, so a natural question arises as to whether this factor provides new insights into cosmology. We claim that it does, but these insights may not be readily apparent on the surface. We need to delve deeper into quantum gravity and quantum cosmology to uncover their significance.

3 The Compton wavelength

Before we dig into quantum cosmology we need to shortly discuss the relation between the Compton wavelength and mass. Compton [24] gave the following formula for what today is know as the Compton wavelength:

$$\bar{\lambda} = \frac{\hbar}{m}. \tag{19}$$

If we solve the Compton [24] wavelength formul with respect to the mass we get

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c}. \tag{20}$$

That formula, as we have claimed in multiple papers, can be used to describe the kilogram mass in any context and even for the critical mass of the universe. Some may possibly protest here and say that the Compton wavelength is only related to electrons, as it was found indirectly through Compton scattering of electrons. First of all, there are also multiple papers on the potential Compton wavelength of the proton, as seen in [25] and [26]. It has been demonstrated in multiple papers [27, 28] that even composite masses can be described by equation 20. We believe that only elementary particles have a physical Compton wavelength, but composite masses have an aggregated Compton wavelength of the form:

$$\begin{aligned} m &= \sum_i^n m_i + \sum_i^j \frac{E_i}{c^2} \\ \frac{h}{\lambda} \frac{1}{c} &= \sum_{i=1}^n \frac{h}{\lambda_i} \frac{1}{c} + \sum_{j=1}^N \frac{h \frac{c}{\lambda_j}}{c^2} \\ \lambda &= \frac{1}{\sum_{i=1}^n \frac{1}{\lambda_i}} + \frac{1}{\sum_{j=1}^N \frac{1}{\lambda_j}}. \end{aligned} \tag{21}$$

Here, i indicates the different elementary particles making up the mass m, and j indicates the different energies contributing to m, such as binding energy. However, even pure energy can be seen as mass equivalent since we have $m = E/c^2$, so even pure energy can be treated in this way. For masses larger than the Planck mass, this means we will obtain an aggregated Compton wavelength smaller than the Planck length. Even if we consider the Planck length to be the smallest meaningful length of a physical Compton wavelength, this poses no issues because a Compton wavelength of a composite mass shorter than a Planck length is simply a mathematical aggregate useful for calculations, where none of the physical Compton wavelengths for elementary particles will be below the Planck length.

4 Finding the Planck length as well as the Compton wavelength of the critical mass from CMB and H_0

We do the following derivation starting out from equation 11

$$\begin{aligned}
T_{CMB} &= \frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_H}} \\
T_{CMB} &= \frac{\hbar c}{l_p k_b 8\pi}\sqrt{\frac{2l_p}{\frac{2l_p^2}{\bar{\lambda}_c}}} \\
T_{CMB} &= \frac{\hbar c}{l_p k_b 8\pi}\sqrt{\frac{\bar{\lambda}_c}{l_p}} \\
\frac{T_{CMB}^2 k_b^2 64\pi^2}{\hbar^2 c^2} &= \frac{\bar{\lambda}_c}{l_p^3} \\
\frac{l_p^3}{\bar{\lambda}_c} &= \frac{1}{T_{CMB}^2}\frac{\hbar^2 c^2}{k_b^2 64\pi^2}.
\end{aligned} \tag{22}$$

Furthermore, it has been demonstrated by Haug [29, 30] that the Hubble constant can be expressed as

$$H_0 = \frac{\bar{\lambda}_c c}{2l_p^2}. \tag{23}$$

This implies that the Planck length can be calculated as:

$$l_p = \frac{H_0}{T_{CMB}^2}\frac{\hbar^2 c}{k_b^2 32\pi^2}. \tag{24}$$

Both H_0 and T_{CMB} can be determined without any knowledge of G. We can find H_0 from cosmological red-shift [31]

$$H_0 \approx \frac{zc}{d} \tag{25}$$

where d is the distance to the object emitting light and z is the observed cosmological red-shift. This naturally mean we also have

$$l_p = \frac{z}{dT_{CMB}^2}\frac{\hbar^2 c^2}{k_b^2 32\pi^2}. \tag{26}$$

So, this clearly offers another method to find the Planck length independently of G from observations in the cosmos. This method is considerably simpler to implement in practice than the one we described by Haug [30] in 2022. This is not aimed at achieving a more precise measurement of the Planck length compared to existing methods, but it is of great importance as it clearly demonstrates that the Planck length must also be evident in cosmological observations, if not we could not extract it from there.

