Dark Energy & Dark matter: The Theory of Nothing for Everything

Alaya Kouki

Alayakouki03@gmail.com

Abstract

Vacuum is the fundamental state of the quantum field & this is the dark energy. Dark Energy has a characteristic frequency which is an universal constant. Every point of space-time is considered as an oscillator in its fundamental state. Vacuum has also a density of entropy.

Dark Matter is a high density of vacuum energy.

1.Introduction :

The energy density of vacuum as given by General Relativity is as follows[1] :

$$U_0 = \frac{\Lambda . c^4}{8\pi G} \approx 10^{-9} Joule.m^{-3} \qquad (1)$$

With :

$\Lambda = 1.088\ 10^{-52} m^{-2}$: cosmological constant ;

$c = 3\ \ 10^{8} m.s^{-1}$: relativity constant ;

$G = 6.67\ \ 10^{-11}\ SI\ units$: gravitationnel constant ;

As per the quantum theory the energy of an oscillator is quantified as follows :

$$E = \left(n + \frac{1}{2}\right).\hbar\omega \qquad (2)$$

With :

n:natural integer ;

$\hbar = \frac{h}{2\pi} = 1.054\ 10^{-34} Joule.seconde$:reducedPlanck constant ;

$\omega = 2\pi.\nu$: the oscillator frequency.

The ground state of the oscillator corresponds for $n = 0$ and in this case the energy of the oscillator is not zero according to equation (2) which contradicts classical mechanics which provides for a zero total energy therefore zero vacuum energy.

The quantification of energy according to Poincare who wants to be consistent with the ideas of Planck is that “A physical system is only susceptible to a finite number of distinct states; it jumps from one state to another without going through a continuous series of intermediate states. Planck introduced an action element h, which corresponds to the smallest existing volume of phase space. Qualitatively, this volume corresponds to an elementary probability domain. The systems – and therefore the oscillators – corresponding to this domain are indistinguishable. This assumption makes it possible to introduce certain limitations in the possible values of p and q (momentum and position) which then makes it possible to limit the number of independent variables (i.e. the number of degrees of freedom of the system to avoid when the wavelength decreases the expression of the energy density of the black body in the formula of Jeans does not explode for the small wavelengths).

The energy quantity ε corresponding to h verifies:

$$h = \int_E^{E+\varepsilon} dqdp \qquad (3)$$

The ellipse of the phase space of equation $E = \frac{1}{2}Kq^2 + \frac{p^2}{2L}$has a surface $S(E) = 2\pi E\sqrt{\frac{L}{K}}$ so :

$h = S(E+\varepsilon) - S(E) = 2\pi\varepsilon\sqrt{\frac{L}{K}} = 2\pi\varepsilon.\frac{1}{\omega}$than :

$$\varepsilon = h\nu \qquad (4)$$ » [2]

The magnitude of power Wcorresponding for this same quantity of energyε verify :

$W = \int_E^{E+\varepsilon}\frac{K}{L}dqdp = \frac{K}{L}\frac{2\pi}{\omega}\varepsilon = 2\pi\omega\varepsilon$because$K = L\omega^2$, this is possible (to have the dimension of a power) if ε verify equation (4) or also there is a constant $\alpha_0$having the dimension of a power such that :

$$\varepsilon = \alpha_0.\tau \qquad (5)$$

withτ : a characteristic time of the oscillator.

Constants h &α_0 are declared universal constants .

There are two others solutions for the power W but we decline because the correspondentproportional coefficients can be deduce from the absolute system of unities where $h = c = \alpha_0 = 1$.

One can define the universal constant "a"as :

$$a = \frac{\alpha_0}{c^2} \qquad (6)$$

The constant "a" has the dimension of a mechanical impedance, in other words space-time cannot be conceived as completely empty: it is a superfluid of coefficient of friction "a" and having a negative pressure so as to cancel the viscosity effect.

In the ground state an oscillator in this space-time absorbs a certain amount of energy and returns it to space-time in a perpetual fashion.

The electromagnetic field can be considered as an oscillator and in its fundamental state it fills all space-time. Thus the vacuum can be defined as being the fundamental state of the electromagnetic field: all the points of space-time are oscillators in their fundamental states. The vacuum energy predicted by General Relativity actually comes from this conception of the electromagnetic field in its ground state.

The fundamental constant of Nature α_0 has a connection with the notion of the energy of the vacuum, a thing encountered in the theory of General Relativity whose equations of the gravitational field on a cosmic scale are:

$$R_{ik} - \frac{1}{2}.R.g_{ik} = -\frac{8\pi G}{c^4}.T_{ik} - \Lambda.g_{ik} \qquad (7)$$

with : R_{ik} : curvature tensor;

R : scalar (curvature of space-time)

T_{ik} : momentum-energy tensor of matter ;

g_{ik} : metric tensor with signature (+,-,-,-) ;

Λ : a constant having the dimension as L^{-2} ;

$G = 6{,}67\ 10^{-11}\ SI\ units$: newtonien gravitationnel constant.

$i, k = 0{,}1{,}2{,}3$ tensors indices

In equation (7) we are looking for a form of energy that the energy-momentum tensor describing ordinary energy would not contain:

$$R_{ik} - \frac{1}{2}.R.g_{ik} = -\frac{8\pi G}{c^4}.\left(T_{ik} + \frac{\Lambda.\mathrm{c}^4}{8\pi G}.g_{ik}\right) \qquad (8)$$

At $T_{ik} = 0$the desired tensor is:

$$T_{ik}^{void} = -\frac{\Lambda.\mathrm{c}^4}{8\pi G}.g_{ik} \qquad (9)$$

Equation (9) is that of a perfect fluid whose volume energy density is:

$$U_0 = \rho_0.c^2 = \frac{\Lambda.\mathrm{c}^4}{8\pi G} \qquad (10)$$

And we have in the absence of any form of ordinary energy:

$$\nabla^i T_{ik}^{void} = 0 \qquad (11)$$

It is certain that the new constant $\alpha_0$has a direct relationship with the constantΛ .

"We must have $\Lambda \ll \mathcal{L}^{-2}$ where $\mathcal{L}$ is the characteristic length over which, to the Newtonian approximation, the gravitational potential φ can varies , so as to find the Poisson law:

$$\nabla^2\varphi \approx 4\pi G\rho - \Lambda.c^2 \qquad (12)$$

With: : density of matter;

∇^2: Laplace operator;

This constant is negligible and will therefore only play a role in large systems much larger than stellar or even galactic systems, hence its name of cosmological constant. » [3]

For a spherical mass M the gravitational field to the classical approximation is:

$$\boldsymbol{g} = -\frac{G.M}{r^2}\,.\boldsymbol{u_r} + \frac{\Lambda.c^2}{3}.r.\boldsymbol{u_r} (13)$$

It is clear that one sees in the second term of the gravitational field the aspect arises at least a negative constant Λ. For a positive constant Λ we will have a repulsive gravitational field.

In other words, gravitation is a manifestation of the vacuum and the Newtonian field is nothing but a limit where a certain volume of space (of mass M) interacts with the totality of a spatial geometry which extends to the infinite and modelled at each point by an oscillator.

The similarity between the model of gravity and the black body theory is clear. A black body is a cavity whose wall supposedly formed of an infinity of oscillators which exchange energy with the radiation inside. If we still continue to assume that the electromagnetic field inside is an oscillator, all the energy exchanged is between the oscillators and the vacuum. For the atoms of the wall of the black body each electron is on the one hand maintained in connection with the atomic nucleus thanks to the electrostatic force on the one hand and thanks on the other hand to a vacuum energy which pushes it towards the exterior (expansion of the atom): the balance is maintained between a movement of attraction towards the interior and a movement of repulsion towards the exterior i.e. in permanent oscillations. An accelerated electrical charge radiates energy and this energy is harvested from the vacuum.

2.Vacuum energy :

With this conception of the vacuum one can think of using the theory of black body radiation to calculate the energy density of the vacuum [3]. The vacuum will be considered as a black body containing an infinity of oscillators in their fundamental states.

Instead of characterizing the vacuum by a classical average energy "kT" for a two degree of freedom, it will be characterized by an average energy "$\frac{1}{2}h\nu_0$" a fundamental state of the electromagnetic field where "ν_0"is a universal constant which can be a measure of the energy density of the vacuum consistent with the value given by General Relativity and with that of the model of the oscillating point in its ground state.

Moreover, the Planck model of the black body is an enclosure composed of several resonators located on the wall and kept in motion by the electromagnetic field inside:

« Let's the state of such an oscillator be completely determined by its moment $f(t)$, that is , by the product of the electric charge of the pole situated on the positive side of the axis and the pole distance, and by the derivative of f with respect to time or

$$\frac{df}{dt} = \dot{f}(t)$$

Let the energy of the oscillator be of the following simple form:

$$U = \frac{1}{2}\,K\,f^2 + \frac{1}{2}L\dot{f}^2$$

Where K and L denote positive constants "

"If during its vibration an oscillator , neither absorbed nor emitted an energy, its energy of vibration U will remain constant and we would have….periodical vibration:

$$f = C\cos(2\pi\nu t - \theta)$$

Where C and θ denotes constants of integration……

$$\nu = \frac{1}{2\pi}.\sqrt{\frac{K}{L}}$$

"136. If now the assumed system of oscillators is in a space traversed by heat rays, the energy of vibration, U, of an oscillator will not remain constant , but will be always changing by absorption and emission of energy"

"The first question is : what determines the thermodynamic state of the system considered? For this purpose , according to section 124, the numbers N1, N2, N3…etc. of the oscillators which lie in the region elements 1, 2, 3……of the 'state space' must be given. The state space of an oscillator contains those coordinates which determine the microscopic state of the an oscillator. In the case in question , these are two in number, namely, the moment f and the rate at which it varies , $\dot{f}$, or in stead of the latter the quantity

$\psi = L\dot{f}$,

Which is the dimension of an impulse. The region element of the state plane , is according to the hypothesis of quanta (section 126), the double integral

$\iint d\psi df = h.$

The quantity h is the same for all region elements"

"In the first place , as regards the shape of the region elements , the fact that in the case of undisturbed vibrations of an oscillator the phase is always changing whereas the amplitude remains constant leads to the conclusion that for the macroscopic state of the oscillators, the amplitudes only not the phases must be considered, or in other words the region elements of the $f\psi$ plane are bounded by the curves C= constant, that is by the ellipses….:

$$\left(\frac{f}{C}\right)^2 + \left(\frac{\psi}{2\pi\nu LC}\right)^2 = 1$$

The semi-axes of such an ellipse are:

$$a = C \ and \ \ b = 2\pi\nu LC$$

Accordingly the region elements 1,2,3…..n are the concentric similar and similarly situated elliptic rings which are determined by the increasing values of C:

$$0, C_1, C_2, \ldots\ldots C_n$$

The n^{th} *region element is that which is bounded by the ellipses* $C = C_{n-1}$ *and* $= C_n$ *. The first region is the full ellipse* C_1 *. All these rings have the same area h which is found by subtracting the area of the ellipse* C_{n-1} *from the full ellipse* C_n *hence….:*

$$C_n^2 = \frac{nh}{2\pi^2 \nu L}$$

Thus the semi-axes of the bounding ellipses are in the ration of the square roots of the integral numbers" [4]

The surface of the Planck ellipse is:

$$S_n = \pi ab = \pi C 2\pi \nu L C = \pi.\frac{K}{\omega}.C^2 = \frac{2\pi E}{\omega} = n.h$$

And we have always :

$$S_n - S_{n-1} = h$$

The energy of the oscillator is :

$$E = \frac{1}{2}K.C^2 = n.\hbar.\omega$$

At$T = 0K$ all Planck oscillators lie in the full ellipse C_0 as already explained. They are all indistinguishable and do not interact with each other.

