

The Dark Energy

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Abstract

Vacuum is the fundamental state of the electromagnetic field & this is the dark energy. Dark Energy has a characteristic frequency which is an universal constant. Every point of space-time is considered as an oscillator in its fundamental state. Vacuum has also a density of entropy.

1.Introduction :

The energy density of vacuum as given by General Relativity is as follows[1] :

$$U_0 = \frac{\Lambda.c^4}{8\pi G} \approx 10^{-9} \text{Joule.m}^{-3} \quad (1)$$

With :

$\Lambda = 1.088 \cdot 10^{-52} \text{m}^{-2}$: cosmological constant ;

$c = 3 \cdot 10^8 \text{m.s}^{-1}$: relativity constant ;

$G = 6.67 \cdot 10^{-11} \text{SI units}$: gravitationnel constant ;

As per the quantum theory the energy of an oscillator is quantified as follows :

$$E = \left(n + \frac{1}{2}\right) \cdot \hbar\omega \quad (2)$$

With :

n :natural integer ;

$\hbar = \frac{h}{2\pi} = 1.054 \cdot 10^{-34} \text{Joule.seconde}$: reducedPlanck constant ;

$\omega = 2\pi \cdot \nu$: the oscillator frequency.

The ground state of the oscillator corresponds for $n = 0$ and in this case the energy of the oscillator is not zero according to equation (2) which contradicts classical mechanics which provides for a zero total energy therefore zero vacuum energy.

The quantification of energy according to Poincare who wants to be consistent with the ideas of Planck is that "A physical system is only susceptible to a finite number of distinct states; it jumps from one state to another without going through a continuous series of intermediate states. Planck introduced an action element h , which corresponds to the smallest existing volume of phase space. Qualitatively, this volume corresponds to an elementary probability domain. The systems – and therefore the oscillators – corresponding to this domain are indistinguishable. This assumption makes it possible to introduce certain

limitations in the possible values of p and q (momentum and position) which then makes it possible to limit the number of independent variables (i.e. the number of degrees of freedom of the system to avoid when the wavelength decreases the expression of the energy density of the black body in the formula of Jeans does not explode for the small wavelengths).

The energy quantity ε corresponding to h verifies:

$$h = \int_E^{E+\varepsilon} dqdp \quad (3)$$

The ellipse of the phase space of equation $E = \frac{1}{2}Kq^2 + \frac{p^2}{2L}$ has a surface $S(E) = 2\pi E \sqrt{\frac{L}{K}}$ so :

$$h = S(E + \varepsilon) - S(E) = 2\pi\varepsilon \sqrt{\frac{L}{K}} = 2\pi\varepsilon \cdot \frac{1}{\omega} \text{ then :}$$

$$\varepsilon = h\nu \quad (4) \gg [2]$$

The magnitude of power W corresponding for this same quantity of energy ε verify :

$W = \int_E^{E+\varepsilon} \frac{K}{L} dqdp = \frac{K}{L} \frac{2\pi}{\omega} \varepsilon = 2\pi\omega\varepsilon$ because $K = L\omega^2$, this is possible (to have the dimension of a power) if ε verify equation (4) or also there is a constant α_0 having the dimension of a power such that :

$$\varepsilon = \alpha_0 \cdot \tau \quad (5)$$

with τ : a characteristic time of the oscillator.

Constants h & α_0 are declared universal constants .

There are two others solutions for the power W but we decline because the correspondent proportional coefficients can be deduce from the absolute system of unities where $h = c = \alpha_0 = 1$.

One can define the universal constant " a " as :

$$a = \frac{\alpha_0}{c^2} \quad (6)$$

The constant " a " has the dimension of a mechanical impedance, in other words space-time cannot be conceived as completely empty: it is a superfluid of coefficient of friction " a " and having a negative pressure so as to cancel the viscosity effect.

In the ground state an oscillator in this space-time absorbs a certain amount of energy and returns it to space-time in a perpetual fashion.