It is worth noting that as early as 1984, Cahill [32, 33] suggested simply to solve the Planck mass formula, $m_p = \sqrt{\frac{\hbar c}{G}}$, with respect to G and then expressed G from the Planck mass as $G = \frac{\hbar c}{m_p^2}$. However, in 1987, Cohen [34] that did a similar derivation also pointed out that this would lead to an unsolvable circular argument, as there was no known method at the time to find the Planck units independent of calculating them from G, $\hbar$, or c. As recently as 2016, in an interesting paper by McCulloch [35], he highlighted the circular problem. In 2017, Haug [36] was the first to publish a method for finding the Planck length independently of G, and multiple publications on this topic have appeared since then [27, 37].

Additionally, the reduced Compton wavelength of the critical mass can be determined from the CMB temperature and the Hubble constant.

$$\bar{\lambda}_c = \frac{H_0^3}{T_{CMB}^4} \frac{\hbar^4 c}{k_b^4 512\pi^4} \approx 3.79 \times 10^{-96}\ m \tag{27}$$

this is much smaller than the Planck length, but then this is not a physical Compton wavelength, but an aggregates of Compton wavelength in fundamental particles and energies making up the rest mass of the critical Friedmann mass M_c. We actually do not need to distinguish between energy and mass as energy is treated as res-mass equivalent according to $m = E/c^2$.

Alternatively we can also find the the reduced Compton wavelength from one cosmological red-shift observation plus the CMB temperature, this gives

$$\bar{\lambda}_c = \frac{z^3}{d^3 T_{CMB}^4} \frac{\hbar^4 c^4}{k_b^4 512\pi^4}. \tag{28}$$

That we can extract the Planck length and the reduced Compton wavelength directly from two cosmological observations is, in our view, much more than a coincidence. It means cosmology is ultimately linked to the Planck scale and Compton scale of matter and energy, something that will become much clearer in the next section.

5 The deeper meaning of the relation between Hawking, CMB and the Planck scale and Quantum gravity

For the critical mass of the universe we will use notation $\bar{\lambda}_c$ to point out it is the reduced Compton wavelength of the critical mass. Next we insert M_c into formula below:

$$\begin{aligned}
\frac{T_{CMB}^2}{T_{Hw}^2} &= \frac{R_H}{2l_p} \\
\frac{T_{CMB}^2}{T_{Hw}^2} &= \frac{\frac{c^2 R_H}{2G}}{\frac{c^2 l_p}{G}} \\
\frac{T_{CMB}^2}{T_{Hw}^2} &= \frac{M_c}{m_p} \\
\frac{T_{CMB}^2}{T_{Hw}^2} &= \frac{l_p}{\bar{\lambda}_c}.
\end{aligned} \tag{29}$$

and naturally we then also have that $\frac{T_{CMB}}{T_{Hw}} = \sqrt{\frac{l_p}{\bar{\lambda}_c}}$. Important here is that we can also find the reduced Compton wavelength of the critical Friedmann mass-energy without any knowledge of $\hbar$ or knowledge of the kilogram mass M_c, as demonstrated by Haug [30]. Also, the Planck length can be found independently of any knowledge of G, or c, as demonstrated in this paper as well as in [27, 37].

The last line of equation 29 is, in our view, a very important result as it demonstrates why $\frac{T_{CMB}^2}{T_{Hw}^2}$ represents at the deepest level and why, because what does $\frac{l_p}{\bar{\lambda}_c}$ represent? It is the reduced Compton frequency in the universe mass (energy) per Planck time. We [38] have recently demonstrated that the reduced Compton wavelength is even mathematically identical to the rest-mass energy photon wavelength so even energy can be treated this way as energy can be treated as rest-mass equivalent, as it is often done. This should also be seen in line with the fact that we have been able to quantize general relativity theory without altering any outputs from general relativity theory; see [39], where Einstein's [40, 41] field equation is re-written as:"

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi l_p^2}{\hbar c}T_{\mu\nu}. \tag{30}$$

This gives a Schwarzschild solution of:

$$ds^2 = -\left(1 - \frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)c^2dt^2 + \left(1 - \frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}dr^2 - r^2 g\Omega^2 \tag{31}$$

where $\bar{\lambda}_M$ is the reduced Compton wavelength of the mass M, and $g\Omega^2 = (d\theta^2 + \sin^2\theta d\phi^2)$. This provides exactly the same predictions as the standard Schwarzschild solution, but it offers a deeper insight in our view.

