N=1 Supersymmetric Dual Quantum Field Model

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Abstract*: This paper introduces a supersymmetric dual-matter atomic model based on two intersecting fields that periodically vary in either the same or opposite phases, forming a shared nucleus of two transversal and two vertical subfields that represent the particles and antiparticles of the dual atomic nucleus. The bosonic or fermionic characteristics of the nuclear subfields are determined by their topological transformations, caused by the pushing forces generated by the negative or positive curvatures of the intersecting fields during their contraction or expansion.*

With a mainly visual and conceptual approach, the model employs a set of 2x2 complex rotational matrices of eigenvectors related in a modular way to Sobolev interpolations and to Tomita-Takesaki theory, illustrating problems as reflection positivity, the mass gap, or the arising of a purely imaginary time, between others.

The article first presents the fields model in a general way, then it introduces the mathematical formalisms, translates the general system to the atomic terminology, and finally compares the model with already known developments.

Keywords: Mass gap; reflection positivity; de Sitter vacuum; anti de Sitter; zero-point energy; Sobolev interpolation; supersymmetry; supersymmetric quarks; Dirac spinors; gauge transformations; rotational gauge transformations; 2x2 rotational matrix; Hermitian; fractional derivatives; complex and conjugate functions; mirror symmetry; antisymmetry; Pauly exclusion; superposition; entanglement; two-time dimensions; Calabi-Yau manifolds; elliptic fibrations; string theories; Ramond Ramond field; conformal theory; dark energy; Lobachevsky; Fourier; convolution; involution; Wick rotation; Minkowski; Tomita-Takesaki; modular matrices; polar decomposition; Langlands dual group; Cartan-Killing pairing; Bethe matrices; Higgs bundle; exotic matter; positronium; protonium; Wey semimetals; hidden sector; dark matter; Hitchin theory; Hyperkahler surfaces; Kummer quartic; Redox; acid-base reactions; Riemann-Silberstein vector.

1. Introduction.

The proposed dual atomic model is based on two intersecting fields vibrating with same or opposite phases that synchronize and desynchronize periodically. Their intertwinement gives rise to a composite manifold of 2 vertical subfields – one in the concave side of the intersection and another in its convex side – and two ½ handed transversal subfields – left and right.

The shape, mass, charge, inner kinetic energy, and spatial displacements of the 4 subfields will be determined by the pushing forces that the negative or positive curvatures of the intersecting fields generate while contracting or expanding respectively.

First, we are going to introduce the fields model in a general way, to later translate it to the quantum mechanics terminology.

1.1 Antisymmetric system.

When the phases of vibration of the intersecting fields are opposite, the left and right-handed transversal subfields will be mirror antisymmetric at the same time: when the left transversal subfield expands the right one will contract and conversely. They will be also noncommutative subfields at the same time.

The top vertical subfield will move right to left, getting a negative sign, or left to right, getting a positive sign, towards the side of the intersecting field that contracts; moving in a pendular way left or right, this vertical subfield will be its own anti-subfield at different times.

The mirror transversal subfields are characterized by an antiphase relationship with each other, yet each of them maintains phase coherence with the intersecting space that encompasses it."

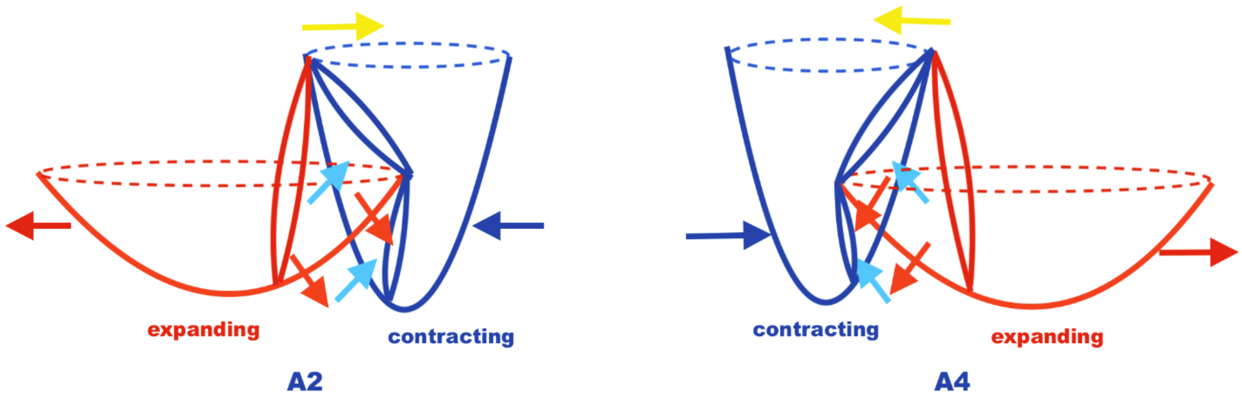

Fig. 1. Antisymmetric system vibrating with opposite phases

Fig. 1 shows the antisymmetric system in two different moments. In the first moment represented by A2, the left and right transversal subfields are mirror antisymmetric – when one contracts the other one expands - and the top vertical subfield moves towards right. A moment later represented by A4, the left and right transversal subfields remain antisymmetric, having commuted their expanding or contracting states, and the top vertical subfield moves towards left.

The left transversal subfield at moment A2 is mirror symmetric of the right transversal subfield at moment A4. And the top vertical subfield moving right at A2 is mirror symmetric of the top vertical subfield moving left at A4. (When the top vertical subfield moves towards right at A2, it can be considered it exists in a virtual way also at A2, in the sense it has the potential of effectively existing a moment later (at A4) moving towards left).

Owing to their mirror anti-symmetry, the states of the left and right transversal subfields at moment A2 (or at moment A4) are mutually exclusive: when the left transversal subfield contracts, thereby increasing its density and inner orbital kinetic energy, the right-handed transversal subfield will expand, leading to a decrease in its density and inner kinetic energy.

In this sense, their states can be said to be governed by an “Exclusive” principle.

The opposite states of the left and right transversal subfields are not superposed because, being mirror reflective subfields, they are different subspaces that reflect each other with a delayed or advanced phase of time.

Within the framework of a dual composite system such as the one proposed in this model, both “superposition” and “exclusion” must be interpreted in terms of mirror symmetry or anti-symmetry.

1.2 Symmetric system.

In contrast, when the phases of vibration of the intersecting fields are equal, the transversal subfields will exhibit mirror symmetry simultaneously. Their states will be “entangled”. Sharing the same phase, they will show a phase opposition relative to the intersecting fields.

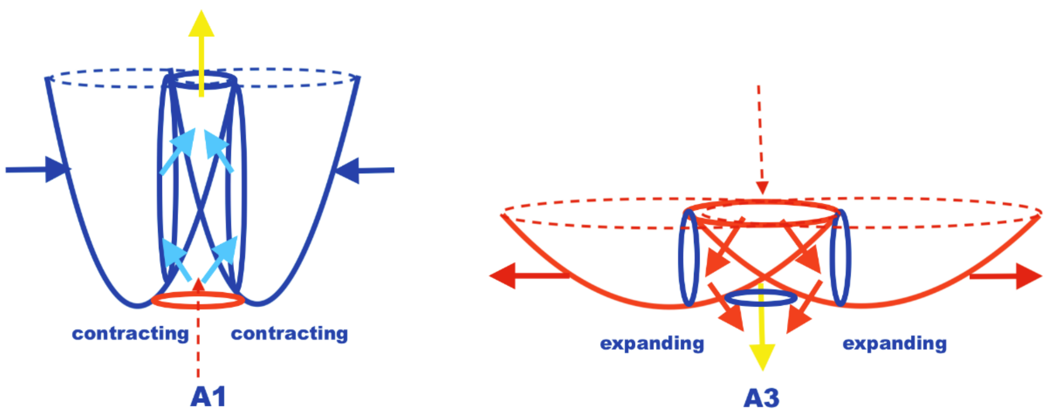

Fig. 2. Symmetric system vibrating with same phases

Once the system exhibits mirror symmetry, the top vertical subfield aligns with the phase of the intersecting fields: when both intersecting fields contract, the top vertical subfield also contracts, ascending upwards while emitting a pulsating force.

Subsequently, when both intersecting fields expand, the previously ascending subfield will decay while expanding. When such a decay occurs, an inverted pushing force is generated on the convex side of the system by the positive (or convex) curvatures of the expanding intersecting fields.

For a detector placed in the concave side of the system, the mass and energy that occurs in the convex side of the intersection of the curved fields will be "dark" as directly undetectable.

1.3 Vectors of force and types of interactions.

The pushing forces created by the contracting or expanding fields, with their inner negative curvatures or their outer positive curvatures, can be represented in a two-dimensional frame by means of 4 vectors.

Two vectors compressing a subfield imply a stronger force experienced by the contracting subfield whose volume decreases while its density increases, and its inner orbital motion experiences an acceleration or boost. The increased inner kinetic energy represents a greater bond that intertwines the intersecting fields in a stronger way.

Two vectors decompressing a subfield represent a weaker force experienced by the subfield, which expands in volume, decreases in density, and decelerates in inner kinetic energy. The decreased inner kinetic energy implies a weaker bond between the two intersecting fields.

In that sense, we can speak about a strong and a weak interaction between the two intersecting fields that let the subfields of their shared nucleus remain united with a greater or weaker bond.

Alongside these strong and weak interactions, we are going to consider electric to the pushing force caused by the displacement of the vertical subspace when moving left or right in the antisymmetric system, and upwards or downwards in the symmetric system, and magnetic to their inner orbital motions.

We consider gravitational the curvatures of the intersecting fields as they determine the mass and energy of their related subspaces.

Additionally, when considering the vibration of the intersecting fields, it can be noted that the outer force of pressure produced by the positive curvature of an expanding field will be weaker than the inner pushing force caused by the negative curvature of a contracting field, because the density of the expanding field will be lower than the density of the contracting field, experiencing a different rate of resistance while expanding or contracting.

2. Mathematical formalisms.

2.1 Rotational 2x2 complex matrices

The symmetric and antisymmetric manifolds can be considered either as two separate and independent systems, as two systems related by supersymmetric partners, or as two topological systems that are periodically transformed into each other by the periodical synchronization and desynchronization of the phases of vibration of the intersecting fields, forming a supersymmetric manifold. Here we only consider the latter case.

The system gets an additional complexity if it is a rotational structure. Let’s examine the rotation of the system around its axis by means of a group of complex 2x2 rotational matrices with a 90-degree rotation operator.

The four elements of the matrices are visually represented as eigenvectors, with eigenvalue 1 or –1, that reverse their directions when rotating the complex plane.

The four vectors actually change their position each time the whole plane rotates 90-degrees, but we can only distinguish that they have changed their direction. Such a change occurs when they commute their sign, which implies a multiplication of the eigenvector magnitude by 1 or –1, being flipped or reflected across its origin, or being permuted 180 degrees.

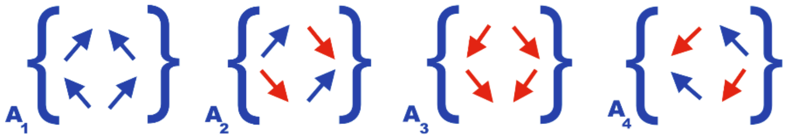

Fig. 3. Set of transformation matrices of eigenvectors

Fig. 3 shows the collection of the transformation matrices, that result when rotating 90º the complex plane four times, performing the operations of transposition, inversion and complex conjugation.

The identity matrix A1 represents the position of the eigenvectors of the symmetric system when the two intersecting fields, in phase, simultaneously contract.

- Rotating A1 by 90 degrees gives us the transpose matrix A2, whose eigenvectors represent the forces of pressure in the antisymmetric system when the left intersecting field expands and the right one contracts. A2 is also the partial conjugation of A1

- Rotating A2 by 90 degrees gives us the negative of A1, or A3. The A3 eigenvectors represent the forces of pressure in the symmetric system when the two intersecting fields expand with the same phase.
- Rotating A3 by 90 degrees gives us the transpose of A3, or A4. The A4 eigenvectors represent the forces of pressure in the antisymmetric system when the left intersecting field contracts and the right one expands. A4 is also the negative of A2 and the second partial conjugation of A1.
- Completing a 360-degree rotation by rotating A4 by 90 degrees gives us A1, which represents the initial situation when both intersecting fields simultaneously contract.

2.2 Fractional differentiation.

The mentioned matrix operations can also be expressed in terms of differential equations.

That can be visually represented by considering the eigenvectors as being tangent to a point of a symbolic unit circle of radius 1.

The slope of the tangent eigenvectors will represent a derivative, or an antiderivative, related to the complex function represented by A1 and A3 matrices, or related to the conjugate function represented by A2 and A4 matrices.

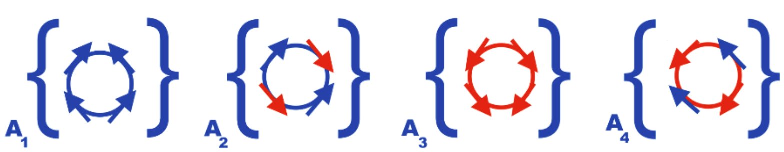

Fig. 4. Eigenvectors as tangent slopes of a unit circle in the rotational matrices

Each 90º rotation only changes the sign of two eigenvectors with respect to the previous matrix. In that sense, the transposition performed by A2 represents a fractional number of derivatives, as only ½ of the eigenvectors – the ones related to its main diagonal –have commuted their sign with respect to A1.

A2 is a partial ½ conjugate transformation of the complex matrix A1.

If the commuted eigenvectors represent the spin of the subfields in the antisymmetric system related to matrix A2, those subfields will have a noninteger spin with respect to A1.

In this case, their mirror counterparts will be governed by an exclusion principle.

A3, the negative reflection of A1, represents an integer number of derivatives with respect to A1, as four of four eigenvectors have commuted their sign, although it only encodes a fractional number of derivatives ½ with respect to A2, the ones related to the second diagonal (upper left and bottom right eigenvectors) with eigenvalue -1.

Therefore, to obtain the complete first order derivation of A1, it is necessary to rotate the matrix 90 degrees twice, having previously performed a partial conjugation of A1.

If the commuted eigenvectors represent the spin of the subfields in the symmetric system related to matrix A3, those subfields will have an integer spin with respect to A1.

In this case, the transversal mirror symmetric subfields are not governed by an exclusion principle.

However, the top vertical subfield with integer spin and its reflection counterpart located at the convex side of the system still will be ruled by an exclusion principle because when the top subfield contracts at the concave side of the manifold, its mirror reflection subfield will expand at the convex side.

A4 encodes two positive eigenvectors with eigenvalue +1. They represent ½ number of antiderivatives with respect to A3.

A4 also represents the whole first order derivative of A2, and A2 and A4 together form a whole conjugation with respect to A1.

Therefore, to obtain the complete conjugation of A1, it is necessary to rotate the matrix 90 degrees three times.

A1 represents the first order antiderivative of A3 and the half-antiderivative of A4.

Therefore, to revert A1, the matrix must be rotated 90 degrees four times.

The rotational dynamic of the eigenvectors represented in this group of complex matrices, seems to imply that the smooth evolution of the symmetric system represented by A1 (when the intersecting fields contract) and A3 (when a moment later the intersecting fields expand), loses its linear continuity by being interpolated in between of A2 and A4.

In this sense, if the symmetric system is described by a complex ordinary differential equation and the antisymmetric system is described by the conjugate solution of the differential equation, then those separate equations can only describe the evolution of the physical states and displacements of half of the dual system.

In that case, the system may be incompletely described and could only be defined by statistical methods.