The electromagnetic field can be considered as an oscillator and in its fundamental state it fills all space-time. Thus the vacuum can be defined as being the fundamental state of the electromagnetic field: all the points of space-time are oscillators in their fundamental states. The vacuum energy predicted by General Relativity actually comes from this conception of the electromagnetic field in its ground state.

The fundamental constant of Nature α_0 has a connection with the notion of the energy of the vacuum, a thing encountered in the theory of General Relativity whose equations of the gravitational field on a cosmic scale are:

$$R_{ik} - \frac{1}{2} \cdot R \cdot g_{ik} = -\frac{8\pi G}{c^4} \cdot T_{ik} - \Lambda \cdot g_{ik} \quad (7)$$

with : R_{ik} : curvature tensor;

R : scalar (curvature of space-time)

T_{ik} : momentum-energy tensor of matter ;

g_{ik} : metric tensor with signature (+,-,-,-) ;

Λ : a constant having the dimension as L^{-2} ;

$G = 6,67 \cdot 10^{-11} SI \text{ units}$: newtonien gravitationnel constant.

$i, k = 0,1,2,3$ tensors indices

In equation (7) we are looking for a form of energy that the energy-momentum tensor describing ordinary energy would not contain:

$$R_{ik} - \frac{1}{2} \cdot R \cdot g_{ik} = -\frac{8\pi G}{c^4} \cdot (T_{ik} + \frac{\Lambda \cdot c^4}{8\pi G} \cdot g_{ik}) \quad (8)$$

At $T_{ik} = 0$ the desired tensor is:

$$T_{ik}^{void} = -\frac{\Lambda \cdot c^4}{8\pi G} \cdot g_{ik} \quad (9)$$

Equation (9) is that of a perfect fluid whose volume energy density is:

$$U_0 = \rho_0 \cdot c^2 = \frac{\Lambda \cdot c^4}{8\pi G} \quad (10)$$

And we have in the absence of any form of ordinary energy:

$$\nabla^i T_{ik}^{void} = 0 \quad (11)$$

It is certain that the new constant α_0 has a direct relationship with the constant Λ .

"We must have $\Lambda \ll \mathcal{L}^{-2}$ where \mathcal{L} is the characteristic length over which, to the Newtonian approximation, the gravitational potential φ can varies , so as to find the Poisson law:

$$\nabla^2 \varphi \approx 4\pi G \rho - \Lambda \cdot c^2 \quad (12)$$

With: ρ : density of matter;

∇^2 : Laplace operator;

This constant is negligible and will therefore only play a role in large systems much larger than stellar or even galactic systems, hence its name of cosmological constant. » [3]

For a spherical mass M the gravitational field to the classical approximation is:

$$\mathbf{g} = -\frac{G.M}{r} \cdot \mathbf{u}_r + \frac{\Lambda.c^2}{3} \cdot r \cdot \mathbf{u}_r \quad (13)$$

It is clear that one sees in the second term of the gravitational field the aspect arises at least a negative constant Λ . For a positive constant Λ we will have a repulsive gravitational field.

In other words, gravitation is a manifestation of the vacuum and the Newtonian field is nothing but a limit where a certain volume of space (of mass M) interacts with the totality of a spatial geometry which extends to the infinite and modeled at each point by an oscillator.

The similarity between the model of gravity and the black body theory is clear. A black body is a cavity whose wall supposedly formed of an infinity of oscillators which exchange energy with the radiation inside. If we still continue to assume that the electromagnetic field inside is an oscillator, all the energy exchanged is between the oscillators and the vacuum. For the atoms of the wall of the black body each electron is on the one hand maintained in connection with the atomic nucleus thanks to the electrostatic force on the one hand and thanks on the other hand to a vacuum energy which pushes it towards the exterior (expansion of the atom): the balance is maintained between a movement of attraction towards the interior and a movement of repulsion towards the exterior i.e. in permanent oscillations. An accelerated electrical charge radiates energy and this energy is harvested from the vacuum.