This factor $\frac{l_p}{\bar{\lambda}}$ then appears in every gravitational prediction derived from the theory of general relativity that can be empirically tested, as demonstrated in Table 2. This implies that we may have a comprehensive quantum gravity theory, along with its associated quantum cosmology. While this is a bold claim and should not be automatically accepted, we believe it merits sufficient attention from the physics community. Over time, multiple researchers can collectively assess whether this represents a breakthrough in our understanding of gravity and cosmology or not.

Table 1 summarizes the relationships between the Hubble scale, the Planck scale, and the factor $\frac{T_{CMB}^2}{T_{Hw}^2}$. In the rightmost column, we summarize that these formulas, when viewed from the deepest level, indicate that the Hubble scale is simply the reduced Compton frequency per Planck time $\frac{l_p}{\bar{\lambda}_c}$ multiplied by the various Planck units corresponding to the dimension we are examining within the Hubble sphere. The formulas in the far right column have independently been derived by an alternative approach related to the same quantum cosmology described in [42]. That we can get to the same formulas by starting out from different aspects in terms of observations etc. is in our view strengthening our view that this is a fully consistent theory.

Table 1: This table shows relationship between Hawking temperature and CMB temperature and the large scale properties of the Hubble sphere, in addition we show the deeper level of quantum cosmology.

Entity	From T_{Hw} and T_{CMB}	Deeper level
Hubble radius	$R_H = 2l_p \frac{T_{CMB}^2}{T_{Hw}^2}$	$R_H = 2l_p \frac{l_p}{\bar{\lambda}_c}$
Critical mass	$M_c = m_p \frac{T_{CMB}^2}{T_{Hw}^2}$	$M_c = m_p \frac{l_p}{\bar{\lambda}_c}$
Critical energy	$M_c c^2 = E_p \frac{T_{CMB}^2}{T_{Hw}^2}$	$M_c c^2 = E_p \frac{l_p}{\bar{\lambda}_c}$
Hubble time	$t_H = 2t_p \frac{T_{CMB}^2}{T_{Hw}^2}$	$t_H = 2t_p \frac{l_p}{\bar{\lambda}_c}$
Hubble constant	$H_0 = \frac{1}{2} f_p \frac{T_{Hw}^2}{T_{CMB}^2}$	$H_0 = \frac{1}{2} f_p \frac{\bar{\lambda}_c}{l_p}$
Hubble entropy	$S_H = k_b 4\pi \frac{T_{CMB}^4}{T_{Hw}^4}$	$S_H = k_b 4\pi \frac{l_p^2}{\bar{\lambda}_c^2}$

Table 2 shows standard gravity predictions derived from the quantized Planck form of general relativity theory, they give all the same predictions as general relativity theory. We show this to demonstrate that also in all these formulas the reduced Compton frequency per Planck time, $\frac{l_p}{\bar{\lambda}_M}$ appears. This is in our view the corner stone in quantization of gravity.

Table 2: The table shows a series of gravity predictions given by general relativity theory, that one get from the normal way to write the field equatiion, but also from the new quantized way to write the field equation. Again we see the term $\frac{l_p}{\bar{\lambda}}$ in every formula. This is the reduced Compton frequency in the mass M per Planck time.

Prediction	**Formula:**
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{c^2 l_p}{R^2} \frac{l_p}{\bar{\lambda}_M}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R}{c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}}$
Gravitational red shift	$z = \frac{\sqrt{1-\frac{2GM}{R_1 c^2}}}{\sqrt{1-\frac{2GM}{R_2 c^2}}} - 1 = \frac{\sqrt{1-\frac{2l_p}{R_1}\frac{l_p}{\bar{\lambda}_M}}}{\sqrt{1-\frac{2l_p}{R_2}\frac{l_p}{\bar{\lambda}_M}}} - 1$
Time dilation	$T_R = T_f\sqrt{1-\frac{2GM}{Rc^2}} = T_f\sqrt{1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Gravitational deflection (GR)	$\theta = \frac{4GM}{c^2 R} = 4\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}$
Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2} = \frac{6\pi l_p}{a(1-e^2)}\frac{l_p}{\bar{\lambda}_M}$
Schwarzschild radius	$R_s = \frac{2GM}{c^2} = 2l_p\frac{l_p}{\bar{\lambda}}$