The surface of the Planck ellipse surrounded by C_0 is zero and yet all the Planck oscillators are reduced to this ellipse at $T = 0K$ and they each have an energy $E = \frac{1}{2}h\nu$ so we can't speak about vacuum energy because there is no oscillators in a phase area equal to zero.

According to Bohr quantum mechanics for great level quantum state the classical measurement and the quantum measurement are the same: this is called Bohr principle correspondence. But Bohr consider the Principle of Correspondence as a quantum principle i.e. for any level quantum state there is a corresponding classical measurement[7]. To adopt this principle than for Planck resonator at low quantum level we should get:

$$C_n^2 = \frac{(n + \frac{1}{2})h}{2\pi^2 \nu L}$$

This is a mathematical forcing to be coherent with the Bohr principle of correspondence[6]. So the energy of the resonator is:

$$E = \frac{1}{2}K.C_n^2 = (n + \frac{1}{2}).\hbar.\omega$$

Which correspond to the equation (2) as given by the De Broglie-Schrödinger quantum mechanics . Of course the last equation tends to Planck equation $E = n.\hbar.\omega$for great n.

At $T = 0K$ all Planck oscillators lies in the area $S_0 = \frac{h}{2}$.

This is only possible if this energy is external to the Planck oscillators i.e. comes from the void. The Planck oscillators interacting with this energy will each have a certain average energy with reference to a base energy $\frac{1}{2}h\nu_0$. Thus we can conclude straight away that the entropy of the vacuum is not zero.

The electromagnetic field can also be considered as an oscillator and in its fundamental state it will have the energy $E = \frac{1}{2}h\nu$ and then it can maintain the vibration of the Planck oscillators. Each of these Planck oscillators will certainly have an average energy by reference to an energy $\frac{1}{2}h\nu_0$ where ν_0 is a constant. This average energy of the electromagnetic field in its fundamental state replaces in classical thermodynamics the energy kT of an oscillator two-dimensional. The average energy of the Planck oscillator is calculated by Boltzmann statistics. In fact the Planck oscillators do not emit energy at $T = 0K$ and then the total energy density of the Planck oscillators is none other than the vacuum energy density. Its effects are among others gravitational as a cause of the expansion of the Universe.

Thus the number of electromagnetic oscillators in their ground states with frequencies between ν and $\nu + d\nu$ is as in the black body theory:

$$N = \frac{1}{\exp\left(\frac{\frac{1}{2}h\nu}{\frac{1}{2}h\nu_0}\right)-1} = \frac{1}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \qquad (14)$$

The average energy of the oscillators in this frequency interval is:

$$E_\nu = N.\frac{1}{2}h\nu = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \qquad (15)$$

The total energy density of the vacuum is then according to the black body theory:

$$U_0 = \int_0^\infty \frac{8\pi\nu^2}{c^3}.E_\nu d\nu = \int_0^\infty \frac{4\pi h}{c^3}.\frac{\nu^3}{\exp\left(\frac{\nu}{\nu_0}\right)-1}d\nu = \frac{4\pi^5 h}{15.c^3}.\nu_0^4 \qquad (16)$$

If we equalize this expression of the energy density of the vacuum with that given by General Relativity we obtain that:

$$\nu_0 = \left[\frac{15\,\Lambda.c^7}{32.\pi^6.G.h}\right]^{\frac{1}{4}} \approx 0.7\ 10^{12}Hz \qquad (17)$$

Which proves that $\nu_0$is an universal constant.

We will take by definition as power quantum:

$$\alpha_0 = h.\nu_0^2 \qquad (18)$$

3.The origin of the frequencyν_0 :

The theory of the black body is based on the modelling of it by a cavity whose walls are oscillators which absorb and emit energy in thermal equilibrium with the electromagnetic radiation inside. These oscillators are actually the electrons elastically bound to their atoms.

Planck in his law of distribution of radiant energy does not take into account the energy of the vacuum because at his time quantum mechanics was not developed enough to predict the energy at zero point (i.e. at zero Kelvin) of an oscillator that is not zero.

Planck's law of distribution of black body radiation is as follows [5]:

$$u_{\nu} = \frac{8\pi h.\nu^3}{c^3}.\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (19)$$

At$T = 0K$Planck's law gives zero energy density when in fact the Planck oscillators each have non-zero ground state energy. Planck should already have corrected this situation by fixing a cut-off frequency ν_0 below which his law (19) is no longer valid but also this hypothesis also poses a problem since for a low $\nu_0$brings Planck's law back to that of Jeans which is not valid in this frequency interval. Moreover Planck indirectly refuses the quantification of the energy of an oscillator. For him the energy of an oscillator varies between$E_n = nh\nu$and $E_{n+1} = (n+1)h\nu$in a continuous way like Boltzmann's distribution law for ideal gases and then an oscillator has an average energy equal to$\frac{E_n+E_{n+1}}{2} = \left(n+\frac{1}{2}\right)h\nu$ .This is how the energy $\frac{1}{2}h\nu$appears for the first time in Planck theory of heat radiation.

If we integrate Planck's law we will have:

$$U = \frac{8.\pi^5.k^4}{15.h^3.c^3}.T^4 \qquad (20)$$

The total energy of the black body can be taken as the sum of the two energies (20) and (16) so that at $T = 0K$this energy is not zero and the most preponderant fundamental state of the electromagnetic field is the frequency ν_0 .

The energy and momentum of a mass corpuscle $m$is according to Special Relativity:

$$\varepsilon = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (21)$$

$$\boldsymbol{p} = \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (22)$$

This corpuscle is represented by the 4-vector $p^i = (\frac{\varepsilon}{c}, \boldsymbol{p})$. A corpuscle can also have a wave behaviour represented by the 4-vector $k^i = (\frac{\omega}{c}, \boldsymbol{k})$as :

$$p^i = \hbar.k^i \qquad (23)$$

For a mixed behaviour (no distinction between wave or corpuscle) one can model by a 4-vector state or 4-vector identity $s^i = (c.\tau, \boldsymbol{v}.\tau)$as :

$$p^i = \hbar.k^i = a.s^i = m.c.u^i = c.\xi^i \qquad (24)$$

Where u^i:4-vector of the speed of the corpuscle.

$\xi^i = m.u^i$: 4-vector inertia of the corpuscle.

A corpuscle have an identity in time as :

$$s^0 = c.\tau$$

And an identity in space as:

l= $\tau.\boldsymbol{v}$

For low massive particles at high speeds those identities can interact with each other and so can be entangled. The wave-corpuscle duality is due to this entanglement.

The group speed of the packet of waves (modeling the particle) is:

$\frac{1}{v_g} = \frac{dk}{d\omega} = \frac{dk}{dv}.\frac{dv}{d\omega}$ with $\hbar.k = p = \frac{m.v}{\sqrt{1-\frac{v^2}{c^2}}} = al\&\hbar\omega = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

It is easy to get: $\frac{1}{v_g} = \frac{1}{v}$: the group speed is the same of the particle speed.

For phase speed we have:

$v_g.v_f = c^2$ always

So : $v_f = \frac{c^2}{v} = (2\pi\nu_0)^2 \frac{d(\frac{l}{2\pi})}{d\omega}$: speed of one wave forming the packet (plans equal phase).

One can model the interaction of the electron with the vacuum as being a classic corpuscle moving in a fluid of coefficient of friction"a".The maximum vibration speed of the electron will be that of the Bohr model of the atom i.e."αc"where $\alpha = \frac{1}{137}$ is the fine structure constant.

The energy transferred by the electron to vacuum is:

$$\varepsilon = \int_0^{\alpha c} a\boldsymbol{v}.\boldsymbol{v}dt = \int_0^{\alpha c} a.v^2.dt \qquad (25)$$

For every corpuscle we have :

$$\hbar.\omega = \alpha_0.\tau = a.c^2.\tau = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (26)$$

At the classical approximation ($v \ll c$) we get :

$$dt = d\tau \approx \frac{m}{a} d\left(1 + \frac{1}{2}.\frac{v^2}{c^2}\right) = \frac{m}{a}.\frac{vdv}{c^2} \qquad (27)$$

And than according to equation (27):

$$\varepsilon = \frac{1}{c^2}[\frac{1}{4}m.v^4]_0^{\alpha c} = \frac{\alpha^4}{4}.mc^2 \qquad (28)$$

Here $m = 9.1\ 10^{-31} Kg$is the mass of the electron.

The energy of the electron in equilibrium with the ground state radiation of the electromagnetic field is:

$$\varepsilon = \frac{1}{2} . \frac{h\nu}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} \qquad (29)$$

For low oscillation frequencies (classical approximation) we will have:

$$\varepsilon \approx \frac{1}{2} h. \nu_0 \qquad (30)$$

From equation (29) and equation (30) we get :

$$\nu_0 = \frac{\alpha^4}{2h} . mc^2 \approx 0.17\ 10^{12} Hz \qquad (31)$$

This value is close to the value obtained by General Relativity but not quite.

In fact the electron is in equilibrium with the maximum of the vacuum energy distribution. This maximum is obtained when:

$$\frac{\nu}{\nu_0} = 3 + W(-3. e^{-3}) \approx 2.8214 \qquad (32)$$

Where : W: is Lambert function

So we get :

$$\nu = 0.48\ 10^{12} Hz \qquad (33)$$

Which is a value closer to that of General Relativity. It can be said that the majority of the energy density of the vacuum is of the electromagnetic type.

If we do not make a classical approximation in equation (25) we obtain:

$$\varepsilon = \int_0^{\alpha c} a. v^2 d\tau \quad \text{with} \tau = \frac{\frac{m}{a}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (34)$$

So :

$$\varepsilon = \frac{m.c^2}{\sqrt{1-\alpha^2}} . (1 - \sqrt{1 - \alpha^2})^2 \qquad (35)$$

At classical approximation :

$$\varepsilon \approx \frac{1}{4} . \alpha^4 . mc^2 + \frac{1}{8} . \alpha^6 . mc^2 + \cdots = \frac{1}{2} h. \nu_0 \qquad (36)$$

With equation (35) we can already improve the value of ν_0 .

If we know ν_0 than we can search ν as we get:

$$\frac{m.c^2}{\sqrt{1-\alpha^2}} . (1 - \sqrt{1 - \alpha^2})^2 = \frac{1}{2} . \frac{h\nu}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} \qquad (37)$$

Note that the constant "a" is eliminated in the integration of (34).

Note that also all those calculations are in a semi-classical approach.

Let's have an approximate value of ν_0 in this way.

We take Bohr model of the atom which is of course is a wrong model even thought he gives a good result in ionisation energy of the Hydrogen.

