The electron interaction with the Dirac delta pulse

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Abstract

After the clasical approach to acceleration of a charged particle by δ-form impulsive force, we consider the corresponding quantum theory based on the Volkov solution of the Dirac equation. We determine the modified Compton formula for frequency of photons generated by the scattering of the δ-form laser pulse on the electron in a rest. The article follows the physical ideas involved in the author text with the title **Electron in an Ultrashort Laser Pulse** (Pardy, 2003).

1 Introduction

The interaction of ultra-short strong laser pulses with extended systems led recently to an advance from the attosecond to few femtosecond electronic and nuclear dynamics. The interaction of clusters with strong ultrafast laser fields leads to the formation of nanoplasmas in which there is a high degree of charge localisation with the emission of energetic electrons and highly charged ions (Ciappina et al., 2016)

Most recently use of short pulses ($\sim$ 10 fs) has succeeded in isolating the electron dynamics from the longer timescale essentially frozen ion dynamics revealing a higher degree of fragmentation anisotropy in both electrons and ions compared to the isotropic distributions found from longer pulses ($\sim$ 100 fs) (Ciappina et al., 2016).

On the othe hand, the problem of interaction an elementary particles with the laser field is, at present time, one of the most prestigeous problem in the particle physics. It is supposed that, in the future, the laser will play the same role in particle physics as the linear or circle accelerators working in today particle laboratories.

One of the problem is acceleration of charged particles by the laser. The acceleration effectiveness of the linear or circle accelerators is limitied not only by geometrical size of them but also by the energy loss of accelerated particles which is caused by bremsstrahlung during the acceleration. The amount of radiation follows from the Larmor formula for emission of radiation by accelerated charged particle (Maier, 1999).

In case of laser acceleration the classical idea is that acceleration is caused by laser light as the periodic electromagnetic field. However, it is possible to show that periodic electromagnetic wave does not accelerate electrons in classical and quantum theory, because the electric and magnetic components of the light field are mutually perpendicular and it means the motion caused by the classical periodic electromagnetic field is not linear but periodic (Landau and Lifshitz, 1962). Similarly, the quantum motion of electron in such a wave firstly described by Volkov (1935) must coincide in the classical limit with the clasical solution.

The situation changes if we consider laser beam as a system of photons and the interaction of electron with laser light is via the one-photon Compton process

$$\gamma + e \rightarrow \gamma + e, \tag{1a}$$

or, the multiphoton Compton process

$$n\gamma + e \rightarrow \gamma + e, \tag{1b}$$

where n is a natural number.

The equation (1b) is the symbolic expression of the two different physical processes. One process is the nonlinear Compton effect in which several photons are absorbed at a single point, but only single high-energy photon is emitted. The second process is interaction where electron scatters twice or more as it traverses the laser focus. In our article the attention is devoted to the nonlinear Compton process.

It is evident that acceleration by laser can be adequately described only by quantum electrodynamics. Such viewpoint gives us the motivation to investigate theoretically the effectiveness of acceleration of charged particles by laser beam. The acceleration of charged particles by laser beam has been studied by many authors (Tajima, 1979; Katsouleas et

al., 1983; Scully et al., 1991; Baranova et al., 1994) . Many designs for such devices has been proposed. Some of these are not sufficiently developed to be readily intelligible, others seem to be fallacious and others are unlikely to be relevant to ultra high energies. Some designs were developed only to observe pressure of laser light on microparticles in liquids and gas (Askin, 1970; Askin, 1972).

It is necessary to say that acceleration by the Compton effect differs from the effect of light pressure which was considerred in past for instance by Russian physicists Lebedev and Nichols and Hull (1903) and which was also verified experimentally by these scientists. The measurement consisted in determination of force acting on the torsion pendulum. It was confirmed that the pressure is very small. Only after invention of lasers the situation changed because of the very strong intensity of the laser light which can cause the great pressure of the laser ray on the surface of the condensed matter.

