Gravity-time and the inside of a Black hole
: Another way to define time

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Abstract

Time is a concept to explain change. Since gravity is a force that causes change, time can be defined using the rate at which gravity is propagated. (gravity-time)

The speed of gravity is invariant in the pan-inertial frame and is not affected by other gravity, gravity-time is homogeneous even in the gravitational field. (In an inertial frame, gravity-time is equal to the time of the special theory of relativity)

Gravity, created by a point mass, can be removed by using a reference frame of uniform circular motion (speed v) so that the speed of light in the gravitational field can be calculated. Calculate the speed of light with gravity-time as follows:

$C_G = \sqrt{c_0^2 - 2v^2}$ (c_0 : the speed of gravity)

Here, if Newtonian mechanics is applied as an approximation and the above equation is converted into an equation of radius R, then the speed of light (time dilation, $c_0\sqrt{1-\frac{2GM}{Rc_0^2}}$) and the size of the black hole ($R= \frac{2GM}{c_0^2}$) exactly match the values calculated by the general theory of relativity.

The principle of constancy of the speed of gravity establishes the Lorentz transformation. When viewed from a moving object in respect to a star, gravity-time in the star passes slowly. Due to the difference in the time flow between the star and the object, the gravity transmitted from the star for a certain time reaches the moving object in a longer time, so the gravity works weakly compared to a stationary object.

The gravity-time equations of motion could be represented using the momentarily co-moving pan-inertial frame. With this equation, we can show that even inside a black hole, the acceleration or the acting force does not become infinite. Also, the size of a black hole is smaller than the Schwarzschild radius.

The theory of gravity-time predicts that gravitational wave is faster than light in a gravitational field, and that gravity works weakly on a moving object compared to a stationary one.

1. The meaning of time

'What is time ?' is a philosophical question, but time in physics is a concept to explain any change. If there were no changes, there would be no need for the concept of time.

A space is needed for change to occur, so a space is needed to define time. In order to measure the time T_A at location A, a space S_A including location A is required. If there is a change in any space S_A containing location A, we can measure the time at location A.

In the same way, the time T_B at location B could be measured. Then, what is the relationship between T_A and T_B ? Since the nature of time is closely related to change, how a change at location A affects location B determines the relationship between T_A and T_B.

In a world where all changes are transmitted immediately, which is assumed by Newtonian mechanics, the time T_A and the time T_B are always the same because everyone can observe the same event at the same time.

In a world of the special theory of relativity, light can be used to measure time in an inertial reference frame. An observer (location A) in an inertial frame can measure the time T_A by converting the distance traveled by light into time using the principle of constancy of the speed of light. That is, *light velocity = light path / time interval ([1]Einstein, Albert (1905)).* In the same way, an observer (location B) in an inertial frame can measure the time T_B , the relationship between T_A and T_B can be explained by the Lorentz transformation.

However, according to the general theory of relativity, light slows down (or time slows down) in a space with strong gravity, and time stops when it reaches the event horizon of a black hole. In this situation, time loses its function to explain change. Nevertheless, even inside a black hole, matter has some action, so we need a new concept of time to explain it.

2. Gravity-time

2.1 Definition

Gravity-time denotes a time measured based on the speed of gravity. Calculate gravity-time interval as follows:

gravity velocity = gravity path / gravity-time interval

The propagation of gravitational wave is an instance of gravity's propagating, one way to measure gravity path is to measure the path of gravitational wave.

2.2 Pan-inertial frame

A reference frame that is stationary or moving at constant velocity in a space with little gravity on the outer periphery of the universe can be regarded as an inertial frame. In such a space, we can find various inertial frames. Let us now define a pan-inertial frame.

A pan-inertial frame denotes a reference frame of which the relative velocity with some inertial frame is zero. From the definition, an inertial frame is a pan-inertial frame. The practical benefit of a pan-inertial frame is that it can introduce a reference frame similar to an inertial frame even in the gravitational field.

2.3 The principle of constancy of the speed of gravity, and the principle of gravity independency

The principle of constancy of the speed of gravity means that the speed at which gravity travels is constant as viewed by any observer in all pan-inertial frames.

The principle of gravity independency means that the speed of gravity is not affected by other gravity.

By the two principles above, observers in the universe experience the same flow of time even in the gravitational field. That is, gravity-time becomes homogeneous throughout the universe, including the gravitational field.

2.4 Does gravity-time exist ?

Assuming the principle of constancy of the speed of gravity, Lorentz transformation is established. Let S, S' be pan-inertial frames with relative velocity $\vec{v}$. As viewed by an observer of S, the time of S' becomes slow. Futhermore if the relative speed($|\vec{v}|$) increases, the time of S' slows down even more. That implies, in an accelerated reference frame, the farther away from the direction of acceleration, the slower the time. This process is difficult to avoid because it is a simple calculation.