The speed of the electron is $v = \alpha . c$ with $\alpha = \frac{e^2}{\hbar . c}$ in the cgs system.

Of course according to this model the electron is in circular motion so according to classical electrodynamics it will radiate a power as [10]:

$$W = \frac{1}{3c^3} . \omega^4 . (ea_0)^2 \qquad (38)$$

With: $a_0 = \frac{m.e^2}{\hbar^2}$: Bohr radius of the atom hydrogen

ω : frequency of the radiate electromagnetic wave with the condition that $\frac{a_0 \omega}{c} \ll 1$

The frequency ω is also the same speed of rotation of the electron around the nucleus so we have:

$$\omega = \frac{\alpha c}{a_0} \qquad (39)$$

It is clear that the condition $\frac{a_0 \omega}{c} \ll 1$ is verified i.e. that $\alpha = \frac{1}{137} \ll 1$.

If the atom radiate all time energy than the electron will fall on the nucleus . To avoid this falling the electron should always absorb from vacuum the same energy per unit time. So:

$$h\nu_0^2 = W \qquad (40)$$

It comes that $\nu_0 = 432\ 10^{12}\ Hz$: It is a very high value compared to the cosmology value because the Bohr model is wrong.

The time τ for the atom to emit a quantity of energy equal to $\hbar\omega$ is [10]:

$$\frac{1}{\tau} = \frac{W}{\hbar\omega} = \frac{e^2 a_0^2 \omega^3}{3\hbar c^3} \qquad (41)$$

From equation $\hbar\omega = \alpha_0 \tau$ one can deduce that:

$\alpha_0 = \frac{e^2 a_0^2 \omega^4}{3c^3} = \frac{\alpha^4 e^2 c}{3a_0^2} = \frac{4.8^2 x 10^{-20} x 3 x 10^{10}}{137^4 x 3 x 0.529^2 x 10^{-16}} = 0.233\ erg.s^{-1} = 233\ 10^{-10} Watts$ in semi-classical mechanics.

3.**Constant ν_0 from the black body theory:**

The number of photons per unit volume in a black body at a temperature T is:

$$N = \frac{16.\pi.k^3.\zeta(3)}{h^3.c^3} . T^3 \qquad (42)$$

Where $\zeta(3) = 1.2$ function Zeta.

The average energy by photon is :

$$\bar{E}=\frac{U}{N}=\frac{\pi^4.k}{30.\zeta(3)}.T\approx h.\bar{\nu} \qquad (43)$$

The density of energy per unit volume and per unit frequency of a black body is:

$$u_\nu=\frac{8\pi.\nu^2}{c^3}\left(\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}+\frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1}\right) \qquad (44)$$

Constant ν_0 is due to quantum effects so we should go to Wien zone of radiation.

We are in this case of approximation: $h\nu\gg kT\&\nu\ll\nu_0$ so:

$$u_\nu\approx\frac{8\pi h\nu^3}{c^3}.\exp\left(\frac{-h\nu}{kT}\right)+\frac{4\pi h\nu^2}{c^3}.\nu_0 \qquad (45)$$

The total energy per unit volume is:

$$U=\int_0^{\bar{\nu}}u_\nu d\nu=\frac{8\pi k^4T^4}{c^3h^3}\left[-e^{-\bar{\nu}}(\bar{\nu}^3+3\bar{\nu}^2+6\bar{\nu}+6)+6\right]+\frac{4\pi h\nu_0}{3c^3}.\bar{\nu}^3 \quad (46)$$

We take the case of F.Kurlbaum experiment:

"§11. The values of both universal constants h and kmay be calculated rather precisely with the aid of available measurements. F. Kurlbaum, designating the total energy radiating into air from 1 sq cm of a black body at temperature t ○ C in 1 sec by S_t*, found that:*

$S_{100}-S_0=0.0731\,\frac{Watt}{cm^2}=7.31\;10^5\,\frac{erg}{cm^2.sec}$*"[5]*

For $T=373\ K$ we have $\bar{\nu}_{373}=210.4\ 10^{11}\ Hz$

For $T=273\ K$ we have $\bar{\nu}_{273}=154\ 10^{11}\ Hz$

We have also:

$$\frac{c}{4}.\Delta U=measurement\ of\ F.Kurlbaum=731\ Watt.m^{-2}$$

After calculation we get:

$$\nu_0\approx 6.9\ 10^{12}\ Hz \qquad (46)$$

It is a value far from which given by General Relativity but one can some manages to approach the good value.

The most method which we can trust in is to study the behavior of gases and crystals near the absolute zero.

Note that we will never determine the constant ν_0 only from Planck model of black body radiation.

Thiesen equation of the density of energy of black body radiation is [11]:

$$E_\lambda d\lambda=T^5\psi(\lambda T)d\lambda \qquad (47)$$

Where λ is the wavelength , $E_\lambda d\lambda$ the spatial density of energy of black body radiation in the interval λ and $\lambda + d\lambda$,T the temperature and $\psi(x)$ an universal function of the single argument x.

Introducing the density of energy in the frequency interval ν and $\nu + d\nu$ as $u_\nu d\nu$ and doing the substitution $E_\lambda d\lambda$ by $u_\nu d\nu$ and $\lambda, \lambda + d\lambda$ by , $\nu + d\nu$, replace $d\lambda$ by $\frac{c}{\nu^2} d\nu$ Thiesen gives:

$$u_\nu d\nu = T^5 \psi(\frac{cT}{\nu}) \frac{c}{\nu^2} d\nu \qquad (48)$$

Let's introduce the density of power radiation in a black body as:

$$p_\nu = u_\nu \frac{d\nu}{dt} = \frac{\alpha_0}{h} u_\nu \qquad (49)$$

Because we have that$h\nu = \alpha_0 \tau$ and $dt = d\tau$ when varying frequency.

It comes that:

$$p_\nu = \frac{T^5 c}{\nu^2} \psi(\frac{cT}{\nu}) \frac{\alpha_0}{h} \qquad (50)$$

According to Kirchoff-Clausiuslaw the rate of emission of energy of a black body surface in a thermal medium at temperature T is inversely proportional to c^2 so :

$$\int p_\nu d(\frac{1}{\nu}) \sim \frac{1}{c^2} \qquad (51)$$

$$\int u_\nu d\nu \sim \frac{1}{c^2} \qquad (52)$$

In equation (51) we have eliminate the ordinary time. Time is change and there is change when there is frequency change.

Combining the thermodynamic law of Stefan-Blotzmann $E = \sigma . T^4$ and Wien empirical displacement law $\lambda_{max} T = Constant$ implying the maximum of energy emission we get that:

$$E_\lambda = \frac{dE}{d(\lambda T)} = \sigma . T^4 \psi(\lambda T) \qquad (53)$$

Where $\psi(\lambda T)$ is an universal function of the unique argument $x = \lambda T$.

Than the energy density is inversely proportional to c^3 and we get:

$$u_\nu = \frac{T^5}{c^3 \nu^2} g_1\left(\frac{T}{\nu}\right) = \frac{\nu^3}{c^3} g_2\left(\frac{T}{\nu}\right) \qquad (54)$$

Whereg_1&g_2 are universal functions independent from c i.e. $u_\nu \lambda^3$ is the same at $T = Constant.$

Also the power density is inversely proportional to c^3 and we get:

$$p_\nu = \frac{T^5}{\nu^4 c^3} f_1\left(\frac{T}{\nu}\right) = \frac{\nu}{c^3} f_2(\frac{T}{\nu})(55)$$

Wheref_1&f_2 are universal functions independent from c i.e. $p_\nu \lambda$ is the same at $T = Constant.$

Let's continue with Planck for the intensity of radiation as:

$$I = \frac{\nu^2}{c^2}U \qquad (56)$$

Where U the mean energy of the oscillator with resonant frequency ν in the radiation field.

The density of energy per unit volume and per unit frequency is:

$$u_\nu = \frac{8\pi I}{c} = \frac{8\pi\nu^2}{c^3}U \qquad (57)$$

There for:

$$p_\nu = \frac{8\pi\nu^2\alpha_0}{hc^3}U \qquad (58)$$

From equation (55)& (58) we deduce that:

$$U = \frac{h}{8\pi\alpha_0\nu}f_2(\frac{T}{\nu}) \qquad (59)$$

Where c does not appear.

We can write (59) as:

$$T = \nu f_3(U\nu) \qquad (60)$$

With f_3 is another universal function.

Introducing the entropy of the oscillator as Planck did:

$$\frac{1}{T} = \frac{dS}{dU} \qquad (61)$$

Where$S$ is the entropy of the oscillator. So:

$\frac{dS}{dU} = \frac{1}{\nu}f_4(U\nu)$and replacing ν by its expression in (60) than integrating we get:

$$S = f(U\nu) \qquad (62)$$

According to Planck the entropy of the oscillator is:

$S = k\{\left(1+\frac{U}{h\nu}\right)Log\left(1+\frac{U}{h\nu}\right) - \frac{U}{h\nu}Log\left(\frac{U}{h\nu}\right)\}$ which can be written as:

$$S = k\{\left(1+\frac{U\nu}{h\nu^2}\right)Log\left(1+\frac{U\nu}{h\nu^2}\right) - \frac{U\nu}{h\nu^2}Log\left(\frac{U\nu}{h\nu^2}\right)\} \ (63)$$

By substitution in equation (61) we get:

$\frac{1}{T} = \frac{k}{h\nu}Log(1+\frac{h\nu^2}{U\nu})$ or $U\nu = \frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}$ which means that:

$$U = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (64)$$

Integrate the density of power we get:

$$\int_0^\infty p_\nu d\nu = \frac{\alpha_0}{h}\int_0^\infty u_\nu d\nu = \frac{\alpha_0}{h}\sigma T^4 \qquad (65)$$

With $b = \frac{8\pi^5 k^4}{15c^3 h^3}$ in $[Joule.m^{-3}.K^{-4}]$

$\int_0^\infty p_\nu d\nu$ will have the dimension of $[Watt.m^{-3}.s^{-1}]$

$\frac{ch}{4\alpha_0}\int_0^\infty p_\nu d\nu$ will have the dimension of $[Watt.m^{-2}]$ which is measured by F.Kurlbaum but does not contain constant α_0 . So we can't determine it according only to Planck model.

Expression (63) of the entropy is like that an oscillator have the mean power as:

$$W_\nu = \frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (66)$$

It is like that every single oscillator (one of the infinite resonator which contribute in the mean value (66)) radiate energy in quantum of power as multiple integer of :

$$\delta = h\nu^2 \qquad (67)$$

The energy radiated is :

$$\varepsilon = -\int \delta d\left(\frac{1}{\nu}\right) = h\nu \quad (68)$$

Taking account of Heisenberg principle of uncertainty for packet of waves as:

$$\Delta\omega.\Delta t \geq 1 \qquad (69)$$

Where :$\Delta\omega$: the uncertainty about the frequency

Δt : the uncertainty in time

We get from (69) :

$$\Delta\nu \geq \sqrt{\frac{\alpha_0}{2\pi h}} \qquad (70)$$

Equation (70) limit the validity of Planck model.

