EXTENDED EINSTEIN FIELD EQUATIONS FOR COMPLEX SPACETIME

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Abstract. In paper on EEFE [1] - that is Extended Einstein Field equations i will explore in short mathematical model behind quantazation of physical field.

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Contents

1. Complex field equations

1.1. **Complex spacetime.** First I do write two possible way of writing spacetime interval that means distance in complex spacetime, then I will combine then into one single real interval:

$$ds^2 = g_{ab} dz^a dz^b \tag{1.1}$$

$$d\overline{s}^2 = \overline{g}_{ab} d\overline{z}^a d\overline{z}^b \tag{1.2}$$

$$d\overline{s}ds = \sqrt{d\overline{s}^2 ds^2} = \sqrt{g_{ab}\overline{g}_{ab} dz^a dz^b d\overline{z}^a d\overline{z}^b} = ds^2 \tag{1.3}$$

Now what is left is to define metric tensor and in general a complex field:

$$g_{ab} = \frac{\partial \chi^i}{\partial z^a} \frac{\partial \chi^j}{\partial z^b} \eta_{ij} \tag{1.4}$$

$$\overline{g}_{ab} = \frac{\partial \overline{\chi}^i}{\partial \overline{z}^a} \frac{\partial \overline{\chi}^j}{\partial \overline{z}^b} \eta_{ij} \tag{1.5}$$

$$\chi^k = a^k (\mathbf{x}) e_k + i b^k (\mathbf{x}) e_k \tag{1.6}$$

$$\overline{\chi}^k = a^k (\mathbf{x}) e_k - i b^k (\mathbf{x}) e_k \tag{1.7}$$

$$\chi^k \left(\chi^k\right)^\dagger = \chi^k \overline{\chi}_k = a^k (\mathbf{x}) a_k (\mathbf{x}) + b^k (\mathbf{x}) b_k (\mathbf{x}) \tag{1.8}$$

Now when i have all fields defined I can move to most used field that is scalar field.

1.2. **Scalar fields.** Let me state that there is a scalar field that integral is equal to some real number N that will be normalization constant of that field. I will write that scalar field just as ψ, all information about it can be written:

$$\int_{\mathbf{M}^4} \psi d^4\mathbf{x} = N \tag{1.9}$$

$$\frac{1}{N} \int_{\mathbf{M}^4} \psi d^4\mathbf{x} = 1 \tag{1.10}$$

$$\frac{1}{N} \int_{x^a(\mathbf{x}) \in \mathbf{M}^4} \psi d^4\mathbf{x} = \varphi \left(x^a (\mathbf{x})\right) \tag{1.11}$$

1.3. **Field equation.** To construct field equation I need only set Riemann tensor to a complex tensor instead of real one that is just done by changing metric tensors. I will define probability function with field equation:

$$R^\dagger_{abcd} = \overline{R}^{abcd} \tag{1.12}$$

$$R^\dagger_{abcd} R_{abcd} = \psi \tag{1.13}$$

$$R^\dagger_{abcd} R_{abcd} = R^\dagger_{abcd} \left(\kappa T_{abcd} + \frac{1}{2} R_{ac} g_{bd}\right) \tag{1.14}$$

$$\psi = R^\dagger_{abcd} \left(\kappa T_{abcd} + \frac{1}{2} R_{ac} g_{bd}\right) \tag{1.15}$$

$$\frac{1}{N} \int_{x^a(\mathbf{x}) \in \mathbf{M}^4} R^\dagger_{abcd} \left(\kappa T_{abcd} + \frac{1}{2} R_{ac} g_{bd}\right) d^4\mathbf{x} = \varphi \left(x^a (\mathbf{x})\right) \tag{1.16}$$

References

[1] https://vixra.org/abs/2309.0054

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