

The Symmetry of N-domain and Numbers Conjectures

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Abstract In this paper, we discuss the symmetry of N-domain and we find that using the symmetry characters of Natural Numbers we can give proofs of the Prime Conjectures: Goldbach Conjecture、Polignac’s conjecture (Twins Prime Conjecture) and Riemann Hypothesis. . We also gave a concise proofs of Collatz Conjecture in this paper.

Keywords $D_{1/2 \times 1/2}$ N domain Riemann Hypothesis Prime numbers Conjectures Collatz Conjecture

1. Division of $D_{1/2 \times 1/2}$

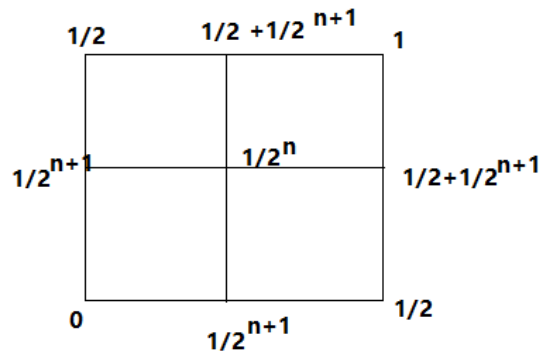
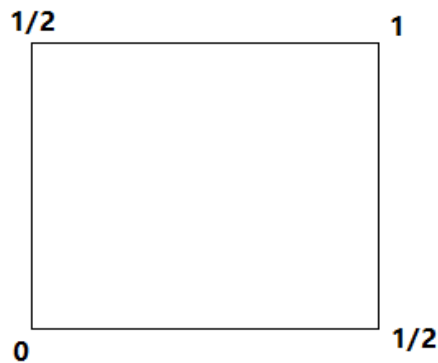


Fig.1. D-domain($D_{1/2 \times 1/2}$)

Fig.2. $D_{1/2 \times 1/2}$ regularization division by $1/2^n$

we can get a square $\begin{bmatrix} 1/2 & 1 \\ 0 & 1/2 \end{bmatrix}$, and we can give a regularization division by $1/2^n$ as show Fig.2 the matrix is:

$$\begin{bmatrix} 1/2 & \frac{1}{2} + 1/2^{n+1} & 1 \\ 1/2^{n+1} & 1/2^n & \frac{1}{2} + 1/2^{n+1} \\ 0 & 1/2^{n+1} & 1/2 \end{bmatrix}$$

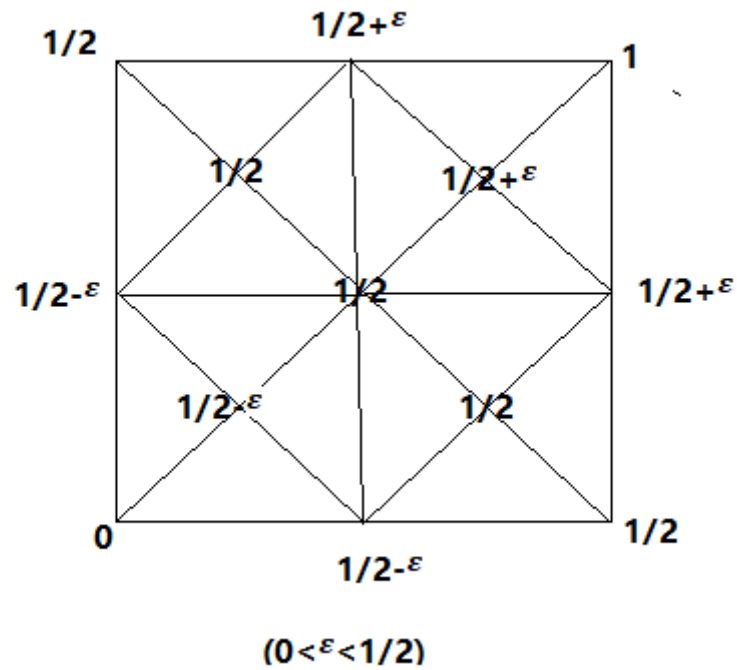


Fig.3. $D_{1/2 \times 1/2}$ diagonalization division by ε ($0 < \varepsilon < 1/2$)
 and we can also give a diagonalization division by ε ($0 < \varepsilon < 1/2$) as show Fig.3
 the matrix is:

$$\begin{bmatrix} 1/2 & \frac{1}{2} + \varepsilon & 1 \\ \frac{1}{2} - \varepsilon & 1/2 & \frac{1}{2} + \varepsilon \\ 0 & \frac{1}{2} - \varepsilon & 1/2 \end{bmatrix}$$

So we can get a division by $1/2^n$ and ε ($0 < \varepsilon < 1/2$) just show as on Fig.4

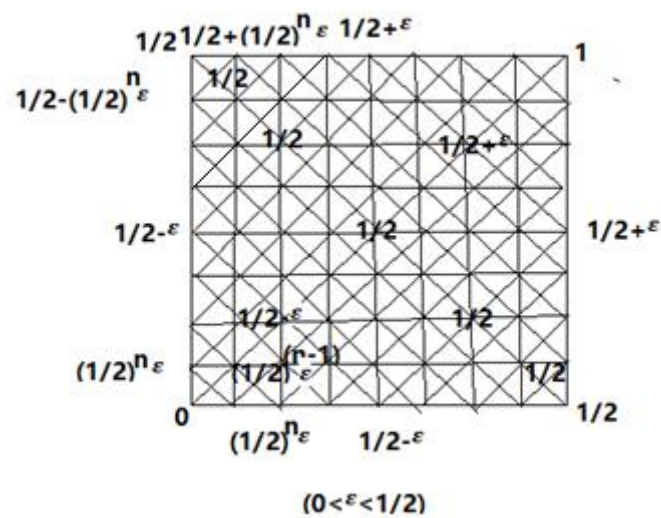


Fig.4. $D_{1/2 \times 1/2}$ division by $1/2^n$ and ε ($0 < \varepsilon < 1/2$)
 the matrix is:

$$\begin{bmatrix} 1/2 & \frac{1}{2} + \frac{1}{2^{n-1}} \varepsilon & \dots & \dots & 1 \\ \frac{1}{2} - \frac{1}{2^{n-1}} \varepsilon & \dots & \dots & \frac{1}{2} + \frac{1}{2^{n-1}} \varepsilon & \dots \\ \dots & \dots & 1/2 & \dots & \dots \\ \dots & \frac{1}{2} - \frac{1}{2^{n-1}} \varepsilon & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 1/2 \end{bmatrix}$$

The $\text{tr}(A) = 1/2 * n$

ε ($0 < \varepsilon < 1/2$) we notice $\varepsilon \rightarrow \frac{1}{n}$ $n \sim (3, 4, \dots)$

All the zero point would be: $\frac{1}{2} \pm \frac{1}{2^{n-1}} \frac{1}{n}$

And when n is a prime number p , the zero point would be: $\frac{1}{2} \pm \frac{1}{2^{p-1}} \frac{1}{p}$

Those are non-trivial zero points.

2. Proof of Riemann Hypothesis.

Riemann Zeta-Function

$$\xi(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod \frac{1}{1 - p^s} \quad (s = a + bi)$$

Riemann Hypothesis: all the Non-trivial zero-point of Zeta-Function $\text{Re}(s) = 1/2$.