2. Vacuum energy :

With this conception of the vacuum one can think of using the theory of black body radiation to calculate the energy density of the vacuum [3]. The vacuum will be considered as a black body containing an infinity of oscillators in their fundamental states.

Instead of characterizing the vacuum by a classical average energy " kT " for a two degree of freedom, it will be characterized by an average energy " $\frac{1}{2} h\nu_0$ " a fundamental state of the electromagnetic field where " ν_0 " is a universal constant which can be a measure of the energy density of the vacuum consistent with the value given by General Relativity and with that of the model of the oscillating point in its ground state.

Moreover, the Planck model of the black body is an enclosure composed of several resonators located on the wall and kept in motion by the electromagnetic field inside:

« Let's the state of such an oscillator be completely determined by its moment $f(t)$, that is, by the product of the electric charge of the pole situated on the positive side of the axis and the pole distance, and by the derivative of f with respect to time or

$$\frac{df}{dt} = \dot{f}(t)$$

Let the energy of the oscillator be of the following simple form:

$$U = \frac{1}{2} K f^2 + \frac{1}{2} L \dot{f}^2$$

Where K and L denote positive constants "

“If during its vibration an oscillator, neither absorbed nor emitted an energy, its energy of vibration U will remain constant and we would have....periodical vibration:

$$f = C \cos(2\pi vt - \theta)$$

Where C and θ denotes constants of integration.....

$$v = \frac{1}{2\pi} \cdot \sqrt{\frac{K}{L}}$$

“136. If now the assumed system of oscillators is in a space traversed by heat rays, the energy of vibration, U, of an oscillator will not remain constant, but will be always changing by absorption and emission of energy”

“The first question is : what determines the thermodynamic state of the system considered? For this purpose, according to section 124, the numbers N1, N2, N3...etc. of the oscillators which lie in the region elements 1, 2, 3.....of the ‘state space’ must be given. The state space of an oscillator contains those coordinates which determine the microscopic state of the an oscillator. In the case in question, these are two in number, namely, the moment f and the rate at which it varies, \dot{f} , or in stead of the latter the quantity

$$\psi = L\dot{f},$$

Which is the dimension of an impulse. The region element of the state plane, is according to the hypothesis of quanta (section 126), the double integral

$$\iint d\psi df = h.$$

The quantity h is the same for all region elements”

“In the first place, as regards the shape of the region elements, the fact that in the case of undisturbed vibrations of an oscillator the phase is always changing whereas the amplitude remains constant leads to the conclusion that for the macroscopic state of the oscillators, the amplitudes only not the phases must be considered, or in other words the region elements of the $f\psi$ plane are bounded by the curves C= constant, that is by the ellipses....:

$$\left(\frac{f}{C}\right)^2 + \left(\frac{\psi}{2\pi vLC}\right)^2 = 1$$

The semi-axes of such an ellipse are:

$$a = C \text{ and } b = 2\pi vLC$$

Accordingly the region elements 1,2,3.....n are the concentric similar and similarly situated elliptic rings which are determined by the increasing values of C:

$$0, C_1, C_2, \dots \dots C_n$$

The n^{th} region element is that which is bounded by the ellipses $C = C_{n-1}$ and $= C_n$. The first region is the full ellipse C_1 . All these rings have the same area h which is found by subtracting the area of the ellipse C_{n-1} from the full ellipse C_n hence.....:

$$C_n^2 = \frac{nh}{2\pi^2\nu L}$$

Thus the semi-axes of the bounding ellipses are in the ration of the square roots of the integral numbers" [4]

The surface of the Planck ellipse is:

$$S_n = \pi ab = \pi C 2\pi\nu L C = \pi \cdot \frac{K}{\omega} \cdot C^2 = \frac{2\pi E}{\omega} = n \cdot h$$

And we have always :

$$S_n - S_{n-1} = h$$

The energy of the oscillator is :

$$E = \frac{1}{2} K \cdot C^2 = n \cdot \hbar \cdot \omega$$

At $T = 0K$ all Planck oscillators lie in the full ellipse C_0 as already explained. They are all indistinguishable and do not interact with each other.