2.3 Rotational interpolation.

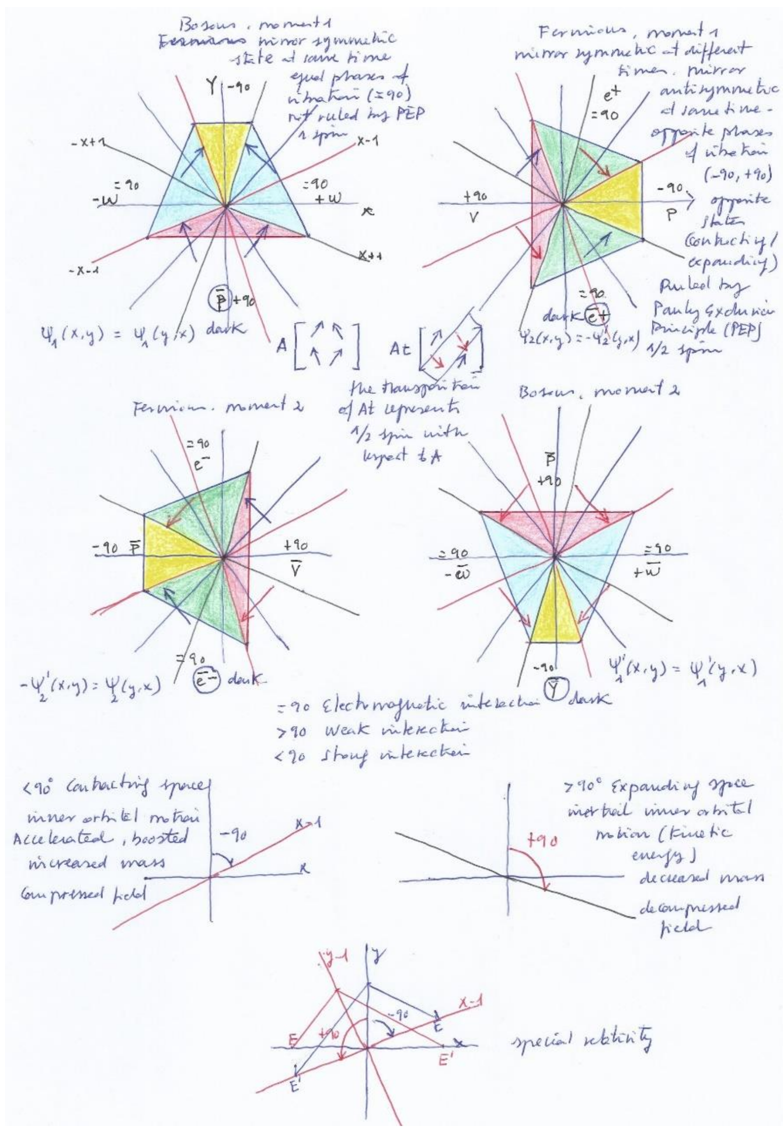

Fig. 5. Rotational system, visual representation

Diagram, Fig. 5, represents the fractional commutation of the eigenvectors embedded in the rotational nucleus of subfields shared by the intersecting fields.

The subfields in the picture would change their shape while the 90-degree rotation is performed, as they contract or expand, move left or right, or ascend or descend – because of the vibration of the intersecting fields – while the system rotates.

However, we will discuss later the conformal or nonconformal nature of the model.

The interpolation between the symmetric and the antisymmetric systems may be related to Sobolev interpolations [1], where “spaces of functions that have a noninteger number of derivatives are interpolated from the spaces of functions with integer number of derivatives”.

In the antisymmetric conjugate system, Sobolev embedding may be represented by the right contracting transversal subfield at moment t (matrix A2), being virtually embedded inside of the right expanding transversal subfield at moment t’ (matrix A4):

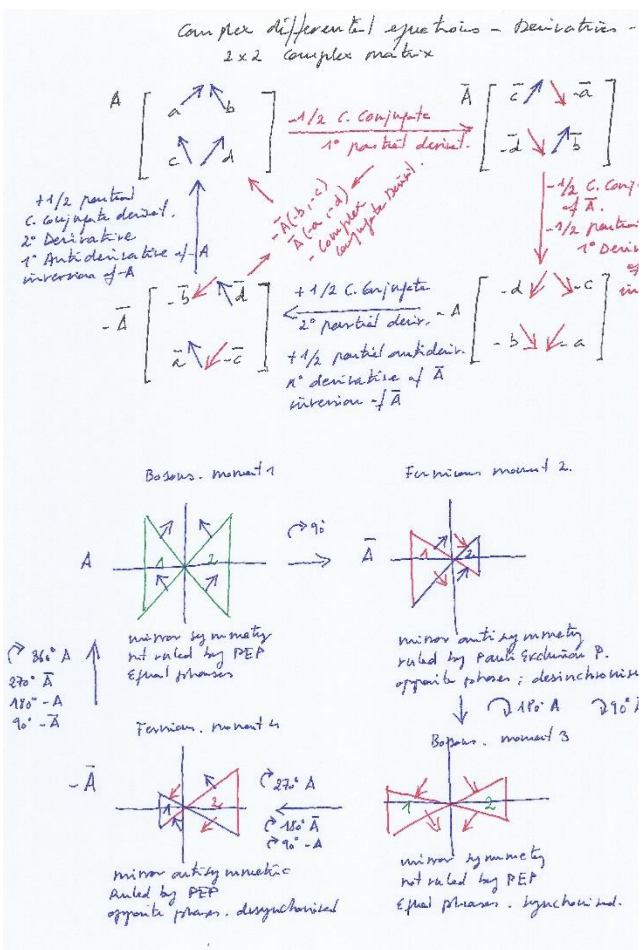

Fig. 6. Matrices interpolation

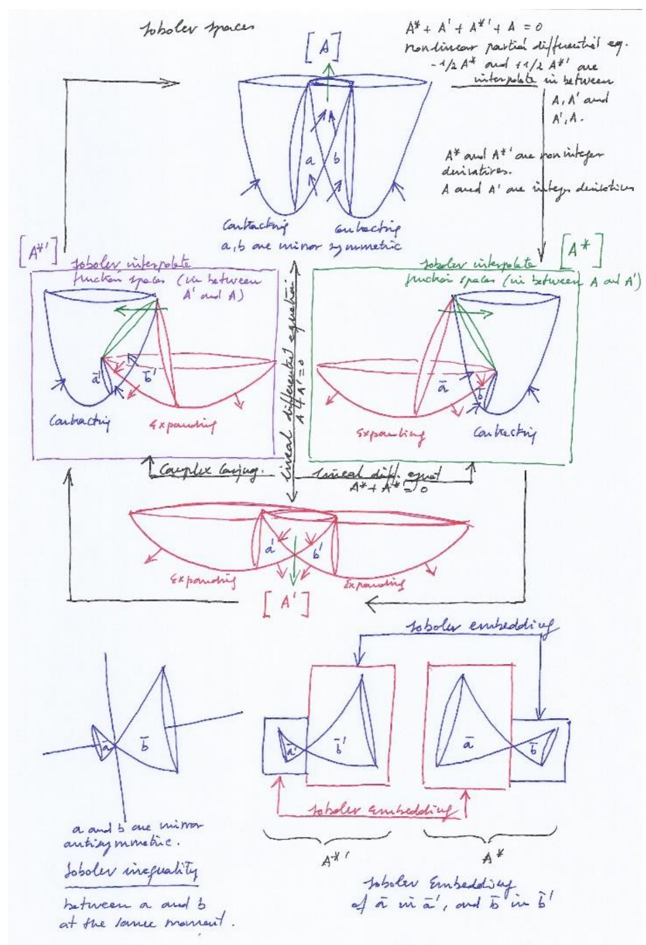

Fig. 7 Sobolev function spaces interpolation

2.4 Convolution.

The combination of the complex ordinary function of four complex variables, represented by matrices A1 and A3, and the complex conjugate function represented by matrices A2 and A4, can be also described as a convolution [2].

Adding the products of the four transformation matrices, their fractional derivatives or antiderivatives that represent partial conjugations of the previous state, the identity matrix that represents the complex function is obtained.

2.5 Harmonic functions.

The conjugate function given by A2 and A4 is a conjugate harmonic of the complex function, and vice versa. The complex and the conjugate function are, in the context of the rotational system, interdependent and cannot exist without each other.

Antisymmetry arises in the conjugate system of A2 and A4 by introducing a change of phase in one of the sides of the reflection, while the other side keeps following the unchanged phase of A1 and A3. That change of phase can occur by a gradual desynchronization, or suddenly, when a rotation occurs changing the sign of ½ eigenvectors.

The addition of the main and harmonic phases can be performed with a Fourier transform [3].

2.6 Bäcklund transformations.

The constructive interdependence of the complex and conjugate functions represented by the mentioned complex A1, A3 and conjugate A2, A4 matrices respectively can be interpreted as well as Bäcklund transformations, where the conjugate function transforms the complex function and vice versa.

The prototypical example of a Bäcklund transform [4] is the Cauchy-Reimann system, where "a Bäcklund transformation of a harmonic function is just a conjugate harmonic function".

2.7 Operator algebras.

The symmetric and antisymmetric systems can also be described as two independent groups of cyclic eigenvectors that form two von Neumann algebras: an antisymmetric automorphic algebra and a symmetric automorphic algebra, which imply antisymmetric and symmetric mirror reflection algebras, respectively.

However, both independent algebras can be related by means of modular combinations, which, in the context of our rotational matrices and interpolated functions, are the combinations of the transform matrices whose operations represent fractional derivatives or antiderivatives.

Modular combinations of von Neumann algebras are studied by Tomita-Takesaki (TT) theory [5].

In TT theory two intersecting algebras form two shared “modular inclusions” (with + - “half sided” subalgebras) and a “modular intersection” (with an “integer sided” subalgebra).

The left and right half handed subalgebras will be the image of each other, when they are commutative, or they will not be their image when they are noncommutative. Mapping the modular inclusion to its reflection image, the left and right subalgebras will be the opposite image of each other (reverting their initial signs) if they are commutative; if they are noncommutative, the initial left sided subalgebra will be the image of the right sided mapped subalgebra, and the initial right-handed subalgebra will be the image of the left sided mapped subalgebra.

TT theory decomposes a linear transformation into its modular building blocks, showing the automorphisms.

Decomposing the bounded operator, it obtains the modular operator and the modular conjugation (or modular involution) which is a transformation that reverses the orientation, preserving distances and angles.

Translating the abstract algebraic terms to our present model, we can say that the two intersecting algebras represent our two intersecting fields vibrating with same or opposite phase.

The half handed subalgebras (or “modular inclusions”) will be the transversal subfields of the nucleus shared by the intersecting fields, while the integer handed subalgebra (or “intersection inclusion”) will be our vertical subfields. In this context, we identify commutativity and noncommutativity with mirror symmetry and mirror antisymmetry, respectively.

The bounded operator that is decomposed will be the 90 degrees rotational matrix; The modular building blocks are the set of matrices that are obtained when applying the operator. The modular operator will be the ½ partial conjugate A2 matrix; And the modular conjugation will be the conjugate matrix A4 (which forms the whole conjugation by adding the fractional conjugations ½+½).

So, separating the conjugate matrix from the complex one the automorphism of the antisymmetric conjugate system is found.

The half sided algebras that form a modular inclusion are noncommutative, it means we are in the antisymmetric system where the left intersecting field contracts while the right one contracts and vice versa; in that system, the left transversal subfield will be the mirror symmetric image (it will be the mapped image) of the right transversal subfield when, later, the left intersecting field expands and the right one contracts.

In that sense we are mapping here a past half handed subalgebra with its future image. A time delay will exist between both subalgebras.

Considering Δ as the modular operator A2, J the modular conjugation A4, and M the intersection of two Von Newmann algebras, $\Delta^{-Yt} M \Delta^{it}$ will represent the positive and negative ½ sided modular inclusions of the modular operator, being t a real time dimension and it an imaginary time dimension given by the partial conjugation of A1 or A3.

It is this different time dimension what makes noncommutative, as non-interchangeable, the modular + and - inclusions related to Δ in the antisymmetric system. We will speak later about the problem of time in a more detailed way.

Applying the modular involution, we have $J^{yt}\ M'\ J^{-it}$.

Δ^{-yt} is transformed into J^{yt} and Δ^{it} is transformed into $J^{-it'}$, being $J^{yt}\ M'\ J^{-it}$ the involutive automorphism of $\Delta^{-yt}\ M\ \Delta^{it}$.

The noncommutative, as non-interchangeable, Δ^{-yt} and Δ^{it} become commutative or interchangeable through time at $J^{yt}\ M'\ J^{-it}$, fixing in that way their antisymmetry.

The same type of operations can be made taking A2 as the identity matrix. In that case, rotating clockwise, A3 would be the modular operator and A1 the modular conjugate automorphism.

2.8 Vertex operators.

Among the numerous mathematical developments relevant to the intersecting model, we can also highlight the vertex operators' formalism [6], where fields are inserted at specific locations of a two-dimensional space.

In our model, the vertex point would be given by the point of intersection between the left and right intersecting fields.

That intersection point would represent a unified coupling gauge point, which will be displaced upwards or downwards in the symmetric system or leftwards or rightwards in the antisymmetric system."

2.9 Reflection positivity.

Related to this delay of time in the antisymmetric system, we can also mention a property that all unitary quantum field theories are expected to hold: "Reflection positivity" (RP). [7]

The positive increasing energy that appears on one side of the mirror system should be reflected as well in the other side. But in the context of the antisymmetric system, we find that positive energy of the contracting right transversal subfield is not simultaneously reflected in the expanding left transversal subfield, which has a negative energy.

Because of that, to obtain a positive energy reflected at the left side, making the sides of the system virtually symmetric, a time reverse operation will be needed.

To see the positive energy reflected at the left side we will need to go back in time to the moment where the left transversal subfield was contracting having a positive energy. That operation is performed by a type of "Wick rotation". [8]

We can represent the main time phase of the symmetric system with the Y coordinate.

Performing from there a partial conjugation that involves a fractional derivative, the time coordinate rotates to the complex plane. At that moment the mirror system becomes antisymmetric as one side of the system keeps following the imaginary time of Y while the other side follows a harmonic phase. A positive or negative time lag has been introduced.

To go back in time on side of the system is a way of virtually making the time phases symmetric again. To achieve that time reverse we make a reverse rotation of the complex time axis X+iY to complete a whole complex conjugation at –X –IY.

That time backwards rotation represents an antiderivative. (If it were forward it would represent a derivative).

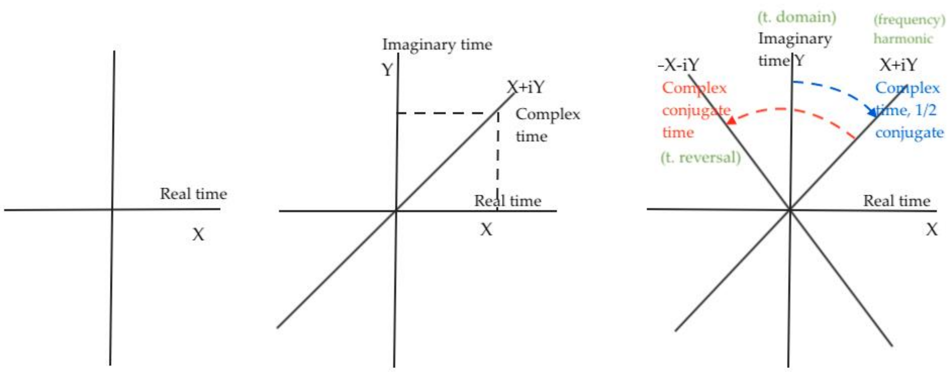

Fig. 8. Rotational time backwards and forwards

When the time reverse has been symbolically completed, we will presence in the left side of the mirror system how the left subfield contracts having an increased positive energy; it is a past reflection of the future positive energy that there will be a moment later in right side.