We can get (66) as per Boltzmann-Maxwell statistics or as to link the entropy of an oscillator to its mean power as Planck do in the case of linking entropy to the mean energy of the oscillator as follows:

The black body is contain N oscillators which radiate energy in irregular manner and does not influence each other. Their amplitudes and phases are not constant

The total energy and entropy of the system are:

$$U_N = NU \qquad (71)$$

With : mean energy of an oscillator;

$$S_N = NS \qquad (72)$$

With S mean entropy of an oscillator

The total power of the system is:

$W_N = NW$ (73)

With W : mean power of an oscillator

To find the probability that the N oscillators have the total power W_N & the total energy U_N we suppose that the power radiated by an oscillator is not continuous but rather radiated as multiple of quanta of power δ and as multiple of quanta of energy εas follows:

$W_N = P\delta$ (74)

$U_N = P\varepsilon$ (75)

Where P is integer.

The number of manner how to distribute P parts of δ among the N oscillators and P parts of ε among the N oscillators is given by combinatory analysis as follows (number of completions: the number of combinations P elements made from $(N + P - 1)$ elements):

$\Omega = C_{N+P-1}^{P} = \frac{(N+P-1)!}{(N-1)!P!}$ (76)

And by Stirling formulae $N! \approx N^N$ we get:

$\Omega \approx \frac{(N+P)^{(N+P)}}{N^N P^P}$ (77)

From Boltzman law about entropy we deduce that:

$S_N = k\ Log(\Omega) = k\{(N + P)Log(N + P) - NLog(N) - PLog(P)\}$ (78)

And using (73) & (74) we get:

$$S_N = k\{N\left(1 + \frac{W}{\delta}\right)Log(N) + N\left(1 + \frac{W}{\delta}\right)Log\left(1 + \frac{W}{\delta}\right) - NLog(N) - N\frac{W}{\delta}Log(N) - N\frac{W}{\delta}Log\left(\frac{W}{\delta}\right)\}$$

So:

$$S_N = k\{N\left(1 + \frac{W}{\delta}\right)Log\left(1 + \frac{W}{\delta}\right) - N\frac{W}{\delta}Log\left(\frac{W}{\delta}\right)\}$$

Than:

$S_N = kN\{\left(1 + \frac{W}{\delta}\right)Log\left(1 + \frac{W}{\delta}\right) - \frac{W}{\delta}Log\left(\frac{W}{\delta}\right)\}$ (79)

It comes that the entropy of an oscillator is :

$S = k\{\left(1 + \frac{W}{\delta}\right)Log\left(1 + \frac{W}{\delta}\right) - \frac{W}{\delta}Log\left(\frac{W}{\delta}\right)\}$ (80)

Using (71) and (75) we get:

$S = k\{\left(1 + \frac{U}{\varepsilon}\right)Log\left(1 + \frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}Log\left(\frac{U}{\varepsilon}\right)\}$ (81)

From equations (54) & (57) we can deduce that:

$$U = \frac{\nu}{8\pi} g_2\left(\frac{T}{\nu}\right) \qquad (82)$$

Where c does not exist.

Equation (82) can be also written as:

$$T = \nu g_3\left(\frac{U}{\nu}\right) \qquad (83)$$

Where g_3 is another universal function of the argument $\frac{U}{\nu}$.

Combining equation (83) with (61) we get:

$$\frac{dS}{dU} = \frac{1}{\nu} g_4(\frac{U}{\nu}) \qquad (84)$$

Integrate (84) and taking in consideration that S is total differential we get:

$$S = g(\frac{U}{\nu}) \qquad (85)$$

And this (equation(85)) what does it mean Wien displacement law for Planck.

According to equation (81)Planck conclude that he should get :

$$\varepsilon = h\nu \qquad (86)$$

And by substituting in equation(61) Planck get for the mean energy U of the oscillator:

$$U = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (87)$$

Where: h, k : are universal constants.

From equation (80) it is clear that to get the same entropy we should have:

$$W = U\nu \qquad (88)$$

$$\delta = \varepsilon\nu \qquad (89)$$

Equation (88) is coherent with equation (62).

It is clear that now we can determine the density of power and energy as follows:

$$E = \int_0^\infty \frac{8\pi\nu^2}{c^3} W d\nu = \int_0^\infty \frac{8\pi\nu^2}{c^3} U\nu d\nu = \int_0^\infty \frac{8\pi\nu^2}{c^3} \frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1} d\nu \quad (90)$$

So:

$$E = \frac{8\pi k^5}{c^3 h^4} T^5 \zeta(5)\Gamma(5) \approx \frac{240 k^5}{c^3 h^4} T^5 \quad (91)$$

With $E[Watt. m^{-3}]$

$$H = \int_0^\infty \frac{8\pi\nu^2}{c^3} u_\nu d\nu = \sigma T^4 \qquad (92)$$

With $H[Joule.m^{-3}]$

Equation (92) is verified by experience: this equation is Boltzmann equation.

Equation (91) should be relayed to experience data .

Wien measurements of black body radiation in 1899 are as follows[12]:

Temperature (K)	λ_{max} (micron)	$\lambda_{max}T$ $[\mu.K]$
621	4.53	2814
723.0	4.08	2950
908.5	2.96	2980
998.5	2.96	2956
1094.5	2.71	2966
1259.0	2.35	2959
1460.4	2.04	2979
1646.0	1.78	2928

Table 01: $\lambda_{max}T$ Wien law verification 1899 by Lummer&Pringsheim

Temperature (K)	$E_{max}d\lambda$ at $\lambda_{max} \pm \frac{d\lambda}{2} x10^{17}[erg.sec^{-1}.cm^{-3}]$	$E_{max}.T^{-5}x10^{17}[erg.sec^{-1}.cm^{-3}.K^{-5}]$
621	1.01	2190
723.0	2.14	2166
908.5	6.83	2208
998.5	10.7	2166
1094.5	17.0	2164
1259.0	34.4	2176
1460.4	74.5	2184
1646.0	135.3	2246

Table 02: $E_{max}d\lambda \ at \ \lambda_{max} \pm \frac{d\lambda}{2}$Lummer&Pringsheim measurements 1899.

In fact equation (90) is wrong because who tells us that all oscillators radiate energy ?. According to Max Planck [15] only a certain percentage of oscillators which radiate power , the other absorb energy and this percentage is equal to $\eta = 1 - \exp(\frac{-h\nu}{kT})$ so we should multiply the formulae in the integral (90) by η but it is not enough because we should comply with experimental data. To do this one can multiply the formulae in the integral (90) by a constant α in order to fit exactly the experimental data [16].

Replace in (90) $\nu \ by \ \frac{c}{\lambda}$ and $d\nu \ \ by - \frac{c}{\lambda^2}d\lambda$ one can get:

$$E=\int_0^{\infty}\alpha.\frac{8\pi hc^2}{\lambda^6}.\frac{1}{\exp\left(\frac{hc}{k\lambda T}\right)-1}.d\lambda=\int_0^{\infty}\alpha.\frac{8\pi hc^2T^5}{x^6}.\frac{1}{\exp\left(\frac{hc}{kx}\right)-1}.dx \text{ with } x=\lambda T$$

So:

$$E_\lambda=\frac{dE}{dx}=\alpha.\frac{8\pi hc^2T^5}{x^6}\frac{1}{\exp\left(\frac{hc}{kx}\right)-1}=\sigma T^4\psi(\lambda T)=\frac{8\pi hc}{\lambda^5}.F\left(\frac{c}{\lambda T}\right) \quad (93)$$

And of course we have always:

$\lambda^5E_\lambda=Constant$ when $\lambda T=Constant.$

Equation (91) of the total energy becomes as follows:

$$E=\alpha.\frac{8\pi k^5}{c^3h^4}T^5\zeta(5)\Gamma(5)\approx\alpha.\frac{240k^5}{c^3h^4}T^5 \qquad (94)$$

In general the "radiancy" of a black body is given by Cardoso & de Castro law as a generalized Stefan-Boltzmann law in $D-Dimentionnal$ Universe [13]:

$$E_T=R_T=\sigma_DT^{D+1} \qquad (95)$$

With $\sigma_D=\left(\frac{2}{c}\right)^{D-1}(\sqrt{\pi})^{D-2}\frac{k^{D+1}}{h^D}D(D-1)\Gamma(\frac{D}{2})\zeta(D+1)$: generalized Stefan-Boltzmann constant .

D : spatial dimension of the Universe.

-For $D=3$ we have $\sigma_3=\sigma=\frac{c}{4}b=\frac{2\pi^5k^4}{15c^2h^3}$: Stefan-Boltzmann constant given in thermo dynamical analysis and proved by experience.

-For $D=4$ equation (95) should be equal to equation (94) so we can deduce easily that:

$$\alpha=\frac{1}{2} \qquad (96)$$

It is logic in a black body oven that at any time only50% of Planck resonators radiate energy, the others (also 50%) are absorbing energy.

If we take in consideration Planck assumption that only η oscillators radiate energy in the frequency interval ν & $\nu+d\nu$ so we should have from (90) that:

$$E=\int_0^{\infty}\frac{8\pi\nu^2}{c^3}\frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}\left(1-\exp\left(-\frac{h\nu}{kT}\right)\right)d\nu=\frac{1}{2}.\frac{8\pi k^5}{c^3h^4}T^5\zeta(5)\Gamma(5)$$

And this leads us after replacing $\frac{h\nu}{kT}$ $by\,x$ to:

$\int_0^{\infty}\frac{x^4.e^{-x}}{e^x-1}dx=12\zeta(5)$ to be verified with machine calculator.

If now we consider Planck theory of heat radiation we have for the mean energy of the oscillator [17]:

$$\frac{dU}{dt}=Constant$$

But we have also that $U = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}$ than we get:

$$\frac{dU}{dt} = \alpha_0 . \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} + h\nu . \frac{-\frac{\alpha_0}{kT} . e^{\frac{h\nu}{kT}}}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

An to be conform with Planck approximation ($h\nu \ll kT$) we get:

$\frac{dU}{dt} \approx -\alpha_0$: Planck had discover the new universal constant one hundred years ago.

We can also get the wave length where the black body get its maximum power. We have:

$E_\lambda = \frac{1}{2} . \frac{8\pi hc^2}{\lambda^6} . \frac{1}{\exp\left(\frac{hc}{k\lambda T}\right)-1}$ (97)

The maximum of the function (97) is when $\frac{dE_\lambda}{d\lambda} = 0$. Replace $\frac{hc}{k\lambda T}$ $by\ x$ and we get:

$x = 6 + W(-6. e^{-6})$ (98)

With W Lambert function defined as: $z = \omega . e^{\omega} \Leftrightarrow \omega = W(z)$.

We have here: $z = -6. e^{-6} = -0.0148725$ with a calculator we get $\omega = -0.015$ so:

$$x = 5.985 = \frac{hc}{k\lambda T}$$

$\frac{h}{k} = 5.8653\ 10^{-11}\ sec. K$ as in Planck theory

It is very easy to get:

$\lambda_{maxpower} . T = 2.439\ 10^{-3}\ m. K$ (99)

It is clear that our Universe can have maximum 5 dimensions :Length , Width and High + two hidden dimensions where probably the Dark Energy exist(one is space the other is time).