6 A speculative approach to a possible solution for the vacuum catastrophe

The vacuum energy than what one estimate from observations is about 5.96×10^{-10} kg/m^3, see the Planck Collaboration [43]. On the other hand quantum field theory predict a vacuum energy of approximately

$$\rho_{vac} = \frac{m_p}{\frac{4}{3}\pi l_p^3} \approx 1.23 \times 10^{96} \text{ kg/m}^3, \tag{32}$$

thus over-estimating the vacuum energy, relative to the observational constraint by more than 120 orders of magnitude, see for example [44, 45]. This is well known as the vacuum catastrophe because the theory's predictions are extremely different from observations. Something is clearly not understood, and the question is, what could it be?

Entropy is related to how energy spreads out over time. In the previous section, we calculated the entropy of the Hubble sphere. If we simply take the vacuum field energy predicted by quantum field theory and divide it by the Hubble entropy (after removing the Boltzmann constant from the entropy to ensure compatible units), we get

$$\rho_{vac} = \frac{\frac{m_p}{\frac{4}{3}\pi l_p^3}}{S_H} k_b = 5.35 \times 10^{-27} \text{ kg/m}^3$$

This is very close to the estimated vacuum energy from observations, which is approximately correct. We acknowledge that our idea is somewhat ad-hoc, and we will not conclude that this is the final answer. Instead, we simply claim that this appears promising. There could be multiple ways to interpret this. One possibility is that the vacuum field has enough energy to cause Planck mass-sized micro black holes to appear and disappear. Another possibility is that this is somehow more closely linked to the Big Bang.

This gives a predicted Cosmological constant of

$$\Lambda = \kappa \rho_{vac} \approx \frac{8\pi G}{c^4} \times 5.35 \times 10^{-27} \text{ kg/m}^3 = 1.00 \times 10^{-52}\ m^{-2} \tag{34}$$

Again this section is somewhat speculative, but it should be interesting enough to be investigated further by other researchers.

7 More alternative ways to express the large scale properties of the universe

In this section, we also demonstrate alternative ways to rewrite the equations from the previous section, relying solely on the CMB temperature the Hawking temperature and the Planck scale, rather than the Hawking temprature. See also [13, 14, 17] as many of these formulas represented below can also simply be re-written from these. As for the Hubble constant, we have

$$H_0 = \frac{T_{CMB}^2}{T_p^2} \frac{32\pi^2}{t_p} \tag{35}$$

where $t_p = \frac{l_p}{c}$ is Planck time. Or we can alternatively write it from the Hawking temperature

$$H_0 = \frac{T_{CMB}^2}{T_p^2} \frac{t_p}{32\pi^2} \tag{36}$$

Table 3: This table shows relationship between Hawking temperature and CMB temperature and the large scale properties of the Hubble sphere, in addition we show the deeper level of quantum cosmology.

