Inversions (Mirror Images) With Respect to the Unit Circle and Division by Zero

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Abstract: In this note, we will consider the interesting inversion formula that was discovered by Yoichi Maeda with respect to the unit circle on the complex plane from the viewpoint of our division by zero: $1/0 = 0/0 = 0$.

David Hilbert:

The art of doing mathematics consists in finding that special case which contains all the germs of generality.

Oliver Heaviside:

Mathematics is an experimental science, and definitions do not come first, but later on.

Key Words: Division by zero, division by zero calculus, point at infinity, inversion, mirror image.

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1 Introduction

In this note, we will consider the interesting inversion formula that was discovered by Yoichi Maeda with respect to the unit circle on the complex plane from the viewpoint of our division by zero: $1/0 = 0/0 = 0$.

2 Y. Maeda's formula

Y. Maeda [1] obtained the following interesting result:

Proposition 4.2 (inverse relation) : *Let U be the unit circle on a complex plane. Let $A(\alpha)$ and $B(\beta)$ be two points on U. Let $-A(\overline{\alpha})$ be the conjugate of $A(\alpha)$. Let $I(zi)$ be the intersection of $-A(\alpha)B(\beta)$ and the real axis. Let $I(ze)$ be the intersectin of $A(\alpha)B(\beta)$ and the real axis. Then, $I(zi) \cdot I(ze) = 1$, that is, $I(zi)$ is the inversion of $I(ze)$ with respect to U.*

Furthermore, he gave the formulas

$$ze = \frac{\alpha\overline{\beta} - \overline{\alpha}\beta}{(\alpha - \overline{\alpha}) - (\beta - \overline{\beta})}$$

and

$$zi = \frac{\alpha\beta - \overline{\alpha}\overline{\beta}}{(\alpha - \overline{\alpha}) + (\beta - \overline{\beta})}.$$

3 Essences of the division by zero: $1/0 = 0/0 = 0$ and the division by zero calculus

S. Takahasi ([6]) discovered a simple and decisive interpretation for the division by zero by analyzing the extension of fractions and by showing the complete characterization in the following:

Proposition: *Let F be a function from $\mathbf{C} \times \mathbf{C}$ to $\mathbf{C}$ satisfying*

$$F(b,a)F(c,d) = F(bc, ad)$$

for all

$$a, b, c, d \in \mathbf{C}$$

and

$$F(b,a) = \frac{b}{a}, \quad a, b \in \mathbf{C}, a \neq 0.$$

Then, we obtain, for any $b \in \mathbf{C}$

$$F(b,0) = 0.$$

Note that the complete proof of this proposition is simply given by 2 or 3 lines.

In the long mysterious history of the division by zero, this proposition seems to be decisive. The paper had been published over fully 9 years ago, but we see still curious information on the division by zero and we see still many wrong opinions on the division by zero with confusions. See the references.

Indeed, Takahasi's assumption for the product property should be accepted for any generalization of fraction (division). Without the product property, we will not be able to consider any reasonable fraction (division).

Following Proposition, we should **define**

$$F(b,0) = \frac{b}{0} = 0,$$

and consider that, for the mapping

$$W = f(z) = \frac{1}{z}, \tag{3.1}$$

the image of $z = 0$ is $W = f(0) = 0$ (**should be defined from the form**). This fact seems to be a curious one in connection with our well-established popular image for the point at infinity on the Riemann sphere. As the representation of the point at infinity on the Riemann sphere by the zero $z = 0$, we will see some delicate relations between 0 and ∞ which show a strong discontinuity at the point of infinity on the Riemann sphere. We did not consider any value of the elementary function $W = 1/z$ at the origin $z = 0$, because we did not consider the division by zero $1/0$ in a good way. Many and many people consider its value at the origin by limiting like $+\infty$ and $-\infty$ or by the point at infinity as ∞. However, their basic idea comes from **continuity** with the common sense or based on the basic idea of Aristotele. However, as the division by zero we will consider its value of the function $W = 1/z$ as zero at $z = 0$. We will see that this new definition is valid widely in mathematics and mathematical sciences, see the references. Therefore, the division by zero will give great impact to calculus, Euclidean geometry, analytic geometry, complex analysis and the theory of differential equations at an undergraduate level and furthermore to our basic idea for the space and universe.