Here, we consider the interaction of an electron with a laser pulse. First, we consider the clasical approach to acceleration of a charged particle by δ-form impulsive force. Then, we discuss the corresponding quantum theory based on the Volkov solution of the Dirac equation. We determine the modified Compton formula for frequency of photons generated by the scattering of the δ-form laser pulse on the electron in a rest.

2 Classical theory of interaction of particle with an impulsive force

We idealize the impulsive force by the dirac δ-function. Newton's second law for the interaction of a massive particle with mass m with an impulsive force $P\delta(t)$ is as follows

$$m\frac{d^2x}{dt^2} = P\delta(t), \tag{2}$$

where P is some constant.

Using the Laplace transform on the last equation, with

$$\int_0^\infty e^{-st}x(t)dt = X(s), \tag{3}$$

$$\int_0^\infty e^{-st}\ddot{x}(t)dt = s^2X(s) - sx(0) - \dot{x}(0), \tag{4}$$

$$\int_0^\infty e^{-st}\delta(t)dt = 1, \tag{5}$$

we obtain:

$$ms^2X(s) - msx(0) - m\dot{x}(0) = P. \tag{6}$$

For a particle starting from rest with $\dot{x}(0) = 0, x(0) = 0$, we get

$$X(s) = \frac{P}{ms^2}, \tag{7}$$

and using the inverse Laplace transform, we obtain

$$x(t) = \frac{P}{m}t \tag{8}$$

and

$$\dot{x}(t) = \frac{P}{m}. \tag{9}$$

Let us remark, that if we express δ-function by the relation $\delta(t) = \dot{\eta}(t)$, then from equation (2) follows $\dot{x}(t) = P/m$, immediatelly. The physical meaning of the quantity P can be deduced from equation $F = P\delta(t)$. After t-itegration we have $\int F\,dt = mv = P$, where m is mass of a body and v its final velocity (with $v(0) = 0$). It means that value of P can be determined a posteriory and then this value can be used in more complex equations than eq. (2). Of course it is necessary to suppose that δ-form of the impulsive force is adequate approximation of the experimental situation.

In case of the harmonic oscillator with the damping force and under influence of the general force, the Newton law is as follows:

$$m\frac{d^2x(t)}{dt^2} + b\dot{x}(t) + kx(t) = F(t). \tag{10}$$

After application of the Laplace transform and with regard to the same initial conditions as in the preceding situation, $\dot{x}(0) = 0, x(0) = 0$, we get the following algebraic equation:

$$ms^2X(s) + bsX(s) + kX(s) = F(s), \tag{11}$$

or,

$$X(s) = \frac{F(s)}{m\omega_1}\frac{\omega_1}{(s + b/2m)^2 + \omega_1^2} \tag{12}$$

with $\omega_1^2 = k/m - b^2/4m^2$.

Using inverse Laplace transform denoted by symbol $\mathcal{L}^{-1}$ applied to multiplication of functions $f_1(s)f_2(s)$

$$\mathcal{L}^{-1}(f_1(s)f_2(s)) = \int_0^t d\tau F_1(t - \tau)F_2(\tau), \tag{13}$$

we obtain with $f_1(s) = F(s)/m\omega_1$, $f_2(s) = \omega_1/((s+b/2m)^2+\omega_1^2)$, $F_1(t) = F(t)/m\omega_1$, $F_2(t) = \exp(-bt/2m)\sin\omega_1 t$.

$$x(t) = \frac{1}{m\omega_1}\int_0^t F(t - \tau)e^{-\frac{b}{2m}\tau}\sin(\omega_1\tau)d\tau. \tag{14}$$

For impulsive force $F(t) = P\delta(t)$ we have from the last formula

$$x(t) = \frac{P}{m\omega_1}e^{-\frac{b}{2m}t}\sin\omega_1 t. \tag{15}$$

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3   Classical interaction of a charged particle with a laser pulse