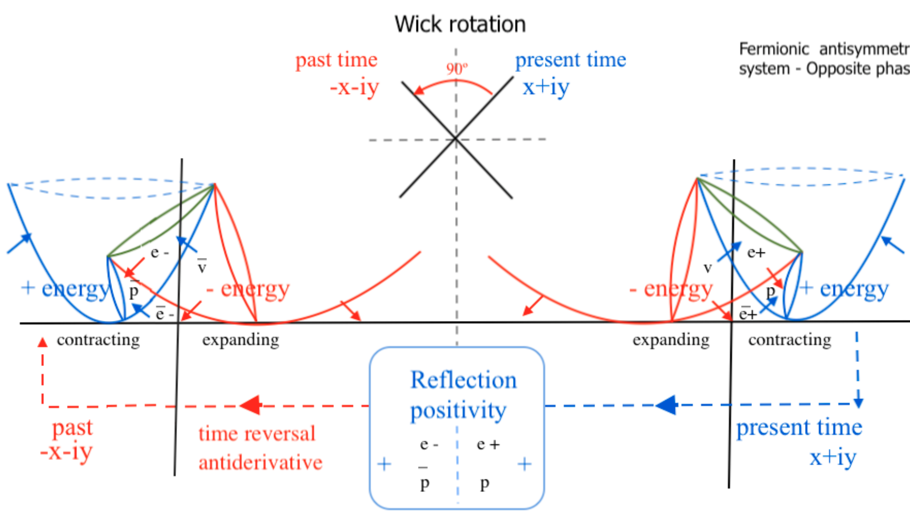

Fig. 9. Reflection positivity in the antisymmetric system

In this past time, at the right side of the system the right subfield will be expanding having a decreased negative energy.

When it comes to the symmetric system, positivity is reflected between the right and left transversal subfields at same time. In that sense, it's not necessary to use the Wick operation to reverse time. Both left and right transversal subfields will be the mirror reflection of each other at the same time.

However, in the case of the strong interaction in the symmetric system, when the contracting vertical subfield has an increased positive energy while ascending to emit a pushing force, it will be necessary to virtually visit a past moment to look for a previous state where positivity could be reflected.

Going back in time, we will find that the vertical subfield is losing its energy while expanding and moving downwards. So, at that moment of time the vertical subfield will not display a positive energy.

Reflection positivity, however, can be found at that past moment in the convex side of the system of the two intersecting fields, where an inverted subfield with convex curvatures will be experiencing an increased energy.

That inverted subfield can be mirroring related to the vertical subfield that in a future state will be ascending in the concave side of the system.

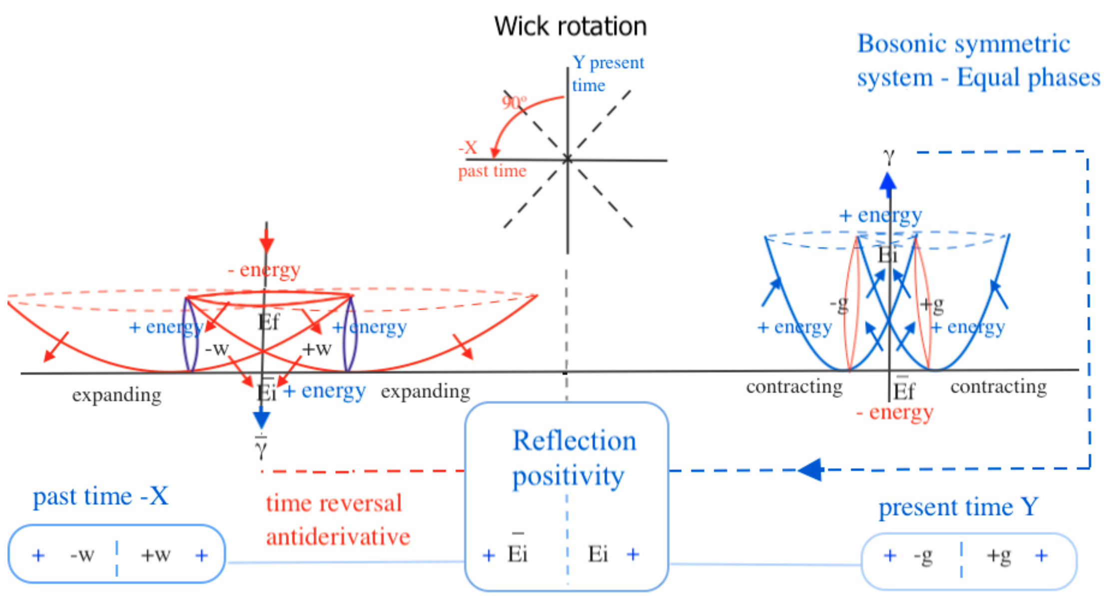

Fig. 10. Reflection positivity in the symmetric system

The missing reflection positivity in the concave side of the system in the strong interaction can be related to a mass gap problem when it comes to the weak interaction.

2.10 Mass gap problem.

There will be a mass gap [9] in the system when the two intersecting fields simultaneously expand, and the vertical subfield experiences a decay of energy.

This case represents the ground state with the lowest possible energy of the vertical subfield, which is going to be always greater than 0 because the highest rate of expansion of the intersecting fields prevents them from having zero curvature.

The zero point of the vacuum, where there should be no energy nor mass, is placed at the point of intersection of the XY coordinates, and that point is never reached by the descending and expanding subfield that decays.

An "upper" mass gap would be referred to the highest possible mass of a particle in the strong interaction. Its limit would be given by the greatest rate of contraction of the intersecting spaces.

Fig. 11 represents graphically the mass gap in the symmetric system; in a similar way it can be represented for the antisymmetric system, related to the transversal decompressed subfield (for the lower gap) and the transversal compressed subfield (for the upper gap):

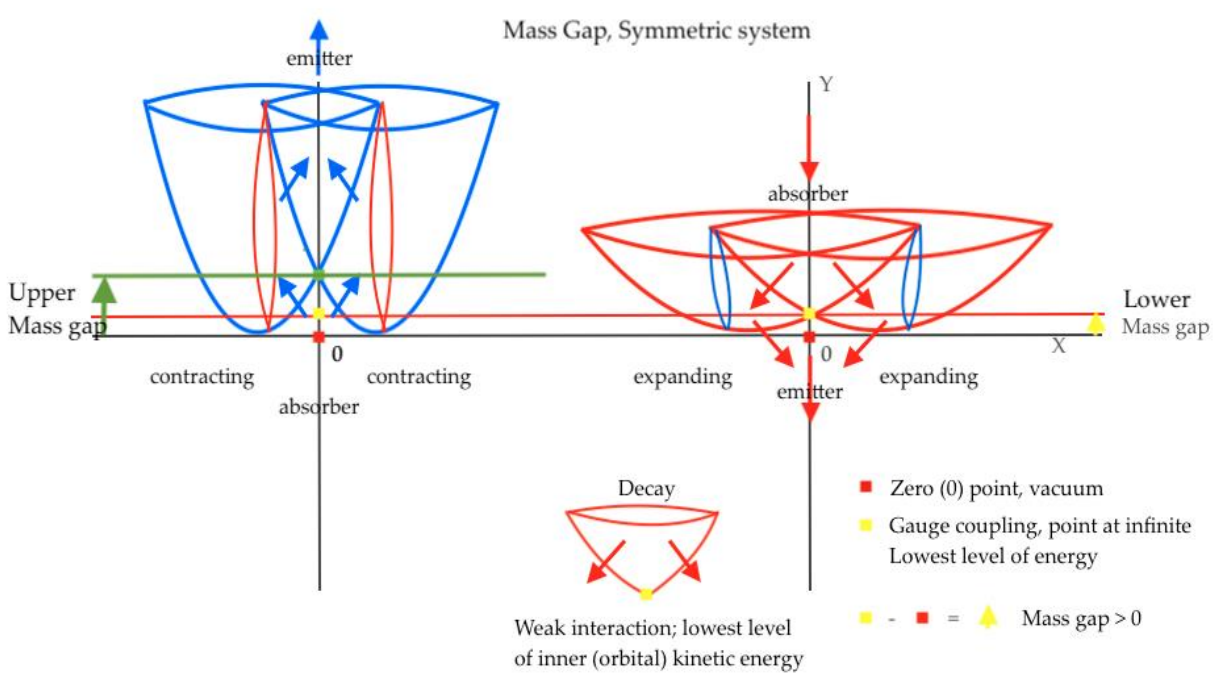

Fig. 11. Mass gap in the symmetric system

The zero point is represented with a yellow point on the above diagram when the intersecting fields expand.

The bottom of the descending subfield with the weakest level of density and inner kinetic energy is represented with a black point. An arrow points out the distance between those critical points.

However, in this model the zero point does not represent a vacuum place where no energy and mass exist.

The zero point at the moment of the weakest energy in the convex side of the symmetric system is where a double pushing force caused by the positive curvature of the expanding intersecting fields arises.

That inverted "dark" pushing force will be equivalent to the force lost by the decaying subfield.

In the antisymmetric system, the lowest energy level occurs when a transversal subfield experiences a double decompression due to the displacement of the concave curvature of the contracting intersecting field and the displacement of the positive curvature of the expanding intersecting field.

The corresponding double compression is then experienced by its mirror antisymmetric transversal subfield.

Fig. 12. Represents visually the map gap in the antisymmetric system, with the left and right displacements of the point of intersection:

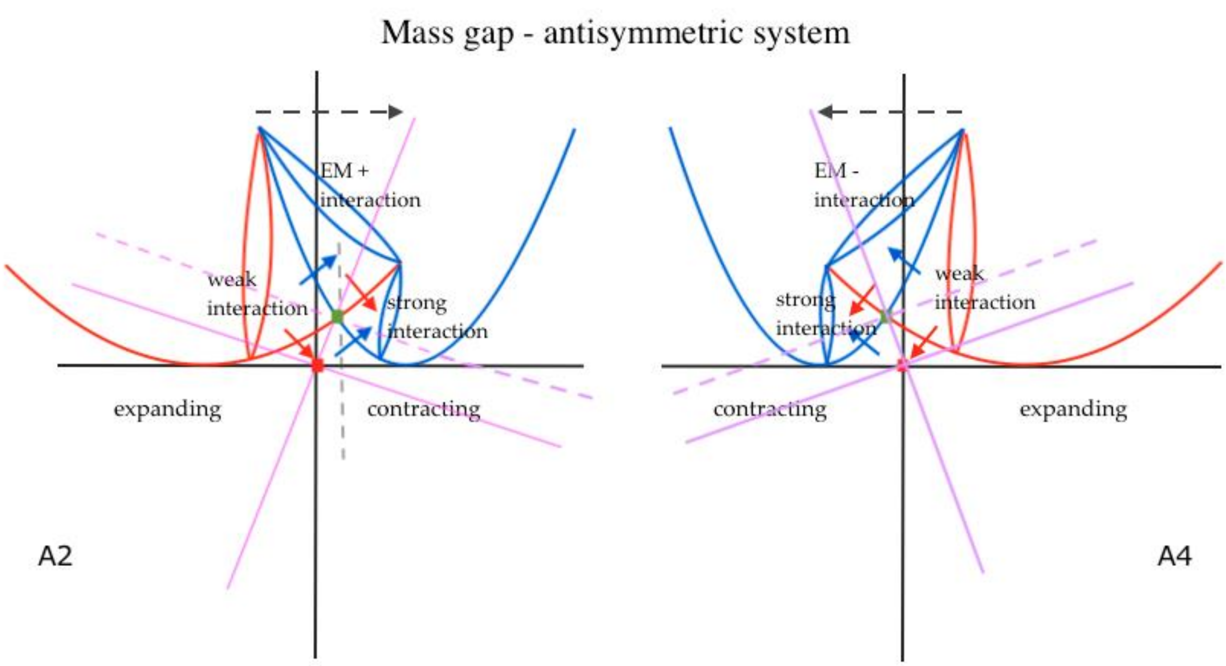

Fig. 12. Mass gap in the antisymmetric system

2.11 Representation theory.

In the context of representation theory, we may also interpret the antisymmetric system of subspaces represented by the matrix A2 as an original vector space, and its negative mirror reflection matrix A4 as a dual vector space.

To connect A2 and A4, forming their isometric automorphism through time, it's necessary to pass through A1 (the identity matrix) and A3 by means of the fractional differentiation or partial conjugation given by the transformation matrices when applying the 90 degrees rotational operator.

The bridge that fixes the gap between A2 and A4 will be related to the Langlands parametrization.

The Langlands dual group that allows the creation of the automorphism between the subspaces of the antisymmetric system will be represented by the conjugate matrices A2 (90º) and A4 (270º), and the complex matrices A1 (0º) and A3 (180º).

The two eigenvectors that determine the curvature of each transversal subspace and the sign commutation of ½ of each pair that causes the transformations of the system would be related to Cartan-Killing pairing.

Additionally, considering as an example the antisymmetric system where the left transversal subspace L contracts at t1 while the right transversal subspace R expands at t2, and the left transversal subspace expands at t1' while the right one contracts at t2', it can be suggested that the expanding Lt1' will be the functional Galois extension of Lt1, and Rt2 will be the functional Galois extension of Rt2', connecting the intersecting fields model as well with the Langlands program.

The collection of 2x2 complex matrices may also be related to Bethe [10] transfer matrices of eigenvectors related to complex and conjugate functions in the context of quantum integrable systems.

The unit circle mentioned at the beginning may be interpreted as a flattened version of the unit sphere, which is a visual way of representing geometrically the rotations of nuclear spins in ½ particles.

That unit sphere is also related to Bloch theory. [11]

2.12 The role of pictures in Klein's research.

On the other hand, it can be interesting to mention that "a significant role in Klein's research" - when he developed the geometric theory of automorphic functions, combining Galois and Riemann ideas - "was played by pictures" related to "transformation groups (linear fractional transformations of a complex variable)". [12]

Klein's hand-drawn diagram [13] related to elliptic modular functions is the same two-dimensional figure of four subspaces that we have seen forms the cobordant nucleus shared by the two intersecting fields.

The left and right transversal subspaces show half convex and half concave curvatures, the top vertical subspace if formed by a concave curvature, and the vertical inverted subspace exhibits a double convex curvature.

3. Conceptual translation to a quantum mechanics model.

Once this model has been presented in a general way, we will try to describe it in terms of an atomic nucleus using a hypothetical approach that follows the symmetry or antisymmetry implicit in the Pauli Exclusion principle.

Our atomic antisymmetric nucleus will be formed by a proton, a positron and a neutrino, or by an antiproton, an electron and an antineutrino, depending on the moment we observe the evolution of the system:

3.1 Antisymmetric system, the left intersecting field expands while the right one contracts {A2}:

- The right contracting transversal subspace will represent a proton.
- The left expanding transversal subspace will represent a neutrino.
- The vertical subspace moving towards right will represent a positron.

3.2 Antisymmetric system, the left intersecting field contracts while the right one expands {A4}:

- The right contracting proton will expand becoming a right expanding antineutrino.
- The left expanding neutrino will contract becoming a left-handed contracting antiproton.
- The vertical positron will move towards left becoming an electron.

Fig. 13 represents the limit states of the evolution of the antisymmetric system, but it does not reflect the moment when the top vertical subfield passes through the central axis, which is the referential center of symmetry of the system, having a neutral charge.

That will occur during the intermediate expansion or contraction of the intersecting fields.

In that case, the proton or antiproton transversal subfield that is expanding, and the neutrino or antineutrino transversal subfield that is contracting, will have an isomorphic shape.

It may be then when the right and left transversal subfields may be identified as neutron and antineutron.