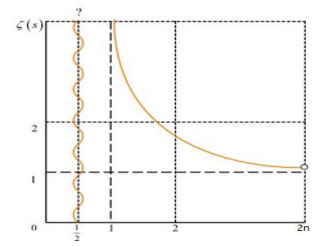
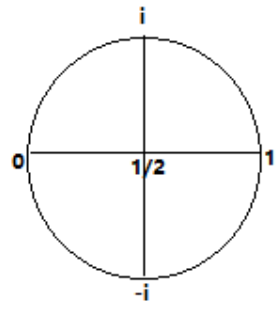
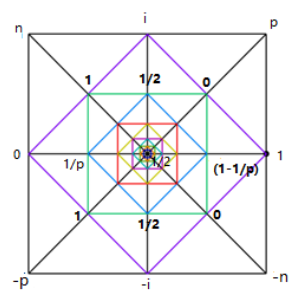


Figure.5. Riemann Hypothesis: all the non-trivial Zero points of Riemann zeta-function are on the $1/2$ axis.



(a) $L^{1/2}_{(0, 1/2, 1)}$ space



(b) N-domain division by $1/2^n$ and $1/p$

Fig.6. N-domain analytic continuation with $1/p$ in $L^{1/2}_{(0, 1/2, 1)}$ space

We have

$$1/2 = 1/2 \quad 0 = 1/2 - 1/2 \quad 1 = 1/2 + 1/2 \quad i^2 = -1$$

$$1/2 = (1/2 + 1/2 \cdot i) (1/2 - 1/2 \cdot i)$$

So we can construct a space with a 1/2 Fixed Point, we call it $L^{1/2}_{(0 \ 1/2 \ 1)}$
 We also have

$$\frac{1}{p} \rightarrow 0$$

$$1 - \frac{1}{p} \rightarrow 1$$

$$i^{2n} = \pm 1 \quad i^n = (i \ -1 \ -i \ 1)$$

$$i^p = \pm i$$

$$z_{p0} = \frac{1}{2} + (1 - \frac{2}{p})i$$

$$z_{pn} = \frac{1}{2} - (1 - \frac{2}{p})i$$

$$\begin{bmatrix} n & i & p \\ 0 & 1/2 & 1 \\ -n & -i & -p \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1/2 & 0 \\ 1/p & 1/2 & 1-1/p \\ 1 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & \dots\dots\dots & \frac{1}{2^n} [1+(1-2/p)i] \\ \dots\dots\dots & 1/2 & \dots\dots\dots \\ \frac{1}{2^n} [1-(1-2/p)i] & \dots\dots\dots & 1/2 \end{bmatrix}$$

The $\text{tr}(A) = 1/2 * n$

This is the proof of Hilbert–Pólya conjecture. This is mean that all the non-trivial Zero points of Riemann zeta-function are on the 1/2 axis just show as Fig.6. So we give a proof of Riemann Hypothesis.

In fact, we have

$$1 + \frac{e^{ip\pi} - e^{i2N\pi}}{(1+i)(1-i)} = \sum \frac{1}{2^N} = 2$$

$N \sim (0, 1, 2, 3, 4, \dots\dots\dots)$ all the natural numbers.

$p \sim (3, 5, 7, \dots\dots\dots)$ all the odd prime numbers.

3. proofs of the Prime Conjectures: Goldbach Conjecture、 Polignac's conjecture and Twins prime conjecture

We have

$n \sim (1, 2, 3, 4, \dots\dots\dots)$ All natural numbers excepted 0

$P \sim (2, 3, 5, 7, \dots\dots\dots)$ All prime numbers

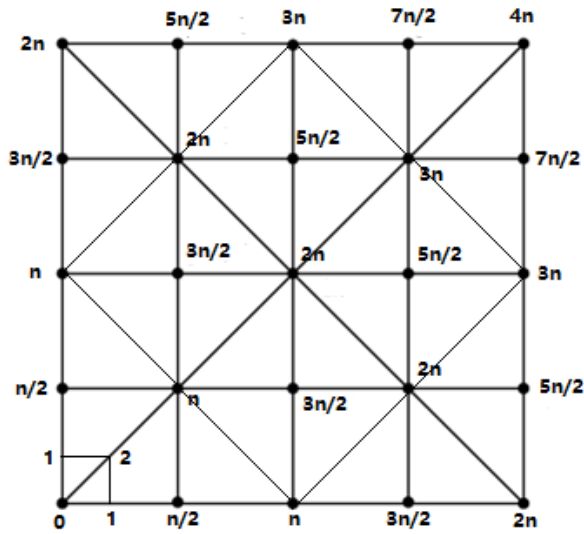


Fig.7. N-domain ($2n \times 2n$) Regularization division by $n/2$
we can get figure.7

