“Wave” Arithmetic versus “Standard” Arithmetic

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Abstract In this paper, a comparison of “standard” arithmetic (S-arithmetic) [1] and newly proposed “wave” arithmetic (W-arithmetic) is going to be performed. S-arithmetic stands for the arithmetic that is commonly used in mathematics, while the W-arithmetic is arithmetic based on slightly modified axioms of S-arithmetic. The basic change that is introduced in W-arithmetic, is that operation of addition does not have only one “global” neutral element, but rather two neutral elements - one global and one “local” (one specific to each number). In the text that follows, only the set of natural number together with zero is going to be considered.

1 Introduction

In this paper, a new type of arithmetic is going to be introduced. It is going to be called “Wave” or W-arithmetic. In W-arithmetic operation of addition has a new feature – instead of one neutral element, two neutral elements for each number are proposed (with the exception for number 0). In more detail, result of summation of any natural number n and 0 would be n (and that is the case in S-arithmetic, too), but also the summation of $n + n$ will again produce result n. It can be seen that there is a global neutral element 0, and each natural number will have a “local” neutral element that is equal to itself. Here, a few examples from real life that could used as a motiviation for discussion about W-arithmetic are going to be presented. For instance, when some blue, green or any other color is combined with the same color, nothing is going to be changed qualitatively, independently of how much color is added to the initial amount. Similarly, if there is some amount of hydrogen and some additional hydrogen is added nothing is going to be changed, qualitatively (here, the extremely high temperature and/or pressure are not analyzed). The same would hold for any other chemical element. Another example would be normalized sinusoidal wave (and name comes from this example) to which the "same" sinusoidal wave is added (same frequency and phase), and the sum is normalized. There are other examples, but they are not going to be mentioned here. In W-arithmetic the numbers do not represent quantities, but rather different qualities. Different labels represent different qualities. Basically, it cannot be said if some number is bigger or smaller than some other number – only thing that can be said is that they are different. However, relation "bigger" or "smaller" can be introduced axiomatically. In W-arithmetic, when we are talking about natural numbers, it is not done in the same sense like in S-arithmetic. Rules of mapping two natural numbers to a sum will hold for numbers in W-arithmetic, too, with the exception of summing number with itself.

In this paper, it is going to be shown that interpretation of the numbers in the context of the proposed W-arithmetic fits much better some axioms of the currently accepted set theory. The existence of such arithmetic is suggested in [2]. Also, it will be shown that with currently adopted interpretation of numbers in S-arithmetic, it seems inconsistent to state that the number of even numbers is equal to the number of natural numbers, or once the even numbers are removed from the set of natural numbers, all numbers are still inside the set of natural numbers. In the case of numbers in the context of W-arithmetic, that is quite reasonable.

2 Analysis of characteristics of W-arithmetic

In this section, some characteristics of W-arithmetic are going to be presented and compared to characteristics of S-arithmetic.

1. All natural numbers can be generated from number 1 and implementation of the operation of addition in the context of S-arithmetic – every natural number can be generated from single "generator". In the case of W-arithmetic, that is not possible – it is necessary to have at least 2 "generators", 1 and 2 (it is possible to start with more "generators", but it cannot be less than 2). Then, by implementation of addition in the context of W-arithmetic, it is possible to generate all natural

numbers. It is simple to understand why the second generator is necessary: 1+1=1 in W-arithmetic.

2. In W-arithmetic subtraction is not always defined and will not always have a unique result. It is clear that 2-1 is not defined - it is not easy task to create some meaningful definition. Also, when the the number is subtracted from itself (e.g. 1-1) there are two possible outcomes: 0 and the number itself.

3. In W-arithmetic addition is not always associative operation. For instance: 3+1+1+1+1 can produce any number between 4 and 7 depending on the "position of brackets" - 3+(1+1+1+1) = 4, (3+1)+(1+1+1) = 5 , ((3+1)+1)+(1+1) = 6, (((3+1)+1)+1)+1 = 7.

4. In W-arithmetic multiplication can be introduced only axiomatically, since the way of introduction of multiplication in the context of S-arithmetic cannot be applied in W-arithmetic. That means that division can be introduced only axiomatically, too. Also, division will not always produce unique result, since, like in the case of addition, multiplication in W-arithmetic has two neutral elements – number 1 and every number is neutral element for itself. It is interesting to notice that in W-arithmetic the number of primes is bigger – squares of prime numbers are primes, too .

5. Since the numbers in W-arithmetic represent different qualities, numbers cannot be compared in the sense it is done in S-arithmetic. So, here we cannot say that 2 is bigger than 1, or that a million is bigger than 2 – the only thing that can be said is that they are different. However, those relations can be introduced axiomatically.

3 Set theory and S- and W-arithmetic

Here, the idea that number of even numbers is equal to number of natural numbers is going to be investigated in the context of S- and W-arithmetic. It is going to be shown that in the case of W-arithmetic that creates no inconsistencies, while in the case of S-arithmetic it seems that it creates some inconsistencies. Even numbers in W-arithmetic are those that are even in S-arithmetic.

If it is assumed that the number of odd and even numbers is equal to the number of natural numbers, the following “situation” is going to be created in S-arithmetic: since it is claimed that it is possible to create bijection between the natural numbers and even numbers (trough relation $n \rightarrow 2n$) and between the natural numbers and odd numbers (through relation $n \rightarrow 2n - 1$), it would be possible to map set of natural numbers to itself (set of natural numbers) trough 1 on 2 map (through simultaneous relations $n \rightarrow 2n$ and $n \rightarrow 2n-1$), which could be seen as inconsistent. The inconsistency comes from the fact that it is adopted that two sets have the same number of elements if it is possible to establish one-to-one mapping from one set to the other, and here is stated that they have the same number of numbers even in the case when the mapping is 1 on 2 (it is easy to understand that it will be the case for mapping 1 on 3, or 1 on google, or 1 on any finite number). In the case of the W-arithmetic this does not create a problem since every number in that context represent a set of the copies of the same number. In the context of S-arithmetic that idea is, incorrectly, used, too. The fact that the result of the operation of addition/subtraction of two equal numbers will result in the same number is not justifiable in the context of S-arithmetic – it is not justifiable to have copies of the same number and count it only once, and it should not be possible to remove some number from the set and still claim that it is somehow inside. In the context of W-arithmetic, both those things are quite reasonable to be considered.

References

[1] P. Lockhart (2019). *Arithmetic*, Belknap Press.

[2] M.V. Jankovic (2023). Proof that Exists Infinitely Many Primes of the Form n^2+1, viXra: 2303.0065.