The surface of the Planck ellipse surrounded by C_0 is zero and yet all the Planck oscillators are reduced to this ellipse at $T = 0K$ and they each have an energy $E = \frac{1}{2} h\nu$ so we can't speak about vacuum energy because there is no oscillators in a phase area equal to zero.

According to Bohr quantum mechanics for great level quantum state the classical measurement and the quantum measurement are the same: this is called Bohr principle correspondence. But Bohr consider the Principle of Correspondence as a quantum principle i.e. for any level quantum state there is a corresponding classical measurement[7]. To adopt this principle than for Planck resonator at low quantum level we should get:

$$C_n^2 = \frac{(n + \frac{1}{2})h}{2\pi^2\nu L}$$

This is a mathematical forcing to be coherent with the Bohr principle of correspondence[6]. So the energy of the resonator is:

$$E = \frac{1}{2} K \cdot C_n^2 = (n + \frac{1}{2}) \cdot \hbar \cdot \omega$$

Which correspond to the equation (2) as given by the De Broglie-Schrödinger quantum mechanics . Of course the last equation tends to Planck equation $E = n \cdot \hbar \cdot \omega$ for great n .

At $T = 0K$ all Planck oscillators lies in the area $S_0 = \frac{h}{2}$.

This is only possible if this energy is external to the Planck oscillators i.e. comes from the void. The Planck oscillators interacting with this energy will each have a certain average energy with reference to a base energy $\frac{1}{2} h\nu_0$. Thus we can conclude straight away that the entropy of the vacuum is not zero.

The electromagnetic field can also be considered as an oscillator and in its fundamental state it will have the energy $E = \frac{1}{2} h\nu$ and then it can maintain the vibration of the Planck oscillators. Each of these Planck oscillators will certainly have an average energy by reference to an energy $\frac{1}{2} h\nu_0$ where ν_0 is a constant. This average energy of the electromagnetic field in its fundamental state replaces in classical thermodynamics the energy kT of an oscillator two-dimensional. The average energy of the Planck oscillator is calculated by Boltzmann statistics. In fact the Planck oscillators do not emit energy at $T = 0K$ and then the total energy density of the Planck oscillators is none other than the vacuum energy density. Its effects are among others gravitational as a cause of the expansion of the Universe.

Thus the number of electromagnetic oscillators in their ground states with frequencies between ν and $\nu + d\nu$ is as in the black body theory:

$$N = \frac{1}{\exp\left(\frac{\frac{1}{2}h\nu}{\frac{1}{2}h\nu_0}\right)-1} = \frac{1}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \quad (14)$$

The average energy of the oscillators in this frequency interval is:

$$E_\nu = N \cdot \frac{1}{2} h\nu = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \quad (15)$$

The total energy density of the vacuum is then according to the black body theory:

$$U_0 = \int_0^\infty \frac{8\pi\nu^2}{c^3} \cdot E_\nu d\nu = \int_0^\infty \frac{4\pi h}{c^3} \cdot \frac{\nu^3}{\exp\left(\frac{\nu}{\nu_0}\right)-1} d\nu = \frac{4\pi^5 h}{15 \cdot c^3} \cdot \nu_0^4 \quad (16)$$

If we equalize this expression of the energy density of the vacuum with that given by General Relativity we obtain that:

$$\nu_0 = \left[\frac{15 \Lambda \cdot c^7}{32 \cdot \pi^6 \cdot G \cdot h} \right]^{\frac{1}{4}} \approx 0.7 \cdot 10^{12} \text{ Hz} \quad (17)$$

Which proves that ν_0 is an universal constant.

We will take by definition as power quantum:

$$\alpha_0 = h \cdot \nu_0^2 \quad (18)$$

3.The origin of the frequency ν_0 :

The theory of the black body is based on the modelling of it by a cavity whose walls are oscillators which absorb and emit energy in thermal equilibrium with the electromagnetic radiation inside. These oscillators are actually the electrons elastically bound to their atoms.