as the matrix is :

$$\begin{bmatrix} 2n & \frac{5n}{2} & 3n & \frac{7n}{2} & 4n \\ \frac{3n}{2} & 2n & \frac{5n}{2} & 3n & \frac{7n}{2} \\ n & \frac{3n}{2} & 2n & \frac{5n}{2} & 3n \\ \frac{n}{2} & n & \frac{3n}{2} & 2n & \frac{5n}{2} \\ 0 & \frac{n}{2} & n & \frac{3n}{2} & 2n \end{bmatrix}$$

$p_0 \in P \sim (0, n)$ $p_n \in P \sim (n, 2n)$

$P \sim (2, 3, 5, 7, \dots)$ All prime numbers.

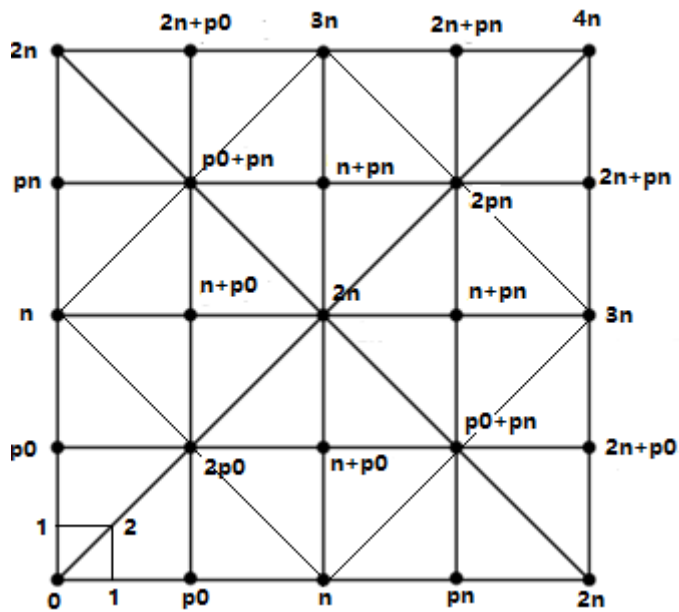


Fig.8. N-domain ($2n \times 2n$) Regularization division by n and P

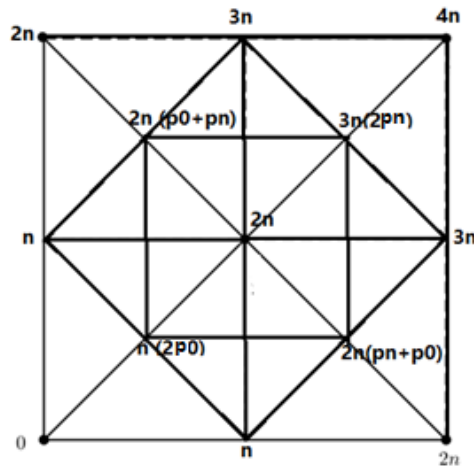
we can get figure.2 as the matrix is:

$$\begin{bmatrix} 2n & 2n + p_0 & 3n & 2n + pn & 4n \\ pn & p_0 + pn & n + pn & 2pn & 2n + pn \\ n & n + p_0 & 2n & n + pn & 3n \\ p_0 & 2p_0 & n + p_0 & p_0 + pn & 2n + p_0 \\ 0 & p_0 & n & pn & 2n \end{bmatrix}$$