However we don’t see appear at all the constant ν_0 .

Let’s read & exam again the history of the quantum theory [14].

For Planck the density of energy of a black body per unit volume and per unit frequency is given by equation (57).

For high frequency we have Wien law:

$u_\nu = \frac{8\pi h\nu^3}{c^3} . \exp(\frac{-h\nu}{kT})$ (97)

For low frequency we have Stefan-Boltzmann law:

$u_\nu = \frac{8\pi h\nu^2}{c^3} kT$ (98)

In a first step we can put for the mean energy of the oscillator:

$U = C. T + D$ (99)

Which is coherent with equation (98) if we neglect D.

From equation (61) we deduce for the entropy of an oscillator that:

$$\frac{dS}{dU} = \frac{1}{T} = \frac{C}{U-D} \quad (100)$$

So we get from (100):

$$\frac{d^2S}{dU^2} = -\frac{C}{(U-D)^2} \quad (101)$$

Which is valid for large U as given by Stefan-Boltzmann law.

For small U as given by Wien law we have from (81) after approximation and replacing $U\ by\ U-D$(we suppose that Wien law is also approximate):

$$\frac{d^2S}{dU^2} = -\frac{1}{a\nu(U-D)} \quad (102)$$

With a a constant.

Combining those formulae as Planck did to be valid in all limits we get:

$$\frac{d^2S}{dU^2} = -\frac{1}{a\nu(U-D)+\frac{(U-D)^2}{C}} = -\frac{C}{(U-D)(aC\nu+U-D)} \quad (103)$$

Now integrate (103):

$$\frac{dS}{dU} = \int -\frac{CdU}{(U-D)(aC\nu+U-D)} = -\int \frac{Cd(U-D)}{(U-D)(aC\nu+U-D)}$$
$$= -\frac{C}{aC\nu} Log\left(\frac{U-D}{aC\nu+U-D}\right) = \frac{1}{T}$$

So:

$$\frac{U-D}{aC\nu+U-D} = \exp(-\frac{a\nu}{T})$$

Than:

$$U = \frac{aC\nu}{\exp\left(\frac{a\nu}{T}\right)-1} + D \quad (104)$$

Than the density of energy is:

$$u_\nu = \frac{8\pi\nu^2}{c^3}.U = \frac{8\pi\nu^3}{c^3}.\frac{aC}{\exp\left(\frac{a\nu}{T}\right)-1} + \frac{8\pi\nu^2}{c^3}D \quad (105)$$

Which should be in the general form $u_\nu(T) = \nu^3\varphi(\frac{\nu}{T})$ as given in (54).

This is possible only when $\frac{8\pi\nu^2}{c^3}D \ll 1$ (we mean that the second term in the equation (105) is negligible compared to the first one).

So the term D contain the universal constant ν_0 .

From (105) the universal constant ν_0 can be easily determined when $T \to 0K$.

In quantum theory of the rigid rotator applied to the specific heat of hydrogen (Ritz 1919) we have for the mean energy:

$$\bar{E} = \frac{1}{2}J.(2\pi\nu)^2 = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1} + \frac{1}{2}.\frac{h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \qquad (106)$$

It is evident that when $T \to 0K$& we suppose $\nu \ll \nu_0$ the excitation frequency (the rotation frequency of the rotator hydrogen) one can get:

$$\nu_0 = \frac{h}{4\pi^2.J} \qquad (107)$$

The same formulae (107) is given by Fritz Reiche for the fundamental states of the rotation frequency of some molecules of gases introducing hydrogen [18].

Schrödinger (1924) had measured a value for $J = 1.43\ to\ 1.48\ 10^{-41}\ g.cm^2$. Let's take a mean value as $J = 1.455\ 10^{-48}\ Kg.m^2$ so:

$$\nu_0 = 11.547\ \ 10^{12}\ \ Hz \qquad (108)$$

Equation (106) signify also that the moment of inertia J is independent from the temperature when it is very low.

For Ehrenfest this moment is $J = 6.9\ 10^{-48}\ Kg.m^2$ and so:

$$\nu_0 = 2.435\ 10^{12}\ \ Hz(109)$$

One can derive equation (92) according to the frequency to know its maximum:

$$\frac{d\bar{E}}{d\nu} = \frac{h}{\exp\left(\frac{h\nu}{kT}\right)-1} - \frac{h^2\nu^2}{kT}.\frac{\exp(\frac{h\nu}{kT})}{(\exp\left(\frac{h\nu}{kT}\right)-1)^2} + \frac{h}{2}.\frac{1}{\exp\left(\frac{\nu}{\nu_0}\right)-1} - \frac{h}{2}.\frac{\nu^2}{\nu_0}.\frac{\exp(\frac{\nu}{\nu_0})}{(ex\left(\frac{\nu}{\nu_0}\right)-1)^2} = 0$$

Pose : $x = \frac{h\nu}{kT}$& $y = \frac{\nu}{\nu_0}$

So one solution is when the first two terms and the second two terms are both equal to zero so we get :

$$1 = x^2.\frac{kT}{h}.\frac{e^x}{e^x-1}\&\ 1 = y^2.\nu_0.\frac{e^y}{e^y-1}$$

It is possible when :

$$kT = h\nu_0 \qquad (110)$$

We can deduce that (106) admit a maximum and so the calorific capacity of hydrogen $C_v = d\bar{E}/dT$ admit a maximum (for a mole multiply (106) by Avogadro number)[8](Figure01). That is why I suppose that I had make a discovery when I had calculated the constant of dark energy ν_0 directly from the $C_v\ in\ [Joule.K^{-1}.mole^{-1}]$ of any chemical product when its temperature rises for $one\ Kelvin$: I had supposed that every molecule absorb $h\nu_0$ energy to rise the temperature of the mole about $1K$.

From (96) we get: $T = \frac{h\nu_0}{k} = \frac{6.626\ 10^{-34}x2\ 10^{12}}{1.38\ 10^{-23}} = 9.6\ x10 = 96K \approx 1K$: **WRONG**.

In fact my supposition is approximately good because for all gases there is equi-partition of energy at a temperature between 120 K and 250 K .

For Helium at absolute zero we can consider it as in crystal with a cubic structure of an edge a where placed in every corner an atom of Helium the frequency of vibration of atoms is [9]:

$\frac{1}{2}h\nu_0 = \frac{h^2}{8ma^2}$(111)

With : $m = 4x\ atomic\ unit = 4x1.66\ 10^{-27}\ Kg$ the mass of a Helium atom

$$a = 2x1.85\ \text{Å}$$

So: $\nu_0 = 1.82\ 10^{12}\ Hz \approx 2\ 10^{12}\ Hz$

Let's draw the curve of $\nu_{Rot} = f(J)$ at low temperature for many gases as given by Fritz Reiche in 1920 (Fig02):

Gas	Molecular mass $[atomic\ mass\ u]$	Moment of Inertia $[x10^{-40}gram.cm^2]$	Rotational frequency $\nu_{ROT}[x10^{12}Hz]$
H_{2para}	2	2.21	0.8
H_{2ortho}	2	0.96	1.8
HCl	36.5	2.64	0.6
HBr	81	3.27	0.5
HF	20	1.37	1.2
D_2	4	3.82	0.4

Table03: Experimental values of moment of inertia of different gases at low temperature. The two first valuesare given by Eucken(In fact there is two values given by Eucken the first one is $J_1 = 0.96\ 10^{-40}\ gram\ .cm^2$for the ortho-hydrogen and the second one is $J_2 = 2.21\ 10^{-40}\ gram.cm^2$ for the para-hydrogen . The third other values are given by Fritz Reiche in 1920 in his book "the quantum theory" page 169 & page 180. For the last value I had calculated the value from experiment as the following (Fig03):

We should write equation (106) as the following:

$$E = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1} + \frac{n}{2}.\frac{h\nu}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1} + \frac{n}{2}h\nu'$$

With $\frac{n}{2}$ should be full integer to absorb or radiate a quanta of rotational energy of a frequency $\nu' = \frac{\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1}$ as per Planck model of the quantification of the oscillator.

In the fundamental state $n = 1$ the calorific rotational capacity for one molecule is for the maximum :

$$C_{ROT} = \frac{\partial E}{\partial T} = h\nu.\frac{-\frac{h\nu}{kT^2}.\exp(\frac{h\nu}{kT})}{(\exp\left(\frac{h\nu}{kT}\right) - 1)^2} = k(1 + \frac{h^2}{4\pi^2 JkT})$$

If we suppose that $h\nu \ll kT$ than:

$$C_{ROT} = -k\left(1 + \frac{h\nu}{kT}\right)$$

And by replacing $\nu\ by\ \nu_0$ as given in (107) & taking the absolute value of the calorific capacity than we get:

$$J = \frac{h^2}{4\pi^2 kT(\frac{C_{ROT}}{k} - 1)} \qquad (112)$$

From the curve we have the experimental value: $For\ T = 88K, (\frac{C_{ROT}}{k})_{max} = 1.24$

Verification: $\frac{h\nu_0}{kT} = \frac{6.626\ 10^{-27} x 0.4\ 10^{12}}{1.38\ 10^{-16} x 88} = 0.218 \ll 1$

If $h\nu \gg kT$ than in absolute:

$$C_{ROT} \approx \frac{h^2\nu^2}{kT^2}\exp(-\frac{h\nu}{kT}) \qquad (113)$$

For a mono-atomic gas we should correct its calorific capacity in constant volume R by adding (113) multiplied by Avogadro number.

By replacing $\nu\ by\ \nu_0$ as given in (107) & taking the absolute value of the calorific capacity than we get:

$$J^2 = \frac{h^4}{k^2T^2.16.\pi^4.\frac{C_{ROT}}{k}}\exp(-\frac{h^2}{4\pi^2 JkT})$$

And we can resolve this equation graphically or by machine calculator.

In fact our hypothesis of a rigid rotator can be realized only at $= 0K$, when the temperature rise the moment of the rotator hydrogen rise also because of the vibration of the atoms along their link axis. The molecule hydrogen is very tinny and in order to take an idea about its dimension, the wave length of visible light radiated from a molecule when excited is about 1000 the radius of an atom. From $T = 0K$ to another low temperature the distance separate the two centers of the atoms grow and so the moment of inertia of the molecule grow until its average rich the first value given by equation (107): at this moment the energy of the molecule change dramatically as it absorb a quantum of rotational energy and so on it grow until the energy of the molecule change as it absorbs two quantum of rotational energy as was explained by Fritz Reiche .Which is not explained by Fritz Reiche that the initial quantum of rotational energy is an universal constant.

It is clear that we expect that the rotational temperature when the molecule absorb a multiple of quantum rotational energy will high for molecules which have light weight.

In the following table04 [20] there is experimental values of rotational temperatures:

Gas	H_2	HD	D_2	HCl	N_2	O_2	CO	NH_3
T_{ROT}	85.4	65.7	43.0	15.2	2.86	2.07	2.77	12.3
Molecular mass	2	3	4	36.5	28	32	28	17

Table04: T_{ROT} is the characteristic temperature governing the $C_{v\ ROT}$. The rotational heat capacity of diatomic molecules has the equi-partition value of $R \sim 2\frac{cal}{mole.deg}$ at high temperature . As the temperature is lowered $C_{v\ ROT}$ attains approximate value of $1.1\ R$ at $T = 0.8\ T_{ROT}$ and than drops down steeply.

