An Truly Easy Proof:
Pi is Irrational

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Abstract

Using the derivative of an integer polynomial composed with Euler's formula we prove that π is irrational.

Proof

Proofs of the irrationality of π are numerous [1], but none are as easy and direct as the following.

Theorem 1. *π is irrational.*

Proof. Let $P(z)$ and $P'(z)$ be integer polynomials with roots other than -1. Then $P(-1)$ and $P'(-1)$ are non-zero integers. Consider that

$$P'(e^{iz}) = P'(e^{iz})ie^{iz}. \tag{1}$$

Now assume for a contradiction that $\pi = p/q$. Per Euler's formula $e^{ip/q} = -1$. We have then

$$P'(-1) = P'(-1)i(-1),$$

using (1). But this says $P'(-1)$ is a complex number, a contradiction. $\square$

References

[1] Eymard, P., Lafon, J.-P. (2004). *The Number π*. Providence, RI: American Mathematical Society.