If we consider the δ-form electromagnetic pulse, then we can write

$$F_{\mu\nu} = a_{\mu\nu}\delta(\varphi). \tag{16}$$

where $\varphi = kx = \omega t - \mathbf{kx}$. In order to obtain the electromagnetic impulsive force in this form, it is necessary to define the four-potential in the following form:

$$A_\mu = a_\mu \eta(\varphi), \tag{17}$$

where function η is the Heaviside unit step function defined by the relation:

$$\eta(\varphi) = \left\{ \begin{array}{ll} 0, & \varphi < 0 \\ 1, & \varphi \geq 0 \end{array} \right. . \tag{18}$$

If we define the four-potential by the equation (17), then the electromagnetic tensor with impulsive force is of the form:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (k_\mu a_\nu - k_\nu a_\mu)\delta(\varphi) = a_{\mu\nu}\delta(\varphi). \tag{19}$$

To find motion of an lectron in δ-form electromagnetic force, we must solve immediately either the Lorentz equation, or, to solve Lorentz equation in general with four potential $A_\mu = a_\mu A(\varphi)$ and then to replace the four-potential by the eta function. Following Meyer (1971) we apply his method and then replace $A_\mu(\varphi)$ by $a_\mu\eta(\varphi)$ in the final result.

The Lorentz equation with $A_\mu = a_\mu A(\varphi)$ reads:

$$\frac{dp_\mu}{d\tau} = \frac{e}{m}F_{\mu\nu}p^\nu = \frac{e}{m}(k_\mu a \cdot p - a_\mu k \cdot p)A'(\varphi). \tag{20}$$

where the prime denotes derivation with regard to φ, τ is proper time and $p_\mu = m(dx_\mu/d\tau)$. After multiplication of the last equation by k^μ, we get with regard to the Lorentz condition $0 = \partial_\mu A^\mu = a^\mu \partial_\mu A(\varphi) = k_\mu a^\mu A'$, or, $k \cdot a = 0$ and $k^2 = 0$, the following equation:

$$\frac{d(k \cdot p)}{d\tau} = 0 \tag{21}$$

and it means that $k \cdot p$ is a constant of the motion and it can be defined by the initial conditions for instance at time $\tau = 0$. If we put $p_\mu(\tau = 0) = p_\mu^0$, then we can write $k \cdot p = k \cdot p^0$. At this moment we have:

$$k \cdot p = \frac{mk \cdot dx}{d\tau} = m\frac{d\varphi}{d\tau}, \tag{22}$$

or,

$$\frac{d\varphi}{d\tau} = \frac{k \cdot p^0}{m}. \tag{23}$$

So, using the last equation and relation $d/d\tau = (d/\varphi)d\varphi/d\tau$, we can write equation (20) in the form

$$\frac{dp_\mu}{d\varphi} = \frac{e}{k \cdot p^0}(k_\mu a \cdot p - a_\mu k \cdot p^0)A'(\varphi) \tag{24}$$

giving (after multiplication by a^μ)

$$\frac{d(a \cdot p)}{d\varphi} = -ea^2 A', \tag{25}$$

or,

$$a \cdot p = a \cdot p^0 - ea^2 A. \tag{26}$$

Substituting the last formula into (24), we get:

$$\frac{dp_\mu}{d\varphi} = -e\left(a_\mu - \frac{k_\mu a \cdot p^0}{k \cdot p^0}\right)\frac{dA}{d\varphi} - \frac{e^2 a^2}{2k \cdot p^0}\frac{d(A^2)}{d\varphi}k_\mu. \tag{27}$$

This equation can be immediately integrated to give the resulting momentum in the form:

$$p_\mu = p_\mu^0 - e\left(A_\mu - \frac{A^\nu p_\nu^0 k_\mu}{k \cdot p^0}\right) - \frac{e^2 A^\nu A_\nu k_\mu}{2k \cdot p^0}. \tag{28}$$