Planck in his law of distribution of radiant energy does not take into account the energy of the vacuum because at his time quantum mechanics was not developed enough to predict the energy at zero point (i.e. at zero Kelvin) of an oscillator that is not zero.

Planck's law of distribution of black body radiation is as follows [5]:

$$u_\nu = \frac{8\pi h \cdot \nu^3}{c^3} \cdot \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} \quad (19)$$

At $T = 0K$ Planck's law gives zero energy density when in fact the Planck oscillators each have non-zero ground state energy. Planck should already have corrected this situation by fixing a cut-off frequency ν_0 below which his law (19) is no longer valid but also this hypothesis also poses a problem since for a low ν_0 brings Planck's law back to that of Jeans which is not valid in this frequency interval. Moreover Planck indirectly refuses the quantification of the energy of an oscillator. For him the energy of an oscillator varies between $E_n = nh\nu$ and $E_{n+1} = (n+1)h\nu$ in a continuous way like Boltzmann's distribution law for ideal gases and then an oscillator has an average energy equal to $\frac{E_n + E_{n+1}}{2} = \left(n + \frac{1}{2}\right)h\nu$. This is how the energy $\frac{1}{2}h\nu$ appears for the first time in Planck theory of heat radiation.

If we integrate Planck's law we will have:

$$U = \frac{8\pi^5 \cdot k^4}{15 \cdot h^3 \cdot c^3} \cdot T^4 \quad (20)$$

The total energy of the black body can be taken as the sum of the two energies (20) and (16) so that at $T = 0K$ this energy is not zero and the most preponderant fundamental state of the electromagnetic field is the frequency ν_0 .

The energy and momentum of a mass corpuscle m is according to Special Relativity:

$$\varepsilon = \frac{m \cdot c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (21)$$

$$\mathbf{p} = \frac{m \cdot \mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (22)$$

This corpuscle is represented by the 4-vector $p^i = \left(\frac{\varepsilon}{c}, \mathbf{p}\right)$. A corpuscle can also have a wave behaviour represented by the 4-vector $k^i = \left(\frac{\omega}{c}, \mathbf{k}\right)$ as :

$$p^i = \hbar \cdot k^i \quad (23)$$

For a mixed behaviour (no distinction between wave or corpuscle) one can model by a 4-vector state or 4-vector identity $s^i = (c \cdot \tau, \mathbf{v} \cdot \tau)$ as :

$$p^i = \hbar \cdot k^i = a \cdot s^i = m \cdot c \cdot u^i = c \cdot \xi^i \quad (24)$$

Where u^i : 4-vector of the speed of the corpuscle.

$\xi^i = m \cdot u^i$: 4-vector inertia of the corpuscle.

A corpuscle have an identity in time as :

$$s^0 = c \cdot \tau$$

And an identity in space as:

$$l = \tau \cdot v$$

For low massive particles at high speeds those identities can interact with each other and so can be entangled. The wave-corpuscle duality is due to this entanglement.

The group speed of the packet of waves (modeling the particle) is:

$$\frac{1}{v_g} = \frac{dk}{d\omega} = \frac{dk}{dv} \cdot \frac{dv}{d\omega} \quad \text{with } \hbar \cdot k = p = \frac{m \cdot v}{\sqrt{1 - \frac{v^2}{c^2}}} = \alpha l \quad \& \quad \hbar \omega = \frac{m \cdot c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

It is easy to get: $\frac{1}{v_g} = \frac{1}{v}$: the group speed is the same of the particle speed.

For phase speed we have:

$$v_g \cdot v_f = c^2 \quad \text{always}$$

So : $v_f = \frac{c^2}{v} = (2\pi\nu_0)^2 \frac{d(\frac{l}{2\pi})}{d\omega}$: speed of one wave forming the packet (plans equal phase).