If we plot the T_{ROT} as a function of molecular mass of the gas we get Fig04. The curve slow down when the molecular mass becomes high except for the HCl .The weight of the atom hydrogen is not comparable to the weight of the atom chloride and than we can't speak about a symmetric rotator that's why its rotational temperature don't follow the curve tendency. I am sure that for example for HBr the curve will rise again for the same reasons but I have not the value to plot it in Fig04.

If we plot the rotational frequency as a function of the molecular weight we get Figure02: It indicate that when the molecular mass began high the rotational frequency becomes constant. We can advance the following experimental value of the universal constant as:

$\nu_0 = 0.9\ 10^{12}\ Hz$ (114)

And if we omit the high value of hydrogen we will get a mean value $\nu_0 = 0.7\ 10^{12}\ Hz$ as in General Relativity.

In modern quantum mechanics the rotational energy of a dia-atomic molecule is as follows [21]:

$E_{ROT} = B\ l(l+1) \approx \frac{n}{2} h\nu_0$ (we suppose that $T \to 0K\ \&\ \nu \ll \nu_0$)

With: $B = \frac{\hbar^2}{2J}$: rotational constant

l : integer

$J = m.J_0$: the moment of inertia of molecules are quantified

m: integer

$J_0 = \frac{h}{4\pi^2 \nu_0}$: the fundamental quantum of moment (universal rotator).

It comes that:

$n = \frac{l(l+1)}{m}$

For all possible values of l & m choose only $\frac{n}{2}$ full integer that's why there is many REICHE curves (FIG06).

The variation of the rotational energy is as:

$\Delta E_{ROT}(l+1 \leftarrow l) = 2B\,(l+1)$

$For\ \ l = 1\ than\ \ \Delta E = 4B,\ n = \frac{2}{m}$ & $for\ m = 1\ than\ n = 2\ so\ J = 2\,J_0$ like this manner we can determine the universal rotator J_0 & so we can deduce ν_0.

Vacuum has also entropy as follows from Boltzmann equation :

$\frac{1}{T} = \frac{dS}{dU}$ (115)

Replace in (115) kT by $\frac{1}{2}h\nu_0$ (because the energy of an oscillator is always half Planck constant times the frequency in the fundamental state) than

$S_{vacuo} = \frac{2kU}{h\nu_0} = \frac{8\pi^5 k}{15c^3}.\nu_0^3 \approx 66.735\ 10^{-11}\,Joule.K^{-1}.m^{-3}$ (116)

The density of entropy of vacuum per unit volume and per unit frequency is:

$S_{\nu-vacuo} = \frac{8\pi k\nu^3}{\nu_0 c^3}.\frac{1}{\exp\left(\frac{\nu}{\nu_0}\right)-1}$ (117)

Where : $k = 1.38\ 10^{-23}\,Joule.K^{-1}$: Boltzmann constant.

It is clear that it is possible to extract energy from vacuum if we can vary its entropy. For example lets have an electric circuit RLC which have a resonance frequency near $10^{12}\,Hz$ and if it is excited with an amplifier at the same frequency than dark energy will have influence on the circuit and probably we can extract energy from vacuum in the resonance zone. The problem is can we make those devices?.

The last remark that if constant Λ of General Relativity is not an universal constant than constant$G$ of Newtonian gravitation is not also an universal constant. Only their ration is universal:

$\frac{4\pi^5 h}{15.c^3}.\nu_0^4 = \frac{\Lambda(\mathrm{t}).c^4}{8\pi G(t)}$ so $G(t) = \Lambda(t).\frac{15.c^7}{32\pi^6 h.\nu_0^4}$.

If $\Lambda \sim \frac{1}{R(t)^2}$ than $G = \frac{Constant}{R(t)^2}$

Where $R(t)$ is the radius of the Universe as a function of time.

General Relativity should be revisited.

4.Density of vacuum energy in $D - dimensional$ Universe:

We can always put that the "mean energy" of an oscillator in a black body with n spatial extra-dimensions is :

$$W = \frac{h\nu^{n+1}}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (118)$$

The "quantum of energy" radiated is :

$$\delta = \varepsilon.\nu^n = h\nu^{n+1} \qquad (119)$$

We get always the same entropy.

The "density of energy " is as follows:

$$E = \int_0^\infty \frac{8\pi\nu^2}{c^3} W d\nu = \frac{8\pi k^{n+4} T^{n+4}}{c^3 h^{n+3}}.\zeta(n+4).\Gamma(n+4) \qquad (120)$$

It should be the same "Radiancy" given by Cordoso& de Castro so we have (120) in another expression:

$$E = \alpha_D.\frac{8\pi k^{D+1} T^{D+1}}{c^3 h^D}\zeta(D+1)\Gamma(D+1) \qquad (121)$$

α_D: Planck coefficient of regulation

It comes that:

$D + 1 = n + 4 \,\&\, D = n + 3$ so $n = D - 3$

$$\alpha_D = \frac{2^{D-4}}{c^{D-4}}.\pi^{\frac{D-4}{2}}.\frac{D(D-1)}{\Gamma(D+1)}.\Gamma(\frac{D}{2})$$

$$For\ D = 4\ than\ \alpha_4 = \frac{1}{2}$$

$$For\ D = 3\ than\ \alpha_3 = \frac{c}{4}$$

$$For\ D = 2\ than\ \alpha_2 = \frac{c^2}{4\pi}$$

$$For\ D = 5\ than\ \alpha_5 = \frac{\pi}{4c}$$

$$For\ D = 6\ than\ \alpha_6 = \frac{\pi}{3c^2}$$

$$For\ D = 3\ than\ \frac{E}{\alpha_3}\ is\ the\ energy\ density$$

$$For\ D = 4\ than\ \frac{E}{\alpha_4}\ is\ the\ power\ density$$

$$For\ D = 5\ than\ \frac{E}{\alpha_5}\ is\ the\ varaiation\ of\ the\ variation\ of\ the\ energy\ per\ unit\ volume$$

$$For\ D = 6\ than\ \frac{E}{\alpha_6}\ is\ the\ varaiation\ of\ the\ variation\ of\ the\ variation\ of the\ energy\ per\ unit\ volume$$

$$For\ D = 2\ than\ \frac{E}{\alpha_2}\ is\ the\ \ density\ of\ action$$

The “total density of energy” of a black body in $D - dimentionnal$ Universe is given by Cordoso& de Castro as:

$\rho_T = a_D T^{D+1}$(120)

With $a_D = \left(\frac{2}{hc}\right)^D (\sqrt{\pi})^{D-1} k^{D+1} D(D-1)\Gamma(\frac{D+1}{2})\zeta(D+1)$

The pressure in a black body in $D - dimensional$ Universe is given by H. Alnes & Co as[19]:

$p = \frac{\rho_T}{D} = \left(\frac{2}{hc}\right)^D (\sqrt{\pi})^{D-1} k^{D+1} (D-1)\Gamma(\frac{D+1}{2})\zeta(D+1) T^{D+1}$ (122)

For vacuum we have the mean energy of an oscillator is:

$U_{mean0} = \frac{1}{2}.\frac{h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1}$ (123)

The number of modes in in $D - dimensional$ Universe with frequencies between ν & $\nu + d\nu$ according to Cordoso & de Castro is:

$N(\nu)d\nu = (D-1)V.\frac{2}{\Gamma(\frac{D}{2})}.(\frac{\sqrt{\pi}}{c})^D.\nu^{D-1}d\nu$ (124)

So the density of energy of vacuum in $D - dimensional$ Universe is:

$\varrho_{vacuo} d\nu = (\frac{\sqrt{\pi}}{c})^D.\frac{D-1}{\Gamma(\frac{D}{2})}.\frac{h\nu^D}{\exp\left(\frac{\nu}{\nu_0}\right)-1}d\nu$ (125)

It comes that (following the path of Cordoso & de Castro):

$\varrho_{vacuo} = \left(\frac{2}{c}\right)^D.\left(\sqrt{\pi}\right)^{D-1}.\frac{h}{2}.D.(D-1).\Gamma\left(\frac{D+1}{2}\right).\zeta(D+1).\nu_0^{D+1}$ (126)

The pressure of vacuum in $D - dimensional$ Universe is:

$p_{vacuo} = \frac{\varrho_{vacuo}}{D} = \left(\frac{2}{c}\right)^D.\left(\sqrt{\pi}\right)^{D-1}.\frac{h}{2}.(D-1).\Gamma\left(\frac{D+1}{2}\right).\zeta(D+1).\nu_0^{D+1}$ (127)

To be counted as negative in accordance with General Relativity.

The density of vacuum energy becomes very high when (from equation 126 in MKS system):

$$\varrho_{vacuo} \sim 10^{-34-8D+12D+12} \sim 10^{4D-22}$$

If $4D - 22 \gg 1$ than ϱ_{vacuo} becomes huge (i.e. $D \gg 6$) but this can remember us the Planck system where the density of vacuum is very huge if we take a Planck mass in a volume of cubic Planck length.

In conclusion let's remark that there is an universal rotator of an inertia moment:

$$J_0 = \frac{h}{4\pi^2 \nu_0}$$

The energy of this rotator is:

$$E_0 = \frac{1}{2} J_0 (2\pi\nu_0)^2 = \frac{1}{2} h\nu_0$$

If we suppose that there is an electric charge Q at a distance of $\frac{c}{\nu_0}$ from the same electric charge in module than it will have a potential energy as:

$$\frac{Q^2}{\frac{c}{\nu_0}} \sim \frac{1}{2} h\nu_0$$

So we should have:

$Q^2 \sim hc$ and that is why there is a constant electric charge in Nature.

For gravitation we have:

$$\frac{1}{2} h\nu_0 = \frac{GM^2}{\frac{c}{\nu_0}} = \frac{GM^2 \nu_0}{c}$$

So :

$$\frac{1}{2} h\nu_0^2 = \frac{hGM^2 \nu_0^2}{hc}$$

Replace $h\nu_0^2 \; by \; \frac{c^5}{G}$ we get (the same dimensions):

$$\frac{c^5}{G} \sim \frac{GM^2}{hc} \cdot \frac{c^5}{G}$$

It comes that:

$M \sim \sqrt{\frac{hc}{G}}$ that is why gravitation began at least with Planck mass.

Vacuum energy explain the most common interactions electromagnetism and gravitation.

The electric charge is a characteristic of vacuum. I had wonder earlier in my secondary school: what is the electric charge. Feynman had wonder also about the same question .

The electric interaction is equal to the gravitation interaction if we have :

$Q \sim M$ so it becomes: $G \sim 1$.