Now, if we put into this formula the four-potential (17) of the impulsive force, then for $\varphi > 0$ when $\eta > 1$, we get:

$$p_\mu = p_\mu^0 - e\left(a_\mu - \frac{a^\nu p_\nu^0 k_\mu}{k \cdot p^0}\right) - \frac{e^2 a^\nu a_\nu k_\mu}{2k \cdot p^0}, \tag{29}$$

The last equation can be used to determination of the magnitude of a_μ similarly as it was done in discussion to the eq. (2). It can be evidently expressed as the number of k-photons in electromagnetic momentum. For $\varphi < 0$, it is $\eta = 0$ and therefore $p_\mu = p_\mu^0$

It is still necessary to say what is the practical realization of the δ-form potential. We know from the Fourier analysis that the Dirac δ-function can be expressed by integral in the following form:

$$\delta(\varphi) = \frac{1}{\pi}\int_0^\infty \cos(s\varphi)ds. \tag{30}$$

So, the δ-potential can be realized as the continual superposition of the harmonic waves. In case it will be not possible to realize experimentally it, we can approximate the integral formula by the summation formula as follows:

$$\delta(\varphi) \approx \frac{1}{\pi}\sum_0^\infty \cos(s\varphi). \tag{31}$$

4 Volkov solution of the Dirac equation with Heaviside four-potential

We know that the four-potential is inbuilt in the Dirac equation and we also know that if the potential is dependent on φ, then, there is explicite solution of the Dirac equation which was found by Volkov (1935) and which is called Volkov solution. The quantum mechanical problem is to find solution of the Dirac equation with the δ-form four-potential (17) and from this solution determine the quantum motion of the charged particle under this potential. Let us first remember the Volkov solution of the Dirac equation

$$(\gamma(p - eA) - m)\Psi = 0. \tag{32}$$

Volkov (1935) found the explicit solution of this equation for four-potential $A_\mu = A_\mu(\varphi)$, where $\varphi = kx$. His solution is of the form (Berestetzkii et al., 1989):

$$\Psi_p = R\frac{u}{\sqrt{2p_0}}e^{iS} = \left[1 + \frac{e}{2(kp)}(\gamma k)(\gamma A)\right]\frac{u}{\sqrt{2p_0}}e^{iS}, \tag{33}$$

where u is an electron bispinor of the corresponding Dirac equation

$$(\gamma p - m)u = 0. \tag{34}$$

The mathematical object S is the classical Hamilton-Jacobi function, which was determined in the form:

$$S = -px - \int_0^{kx} \frac{e}{(kp)}\left[(pA) - \frac{e}{2}A^2\right]d\varphi. \tag{35}$$

If we write Volkov wave function Ψ_p in the form (33), then, for the impulsive vector potential (17) we have:

$$S = -px - \left[e\frac{ap}{kp} - \frac{e^2}{2kp}a^2\right]\varphi, \quad R = \left[1 + \frac{e}{2kp}(\gamma k)(\gamma a)\eta(\varphi)\right]. \tag{36}$$

Our goal is to determine acceleration generated by the electromagnetic field of the δ-form which means that the four-potential A_μ is the Heaviside step function (18). To achieve this goal, let us define current density (Berestetzkii et al., 1989) as follows:

$$j^\mu = \bar{\Psi}_p\gamma^\mu\Psi_p, \tag{37}$$

where $\bar{\Psi}$ is defined as the transposition of (33), or,

$$\bar{\Psi}_p = \frac{\bar{u}}{\sqrt{2p_0}}\left[1 + \frac{e}{2(kp)}(\gamma A)(\gamma k)\right]e^{-iS}. \tag{38}$$

After insertion of Ψ_p and $\bar{\Psi}_p$ into the current density, we have with $A_\mu = a_\mu\eta(\varphi), \eta^2 = \eta$:

$$j^\mu = \frac{1}{p_0}\left\{p^\mu - ea^\mu + k^\mu\left(\frac{e(pa)}{(kp)} - \frac{e^2a^2}{2(kp)}\right)\right\}. \tag{39}$$

for $\eta > 0$, which is evidently related to eq. (28).