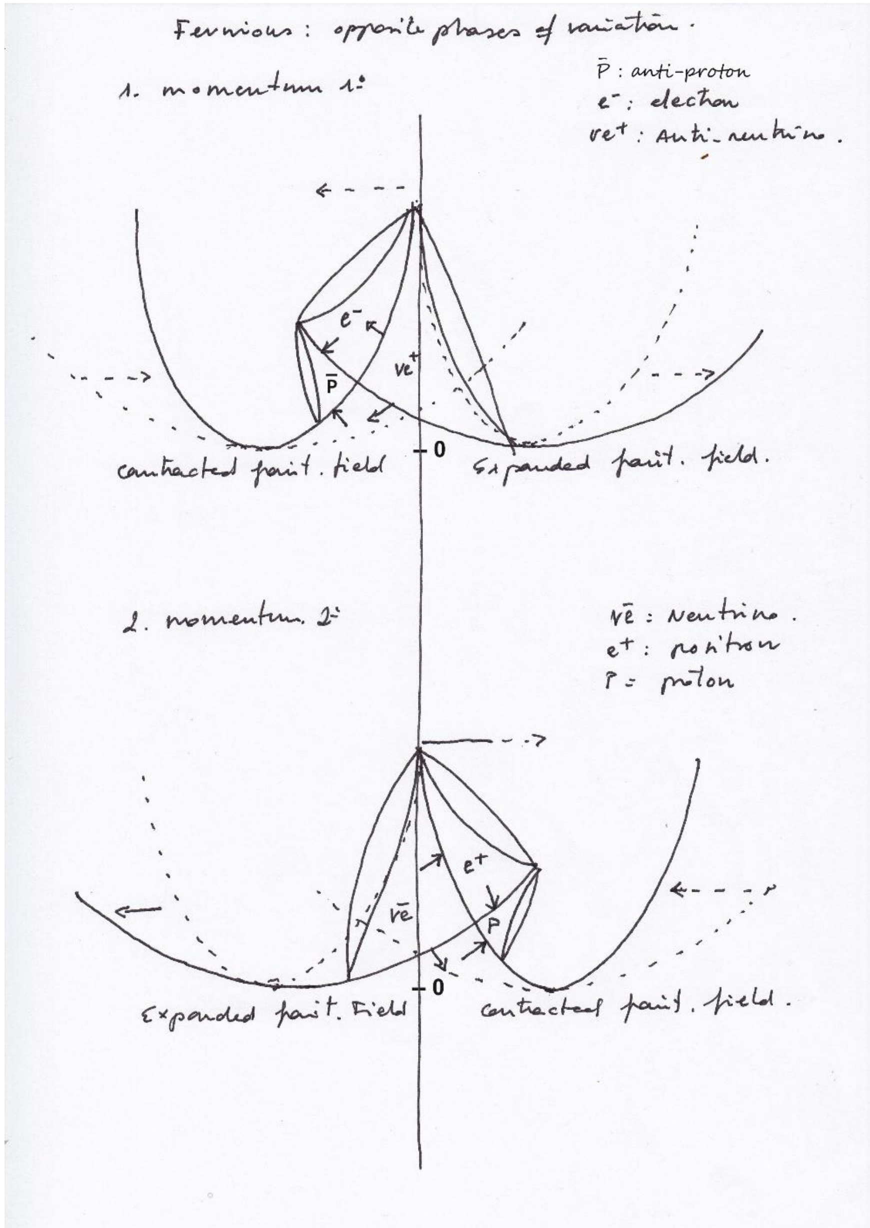

Fig. 13. Antisymmetric system at moment 1 (related to A2) and moment 2 (related to A4)

On the other hand, the diagram shows how the right-handed proton at moment A2 will decay, being virtually embedded in a right-handed antineutrino at moment A4, both in the right side of the mirror system.

Simultaneously, in the left side of the antisymmetric system an antiproton and an electron arise;

Later, the left-handed antiproton of A4 will decay into a left handed neutrino at A2, while in the right side of the mirror system a proton and a positron arise.

Proton and antiproton, and neutrino and antineutrino, will be Dirac antiparticles at different times.

Positron and electron, being their own mirror reflection antimatter as they are the same subfield in different moments, will be Majorana [14] antiparticles.

The existence of an electron and a positron in the same atom, also known as “positronium” [15], was predicted by Dirac in 1928. However, the positronium is formulated as an exotic atom that does not have a proton in its nucleus.

Dirac also predicted the existence of the antiproton. The coexistence of proton and antiproton in the same atom is currently accepted, but also as an exotic structure called “protonium” [16] that does not have electrons nor positrons.

In the dual atomic model, when it comes to the antisymmetric nucleus, isometric matter and antimatter are mutually exclusive at the same time, but they coexist as the chiral antisymmetric – with and advanced or delayed time phase – reflection of each other at the same moment, and as the chiral isometric reflection of each other at different moments.

All the subfields in the antisymmetric system are fermions having a noninteger spin, represented by the commuted eigenvector, being ruled by the Pauli Exclusion principle.

In that sense, they should obey Fermi-Dirac statistics, although this aims to be a causalist non probabilistic model.

Also in that same context, considering an antisymmetric Schrödinger’s cat [17] as a figurative example, it could be said that the right alive contracting cat will be the delay reflection of the left dead expanding cat, and vice versa.

It can be discussed if they are the future or the passed reflection of each other, but that will only be a way to speak.

Anyways, there will not be a single alive and dead cat, but two identical cats with opposite state and position. And their simultaneous states of being “alive” and “dead” only can be considered “superposed” in the context of their mirror antisymmetry.

It is visually represented in Fig.14:

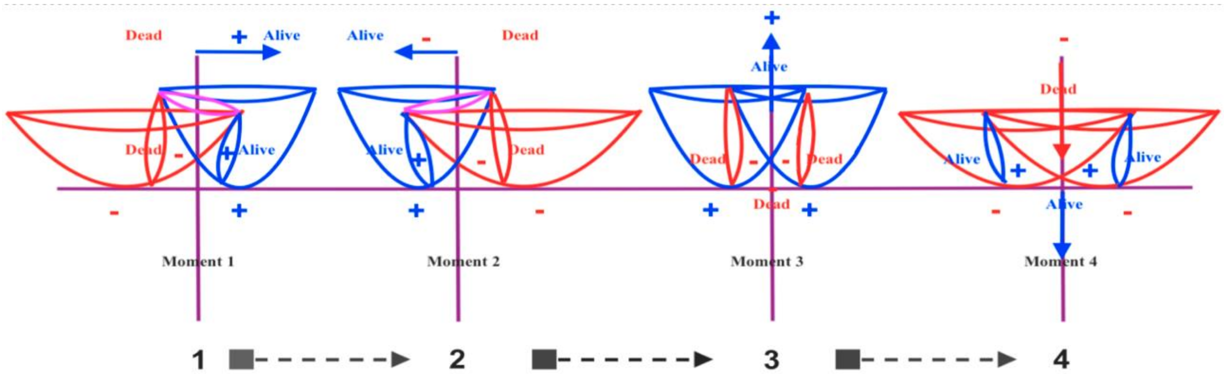

Fig. 14. Dead and or alive Schrodinger cat in mirror antisymmetric and symmetric systems

3.3 Symmetric system, when the left and right intersecting fields contract {A1}:

- The right and left expanding transversal subspaces represent a right positive and a left negative gluon.
- The top vertical ascending subspace that contracts receiving a double force of compression will be the electromagnetic subfield that will emit a photon, while pushing upwards.
- The inverted bottom vertical subspace at the convex side of the system will represent the dark decay of a previous dark antiphoton .

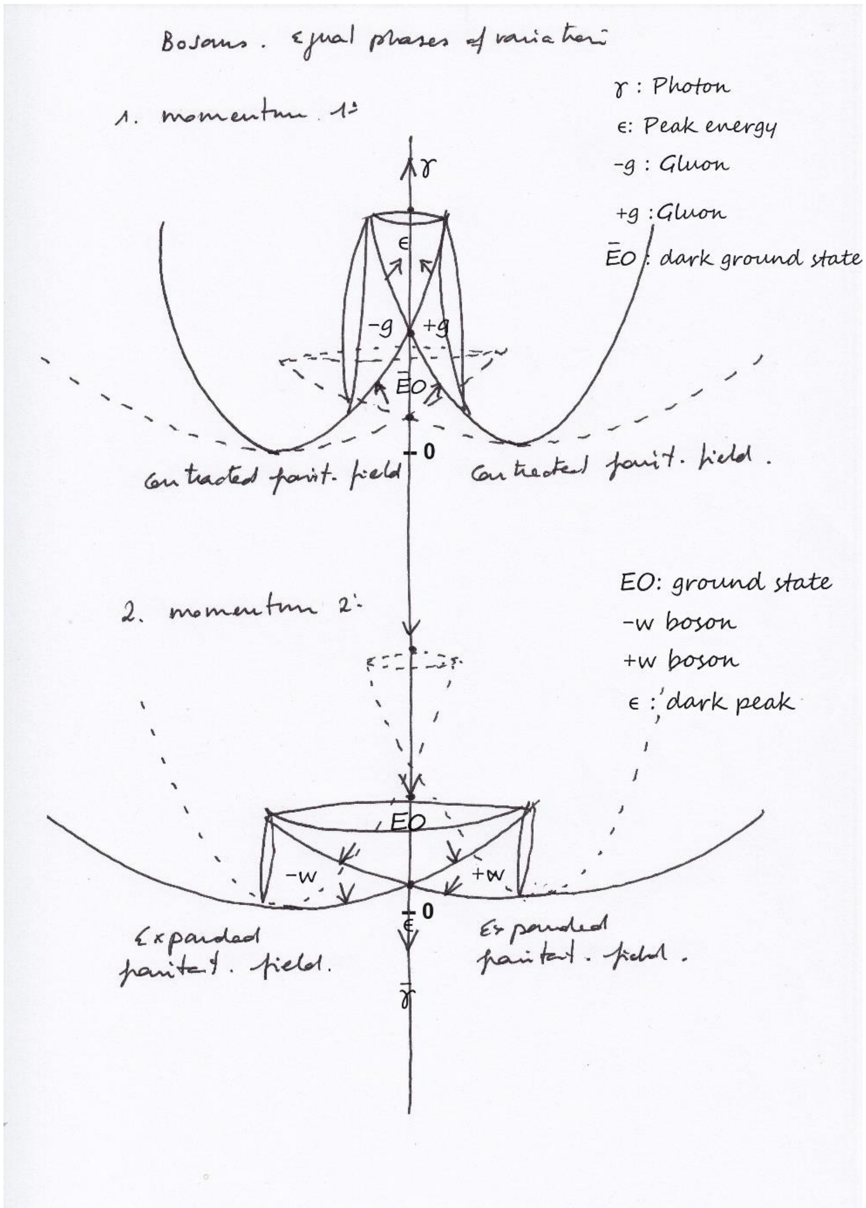

Fig. 15. Antisymmetric system at moment 1 (related to A2) and moment 2 (related to A4)

3.4 Symmetric system, when the left and right intersecting fields expand {A3}:

- The right and left expanding transversal subspaces may represent -W and +W bosons.
- The top vertical descending subspace will be the electromagnetic subfield losing its previous energy, after having emitted a photon.
- The bottom vertical subspace at the convex side of the system will be the dark anti electromagnetic subfield that is going to emit a dark anti-photon.

Examining the above Fig. 15, it can be visually observed that the left and right transversal subspaces will be mirror symmetric antimatters at the same time, being bosons not ruled by the Pauli Exclusion Principle. They should then obey the Fermi-Dirac statistics.

However, photon and dark antiphoton are mutually exclusive and so will be ruled by the Pauli Exclusion principle, although they have an integer spin represented by the two converging eigenvectors.

The identity of the symmetric transversal subfields, labelled before as W bosons and gluons would require more clarification.

Each of those subfields receives a bottom upwards pushing force and a top upwards decompression – in the strong interaction – or a top downwards pushing force and a bottom downward decompression – in the weak interaction.

In the strong interaction, the forces of pressure or decompression of the contracting or expanding intersecting fields will be different, as the contracting fields will have stronger density.

The EM photonic subfield receives a double pushing force from right to left and from left to right caused by the displacement of the negative curvatures of the intersecting fields. Those pushing forces are the same that act decompressing the transversal subfields that expand – labeled as gluons. The emitted photon would have a double helix spin.

Those EM pushing forces are the same that receive the moving right positron and the moving left electron in the antisymmetric system, now converging in the Y axis.

Also, from the perspective of this model, the transversal subspaces are the same topological subfields that contract when the intersecting fields expand in the weak interaction or expand when the intersecting fields contract in the strong interaction.

The strong and weak interactions, then, would be related by the same mechanism. And the mirror transversal subfields that mediate the strong and weak interactions would be the same topological subspaces that are transformed through time.

3.5 Supersymmetry.

The model is N=1 because it relates in a supersymmetric way, through time, each fermionic subfield of the antisymmetric system with a bosonic subfield of the symmetric system.

In that sense:

- The fermionic electron-positron subfield will be the superpartner of the bosonic vertical subfield that emits the photon when ascending.
- The fermionic proton-antineutrino subfield, and the fermionic antiproton-neutrino subfields will be the superpartners of the symmetric transversal right and left subfields respectively, when they contract or expand.

In that way, the symmetry of the system is preserved through time. And the modular Hamiltonian of the system also remains invariant through time.

3.6 Gravity and electromagnetism.

On the other hand, the curvatures of the intersecting fields in this model are considered gravitational in both the symmetric and the antisymmetric systems. This implies that gravitational fields fluctuate or vibrate.

In this model, the electromagnetic charges are considered as the pushing forces caused by the displacements of the subfields of the nucleus. Those displacements are generated by the expansion or contraction of the intersecting gravitational fields that form the nuclear system, while expanding or contracting.

The mass and energy of the nuclear subfields is also caused by the pushing forces derived from the variation of the intersecting gravitational fields.

In a different way, pushing gravity was already considered since Newton’s time by Fatio and later Le Sage [18] [19] and others but was abandoned at the beginning of the XX century since Michelson and Morley demonstrated the nonexistence of a required ether.

However, today the is generally accepted that the Higgs field permeates the whole universe, and that the Higgs mechanism confers mass to the particles, by means of the Higgs bosons, which are the force carrier particles that represent the excitations of the Higgs field.

In that context, the variations of the intersecting gravitational fields may be considered as "gauge bosons", and the dynamics of the intersecting system as a Higgs mechanism.

3.7 Dirac spinors.

A Dirac spinor is a group of four complex vectors that provide information about left or right handedness and about up or down orientation in a 4-dimensional space with 1 time dimension. Three

Using that formalism, it can be said that in the complete description of the dual atomic model four spinors are needed. Two spinors for the fermionic system, and two for the bosonic system.

When it comes to the antisymmetric fermionic system of matrix A2, the spinor will be formed by the four right-handed eigenvectors. In that case, the ½ spin of the mirror reflection left and right transversal subfields or fermionic particles is formed by the right-handed down eigenvectors, that represent the partial conjugation of matrix A1 and a fractional number of derivatives. The state of each transversal subfield is determined by pairs of up and down eigenvectors, in the way described before.

The four eigenvectors of matrix A4 represent the left-handed spinor that is the negative reflection of the spinor of A2.

The dual atomic model uses two-time dimensions to describe the different phases of the antisymmetric dual nucleus. The dynamics of the spinor related to A4 can be interpreted as a time reverse direction with respect to A2.

The four eigenvectors of matrix A1 form a bosonic spinor: two right-handed up eigenvectors and two left-handed up eigenvectors. The negative reflection bosonic spinor of matrix A3 will be formed by two left-handed down eigenvectors and two right-handed down eigenvectors, combined in the ways previously seen.

3.7 Supersymmetric quarks.

The physical pushing forces created by the displacement of the intersecting fields when contracting or expanding, that we have represented as eigenvectors, may be interpreted as “quarks” in the QCD terminology.

We previously saw that when applying the 90 degrees rotation operator, only two eigenvectors change their sign, and the symmetric system becomes antisymmetric, and vice versa.

The cyclic invariance of those eigenvectors through time may be described in terms of supersymmetric quarks.

Fig. 16 shows the way the supersymmetric fermionic and bosonic “quarks” are transformed through time by periodically changing their sign by pairs in each partial conjugation.