One can model the interaction of the electron with the vacuum as being a classic corpuscle moving in a fluid of coefficient of friction "a". The maximum vibration speed of the electron will be that of the Bohr model of the atom i.e. " αc " where $\alpha = \frac{1}{137}$ is the fine structure constant.

The energy transferred by the electron to vacuum is:

$$\varepsilon = \int_0^{\alpha c} a v \cdot v dt = \int_0^{\alpha c} a \cdot v^2 \cdot dt \quad (25)$$

For every corpuscle we have :

$$\hbar \cdot \omega = \alpha_0 \cdot \tau = a \cdot c^2 \cdot \tau = \frac{m \cdot c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (26)$$

At the classical approximation ($v \ll c$) we get :

$$dt = d\tau \approx \frac{m}{a} d \left(1 + \frac{1}{2} \cdot \frac{v^2}{c^2} \right) = \frac{m}{a} \cdot \frac{v dv}{c^2} \quad (27)$$

And than according to equation (27):

$$\varepsilon = \frac{1}{c^2} \left[\frac{1}{4} m \cdot v^4 \right]_0^{\alpha c} = \frac{\alpha^4}{4} \cdot m c^2 \quad (28)$$

Here $m = 9.1 \cdot 10^{-31} \text{Kg}$ is the mass of the electron.

The energy of the electron in equilibrium with the ground state radiation of the electromagnetic field is:

$$\varepsilon = \frac{1}{2} \cdot \frac{h\nu}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} \quad (29)$$

For low oscillation frequencies (classical approximation) we will have:

$$\varepsilon \approx \frac{1}{2} h \cdot \nu_0 \quad (30)$$

From equation (29) and equation (30) we get :

$$\nu_0 = \frac{\alpha^4}{2h} \cdot mc^2 \approx 0.17 \cdot 10^{12} \text{Hz} \quad (31)$$

This value is close to the value obtained by General Relativity but not quite.

In fact the electron is in equilibrium with the maximum of the vacuum energy distribution. This maximum is obtained when:

$$\frac{\nu}{\nu_0} = 3 + W(-3 \cdot e^{-3}) \approx 2.8214 \quad (32)$$

Where W is Lambert function

So we get :

$$\nu = 0.48 \cdot 10^{12} \text{Hz} \quad (33)$$

Which is a value closer to that of General Relativity. It can be said that the majority of the energy density of the vacuum is of the electromagnetic type.

If we do not make a classical approximation in equation (25) we obtain:

$$\varepsilon = \int_0^{\alpha c} a \cdot v^2 d\tau \quad \text{with } \tau = \frac{\frac{m}{a}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (34)$$

So :

$$\varepsilon = \frac{m \cdot c^2}{\sqrt{1 - \alpha^2}} \cdot (1 - \sqrt{1 - \alpha^2})^2 \quad (35)$$

At classical approximation :

$$\varepsilon \approx \frac{1}{4} \cdot \alpha^4 \cdot mc^2 + \frac{1}{8} \cdot \alpha^6 \cdot mc^2 + \dots = \frac{1}{2} h \cdot \nu_0 \quad (36)$$

With equation (35) we can already improve the value of ν_0 .

Note that the constant "a" is eliminated in the integration of (34).

3.Constant ν_0 from the black body theory:

The number of photons per unit volume in a black body at a temperature T is:

$$N = \frac{16.\pi.k^3.\zeta(3)}{h^3.c^3}.T^3 \quad (37)$$

Where $\zeta(3) = 1.2$ function Zeta.

The average energy by photon is :

$$\bar{E} = \frac{U}{N} = \frac{\pi^4.k}{30.\zeta(3)}.T \approx h.\bar{\nu} \quad (38)$$

The density of energy per unit volume and per unit frequency of a black body is:

$$u_\nu = \frac{8\pi.\nu^2}{c^3} \left(\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1} + \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \right) \quad (39)$$

Constant ν_0 is due to quantum effects so we should go to Wien zone of radiation.