The so called kinetic momentum corresponding to j^μ is as follows (Berestetzkii et al., 1989):

$$J^\mu = \Psi_p^*(p^\mu - eA^\mu)\Psi_p) = \bar{\Psi}_p \gamma^0 (p^\mu - eA^\mu)\Psi_p) =$$

$$\left\{ p^\mu - eA^\mu + k^\mu \left(\frac{e(pA)}{(kp)} - \frac{e^2 A^2}{2(kp)} \right) \right\} + k^\mu \frac{ie}{8(kp)p_0} F_{\alpha\beta}(u^* \sigma^{\alpha\beta} u), \tag{40}$$

where

$$\sigma^{\alpha\beta} = \frac{1}{2}(\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha). \tag{41}$$

Now, we express the four-potential by the step function. In this case the kinetic momentum contains the tensor $F_{\mu\nu}$ involving δ-function. It means that there is a singularity at point $\varphi = 0$. This singularity plays no role in the situation for $\varphi > 0$ because in this case the δ-function is zero. Then, the kinetic momentum is the same as j^μ.

5 Emission of photons by an electron moving in the impulsive field

We know that the matrix element M corresponding to the emission of photon by electron in the electromagnetic field is as follows (Berestetzkii et al., 1989):

$$M = -ie^2 \int d^4x \bar{\Psi}_{p'}(\gamma e'^*)\Psi_p \frac{e^{ik'x}}{\sqrt{2\omega'}}, \tag{42}$$

where Ψ_p is the wave function of an electron before interaction with the laser pulse and $\Psi_{p'}$ is the wave function of electron after emission of photon with components $k'^\mu = (\omega', \mathbf{k}')$. The quantity e'^* is the four polarization vector of emitted photon.

We write the matrix element in the more standard form:

$$M = g \int d^4x \bar{\Psi}_{p'} O \Psi_p \frac{e^{ik'x}}{\sqrt{2\omega'}}, \tag{43}$$

where $O = \gamma e'^*$, $g = -ie^2$ in case of the electromagnetic interaction and

$$\bar{\Psi}_{p'} = \frac{\bar{u}}{\sqrt{2p_0'}} \bar{R}(p')e^{-iS(p')}. \tag{44}$$

Using the above definitions, we write the matrix element in the form:

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p_0'2p_0}} \int d^4x \bar{u}(p')\bar{R}(p')OR(p)u(p)e^{-iS(p')+iS(p)}e^{ik'x}. \tag{45}$$

The quantity $\bar{R}(p')$ follows immediately from eq. (33), namely:

$$\bar{R}(p') = \overline{\left[1 + \frac{e}{2kp'}(\gamma k)(\gamma a)\eta(\varphi)\right]} = \left[1 + \frac{e}{2kp'}(\gamma a)(\gamma k)\eta(\varphi)\right]. \qquad (46)$$

Using

$$-iS(p') + iS(p) = i(p' - p) + i(\alpha' - \alpha)\varphi, \qquad (47)$$

where

$$\alpha = \left(e\frac{ap}{kp} - \frac{e^2}{2}\frac{a^2}{kp}\right), \quad \alpha' = \left(e\frac{ap'}{kp'} - \frac{e^2}{2}\frac{a^2}{kp'}\right), \qquad (48)$$

we get:

$$M = \frac{g}{\sqrt{2\omega'}}\frac{1}{\sqrt{2p_0'2p_0}}\int d^4x \bar{u}(p')\bar{R}(p')OR(p)u(p)e^{i(p'-p)x}e^{i(\alpha'-\alpha)\varphi}e^{ik'x}. \qquad (49)$$