We have

$$\begin{aligned} \frac{1}{2}n &\rightarrow p_0 \\ \frac{3}{2}n &\rightarrow pn \\ 2n &\rightarrow (p_0 + pn) \\ n &\rightarrow 2p_0 \\ 3n &\rightarrow 2pn \end{aligned}$$

so we can get figure.9 :



**Fig.9. The Symmetry of N-domain $2n \times 2n$ ($2n=p_0+pn$)
And All the numbers mod $(4n)$, then we get fig.10.**

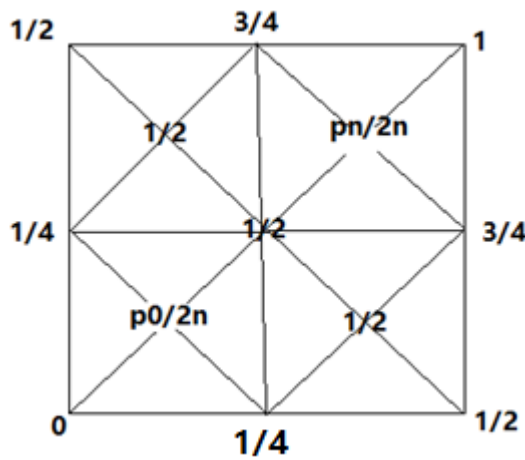


Fig.10. The Symmetry of D-domain $1/2 \times 1/2$

So we have:

$$\frac{pn}{2n} - 1/2 = 1/2 - \frac{p0}{2n} \quad 2n = p0 + pn \quad n \sim (2, 3, 4, \dots)$$

This is the proof of Goldbach conjecture.

$$\frac{pn}{2n} - \frac{p0}{2n} = \frac{3}{4} - \frac{1}{4}$$

$$pn - p0 = 3 \frac{n}{2} - 1 \frac{n}{2} = (3-1) \frac{2k}{2} = 2k \quad k \sim (1, 2, 3, 4, \dots)$$

This is the proof of Polignac's conjecture.

And when

$$k = 1 \\ pn - p0 = 2$$

This is the proof of Twin Primes Conjecture.

In fact we can get a square as show on fig.11.

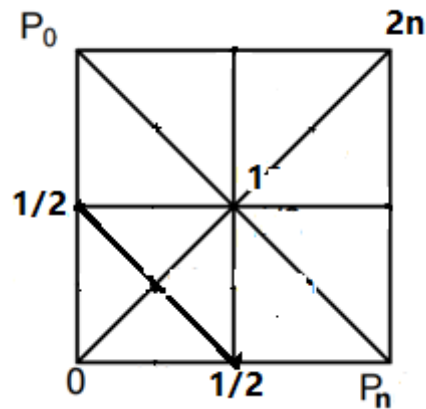


Fig.11. The N-domain ($p0 \times pn$)

the matrix is:

$$\begin{bmatrix} p0 & 2n \\ 0 & pn \end{bmatrix}$$

$$p0 \in P \quad pn \in P$$

$P \sim (2, 3, 5, 7, \dots)$ All prime numbers.

$n \sim (1, 2, 3, 4, \dots)$ All natural numbers excepted 0.

So we have:

$$\frac{2n - p0}{pn - 0} = 1$$

$$2n = p0 + pn \quad n \sim (2, 3, 4, \dots)$$

This is the proof of Goldbach conjecture.

$$\frac{pn - p0}{2n - 0} = 1$$

$$pn - p0 = 2n$$

$$n \sim (1, 2, 3, 4, \dots)$$

This is the proof of Polignac's conjecture.

And when

$$n = 1$$

$$pn - p0 = 2$$

This is the proof of Twin Primes Conjecture.

