Barnett’s resolution of the Minkowski – Abraham dilemma holds, no 4-vector issue

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Abstract

There has been a century long controversy about the momentum density of photons in a medium, with two models getting experimental confirmation: the Minkowski model and the Abraham model. The latter being experimentations targeting the particle aspects of light, while the former focuses on its wave aspects.

In 2010, Barnett proposed a simple resolution to the dilemma: both approaches are correct models, but one needs to distinguish what the momentum densities represent. The model is elegant and consistent with Maxwell’s equations in a medium.

Subsequently, Changbiao Wang published two papers arguing that the reasoning is incorrect because the Abraham light momentum in the medium would not be a 4-vector. This paper shows that such a view is incorrect: the author incorrectly sets the kinetic momentum density of the medium to zero.

Finally, we point out another paper that explains that the controversy is a relativistic effect between the reference frame in proper time for the wave and external observers.

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1. Introduction

For more than a century, the velocity, rather the momentum density of light in media has been controversial, between the formula obtained by Minkowski [1] and the formula obtain by Abraham [2].

[3] provides an overview of the history of analyses and experimentations that have been done to try to decide between the competing theories. Puzzlingly, some results favored one formulation, while others favored the other one. In fact one can summarize, at a high level, that the Minkowski model applies to experiments that rely on the wave facet of light, while the Abraham formula applies when the experiments rely on the dual particle aspect of it.

In 2010, Barnett proposed a resolution to these challenges: both formulations are correct, but they model different physical entities.

That proposed resolution has been challenged in [7,8]. Our paper shows that the challenge is based on incorrect assumptions in terms of inertial reference frame: the proposal of Barnett still stands. Our analysis seems rather trivial, but to our knowledge, since [7,8] have been published, at least [7], they have not been explicitly addressed or challenged in the literature. This paper does so, even if the correction is somehow straightforward.

Then, we bring the reader’s attention to [9], which illustrates how the whole controversy results from relativistic effects. {1,2] are correct, but they come from how we try to model, in an external observer frame of reference, the light in the new media in terms of a new frequency.

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2. The Barnett resolution

As proposed, equation (6) in [4], implies that the total momentum density of the electromagnetic field in a medium, is the sum of the electromagnetic momentum density in the vacuum plus a contribution of the electric polarization, and of the magnetization, of the medium as equation (14) in [5], directly derived from the Maxwell equation in a medium [6]. On the right hand side of equation (14) in [5], the first term is also provided by the photon momentum density used in the Planck/de Broglie equation, and in the corresponding Heisenberg uncertainty principle, i.e., p_{can}, in the convention of [4]. The next two terms represent the effect of the medium respectively due to polarization and magnetization of the medium. The term of the left is the total electromagnetic momentum in the medium and therefore p_{kin}, in the convention of [4]. It is equation (6) in [4]. Per [4], adding all contributions of the medium gives equation (7) of [4], with the Minkowski and Abraham terms identified by definition as in equation (1) in [4].

The difference between the two entities is now clear. The Minkowski term (wave aspect – Planck modified in medium of the photon), is the total momentum of electromagnetic momentum content in the medium (Abraham momentum – what a photon can mechanically impact) plus medium mechanical momentum minus the total contribution of the medium in terms of de Broglie matter wave momentum (canonical). In hindsight, an obvious result, which can be expressed as density of as integrated quantity of the volume of media. We recover equation (7) in [4].

As argued in [4], this can now not only recover the two models, but also shows why experiments encounter one of the other, depending on if measuring the Planck wave momentum impact (per previous paragraph the Minkowski term), or the photon momentum impact (what [4] call kinetic), as the Abraham term.

[4] also explains why the medium introduces a de Broglie wave contribution. It is because in the medium, polaritons are what propagates as quanta, and they include medium matter, along with the photon.

3. Questioning the Barnett proposal

[7,8] argue that the proposal in [4] is incorrect, because equation (7) in [4] would be between 4-vectors, yet p_{Abr} would not be a 4-vector.

The argument comes from putting the medium in a lab, and looking at what happens when an external photon enters the medium. [7] proposes that in such an experimentation, $p_{kin}^{med} = 0$. That is simply incorrect. The lab reference frame does not put the polaritons in an inertial frame where their momentum is constant / null. It is the overall medium that is in such state. The polaritons are created when the photon enters and they carry a non-zero momentum p_{kin}^{med}.

In fact, as explained in [4], and especially as [5] emphasizes, the medium terms can be understood as stress contributions. An inertial frame like the lab, is not the inertial frame for the stress on the medium, that should rather be seen as a field force (think of a gravitation field in examples of the equivalence principle). The equivalence principle does not apply till a similar force field / change in momentum is similarly applied.

So $p_{kin}^{med} \neq 0$

As a result the analysis falls apart and equation (7) in [4] is correct as a relationship between 4-vectors.

4. Why the controversy?

Interestingly, it turns out that the confusions between conservation of energy and momentum, and the Abraham or Minkowski's predictions, can be understood as a relativistic effect, between reference frames in the proper time of the propagating incoming wave, and external reference frames [9].

[9] illustrates it with an interesting approach of accelerated waves, where we can then extrapolate the result both to a wave propagating at c, yet accelerable, and a reference frame attached to the wave. Limits must be considered when reaching *c*, for massless photons. The limit illustrates the source of the problem: energy and momentum are clearly conserved in the proper time reference frame (limit. Momentum appear not conserved to the external observer when modeling what happens just as a change of frequency, which is a modeling shortcut, which requires a change of wavelength inside the new media to conserve energy; at the cost of momentum seemingly no more conserved, and open different options of like Abraham's and Mikowski's proposals for what it should be.

The formalism of [9] also seems to show that in such a context, while Physics is time reversible in proper time, externally only a positive time evolution seems possible, an interesting way to look at the arrow of time between microscopic and macroscopic/many body systems. We plan to revisit this in the context of the multi-fold theory [10-15]. Indeed, it reminds us of the comments made in section 9.13 of [10], where we mentioned that, with particles modeled as microscopic black holes [10,16], time may be perceived differently in both worlds, explaining in that case quantum physics, path integrals with its multiple paths, and its randomness.

5. Conclusions

We have shown the error of reasoning in the papers arguing against the Barnett resolution of the Minkowski Abraham dilemma.

As a result, Barnett's resolution stands.

The whole controversy seems to be the result of a relativistic effect.

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