We are in this case of approximation: $h\nu \gg kT$ & $\nu \ll \nu_0$ so:

$$u_\nu \approx \frac{8\pi h\nu^3}{c^3} \cdot \exp\left(\frac{-h\nu}{kT}\right) + \frac{4\pi h\nu^2}{c^3} \cdot \nu_0 \quad (40)$$

The total energy per unit volume is:

$$U = \int_0^{\bar{\nu}} u_\nu d\nu = \frac{8\pi k^4 T^4}{c^3 h^3} [-e^{-\bar{\nu}}(\bar{\nu}^3 + 3\bar{\nu}^2 + 6\bar{\nu} + 6) + 6] + \frac{4\pi h\nu_0}{3c^3} \cdot \bar{\nu}^3 \quad (41)$$

We take the case of F.Kurlbaum experiment:

“§11. The values of both universal constants h and k may be calculated rather precisely with the aid of available measurements. F. Kurlbaum, designating the total energy radiating into air from 1 sq cm of a black body at temperature t ° C in 1 sec by S_t , found that:

$$S_{100} - S_0 = 0.0731 \frac{\text{Watt}}{\text{cm}^2} = 7.31 \cdot 10^5 \frac{\text{erg}}{\text{cm}^2 \cdot \text{sec}} \text{”}[5]$$

For $T = 373 \text{ K}$ we have $\bar{\nu}_{373} = 210.4 \cdot 10^{11} \text{ Hz}$

For $T = 273 \text{ K}$ we have $\bar{\nu}_{273} = 154 \cdot 10^{11} \text{ Hz}$

We have also:

$$\frac{c}{4} \cdot \Delta U = \text{measurement of F. Kurlbaum} = 731 \text{ Watt} \cdot \text{m}^{-2} \quad (42)$$

After calculation we get:

$$\nu_0 \approx 6.9 \cdot 10^{12} \text{ Hz}$$

It is a value far from which given by General Relativity but one can some manages to approach the good value.

The most method which we can trust in is to study the behavior of gases and crystals near the absolute zero.

In quantum theory of the rigid rotator applied to the specific heat of hydrogen (Ritz 1919) we have for the mean energy:

$$\bar{E} = \frac{1}{2} J \cdot (2\pi\nu)^2 = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1} + \frac{1}{2} \cdot \frac{h\nu}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} \quad (43)$$

It is evident that when $T \rightarrow 0K$ & we suppose $\nu \ll \nu_0$ the excitation frequency (the rotation frequency of the rotator hydrogen) on can get:

$$\nu_0 = \frac{h}{4\pi^2 \cdot J} \quad (44)$$

Schrödinger (1924) had measured a value for $J = 1.43$ to $1.48 \cdot 10^{-41} \text{ g} \cdot \text{cm}^2$. Let's take a mean value as $J = 1.455 \cdot 10^{-48} \text{ Kg} \cdot \text{m}^2$ so:

$$\nu_0 = 11.547 \cdot 10^{12} \text{ Hz} \quad (45)$$

Equation (44) signify also that the moment of inertia J is independent from the temperature when it is very low.

For Ehrenfest this moment is $J = 6.9 \cdot 10^{-48} \text{ Kg} \cdot \text{m}^2$ and so:

$$\nu_0 = 2.435 \cdot 10^{12} \text{ Hz} \quad (46)$$

One can derive equation (43) according to the frequency to know its maximum:

$$\frac{d\bar{E}}{d\nu} = \frac{h}{\exp\left(\frac{h\nu}{kT}\right) - 1} - \frac{h^2\nu^2}{kT} \cdot \frac{\exp\left(\frac{h\nu}{kT}\right)}{\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^2} + \frac{h}{2} \cdot \frac{1}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} - \frac{h}{2} \cdot \frac{\nu^2}{\nu_0} \cdot \frac{\exp\left(\frac{\nu}{\nu_0}\right)}{\left(\exp\left(\frac{\nu}{\nu_0}\right) - 1\right)^2} = 0$$

$$\text{Pose : } x = \frac{h\nu}{kT} \quad \& \quad y = \frac{\nu}{\nu_0}$$

So one solution is:

$$1 = x^2 \cdot \frac{kT}{h} \cdot \frac{e^x}{e^x - 1} \quad \& \quad 1 = y^2 \cdot \nu_0 \cdot \frac{e^y}{e^y - 1}$$