Entity	From T_{Hw} and T_{CMB}	From T_p and T_{CMB}	From T_p and T_{Hw}
Hubble radius	$R_H = 2l_p \frac{T_{CMB}^2}{T_{Hw}^2}$	$R_H = \frac{T_p^2}{T_{CMB}^2} \frac{l_p}{32\pi^2}$	$R_H = \frac{T_p^2}{T_{Hw}^2} \frac{\bar{\lambda}_c}{32\pi^2}.$
Critical mass	$M_c = m_p \frac{T_{CMB}^2}{T_{Hw}^2}$	$M_c = \frac{\hbar}{cl_p} \frac{T_p^2}{T_{CMB}^2} \frac{1}{64\pi^2}$	$\mathrm{M}_c = \frac{\hbar\bar{\lambda}_c}{cl_p^2} \frac{T_p^2}{T_{Hw}^2} \frac{1}{64\pi^2}$
Critical energy	$M_c c^2 = m_p c^2 \frac{T_{CMB}^2}{T_{Hw}^2}$	$M_c = \frac{\hbar}{cl_p} \frac{T_p^2}{T_{CMB}^2} \frac{1}{64\pi^2}$	$\mathrm{M}_c c^2 = \frac{\hbar c\bar{\lambda}_c}{l_p^2} \frac{T_p^2}{T_{Hw}^2} \frac{1}{64\pi^2}$
Hubble time	$t_H = 2t_p \frac{T_{CMB}^2}{T_{Hw}^2}$	$t_H = \frac{T_p^2}{T_{CMB}^2} \frac{t_p}{32\pi^2}$	$t_H = \frac{T_p^2}{T_{Hw}^2} \frac{\bar{\lambda}_c}{c32\pi^2}.$
Hubble constant	$H_0 = \frac{1}{2} f_p \frac{T_{Hw}^2}{T_{CMB}^2}$	$H_0 = \frac{T_{CMB}^2}{T_p^2} \frac{32\pi^2}{t_p}$	$H_0 = \frac{T_{Hw}^2}{T_p^2} \frac{c32\pi^2}{\bar{\lambda}_c}$
Hubble entropy	$S_H = k_b 4\pi \frac{T_{CMB}^4}{T_{Hw}^4}$	$S_H = k_b \frac{T_p^4}{T_{CMB}^4} \frac{1}{1024\pi^3}$	$S_H = k_b \frac{T_p^4}{T_{Hw}^4} \frac{1}{1024\pi^3} \frac{l_p^2}{\bar{\lambda}_c^2}$

and from the Hawking temperature

$$H_0 = \frac{T_{Hw}^2}{T_p^2} \frac{\bar{\lambda}_c}{c32\pi^2}. \tag{37}$$

For the Hubble time we have

$$t_H = \frac{T_p^2}{T_{CMB}^2} \frac{t_p}{32\pi^2}. \tag{38}$$

or from Hawking temperature

$$t_H = \frac{T_p^2}{T_{CMB}^2} \frac{\bar{\lambda}_c}{c32\pi^2}. \tag{39}$$

For the Hubble radius we have

$$R_H = \frac{T_p^2}{T_{CMB}^2} \frac{l_p}{32\pi^2}. \tag{40}$$

or from Hawking temperature

$$R_H = \frac{T_p^2}{T_{CMB}^2} \frac{\bar{\lambda}_c}{32\pi^2}. \tag{41}$$

For the critical mass we get

$$M_c = \frac{\hbar}{cl_p} \frac{T_p^2}{T_{CMB}^2} \frac{1}{64\pi^2}. \tag{42}$$

or from the Hawking temperature

$$M_c = \frac{\hbar\bar{\lambda}_c}{cl_p^2} \frac{T_p^2}{T_{Hw}^2} \frac{1}{64\pi^2}. \tag{43}$$

For Hubble entropy we get

$$S_H = k_b \frac{T_p^4}{T_{CMB}^4} \frac{1}{1024\pi^3}. \tag{44}$$

Table 3 sumarize the equations in this section. However we think it is important to be aware at the deepest level all these formulas represent what is describe in the most right column of table 1.

Hawking [1] derived his temperature consistent with the Schwarzschild [3] metric, this metric is consistent with the critical Friedmann solution, that is when $\Lambda = 0$. The full $\Lambda - CDM$ model naturally have a cosmological constant , still our model is not in conflict with this, it is simply that it appears that the Cosmological constant plays a less role for the CMB temperature. A reason for this could simply be that the CMB often is attributed as a footprint of the earlier stage of the universe.

8 Conclusion

We have demonstrated very simple relations between the Hubble sphere, the CMB, Hawking temperature, and the Planck scale. At the deepest level, we find that $\frac{T_{CMB}^2}{T_{Hw}^2} = \frac{l_p}{\lambda_c}$, which can be interpreted as the reduced Compton frequency of the critical mass and energy in the universe over the Planck time. All the large scale properties of the Hubble sphere are basically this frequency times the Planck unit with the same dimensions as we want to study in the Hubble sphere This is in full consistency with a recent reformulation of the theory of general relativity where also the reduced Compton frequency per Planck time in the gravity mass of interest play a central role. It appears that we have a quantum gravity theory that is fully coherent with quantum cosmology, linking the largest and smallest scales of the universe at the Planck scale. Further the Planck length can be extracted directly from cosmological observations with no knowledge of G.

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Declarations

Conflict of interest

The author declares no conflict of interest.

Availability of data and materials

The study contains no new data.