With regard to the mathematical relation $\eta^2(\varphi) = \eta(\varphi)$, we can put

$$\bar{R}(p')OR(p) = A + B\eta(\varphi). \qquad (50)$$

where

$$A = \gamma e'^* \qquad (51)$$

and

$$B = \frac{e}{2(kp)}(\gamma e'^*)(\gamma k)(\gamma a) + \frac{e}{2(kp')}(\gamma a)(\gamma k)(\gamma e'^*) +$$

$$\frac{e^2}{4(kp)(kp')}(\gamma a)(\gamma k)(\gamma e'^*)(\gamma k)(\gamma a). \qquad (52)$$

The total probability of the emission of photons during the interaction of the laser pulse with electron is as follows:

$$W = \int \frac{1}{2}\sum_{spin.polar.}\frac{|M|^2}{VT}\frac{d^3p'd^3k'}{(2\pi)^6}. \qquad (53)$$

It is evident that the total calculation is complex and involves many algebraic operations with γ-matices and δ-functions. At this moment we rescricte the calculations to the most simple approximation where we replace the term in brackets in eq. (33) by unit and so we write instead of eq. (33):

$$\Psi_p \sim \frac{u}{\sqrt{2p_0}}e^{iS}, \qquad (54)$$

which is usually used in simmilar form for the nonrelativistic calculations as it is discussed by Kreinov et al. (1997). Then, in this simplified situation $\bar{R}OR$ reduces to $A = \gamma e'^*$ and

$$M = \frac{g}{\sqrt{2\omega'}}\frac{1}{\sqrt{2p_0'2p_0}}\int dx^4 \bar{u}(\gamma e'^*)u e^{i(p'-p)x}e^{i(\alpha'-\alpha)\varphi}e^{ik'x} =$$

$$\frac{g}{\sqrt{2\omega'}}\frac{1}{\sqrt{2p_0'2p_0}}\bar{u}(\gamma e'^*)u\delta^{(4)}(lk+p-p'-k'), \tag{55}$$

where

$$l = \alpha - \alpha'. \tag{56}$$

One important step is the determination of W in the determination of trace, because according to the quantum electrodynamics of a spin states it is possible to show that (Berestetzkii et al., 1989)

$$\frac{1}{2}\sum_{spin.polar.}|M|^2 = \frac{1}{2}\mathrm{Tr}\left\{(\gamma p'+m)A(\gamma p+m)\gamma^0 A^+\gamma^0\right\}, \tag{57}$$

In order to determine trace Tr of the combinations of γ-matrix, it is suitable to know some relations. For instance:

$$\mathrm{Tr}(a\gamma)(b\gamma) = 4ab, \quad \mathrm{Tr}(a\gamma)(b\gamma)(c\gamma) = 0, \tag{58}$$

$$\mathrm{Tr}(a\gamma)(b\gamma)(c\gamma)(d\gamma) = 4\left[(ab)(cd) - (ac)(bd) + (ad)(bc)\right]. \tag{59}$$

Then,

$$\mathrm{Tr}\left[(\gamma p'+m)A(\gamma p+m)\bar{A}\right] = S_1 + S_2 + S_3 + S_4; \quad \bar{A} = \gamma^0 A^+ \gamma^0, \tag{60}$$

where (using relations (58) and (59) and $\bar{\gamma}^\mu = \gamma^\mu$ with $e'e'^* = -1$)

$$S_1 = \mathrm{Tr}[\gamma p' A \gamma p \bar{A}] = 4\left[(p'e'^*)(pe') + (pp') + (p'e')(pe'^*)\right] \tag{61}$$

$$S_2 = \mathrm{Tr}[mA\gamma p\bar{A}] = 0 \tag{62}$$

$$S_3 = \mathrm{Tr}[m\gamma p' A\bar{A}] = 0 \tag{63}$$

$$S_4 = \mathrm{Tr}[m^2 A\bar{A}] = 4m^2(e'e'^*) = -4m^2. \tag{64}$$

At this moment we can write probability of the process W in the form:

$$W = \int \frac{1}{2}\sum_{spin.\ polar.}\frac{|M|^2}{VT}\frac{d^3p'd^3k'}{(2\pi)^6} =$$

$$\int \frac{d^3p'd^3k'}{(2\pi)^6}\frac{1}{2}(S_1+S_2+S_3+S_4)\frac{1}{(2\pi)^2}\delta^{(4)}(lk+p-p'-k') =$$

$$\int \frac{d^3p'd^3k'}{(2\pi)^6}\frac{1}{2}\delta^{(4)}(lk+p-p'-k')4\left\{(p'e'^*)(pe') + (pp') - m^2 + (p'e')(pe'^*))\right\}. \tag{65}$$

The presence of the δ-function in the last formula is expression of the conservation law $lk + p = k' + p'$, which we write in the form:

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$$lk + p - k' = p'. \tag{66}$$

If we introduce the angle Θ between $\mathbf{k}$ and $\mathbf{k}'$, then, with $|\mathbf{k}| = \omega$ and $|\mathbf{k}'| = \omega'$, we get from the squared equation (66) in the rest system of electron, where $p = (m, 0)$, the following equation:

$$l\frac{1}{\omega'} - \frac{1}{\omega} = \frac{l}{m}(1 - \cos\Theta); \quad l = \alpha - \alpha', \tag{67}$$

which is modification of the original equation for the Compton process

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1 - \cos\Theta). \tag{68}$$

We see that the substantial difference between single photon interaction and δ-pulse interaction is the factor $s = \alpha - \alpha'$.

We know that the last formula of the original Compton effect can be written in the form suitable for the experimental verification, namely:

$$\Delta\lambda = 4\pi\frac{\hbar}{mc}\sin^2\frac{\Theta}{2}, \tag{69}$$

which was used by Compton for the verification of the quantum nature of light (Rohlf, 1994).

We can express equation (67) in new form. From equation $lk + p = k' + p'$ we get after multiplication it by k in the rest frame of electron:

$$kp' = \omega m - \omega\omega'(1 - \cos\Theta). \tag{70}$$

Then, l in eq. (67) is given by the formula ($a \equiv (v, \mathbf{w})$):

$$l = \frac{2evm - e^2a^2}{2\omega m} - \frac{2eap' - e^2a^2}{2\omega[m - \omega'(1 - \cos\Theta)]}. \tag{71}$$

The equation (67) can be experimentally verified by the similar methods which was used by Compton for the verification of his formula. However, it seems that the interaction of the photonic pulse substantially differs from the interaction of a single photon with electron.

The equation $lk + p = k' + p'$ is the symbolic expression of the nonlinear Compton effect in which several photons are absorbed at a single point, but only single high-energy photon is emitted. The second process, where electron scatters twice or more as it traverses the laser focus is not considerred here. The nonlinear Compton process was experimentally confirmed for instance by Bulla et al. (1996).

6   Discussion

We have presented, in this article, the classical derivation of law of motion of a charged particle accelerated by δ-form mechanical impulsive force in case of the free particle and for the damped harmonic oscillator. Then, we found solution of the Lorentz equation for motion of a charged particle accelerated by the electromagnetic pulse. From the quantum

theory based on the Volkov solution of the Dirac equation, we determined the current density and kinetic momentum of an electron accelerated by the laser pulse. The total probability of emmision of photons during the interaction of the laser pulse with electron was also derived. It involves the relation between initial momenta of particle and photon and final ones.

The present article is continuation of the author discussion on laser acceleration (Pardy, 1998; Pardy, 2001), where the Compton model of laser acceleration was proposed.

The δ-form laser pulses are here considerred as an idealization of the experimental situation in laser physics and we know that absolute zero size of laser pulse cannot be achieved. Nevertheless, it was demonstrated theoretically that at present time the zeptosecond and subzeptosecond laser pulses of duration $10^{-21} - 10^{-22}$ s can be realized by the petawat lasers (Kaplan and Shkolnikov, 2002). It means that the generation of the ultrashort laser pulses is the keen interest in development of laser physics.