4. Concise proof of Collatz Conjecture

Collatz Conjecture:

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \pmod{2} \\ 3n + 1 & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

$$k \in \mathbb{N} \rightarrow f^k(n) = 1$$

We can get figure.10

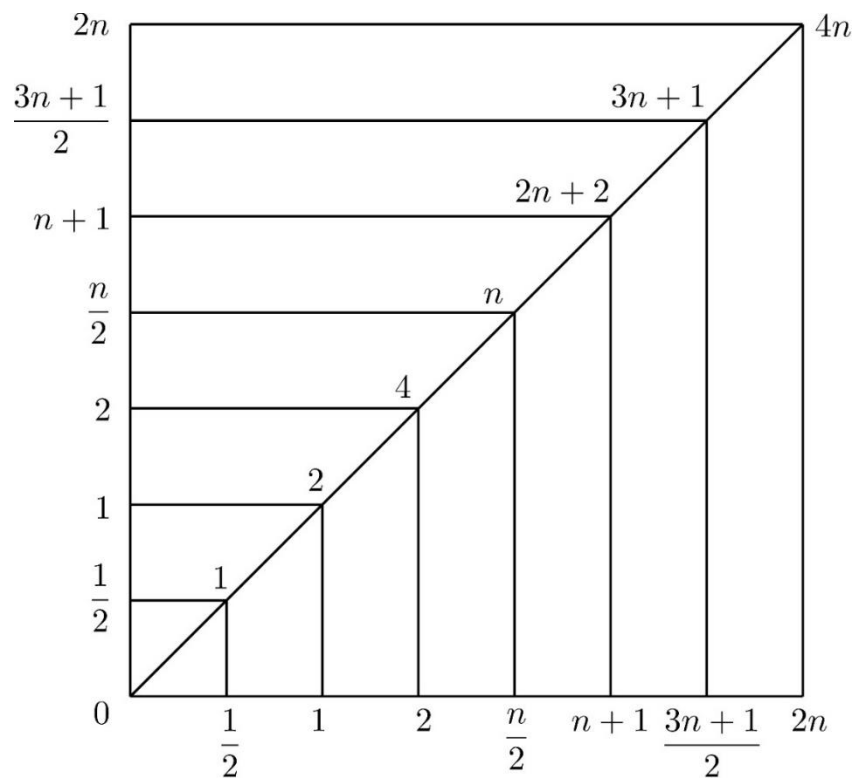


Fig.12 N-domain(2n × 2n) Diagonalization division by n/2

$n \sim (1, 2, 3, 4, \dots)$ all the natural numbers excepted 0

we have:

$$\frac{n}{2} = \frac{3n+1}{2} = \frac{2n+2}{n+1} = \frac{4n+2n+2}{3n+1} = \frac{4n+4}{2n+2} = \frac{4n}{2n} = \frac{4}{2} = \frac{2}{1} = \frac{1}{\frac{1}{2}}$$

$$= 2 = \sum \frac{1}{2^N}$$

$N \sim (0, 1, 2, 3, 4, \dots)$ all natural numbers. This is a concise proof of Collatz Conjecture.

Note

Time quantization

Time is a basic concept in physics. But till now, we have no idea to use mathematical model to describe the structure of “**Time**”. In Newton’s system, Time is an independent existence with space. In Einstein’s system, Time and Space are bonded together just considering the Velocity of Light is a constant **C(m/s)**. And then for a Quantum system, we consider the energy is discrete and then the “**Time contentiousness**” disappeared in this system. But It is that the **Dimension** of Plank’s constant **h(J.s)** is also including the unit of Time . So, we think that if we may construct a Dimension system of Time-Space with energy based on two priori conditions: the velocity of light is a constant **C** and the unit of energy with Time is a constant **h, Plank constant**. And if we can quantized this Time-Space with energy system, Maybe we can get a mathematical model to describe more physics details of the basic structure of Time-space with energy and get a **Unified Field Theory**.