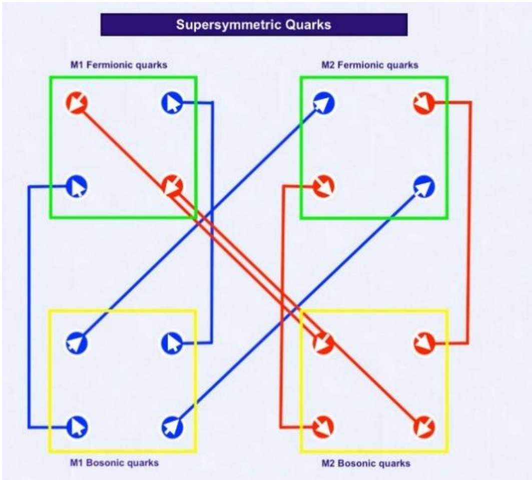

Fig. 16. Supersymmetric bosonic and fermionic quarks

3.8 Spatial topology.

The two possible signs of the unitary eigenvectors may be related to the Hilbert space of dimension 2 we use a simplification of the higher dimensional system.

The space we have described for the bosonic symmetric and the fermionic antisymmetric systems can be considered as a Minkowski space of 4 coordinates for two different frames of reference: x, y, z, t – for a real frame of reference – and x', y', z', t' – for a complex frame of reference whose coordinates are rotated 45 degrees relative to the real ones.

The 45-degree rotation of the coordinate system implies a partial conjugation that introduces the antisymmetry in the system, as we said before.

A different time dimension for the bosonic and fermionic is necessary to describe the different dynamics of those systems, either because the subfields have an opposite phase between them, introducing a time delay in their mutual reflection, or because the subfields have an opposite phase with respect to the vibrations of the intersecting fields.

The dual atomic model suggests that the bosonic system is also antisymmetric when it comes to the different phases of the transversal subfields with respect to the vertical subfields, which follow the phase of the intersecting fields.

The introduced model can be described in terms of moduli spaces of a Higgs bundle.

Higgs bundles [20] were introduced by Nigel Hitchin. In the present model, the two intersecting fields would represent a Riemann geometry system.

The vertical subspace (and its mirror inverted counterpart) would represent the Higgs field that mediates between the left and right transversal subspaces, giving them their masses.

Both the left and right transversal subspaces, together with the intermediate Higgs field, form the moduli space of Higgs bundles.

The transversal subspaces represent the boundary components of the moduli space, while the Higgs field acts as a bridge between them.

Those boundary components are frequently represented by two transversal tori.

Quantum mechanics has been developed mainly in an abstract mathematical way with no visual spatial references. However, the tori geometry is generally used in different ways by many theories to aid visualization and simplify calculations.

The intersecting spaces model can also be thought in terms of the topology of a two genus [21] torus, considering the outer positive and the inner negative curvatures of the torus as the simultaneous representation of the expanding or contracting moments of the vibrating fields when looking at them from above, in an orthographic projection represented in Figs. 17 and 18:

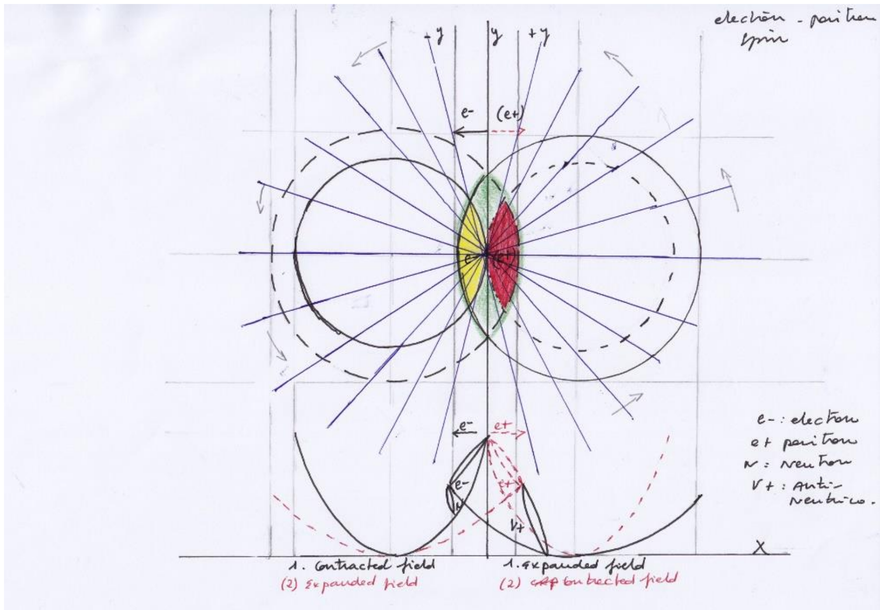

Fig. 17. Two genus torus projection of the antisymmetric system

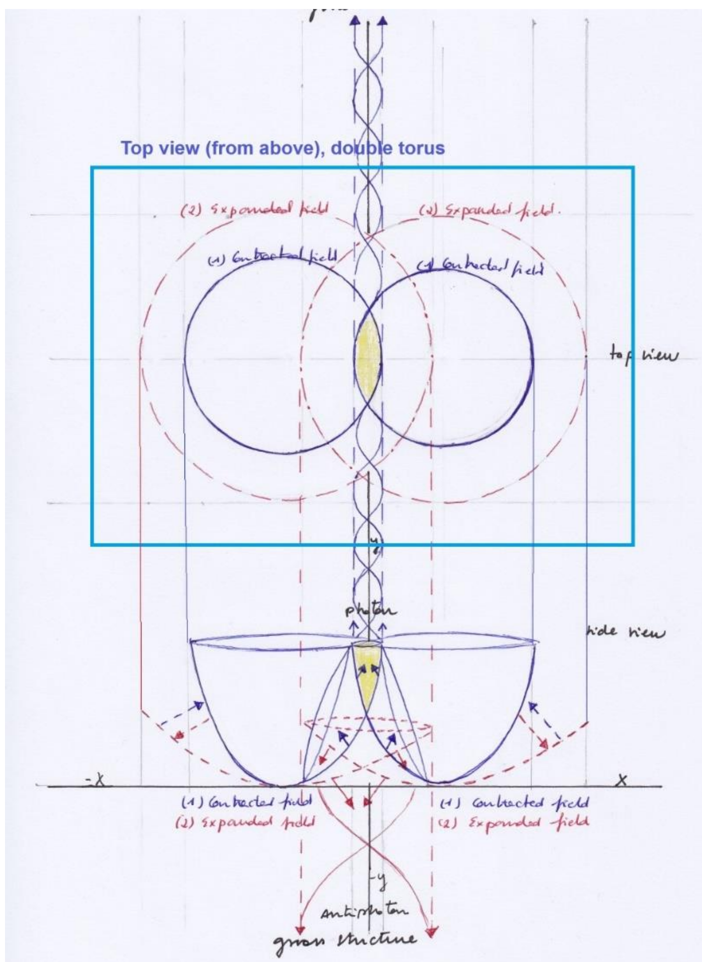

Fig. 18. Two genus torus projection of the symmetric system

The symmetric and antisymmetric subfields can be described as cobordant [22] subspaces. The vertical subspaces share borders with the left and right transversal subspaces, and they all share borders with the two intersecting spaces.

Those borders can be thought of as the unidimensional line that describes the curvatures of the intersecting fields.

The geometry of the intersecting fields model can be also related to Hyperkahler and Kummer quartic surfaces [23].

On the other hand, the shape of the nuclear transversal subfields, on the symmetric or antisymmetric diagrams, reminds the shapes drawn by Lobachevsky [24] when explaining his imaginary geometry.

The topology of the nuclear system may also be described in terms of two double oscillators.

Note that the inner curvature of the transversal subfields is half negative (curved inwards) and half positive (curved outwards). The top vertical subfield in the symmetric and antisymmetric system will have an inner concave curvature, while the bottom inverted vertical subfield will have a double region of positive or convex curvatures.

The topology of the nuclear system may also be described in terms of two double oscillators, four coupled vortices.

The curvatures of the intersecting fields also can be described in terms of longitudinal waves.

3.9 Special Relativity.

Different mathematical transformations, such as Fourier transforms and Wick rotations, are being used to make antisymmetric systems operationally symmetric. Mathematical transformations will also be required to relate the two sets of coordinates associated with the fermionic and bosonic systems.

The main difficulty is that the YX coordinates and the diagonal axis that divides the complex plane are referenced by different metrics. One point at Y cannot be rotated to X + iY without increasing the spatial distances, as it happens in Special relativity.

The same thing happens with the hypotenuse of a right triangle when one tries to measure it with the reference metric of the sides. If the hypotenuse were a rotated side, it would not only be longer than the unrotated sides, but it also could not be precisely determined because of the infinite decimals of irrationality. Two different frames of reference are being measured with the same gauge.

The metric issues between the two reference frames can be managed by adding additional spatial dimensions to separately describe each space-time system or subsystem (in the case of the transversal subfields).

3.10 Möbius transformations.

However, a type of mathematical transformation is needed to relate the coordinates of the two reference frames. This is the subject of Lorentz transformations in special relativity, which are a type of Möbius transformation [25].

Möbius transformations can be used to project the Y and X coordinates to the imaginary points at X' and Y', virtually removing the complex plane while preserving the angles.

We can superpose on a same picture the real and complex planes of the symmetric and antisymmetric systems, representing the result of convolving the complex and conjugate functions, as if the four partial conjugations were taking place at the same time.

The complex plane can therefore be treated as a real plane.

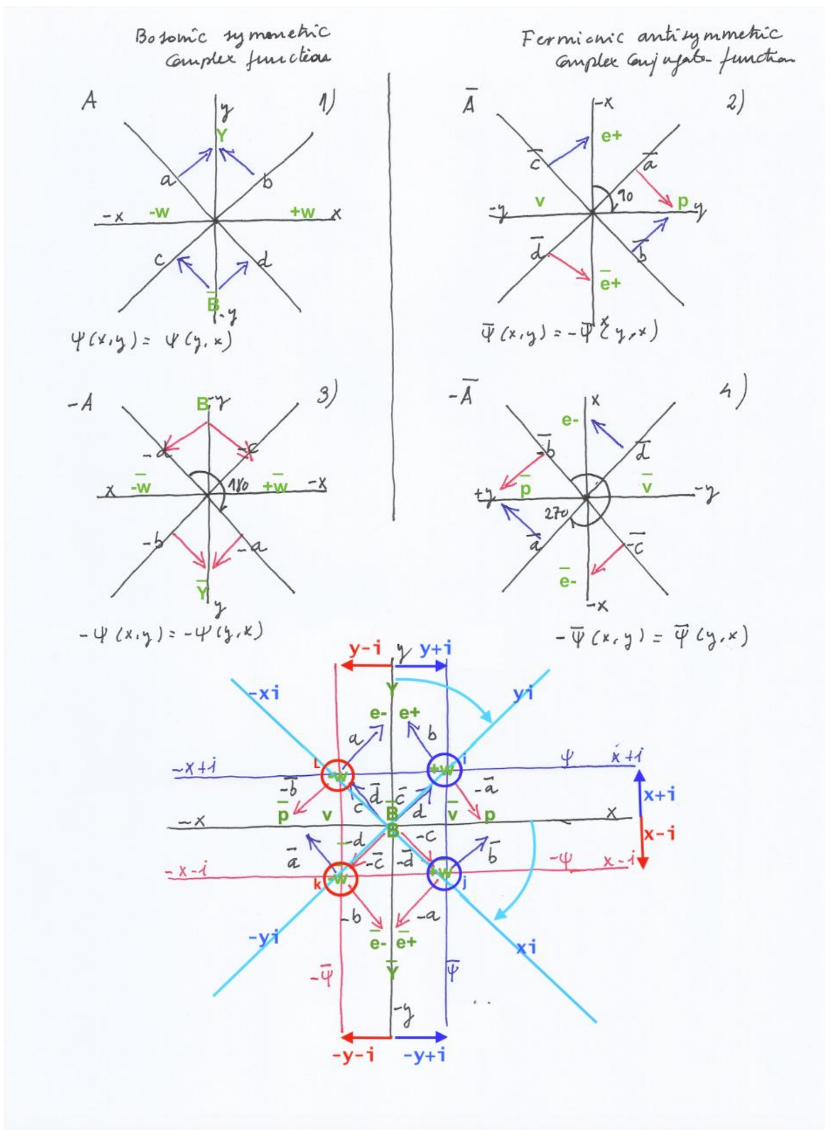

Fig. 19. Superposition of spaces and times evolution.

Fig. 19 shows simultaneously all the possible states the vectors get through time will show the left and right displacements of the fermionic nucleus (from a central axis Y towards a projected + Y' or – Y'), and the upwards and downwards displacements of the bosonic nucleus (from a central axis X towards a projected + X' or – X'), which can be interpreted as a nuclear precession.

By means of the vectors in the diagram, also can be shown the symmetry of both bosonic and fermionic systems is reached over time. The supersymmetric topological transformation of the nuclear subfields, which may take place by means of phase synchronization, the rotation of the whole system, or both, is a spatial and temporal gauge transformation.

3.11 The emergence of imaginary time.

From the perspective of our dual model, time – as the reference to measure a variation – is a notion that cannot be separated from space when it comes to periodically varying folds.

However, between the moment of the highest expansion or contraction, until the opposite contraction or expansion starts, there will be a period of no spatial variation that may be interpreted as no time.

When it comes to a composite manifold with different phases of variation an additional dimension for the emerging time will be needed.

Considering time as a real axis tY in the symmetric modular group A1 A3, the partial conjugation operated by a 90 degrees rotation introduces in the system a new time coordinate ti, that is purely imaginary and causes the emergence of the spatial time antisymmetry in the modular group A2 A4.

Using the previously seen Tomita Takesaki terminology, the handed sides of the modular operator A2 (and also the sides of its automorphic modular conjugate A4) become antisymmetric because half side of the system is going to follow the real time tY, while the mirror side is going to follow the new imaginary time ti, creating a time-gap between both sides of the modular operator and, later, of the automorphic modular conjugate as well.

The imaginary time is not only a symbolic cartesian formalism, but a mathematical way to describe the dynamics of the rotational system that needs a second time dimension to refer to the harmonic ti phase introduced in the modular group by the partial conjugation.

Commuting the sign of half of the eigenvectors while leaving the other half unchanged implies a partial differentiation that can be interpreted as a fractional derivative or antiderivative, as we saw before. It is that fractional derivation what creates the harmonic or fractional imaginary time and the fractional spins in the modular antisymmetric – anticommutative group.

Fractionality is responsible for the break of symmetry that exhibits the commutative group, introducing anticommutativity.

4. Relations with other theoretical models.

To conclude, we can relate the atomic model to other theories and developments, using a visual approach as well.