In the MKS system we have :

$$G = 6.67\ 10^{-11}\ m^3\ Kg^{-1}s^{-2} = 6.67\ 10^{-11}\ L^3M^{-1}T^{-2}$$

Planck mass: $M_{Pl} = 4.3\ 10^{-9}\ Kg$

Time: $T = \frac{L}{c} = \frac{L}{3\ 10^8}$ so:

$G = 6.67\ 10^{-11}.L^3.\frac{M_{Pl}}{4.3\ 10^{-9}}.\frac{9\ 10^{16}}{L^2} = 1.L.M_{Pl}$

It comes that: $L = 7.1\ 10^{-16}\ m$: at this distance gravitation and electromagnetic forces are equivalent.

5.The structure of the Universe:

From the equation (13) we have the gravitational field at great distances as:

$\boldsymbol{g} = (\frac{\Lambda.c^2}{3}.r^2 - \frac{GM}{r}).\frac{\boldsymbol{u_r}}{r}$ (128)

The density of vacuum energy is given by equation (126) as:

$\varrho_{vacuo} = \left(\frac{2}{c}\right)^D.\left(\sqrt{\pi}\right)^{D-1}.\frac{h}{2}.D.(D-1).\Gamma\left(\frac{D+1}{2}\right).\zeta(D+1).\nu_0^{D+1}$ having the dimension of:

$[Joule.s.s^D.m^{-D}.s^{-D-1} = Joule.m^{-D}]$

To get this density as in three dimension universe we should use universals constants where:

$h = c = \nu_0 = 1$

This system of unities is:

$T = \frac{1}{\nu_0}$ $,L = c.T = \frac{c}{\nu_0},$ $M = \frac{h.\nu_0}{c^2}$

So the density of vacuum energy as given by (10) will be:

$\frac{\Lambda.c^4}{8\pi G} = \varrho_{vacuo}.L^{D-3} = \frac{2^D}{c^3}\left(\sqrt{\pi}\right)^{D-1}.\frac{h}{2}.D.(D-1).\Gamma\left(\frac{D+1}{2}\right).\zeta(D+1).\nu_0^4$ (129)

It comes that:

$\Lambda = \frac{4\pi Gh\nu_0^4}{c^7}.2^D\left(\sqrt{\pi}\right)^{D-1}.D.(D-1).\Gamma\left(\frac{D+1}{2}\right).\zeta(D+1)$ (130)

The gravitational field will be very weak when:

$\frac{\Lambda.c^2}{3}.r^2 - \frac{GM}{r} \approx 0$ that is to say the extra-dimension will be curled in the distances:

$R_n = (\frac{3GM}{\Lambda.c^2})^{\frac{1}{3}}$ (131)

With Λ given by equation (130) and $n = D - 3$ the number of extra-dimensions.

When the gravitational field is so weak because of the structure of space the orbital speeds of galaxies surrounding a center of mass will be approximately the same from a certain distance: it is explained that there is Dark matter which maintain the field weak and in fact there is nothing (Fig05).

Every extra-dimension contain energy as :

$$E_{extra} = \varrho_{vacuo}.V_{extra} \qquad (132)$$

With:

$$V_{extra} = \frac{\pi^{\frac{D}{2}}}{\frac{1}{2}\Gamma(\frac{D}{2})}.1^{D} \qquad (133).$$

With V_{extra} is the volume of a sphere of radius equal to one.

The most volume of $D - dimensional$ sphere of radius R is contained in an annulus of width $O(\frac{R}{D})$ near the boundary [23].

The surface area of a unit radius sphere is :

$$A_{extra} = 2\frac{\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})}1^{D-1} \qquad (144)$$

All the energy is near the surface.

So beginning from an universal constant as $M = \frac{h.\nu_0}{c^2}$ so from nothing we can obtain all our Universe.

References :

[1] M. P. Hobson, G. P. Efstathiou and A. N. Lasenby 2006 “General Relativity” page187 , Cambridge University.https://wigner.hu/~barta/GRcourse2020/Hobson%20M.P.,%20Efstathiou%20G.P.,%20Lasenby%20A.N.%20-%20GR%20-%20An%20Introduction%20for%20Physicists(Cambridge,2006).pdf

[2] Roland Abuaf& Simon Thalabard « Une approche historique de la loi du rayonnement noir » 2006

http://www.afhalifax.ca/magazine/wp-content/sciences/AMA2009/GustavKirchhoff/BEAURESUME-tipecorpsnoir.pdf

[3] Nathalie Deruelle “Introduction aux équations d’Einstein de la relativité générale » , http://www.phys.ens.fr/IMG/pdf/docRG.pdf

[4]Max Planck « Theory of heat radiation» pp 162-165, http://www.gutenberg.org/ebooks/40030

[5]Max Planck « A propos de la loi de distribution de l’énergie dans le spectre normal » Comptes Rendus 2 p202-237 (1900).

https://www.equipes.lps.u-psud.fr/Montambaux/histoire-physique/planck-1900-2-fran%C3%A7ais.pdf

[6] ROBERT RYNASIEWICZ : « The correspondance Principle » https://philsci-archive.pitt.edu/9961/1/TheCorrespondencePrinciple.pdf

[7]A.Bokulich : « Three puzzles about Bohr'scorrespondenceprinciple" http://philsci-archive.pitt.edu/4826/1/3Puzzles-BohrCP-Bokulich.pdf

[8]ClaytonGearheart "The Specific Heat of Hydrogen in Quantum Theory" in Research Gate and the link
https://www.csbsju.edu/Documents/Natural%20Sciences/H2_MP06.pdf page 48.
[9]London « On condensed helium at absolute zero"
https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1936.0024

[10]EyvindH.WichmannBerkley cours de physique volume 4 page 130-131, Armand Colin – PARIS 1974.

[11] M.Thiesen , Verhandl.d. Deutch. Phys. Gesellsch. 2,66 (1900)

https://www.lawebdefisica.com/arts/improvementwiens.pdf

[12]ManojK.Harbola « The Genesis of quanta" 23 December 2017 , http://docplayer.net/138888087-The-genesis-of-quanta.html

[13] Cardoso & de Castro "The blackbody radiation in D-dimensional Universes" arxiv 0510002v1 ,

https://arxiv.org/abs/quant-ph/0510002

[14]100 Years of Planck's quantum ,
http://users.df.uba.ar/dmitnik/cursoplasma2014/textos/textoPlanck.pdf

[15] Max Planck "Theory of Heat Radiation" page 269,
https://www.gutenberg.org/files/40030/40030-pdf.pdf

[16]C.E.Mendenhall& F.A. Saunders "The radiation of black body" American Astronomical Society

[17] Max Planck "Theory of Heat Radiation" page 189,
https://www.gutenberg.org/files/40030/40030-pdf.pdf

[18]Fritz Reiche "The quantum theory", page 124,
https://archive.org/details/quantumtheory004289mbp

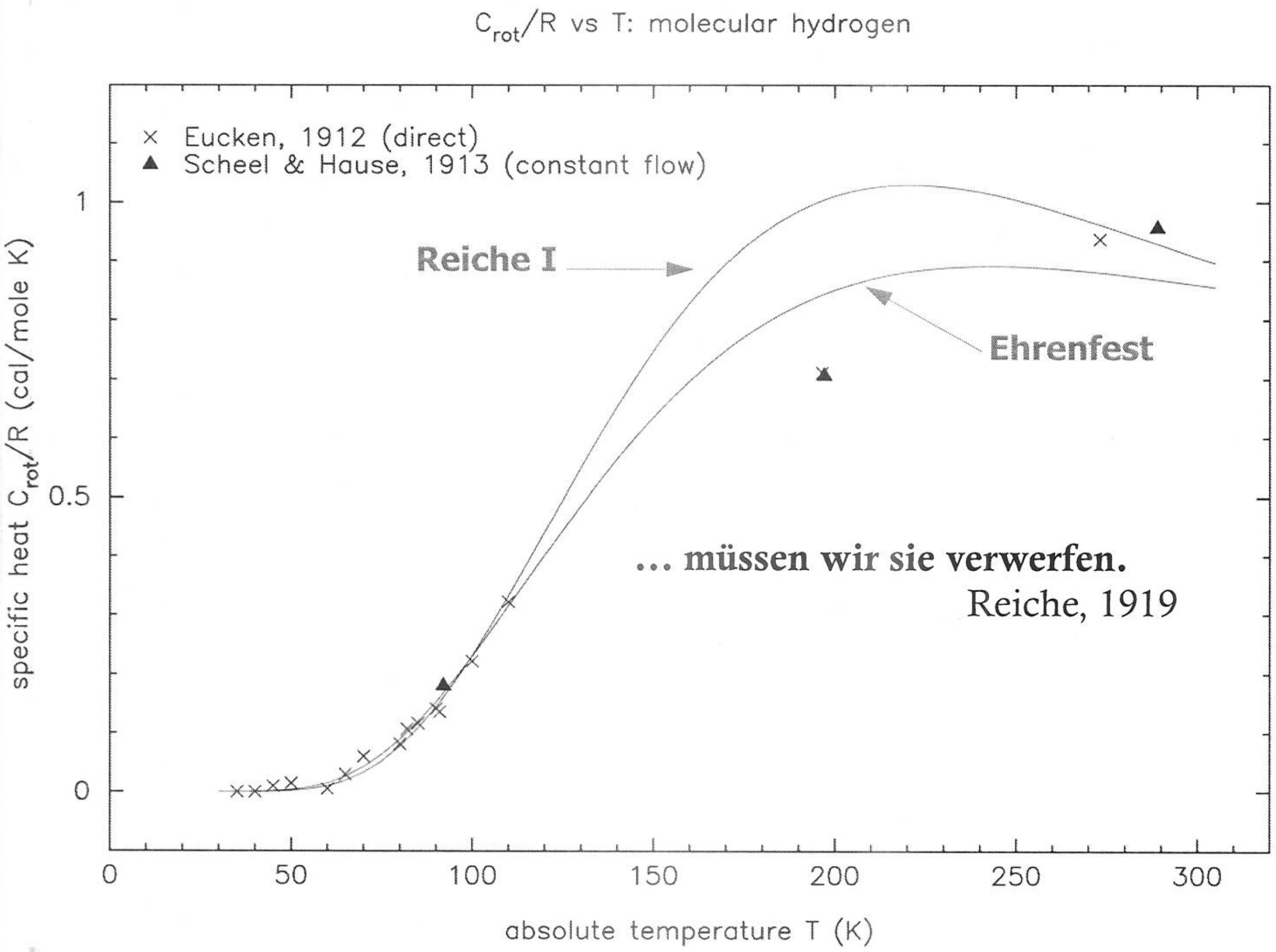
[19] H. Alnes, F. Ravndal and K.Wehushttps://arxiv.org/abs/quant-ph/0506131

[20]E.S.R. Global "Specific heat at low temperatures" page 142, New York 1966,
https://www.eng.uc.edu/~beaucag/Classes/AdvancedMaterialsThermodynamics/Heat%20Capacity%20Books/E.%20S.%20R.%20Gopal%20(auth.)%20-%20Specific%20Heats%20at%20Low%20Temperatures-Springer%20US%20(1966).pdf

[21] Cristina Pizzarini "Introductory Lecture on rotational spectroscopy" , CODECS Summer School 2013 page 24-25. http://kkrk.chem.elte.hu/molim/lectures/Lectures_PUZZARNI.pdf

[22] High-dim-space.pdf, https://www.cs.cmu.edu/~venkatg/teaching/CStheory-infoage/chap1-high-dim-space.pdf