It is possible when :

$$kT = h\nu_0 \quad (47)$$

We can deduce that (43) admit a maximum and so the calorific capacity of hydrogen $C_v = d\bar{E}/dT$ admit a maximum (for a mole multiply (43) by Avogadro number)[8]. That is why I suppose that I had make a discovery when I had calculated the constant of dark energy ν_0 directly from the C_v in [*Joule* · K^{-1} · *mole*⁻¹] of any chemical product when its temperature rises for *ne Kelvin* : I had supposed that every molecule absorb $h\nu_0$ energy to rise the temperature of the mole about 1K.

$$\text{From (47) we get: } = \frac{h\nu_0}{k} = \frac{6.626 \cdot 10^{-34} \cdot 2 \cdot 10^{12}}{1.38 \cdot 10^{-23}} = 9.6 \cdot 10 = 96K \approx 1K : \text{WRONG.}$$

For Helium at absolute zero we can consider it as in crystal with a cubic structure of an edge a where placed in every corner an atom of Helium the frequency of vibration of atoms is [9]:

$$\frac{1}{2} h\nu_0 = \frac{h^2}{8ma^2} \quad (48)$$

With : $m = 4x \text{ atomic unit} = 4x1.66 \cdot 10^{-27} \text{ Kg}$ the mass of a Helium atom

$$a = 2x1.85 \text{ \AA}$$

$$\text{So: } \nu_0 = 1.82 \cdot 10^{12} \text{ Hz} \approx 2 \cdot 10^{12} \text{ Hz}$$

Vacuum has also entropy as follows from Boltzmann equation :

$$\frac{1}{T} = \frac{dS}{dU} \quad (49)$$

Replace in (49) kT by $\frac{1}{2} h\nu_0$ than

$$S_{vacuo} = \frac{2kU}{h\nu_0} = \frac{8\pi^5 k}{15c^3} \cdot \nu_0^3 \approx 66.735 \cdot 10^{-11} \text{ Joule} \cdot K^{-1} \cdot m^{-3} \quad (50)$$

The density of entropy of vacuum per unit volume and per unit frequency is:

$$S_{\nu-vacuo} = \frac{8\pi k \nu^3}{\nu_0 c^3} \cdot \frac{1}{\exp\left(\frac{\nu}{\nu_0}\right) - 1} \quad (51)$$

Where : $k = 1.38 \cdot 10^{-23} \text{ Joule} \cdot K^{-1}$: Boltzmann constant.

It is clear that it is possible to extract energy from vacuum if we can vary its entropy. For example lets have an electric circuit RLC which have a resonance frequency near 10^{12} Hz and if it is excited with an amplifier at the same frequency than dark energy will have influence on the circuit and probably we can extract energy from vacuum in the resonance zone. The problem is can we make those devices?.

The last remark that if constant Λ of General Relativity is an universal constant than constant G of Newtonian gravitation is not also an universal constant. Only their ration is universal:

$$\frac{4\pi^5 h}{15 \cdot c^3} \cdot \nu_0^4 = \frac{\Lambda(t) \cdot c^4}{8\pi G(t)} \quad \text{so} \quad G(t) = \Lambda(t) \cdot \frac{15 \cdot c^7}{32\pi^6 h \cdot \nu_0^4} \cdot$$

$$\text{If } \Lambda \sim \frac{1}{R(t)^2} \quad \text{than} \quad G = \frac{\text{Constant}}{R(t)^2}$$

Where $R(t)$ is the radius of the Universe as a function of time.

General Relativity should be revisited.

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