4.1 String theories.

In the context of string theories, the border of the positive and negative curvatures of the intersecting fields, or of parts of them, may be seen as one-dimensional open or closed strings:

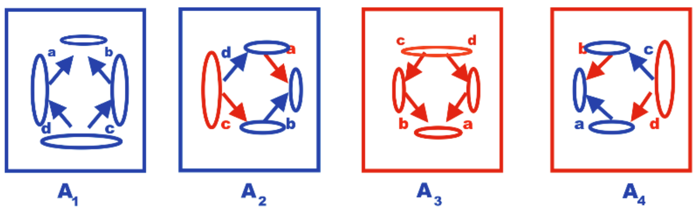

Fig. 22. Unidimensional closed strings

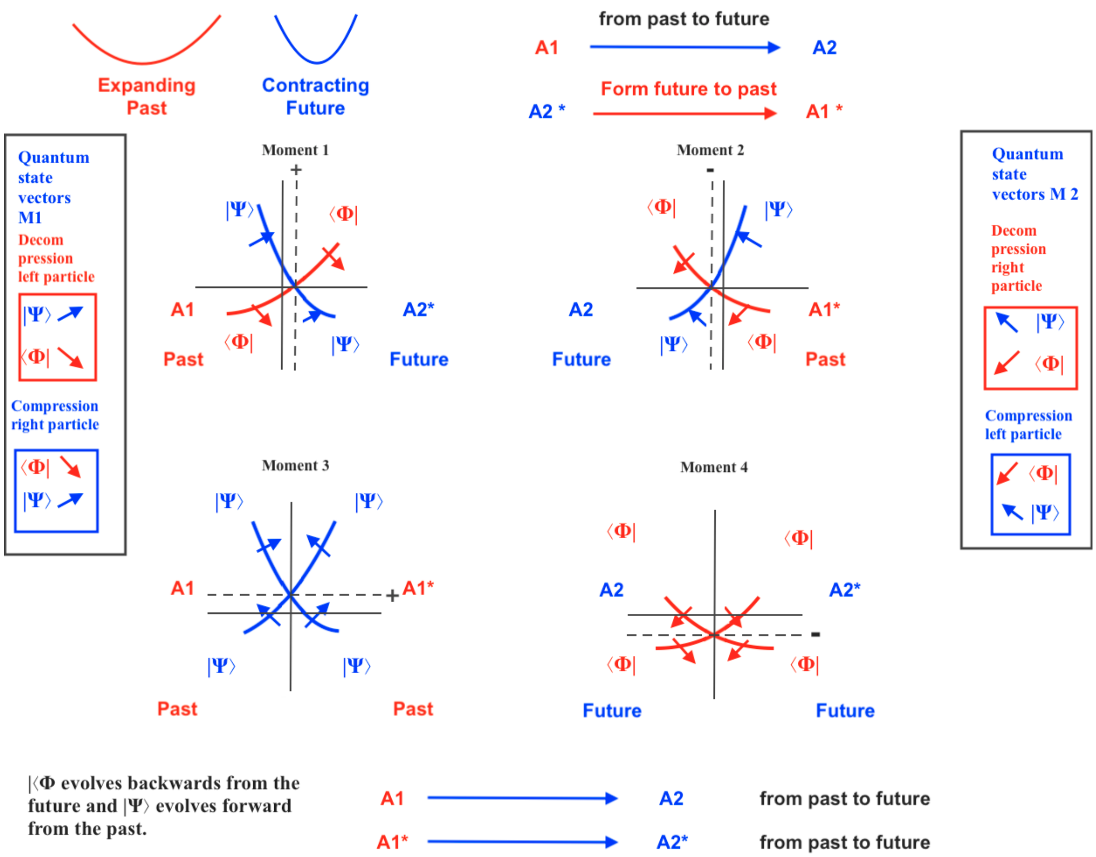

Fig. 20. Unidimensional open strings

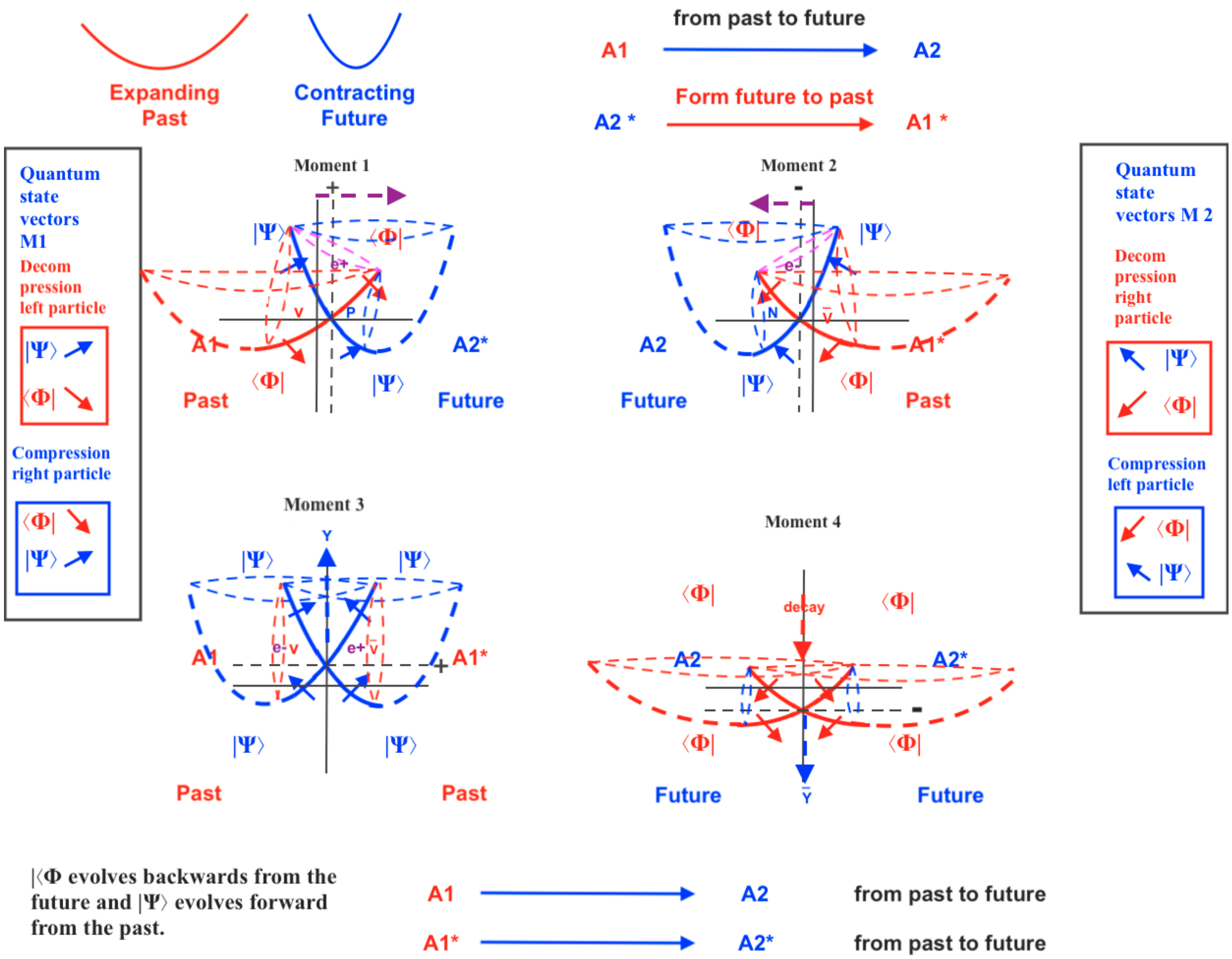

Fig. 21. Unidimensional strings embedded in the dual quantum field model

The transversal subfields of the symmetric and antisymmetric manifolds could be related in some way to Calabi-Yau [26] transversal spaces.

But Calabi-Yau demands smaller sizes for the spatial higher dimensions and compactification.

The elliptic orbits inside of the transversal subfields, caused by their periodical expansion and contraction, can be visually related to the notion of elliptic fibrations used in String theories.

They are represented in Figs. 23 and 24.

As well in the context of string theories, the intersecting fields that interact to form the nucleus of subfields may be related to the positive curvature of de Sitter vacuum spaces, when expand causing an outward pushing force, or to the negative curvature of anti de Sitter [27] vacuum spaces, when they contract causing an inward pushing force.

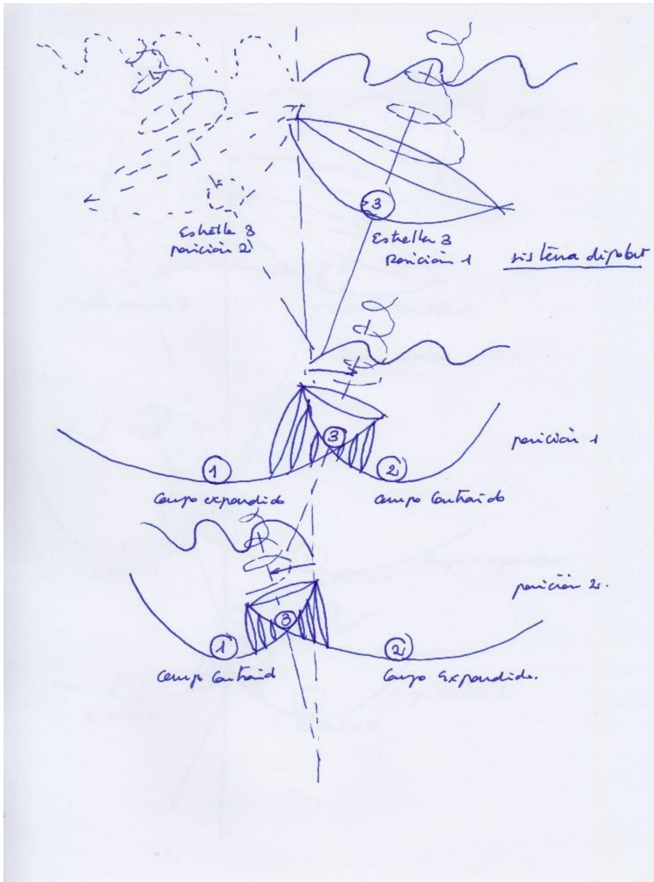

Fig. 23 Elliptic fibrations in antisymmetric system

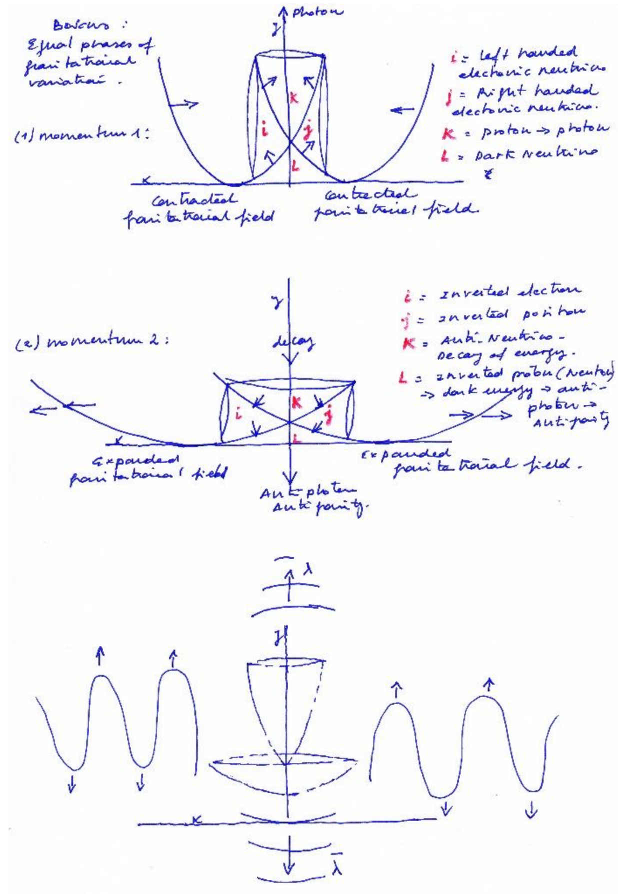

Fig. 24. Elliptic fibrations in the symmetric system

As well in the context of string theories, the intersecting fields that interact to form the nucleus of subfields may be related to the positive curvature of de Sitter vacuum spaces, when expand causing an outward pushing force, or to the negative curvature of anti de Sitter [27] vacuum spaces, when they contract causing an inward pushing force.

The symmetric and antisymmetric systems may also be related to the Ramond-Ramond or the Kalb-Ramond fields. The Ramond-Ramond [28] fields are antisymmetric tensor fields with two spacetime indices, which is a mathematical way to refer to two fields in a dual system.

4.2 Many Worlds interpretation.

Everett's many worlds interpretation [29] of Quantum mechanics proposes multiple worlds that simultaneously coexist.

However, the multiple worlds are independent of each other and do not interact, although there is also an interacting version of the Many Worlds interpretation by H. Wiseman [30].

In the MWI, each world represents a possible state of a particle. In this sense – despite the differences – the "intersecting worlds" model we are introducing in this paper seems conceptually consistent with the idea of multiverses with different states, when it comes to the antisymmetric system.

But, as we saw before, we speak always in terms of the states of the mirror reflection matter and antimatter existing in a nuclear manifold, instead of the states of independent matter or independent antimatter existing in independent folds.

The notion of multiverse is assumed in the present model.

4.3 Wave pilot interpretation.

In 1952 David Bohm, based on De Broglie work, proposed the wave pilot interpretation of quantum mechanics. Previously, the idea that photons or electrons may be guided in some way by a pilot electromagnetic field or by a pilot wave function had been considered by Einstein and Born respectively. [31]

The present model may be interpreted as a two-wave pilot theory, as the displacements of the photonic subfield in the symmetric system and the electron-positron subfield in the antisymmetric system are guided or piloted by the displacements of both intersecting fields while contracting or expanding with same or opposite phases.

The intersecting fields can be characterized as longitudinal waves.

Being a local theory, there are no hidden variables in the dual atomic model. Whatever action on the left or right side of the system will have immediate effects in the shared nucleus.

Action at a distance will have the limit given by the intertwined topology of the dual system.

4.4 Hidden sectors and Hidden valley models.

The hidden sector [32] or hidden valley is a hypothetical collection of quantum fields and their related particles that are not visible to us, being considered related to dark matter.

In the context of the dual matter atom of six folds we introduce in this paper, a hidden sector will be the field or subfield that cannot be directly observed from inside of another field or subfield of the composite atomic manifold.

In that sense, an observer placed inside of the left intersecting field or in the left transversal subfield will not be able to directly detect the right intersecting field nor its related transversal subfield. An observer placed in the vertical subfield with negative curvature will not directly detect the inverse vertical subfield with positive curvature.

In the context of the mirror symmetric or antisymmetric dual system, the "visibility" or invisibility of the "dark" mass or energy will be relative to the position of the observer.

4.5 Transactional interpretation.

The presented model can also be expressed in terms of emitter-absorber transactional models that correlate advanced and retarded waves, taking place a transactional "handshaking" [33] between them when interchanging energy.

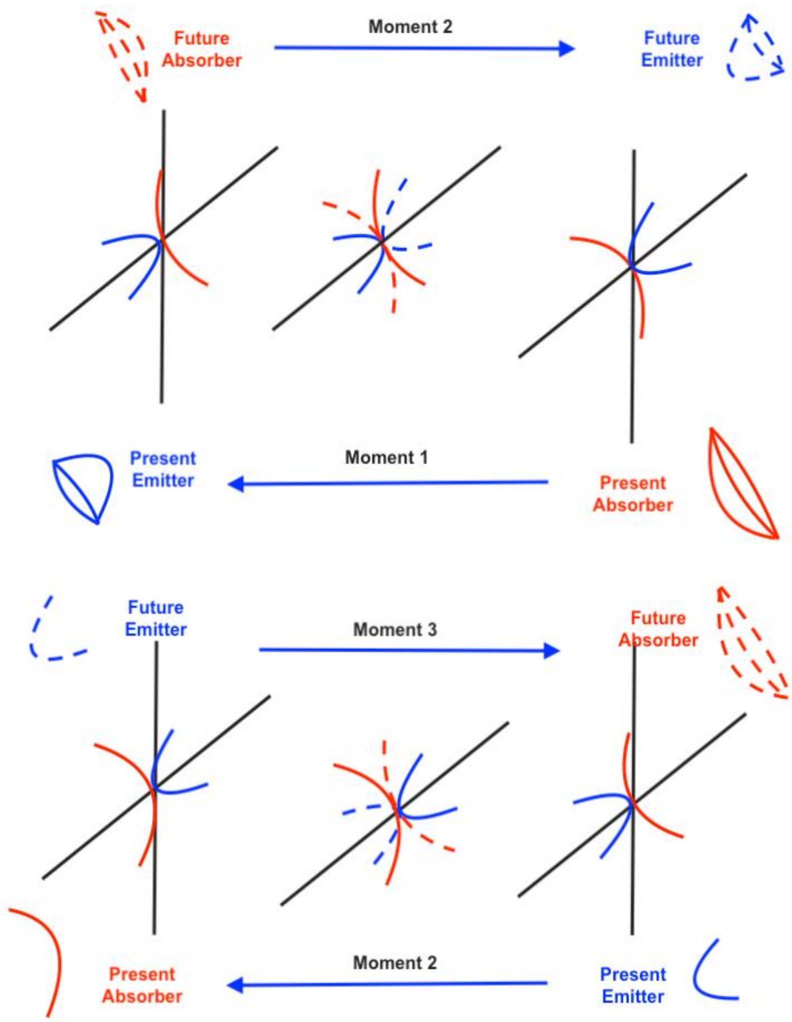

Fig. 25. Transactional handshaking

4.5 Dirac and Weyl semimetals.

Weyl semimetals [34] are considered exotic phases of matter characterized by the presence of Weyl fermions, which are considered ½ spin quasiparticles with chiral handedness and no mass nor charge. They were proposed by H. Weyl in 1929.