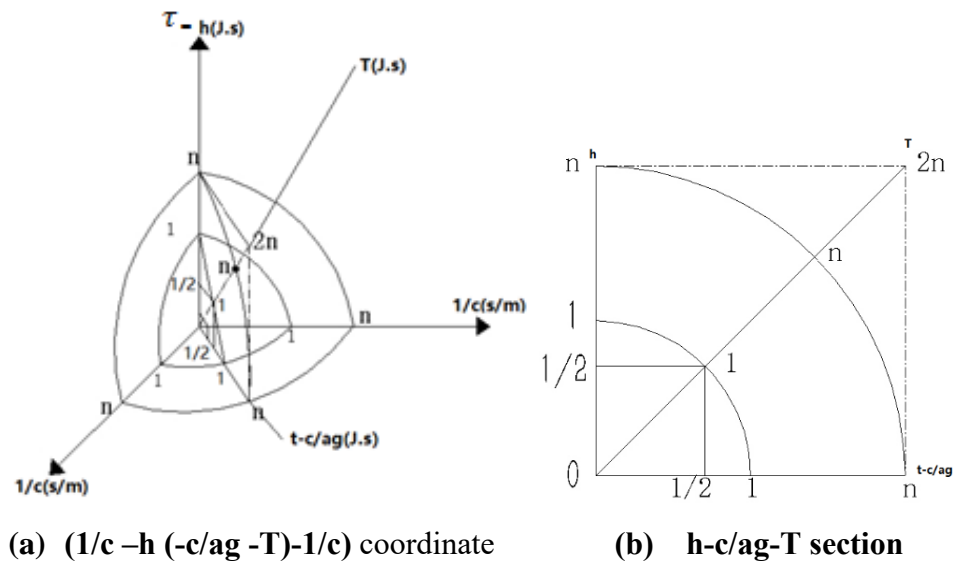


Fig. 13. Time -space with energy coordinate

τ can be defined as

$$\tau \sim nh \text{ (J.s)} \quad n \sim (1,2,3,\dots)$$

h is **Planck constant**.

t can be defined as τ

$$t \sim n \left(\frac{c}{a_g} \right) \text{ (J.s) } n \sim (1, 2, 3, \dots)$$

And

$$T \sim 2n \text{ (J.s)}$$

C is the velocity of Light (m/s), and a_g is the Intensity of field of gravitation (m/s²). So we got a time with energy coordinate system (h-1/c-c/ag-T-1/c) show as Fig.12(a).

For a physic system we can define **mass** as:

$$m_0 \sim \frac{h}{c^2} \text{ (J.s}^3 \cdot \text{m}^{-2})$$

$$m \sim n^3 \frac{h}{c^2} \text{ (J.s}^3 \cdot \text{m}^{-2})$$

and at every moment $T \sim 2n \text{ (J.s)}$ show as **fig.12(b)**

$$\tau = t$$

$$nh = nc/a_g$$

$$\frac{1}{a_g} = h/c \text{ (J.s}^2 \cdot \text{m}^{-1})$$

So we have:

$$m_0 a_g \sim 1/c \text{ (s.m}^{-1})$$

We can define a time space with energy as :

$$T \sim 2n \text{ (J.s) } n \sim (1, 2, 3, \dots)$$

$$S_0 \sim \frac{1}{4} * h * \frac{c}{a_g} \quad S_n \sim n^2 h * \frac{c}{a_g}$$

$$\frac{S_n}{S_0} = 4n^2 \text{ (J}^2 \cdot \text{s}^2)$$

$$\frac{m}{m_0} \sim n^3 \text{ (J}^3 \cdot \text{s}^3)$$

And we notice that if **Goldbach conjecture** $2n = p_0 + p_n$ (n is a nature number , and p_0, p_n are primer numbers) and **Polignac's conjecture** $p_n - p_0 = 2n$ (n is a nature number , and p_0, p_n are primer numbers) be proofed, then

$$T \sim 2n = (p_n \pm p_0)$$

$$\frac{s_n}{s_0} \sim 4n^2 = (pn \pm p_0)^2$$

$$\frac{m}{m_0} \sim n^3 = \left(\frac{pn \pm p_0}{2}\right)^3$$

This will be a model to explain the **randomness** of the nature and **Quantum Entanglement**.

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