The physics with the laser ultrashort laser pulses is analogical to the thermodynamics, vhere absolute zero temperature cannot be achieved, although it is possible to reach temperatures close to it using the use of evaporative cooling, cryocoolers, dilution refrigerators and nuclear adiabatic demagnetization. The use of laser cooling has produced temperatures of less than a billionth of a kelvin. So, while the goal of the thermal physics is the absolute zero temperature, the goal of laser physics laboratories over the world is the δ-form laser pulse.

New experiments can be realized and new measurements performed by means of the laser pulses, giving new results and discoveries. So, it is obvious that the interaction of particles with the laser will form, in the near future, the integral part of the laser and particle physics in such laboratories as ESRF, CERN, DESY, SLAC and so on.

REFERENCES

Ashkin, A. (1970). Acceleration and trapping of particles by radiation pressure, Phys. Rev. Lett. **24**, No. 4, 156-159.

Ashkin, A. (1970). The pressure of laser light, Scientific American **226** (2), 63-71.

Baranova, N. B., Zel'dovich B. Ya. and Scully, M. O. (1994). Acceleration of charged particles by laser beams, JETP **78**, (3) 249-258.

Berestetzkii, V. B, Lifshitz E. M. and Pitaevskii, L. P. (1989). *Quantum electrodynamics*, (Moscow, Nauka).

Bulla, C et al. (1996). Observation of nonlinear effects in Compton scattering, Phys. Rev. Lett. **76**, No. 17, 3116 - 3119.

Ciappina, M. F. et al. (2016). Attosecond physics at the nanoscale, arXiv:1607.01480v1 [physics.atom-ph] 6 Jul 2016.

Kaplan, A. E. and Shkolnikov, P. L. (2002). Lasetron: A proposed source of powerful nuclear-time-scale electromagnetic bursts, Phys. Rev. Lett. **88**, No. 7, 074801-1 – 4.

Katsouleas T. and Dawson, J. M. (1983). Unlimited electron

acceleration in laser-driven plasma waves, Phys. Rev. Lett. **51**, No. 5, 392-395.

Kreinov, V. P., Reiss, H. R. and Smirnov, M. (1997). *Radiative processes in atomic physics*, (John Wiley & Sons, Inc.).

Landau L. D. and Lifshitz, E. M. (1962). *The Classical Theory of Fields*, 2nd ed. (Pergamon Press, Oxford).

Maier, M. (1991). Synchrotron radiation, in: CAS-PROCEEDINGS, ed. Turner, S. CERN 91-04, 8 May, 97-115.

Meyer, J. W. (1971). Covariant classical motion of electron in a laser beam, Phys. Rev. D **3**, No. 2, 621 - 622.

Nichols, E. F. and Hull, G. F. (1903). The pressure due to radiation, Phys. Rev. **17**, 26-50; ibid. 91-104.

Pardy, M. (1998). The quantum field theory of laser acceleration, Phys. Lett. A **243**, 223-228.

Pardy, M. (2001). The quantum electrodynamics of laser acceleration, Radiation Physics and Chemistry **61**, 391-394.

Pardy, M. (2003). Electron in an Ultrashort Laser Pulse, International J. of Theor. Physics, Vol. 42, No. 1, pp 99–110.

Rohlf, J. W. (1994). *Moderm Physics from α to Z^0* (John Wiley & Sons, Inc. New York).

Scully, M. O. and Zubary, M. S. (1991). Simple laser accelerator: Optics and particle dynamics, Phys. Rev. A **44**, No. 4, 2656-2663.

Tajima, T. and Dawson, J. M. (1979). Laser electron accelerator, Phys. Rev. Lett. **43**, No. 4, 267-270.

Wolkow, D. M. (1935). Über eine Klasse von Lösungen der Diracschen Gleichung, Z. Physik, **94**, 250 - 260.