In that context, it is frequent to represent the Weyl fermion as the subcone formed by th intersection of two cones [35].

That type of geometry is closely related to the topology of the dual atomic model introduced in this article. In the context of the dual atomic model, the subcone would the vertical subfield formed by the intersection of a contracting and an expanding field that vary with opposite phases forming an antisymmetric system. That vertical subfield is identified as an electron when moving left toward the side of the intersecting field that contracts or a positron when moving right a moment later.

However, as the Weyl fermion does not have mass and it has a fixed chirality that breaks the CP party that links matter and antimatter, it can also be related to the left or right transversal subfields that this model identifies as and expanding and almost massless neutrino or antineutrino.

The CP parity of the handed neutrino is saved through time, as its mirror reflection counterpart will exist in a future or past time, so to speak, when the left transversal subfield expands at the left side acting as a neutrino, the right transversal subfield will contract acting as a proton, and the vertical subfield will move towards right acting as a positron. A moment later, when the left transversal subfield contracts becoming an antiproton, the right transversal subfield expands acting as an antineutrino, and the vertical subfield moves towards left acting as an electron.

In that sense, the massless transversal “Weyl fermion” would be related to the decay of the proton and antiproton, and to the arising of the electron or positron.

H. Weyl formulated his hypothetical massless fermion in in 1929; The massless neutrino was proposed by Pauli in 1930.

4.6 Redox and acid-base reactions.

The dynamics of the dual nucleus, when it comes to the antisymmetric system, can be conceptualized as a type of reduction-oxidation (redox) reaction, where the right-handed and left-handed sides of the system interchange roles as oxidizing and reducing agents:

- In matrix A2, the right-handed side acts as an oxidizing agent that gains a positron, becoming reduced, while the left-handed side acts as a reducing agent that loses an electron, becoming oxidized.
- Conversely, in matrix A4, the right-handed side acts as a reducing agent that loses a positron, becoming oxidized, while the left-handed side acts an oxidizing agent that gains an electron, becoming reduced.

The interplay of the left- and right-handed sides of the antisymmetric system can be conceptualized as well as an acid-base reaction between the two sides.

- In matrix A2, the left-handed side acts as an acid donor, transferring an antiproton to the right-handed side which acts as a base acceptor receiving a proton.

- In contrast, in matrix A4, the left-handed side acts as a base acceptor, receiving an antiproton from the right-handed side, which acts as an acid proton donor.

The reciprocal transfer of mass, energy and charges between both handed sides of the mirror antisymmetric system is a consequence of their different oscillatory phases.

4.7 Riemann-Silberstein vector.

In the early 20th century, Ludwik Silberstein proposed a novel formulation of Maxwell's equations in which the electromagnetic field is represented by a complex vector, known as the Riemann-Silberstein vector [36]: F = E + iB

The electric field E constitutes the real part of the complex vector F, and the magnetic field B forms its imaginary part. It implies a 45 degrees rotation of the magnetic field B and the introduction of a different time phase in the magnetic field.

The dual atomic model discussed in this article suggests the new time phase affects only to half of the magnetic system at matrix A2 and, later, to the other half at matrix A4. This non-linear evolution would imply that the interaction between the electric and magnetic fields cannot be described by simply adding together the electric and the magnetic fields.

However, as it was previously sawn in Fig.16, it can be useful to represent the evolution of the symmetric (electric) and antisymmetric (magnetic) systems as simultaneously superposed on a same complex space.

In the context of the dual nucleus, the upwards and downwards displacements of the vertical subfield of matrix A1 and A3 cause the electric field, and the rightwards and leftwards displacements of the vertical subfield of matrix A2 and A3 are the source of the magnetic field. By rotating the complex plane four time 90 degrees, the quantized electric and magnetic behavior of the system can be described by the sequence F= A1 + A2 + A3 + A4.

A1 and A3 matrices can be interpreted as representing the electric source of the fields system, while matrices A2 and A3 represent its magnetic source.

A2 would represent the positive monopole and A4 the negative monopole moments of the dipole formed by A2+A4.

The leftwards and rightwards displacements of the subfield that create the lateral pushing electric charge in the magnetic moments of A2 and A4 is caused by the introduction of a time phase shift in half of the system, making it antisymmetric. The symmetry is restored in the electric moments of A1 and A4 when the time phases synchronize.

Fig. 26 tries to visually relate the four vector matrices with the diagrams of quantized EM waves symbolically represented in a sinusoidal way:

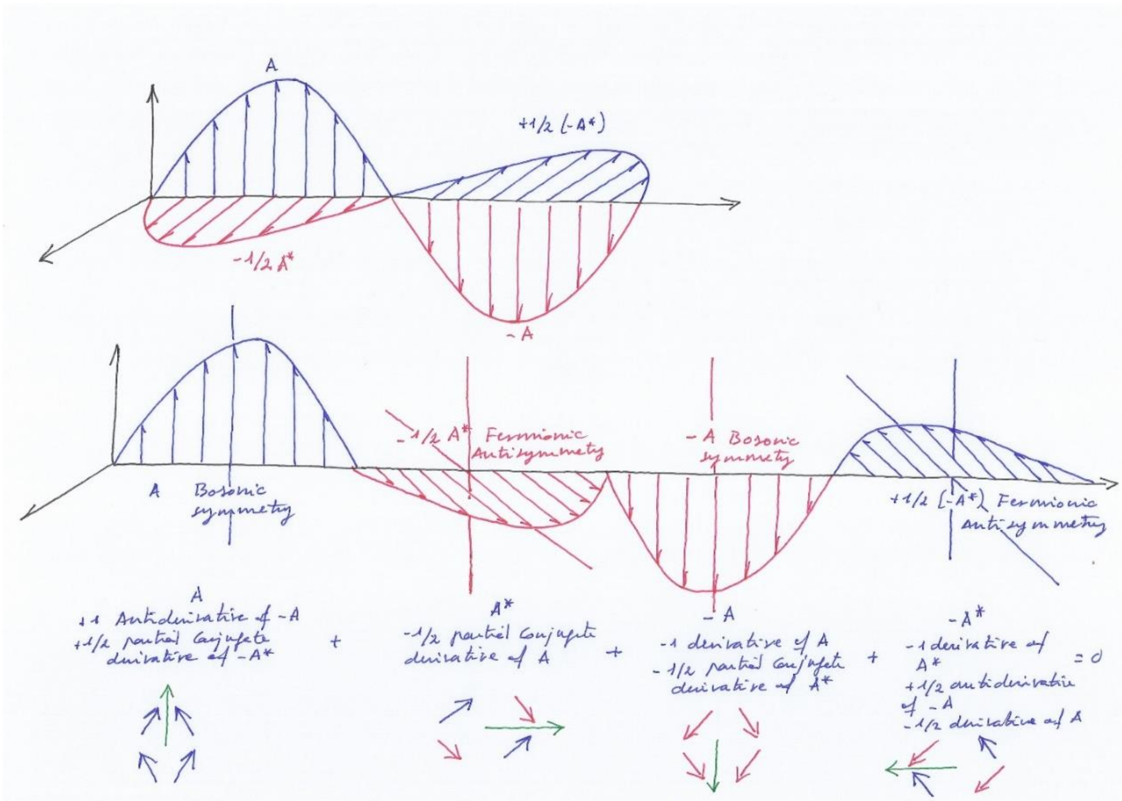

Fig. 26. Fermionic and bosonic representation

4.8 Astrophysics.

Copernicus started to question the geocentric model because of its unexplained asymmetries, considering it as "a monster" formed by the unrelated members taken from different places. [37]

However, his heliocentric model lost its circular symmetry when Kepler realized, based on the more precise measures given by the invented telescopes, that the planetary orbits were elliptic.

Although the Copernican simplicity remained until now, many unexplained asymmetries emerged: different planetary velocities, planetary motions that accelerate and decelerate, different orbital eccentricities, different inclinations, and even planets that rotate in an opposite direction.

Some explanations have been proposed ad hoc, for example for the inverse rotation of Venus, and although the asymmetries of the solar systems can be mathematically predicted and described they are not explained by means of a unique mechanism.

A single and invariant orbital field would not be enough to describe a composite manifold of intertwined spaces and subspaces varying with same or opposite phase.

A system of intersecting universes that fluctuate with same or opposite phase would produce a periodical big bang in the concave side of the symmetric system followed by a big silence when an inverse and dark (as directly undetected from the concave side) big bang occurs in the convex side.

The detected discrepancy in the measures of the rate at which the universe expands, known as the "Hubble tension" [38], could be related to the different pushing forces that are caused by the negative and positive curvatures of intersecting universes that periodically fluctuate.

Also in a rotational multiverse context, after the big bang a new time phase may be introduced in half of the system that will be contracting while the other one will be expanding.

4.9. Biophysics.

The fields and subfields of the model can also be interpreted as fluctuating vortices. Some quantum theories, such as Chern-Simons theory [39], consider vortices instead of fields.

The article, “The Chern-Simons current in systems of DNA-RNA transcriptions” [40], provides an interesting image about a loop space as Hopf fibration [41] over DNA molecule [42], that reminds the diagrams about elliptic fibrations of the transversal subfields previously presented as orbits.

A fundamental role during cell division is played by the rotation of the spindle axis. It’s currently known that abnormal spindle rotation [43] can result in genetic abnormalities.

Considering a cell as a dual system, its inner rotational dynamics may present some similarities with the physics of the dual atomic model we have presented. Abnormal phase changes would affect the processes of cell division and differentiation, accelerating or decelerating their normal paths.

5. Model inconsistencies.

This paper does not aim to provide a rigorous formulation of the dual atomic model it introduces. Its focus is to show the consistency and possibilities of a dual approach to the atomic nucleus.

5.1 The rotation of the whole system, that seems necessary to quantize the classical continuous fields and subfields of the intersecting systems, does not seem to be enough to transform per se the physical properties and phase times of the subfields, synchronizing and desynchronizing the phases of the system.

Misinterpreting the physical meaning of the symbolic formalism given by the eigenvectors' directions seems a possibility in this model.

5.2 The model does not explain how the inner orbital motions of the transversal subfields are affected by the inner orbital motions of the intersecting fields when contracting or expanding.

The inwards pushing forces of the contracting fields with their subsequent compression and orbital acceleration, and their outwards pushing forces when expanding with their subsequent decompression and orbital deceleration, will affect to the orbits of the transversal subfields, especially at the place where their orthogonal convergence occurs forming a shared whirlpool or vortex.

The vortex's interaction with the transversal subfields and the intersecting fields they are embedded within could induce an epicycle-like motion within their inner orbits.

5.3 The paper does not discuss the cause of the periodical curvatures of the intersecting fields, although it relates the system with the Higgs mechanism in a gravitational way.

To describe the periodical curvatures in terms of spatial density and friction, in a thermodynamic way, a source of constant pushing force would be required.

6. Final considerations.

It is not the aim of this work to provide a rigorous formulation of the dual atomic model.

The dual composite geometry of this intersecting fields model and the visual geometric and mathematical simplicity approach of the present work may provide some interesting insights for future clarifications of the fundamentals of quantum mechanics and developments of the physics beyond standard model and astrophysics.

The original aim of the present research was to find a mechanical explanation of the flaws that drive anomalous cell division and differentiation, assuming there must exist a physical mechanism where the biological processes occur with a normal path, and an acceleration or deceleration of those processes are the cause of gene and cell anomalies.

The author suggests that the introduced dual fields model may also provide some clues for further modeling advances in the evolving dynamics of anomalous cell division and differentiation.

This work is dedicated to the memory of Magdalena.