As an algebraic definition of the division by zero, the Yamada field containing the generalized fractionals and containing the division by zero was given completely. Furthermore, our definition is the same of the Moore-Penrose generalized solution of the fundamental equation $az = b$ for the case; that is, $z = b/0 = 0$ that is always uniquely determined the solution.

However, for a function case we need the concept of **the division by zero calculus**.

For a function $y = f(x)$ which is n order differentiable at $x = a$, we will **define** the value of the function, for $n > 0$

$$\frac{f(x)}{(x-a)^n}$$

at the point $x = a$ by the value

$$\frac{f^{(n)}(a)}{n!}.$$

For the important case of $n = 1$,

$$\frac{f(x)}{x-a}|_{x=a} = f'(a). \tag{3.2}$$

In particular, the values of the functions $y = 1/x$ and $y = 0/x$ at the origin $x = 0$ are zero. **We write them as $1/0 = 0$ and $0/0 = 0$, respectively.** Of course, the definitions of $1/0 = 0$ and $0/0 = 0$ are not usual ones in the sense: $0 \cdot x = b$ and $x = b/0$ in the usual sense (however, in the sense of the Moore-Penrose generalized solution, our definition is the same). Our division by zero is given in this sense and is not given by the usual sense as in stated in [2, 3, 4, 5].

In particular, note that

$$\tan\frac{\pi}{2} = 0$$

and for $a > 0$

$$\left[\frac{a^n}{n}\right]_{n=0} = \log a.$$

These will mean that the concept of division by zero calculus is important.

4 Interpretation of the Y. Maeda's formula

First of all, we note that Maeda's formula is the well-known result for the case $A(\alpha) = B(\beta)$ that is not the points i and $-i$.

For the parallel case of two lines $A(\alpha)B(\beta)$ and the real axis that is not $A(\alpha) \neq B(\beta)$ and not the real line, $I(ze) = 0, I(zi) = 0$, by the division by zero.

For the case $A(\alpha) = -1, B(\beta) = 1$, $I(ze) = 0, I(zi) = 0$, by the division by zero.

For the cases $A(\alpha) = B(\beta) = i, -i$, $I(ze) = 0, I(zi) = 0$, by the division by zero.

(For the case $A(\alpha) = B(\beta) = 1, -1$, $I(ze) = 0, I(zi) = 0$, by the division by zero. However, in these cases, by the division by zero calculus or by the classical analysis, we have the desired results $I(ze) = I(zi) = 1, -1$, respectively.)

We can consider the Maeda's formula and result for arbitrary points $A(\alpha)$ and $B(\beta)$ on the unit circle.

Then we see that

$$\frac{1}{0} = \frac{0}{0} = 0.$$

In order to show the importance of the inversion and division by zero, we stated the various related results with one session of the book [3].

Division by zero, division by zero calculus, their essence:

We must not think about the value of the function $f(x) = 1/x$ at the origin, it is a tradition of over 2,000 years since Aristotle. This is the first of the Ten Commandments of Mathematics, and it is still a current common sense. The discovery in 2014 is that its value is zero: $f(0) = 0$. For the first time, after studying the properties surrounding the singularity, we entered the world of the singularity itself, and a new world emerged. We continue to argue that mathematics is fundamentally flawed and that elementary mathematics should be corrected. Situations that cannot be officially acknowledged reveal human weaknesses, and being aware of human weaknesses has great utility in opening up world peace and enlightenment. Textbooks and academic books for mathematics undergraduate students should be changed. Out of love for true wisdom. (2023.10.29.10:30)

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