REICHE CURVE I



… müssen wir sie verwerfen.
Reiche, 1919

FIG01: C_{Rot} as a function of temperature for molecular Hydrogen
(From Clayton Gearhart St. John's University)

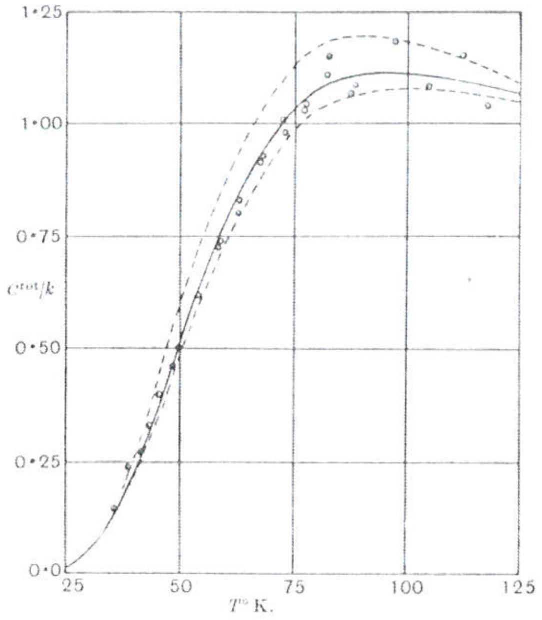


Fig. 3. The rotational molecular heat capacity of D_2.

from Fowler & Guggenheim, *Statistical Thermodynamics*, 1939

Fig 02: Rotational frequency as a function of molecular mass

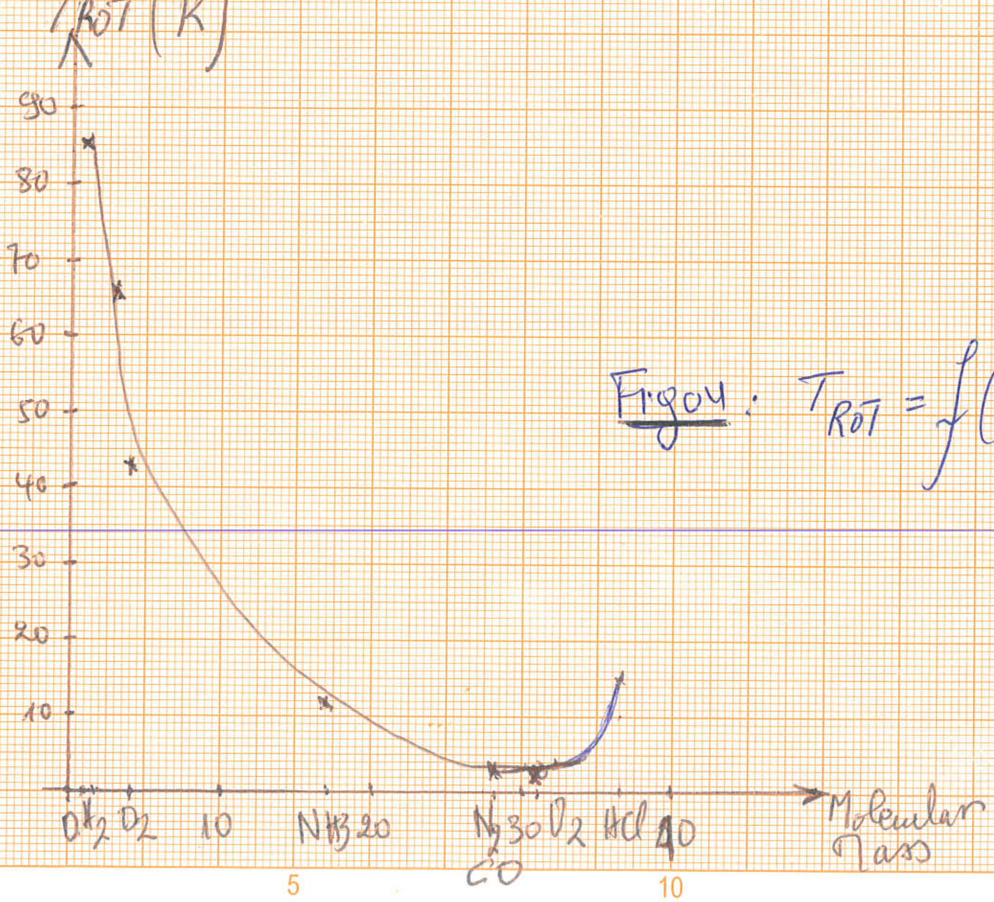


Fig04 : $T_{ROT} = f$(Molecular Mass)

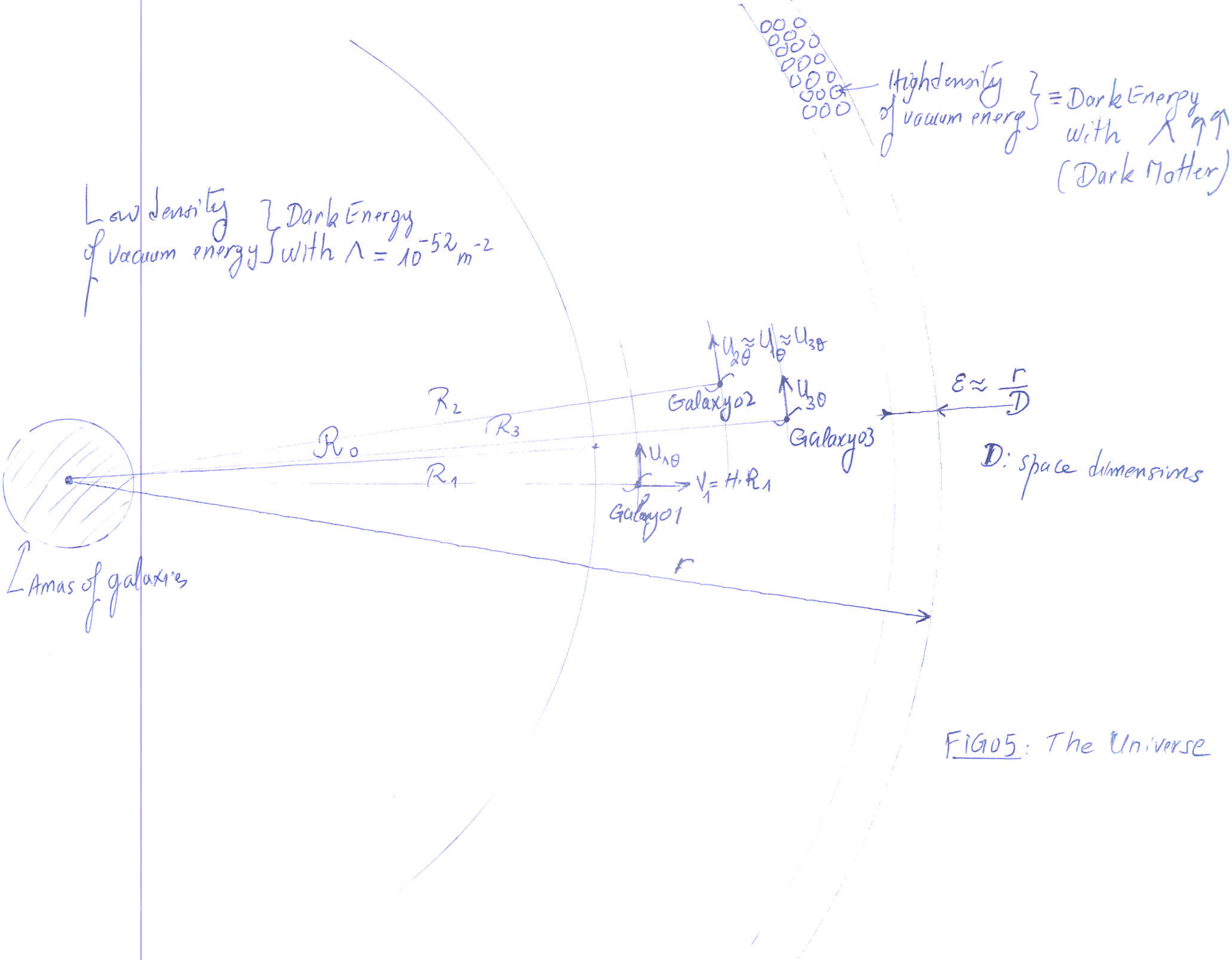


FIG05: The Universe

MOMENTS OF INERTIA

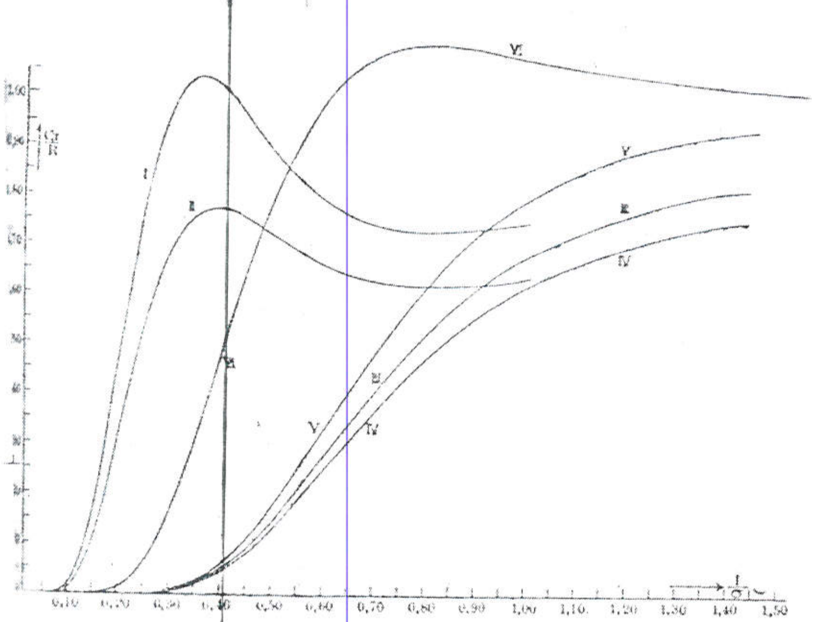


Fig. 3.

from Reiche 1919: plot of

$$C_R/R \text{ vs. } 1/\sigma \quad \text{where } \sigma = \frac{h^2}{8\pi^2 JkT}$$

By early 1920s, spectroscopic measurements began to give independent values for *J*.

This graph from Reiche (1919) shows the effect of the moment of inertia. Reiche and others fit these curves to the data by finding a value of *J* that gave good agreement at low temperatures.

Even though *J* is the only free parameter, the different choices of models correspond to strikingly different values; for example,

	J (in gm-cm^2)
Einstein-Stern	1.47×10^{-41}
Ehrenfest	6.9×10^{-41}
Holm	1.36×10^{-41}
Reiche I	5.21×10^{-41}
Reiche III	2.214×10^{-41}
Reiche V	2.293×10^{-41}
Planck	
Schrödinger (1924)	1.43 to 1.48×10^{-41}

FIG 06: Differents moments of inertia for hydrogen
(From Clayton Gearhart St. John's University Minnesota)