7. Additional Images

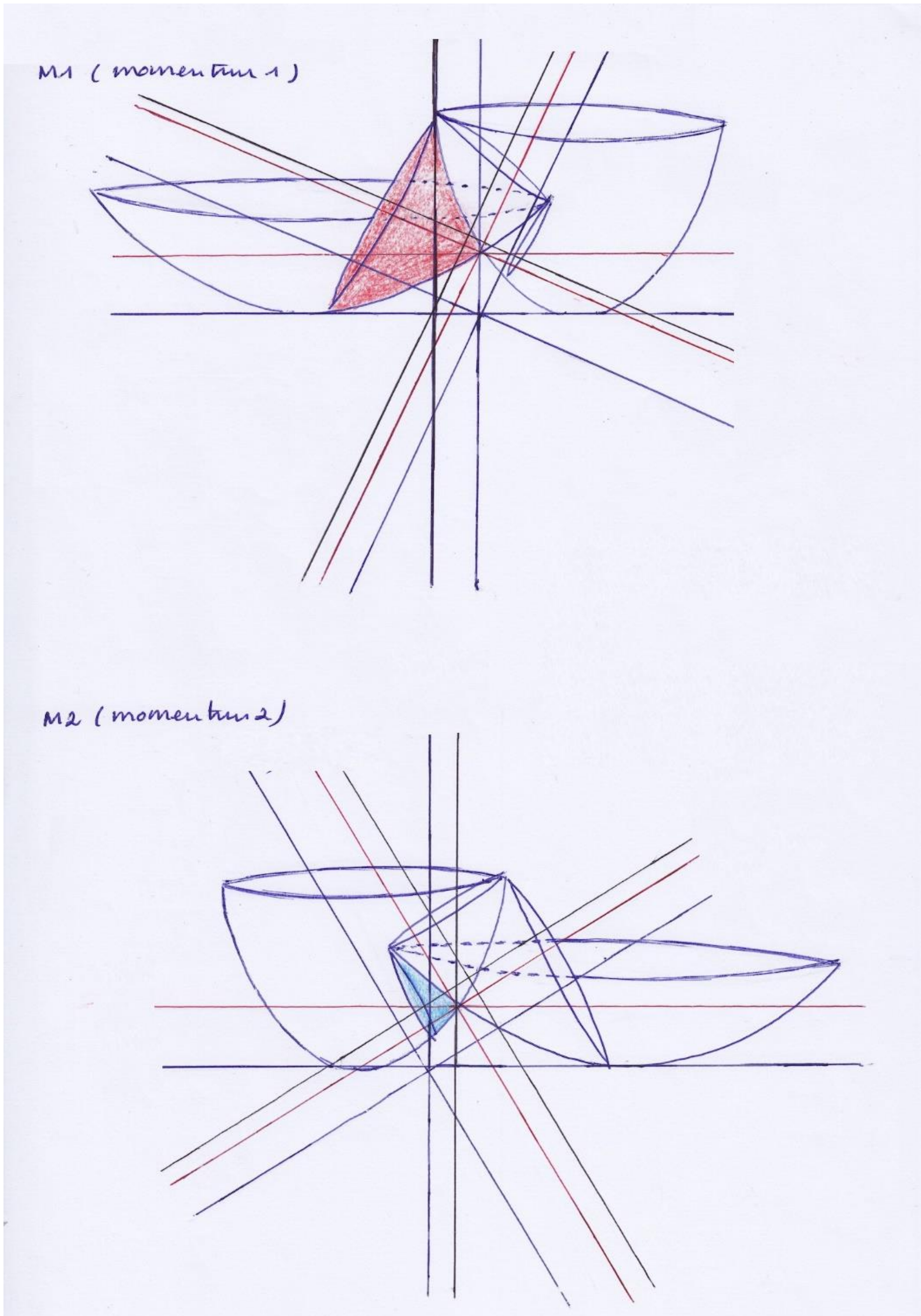

Fig. 27. Frames of reference, antisymmetric system

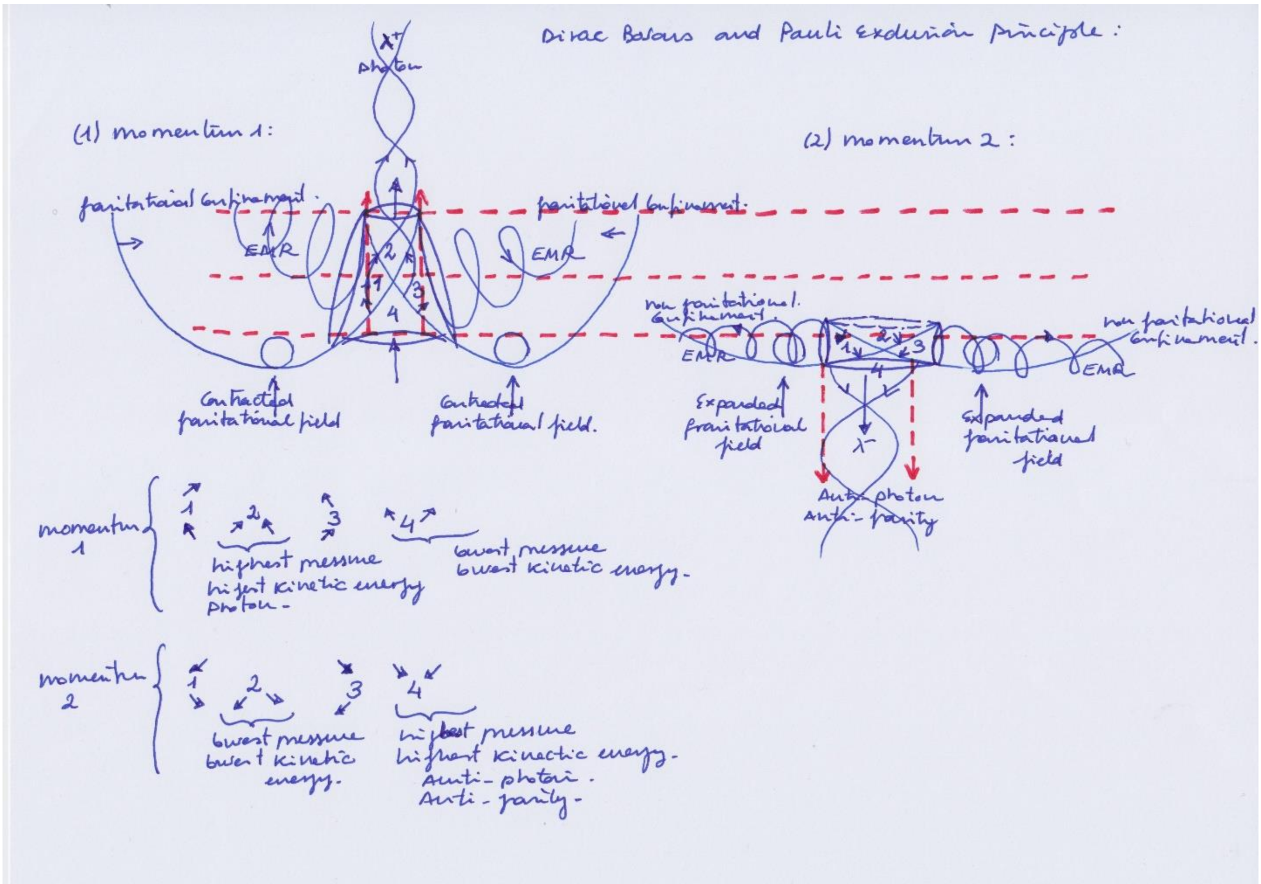

Fig. 28 Symmetric system, (initial diagrams).

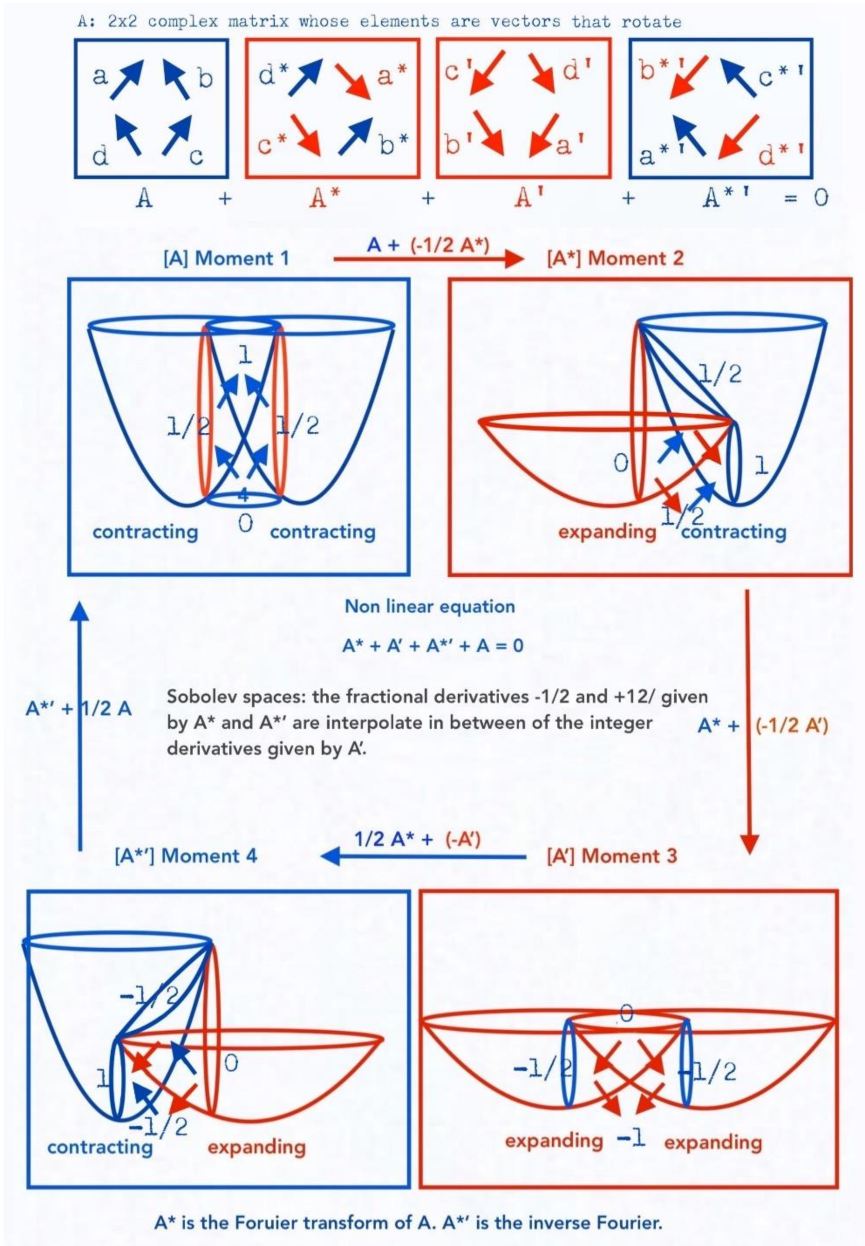

Fig. 29 Fourier transform. Fourier inverse

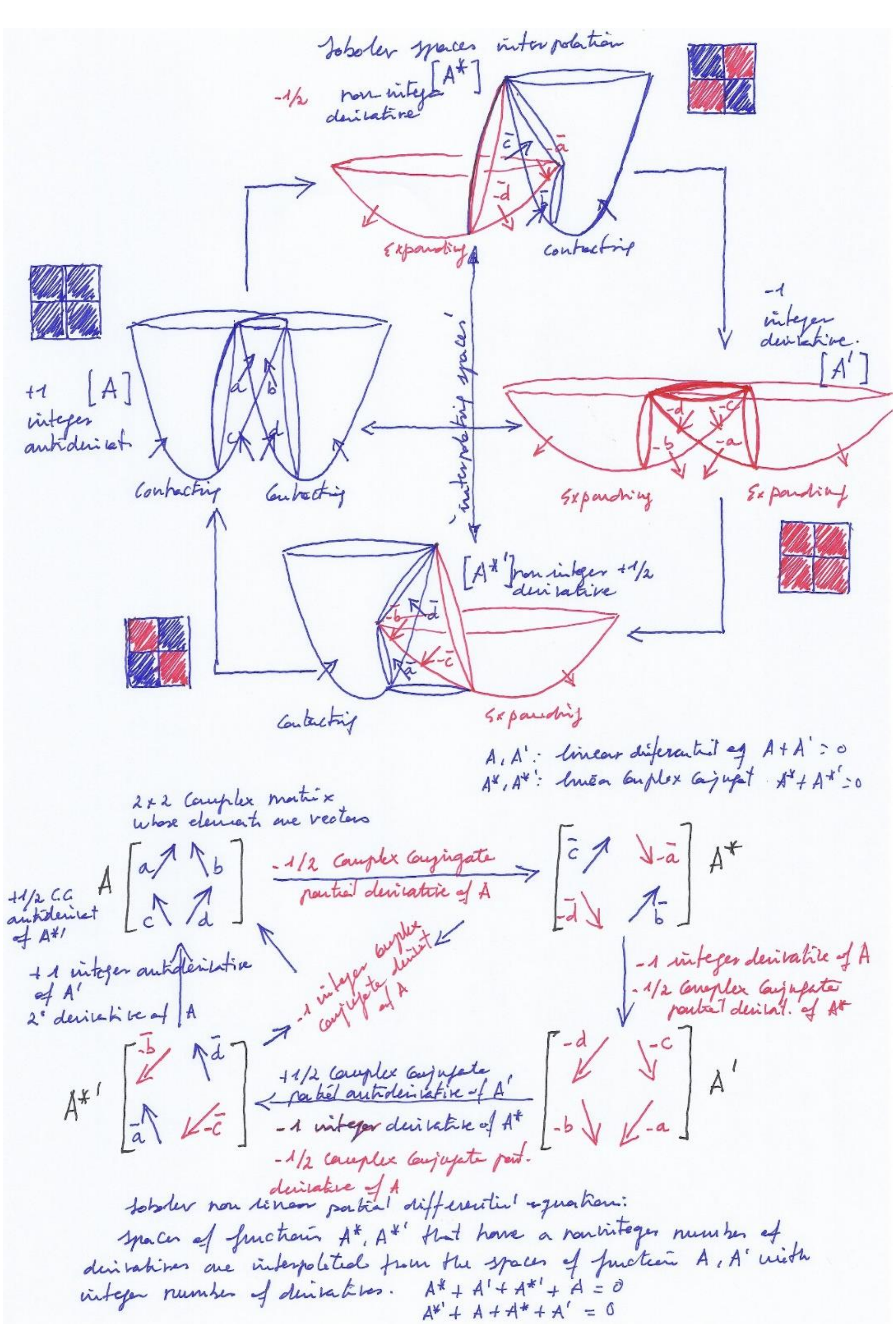

Fig. 30: Modular matrices. Interpolation

8. References

[1] Sobolev interpolation: https://en.wikipedia.org/wiki/Interpolation_space

[2] Convolution; Reflection positivity: https://www.arthurjaffe.com/Assets/pdf/PictureLanguage.pdf

[3] Fourier transform: https://en.wikipedia.org/wiki/Fourier_transform

[4] Bäcklund transform: https://en.wikipedia.org/wiki/B%C3%A4cklund_transform

[5] Tomita Takesaki Theory: https://en.wikipedia.org/wiki/Tomita E2 80 93Takesaki_theory

[6] Vertex operators: https://encyclopediaofmath.org/wiki/Vertex_operator

[7] Reflection positivity: https://www.pnas.org/doi/pdf/10.1073/pnas.1710707114

[8] Wick rotation: https://en.wikipedia.org/wiki/Wick_rotation

[9] Mass gap problem: https://en.wikipedia.org/wiki/Yang E2 80 93Mills_existence_and_mass_gap

[10] Bethe matrices: https://arxiv.org/abs/math/0605015

[11] Bloch sphere: reference request - First occurrence of the Bloch sphere in the scientific literature - History of Science and Mathematics Stack Exchange

[12] "Geometries, Groups, and Algebras in the Nineteenth Century" by I. M. Yaglom, p. 130.

[13] F. Klein, "Vorlesungen über die Entwicklung der Mathematik im 19. Jahrundert", p. 361

[14] Majorana antiparticles: https://en.wikipedia.org/wiki/Majorana_fermion

[15] Positronium: https://en.wikipedia.org/wiki/Positronium

[16] Protonium: https://en.wikipedia.org/wiki/Protonium

[17] Schrödinger's cat: https://en.wikipedia.org/wiki/Schr C3 B6dinger 27s_cat

[18] Le Sage: https://en.wikipedia.org/wiki/Le_Sage 27s_theory_of_gravitation

[19] "Pushing Gravity: New perspectives on Le Sage's theory of gravitation". Ed. Matthew R. Edwards, 2001. ISBN-10 0968368972

[20] Moduli space of Higgs bundle, Fig, 5: The left and right tori on the linked figure would represent the Higgs bundles, the central circles in red would represent the Higgs field, and the blue curved would represent a Riemann space): https://link.springer.com/article/10.1365/s13291-021-00229-1#Fig5

[21] Two genus torus: https://en.wikipedia.org/wiki/Genus_g_surface

[22] Cobordism: https://en.wikipedia.org/wiki/Cobordism

[23] Kummer manifolds: https://en.wikipedia.org/wiki/Kummer_surface

[24] N. Lobachevsky: "*Geometrical Researches on the Theory of Parallels*", p. 24 https://www.google.es/books/edition/Geometrical_Researches_on_the_Theory_of/GJBsAAAAMAAJ?hl=en&gbpv=1&pg=PA22&printsec=frontcover

[25] Möbius transformation: https://en.wikipedia.org/wiki/M C3 B6bius_transformation

[26] Calabi- Yau transversal space: "The Shape of Inner Space" by Shing-Thu Yau" and Steve Nadis. Pp. 12-15. ISHN-10 0465020232

[27] Anti de Sitter space: https://en.wikipedia.org/wiki/Anti-de_Sitter_space

[28] Ramond-Ramond fields: https://en.wikipedia.org/wiki/Ramond%E2%80%93Ramond_field

[29] Evertt's Mani Worlds Interpretation: https://en.wikipedia.org/wiki/Many-worlds_interpretation

[30] H. Wiseman's Many Interacting Worlds: Phys. Rev. X 4, 041013 (2014) - Quantum Phenomena Modeled by Interactions between Many Classical Worlds (aps.org)

[31] Wave pilot history: https://plato.stanford.edu/entries/qm-bohm/#Hist

[32] Hidden sectors: https://en.wikipedia.org/wiki/Hidden_sector

[33] Transactional interpretation of QM. https://en.wikipedia.org/wiki/Transactional_interpretation

[34] Weyl semimetals: https://en.wikipedia.org/wiki/Weyl_semimetal

[35] Diagrams of Weyl fermion: https://tqmatter.org/research/highlights/

[36] Riemann-Silberstein vector: https://en.wikipedia.org/wiki/Riemann%E2%80%93Silberstein_vector

[37] N. Copernicus, "On the Revolutions of the Heavenly Spheres", Preface.

[38] Hubble tension problem: https://webbtelescope.org/contents/early-highlights/webb-confirms-accuracy-of-universes-expansion-rate-measured-by-hubble

[39] Chern Simons theory: https://en.wikipedia.org/wiki/Chern%E2%80%93Simons_theory

[40] https://www.researchgate.net/publication/322783132_The_Chern-Simons_current_in_systems_of_DNA-RNA_transcriptions

[41] Hopf fibration: https://en.wikipedia.org/wiki/Hopf_fibration

[42] https://www.researchgate.net/figure/The-picture-shows-the-Chern-Simons-current-over-CD4-receptor-protein-as-curvature-in_fig4_322783132

[43] Spindle apparatus: https://en.wikipedia.org/wiki/Spindle_apparatus