If a gravitational field is the same as an accelerated reference frame, time slows down as we move into the gravitational field. Since the speed of gravity also slows down inside of a gravitational field, that seems 'constancy of the speed of gravity' and 'gravity independency' become incompatible.

However, the theory of gravity-time derived from 'constancy of the speed of gravity' and 'gravity independency' will show that gravitational field and accelerated reference frame are not the same. In conclusion, gravity-time is well defined, and will show the difference between gravitational field and accelerated reference frame.

2.5 The speed of gravity in an inertial frame

In an inertial frame, the speed of gravity and the speed of light are equal. Therefore, gravity-time and light-time are the same in an inertial frame, and the special theory of relativity can be applied based on gravity-time. The theory of gravity-time reaches the same conclusion as the special theory of relativity in an inertial frame. (Since outer space, where gravity is very weak, can be regarded as an inertial frame, the concept 'the speed of gravity in an inertial frame' is of practical use.)

3. Speed of light in a gravitational field

3.1 Reference frame of gravity free

Suppose a star (point mass) of mass M is in an pan-inertial frame. That is, there is such an inertial frame on the outer periphery of the universe, that this star is stationary when viewed by an observer in the inertial frame .

Now consider an artificial satellite moving in uniform circular motion with respect to this star with a speed v in radius R, as viewed from the star's pan-inertial frame.

Inside the satellite, gravity is offset by centrifugal force, so no force acts on it. Since no forces act on it, light appears to travel at the speed of gravity (=the speed of light in an inertial frame) to an observer inside the satellite.

3.2 Gravity-time in the satellite

Let t be the gravity-time of outside(viewed from the star's pan-inertial frame.), and T be the gravity-time of inside(viewed from an observer stationary in respect to the inside of the satellite).

Since the satellite is moving with a relative speed v to the star, the time of inside always passes slowly as viewed from an observer in the star's pan-inertial frame.

By the Lorentz transformation, $\triangle T = \frac{1}{\gamma}\Delta t$ (where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}}$, c_0 : the speed of gravity)

If the orbital period of the satellite is Θ_t and Θ_T, respectively, then $\Theta_T = \frac{1}{\gamma}\Theta_t$

3.3 The satellite's angular speed

Let θ be angle of orbit, w be angular speed of outside, and W be angular speed of inside. Then, $\frac{d\theta}{dt} = w$, $\frac{d\theta}{dT} = W$.

Since $\int_0^{\Theta_t} wdt = 2\pi$, $\int_0^{\Theta_T} WdT = 2\pi$, then $w\Theta_t = 2\pi$, $W\Theta_T = 2\pi$.

$\therefore\ W = w\Theta_t / \Theta_T = \gamma\omega$

3.4 Speed of light

When light traveling in a gravitational field is observed inside the satellite, it travels at the speed of gravity, so the speed of light observed outside can be calculated using a geometric relationship

Suppose that light travels from a location P with a radius R to the outside in the radial direction. As the light passes through P, the satellite passes by that position.

The light travels diagonally as viewed by an observer inside the satellite, and its speed is the speed of gravity since no force acts inside the satellite. (The fact that light travels diagonally assumes that light slows down in the gravitational field)

Let the satellite be a rectangular parallelepiped whose base is tangent to a sphere that can be drawn from the point mass, and let l be the length in the radial direction.

As shown in Figure 1, when viewed from the outside of the satellite, the light passing through P hits the satellite's ceiling U. Looking at this from inside the satellite, the satellite moves while the light passes through P and reaches U, so P becomes Q, and it seems that the light starts from Q and travels diagonally to reach U.

For an infinitesimal time interval, the figure PQU could be considered as right triangle with hypotenuse QU. Thus, $QU^2 = PQ^2 + PU^2$.

Let $\triangle T$ be the time interval when the light from Q reaches the ceiling U. And let QU be L, and PQ be H.

L is the distance traveled by light, and at this time, the light travels at the speed of gravity, so $L = c_0 \triangle T$.

H is the distance traveled by the satellite as viewed from the inside, so $H = WR \triangle T$ (where W be angular speed of inside, R be the radius of the orbit)

Since the height PU is perpendicular to the direction of motion, there is no change in length, so PU = l.

Therefore, $c_0^2 \triangle T^2 = W^2 R^2 \triangle T^2 + l^2$ $(L^2 = H^2 + l^2)$

On the other hand, let C_G be the speed of light observed from the outside and $\triangle t$ be the time interval required for the light to travel a distance l when observed from the outside, then $l = C_G \triangle t$.

In addition, as seen above, since $\triangle T = \frac{1}{\gamma} \triangle t$,

then $L = c_0 \triangle T = \frac{c_0}{\gamma} \triangle t$

$H = WR \triangle T = wR \triangle t = v \triangle t$

$l = C_G \triangle t$

$L^2 = l^2 + H^2$

$\therefore \quad C_G = \sqrt{c_0^2 - 2v^2}$ (where c_0 be the speed of gravity)

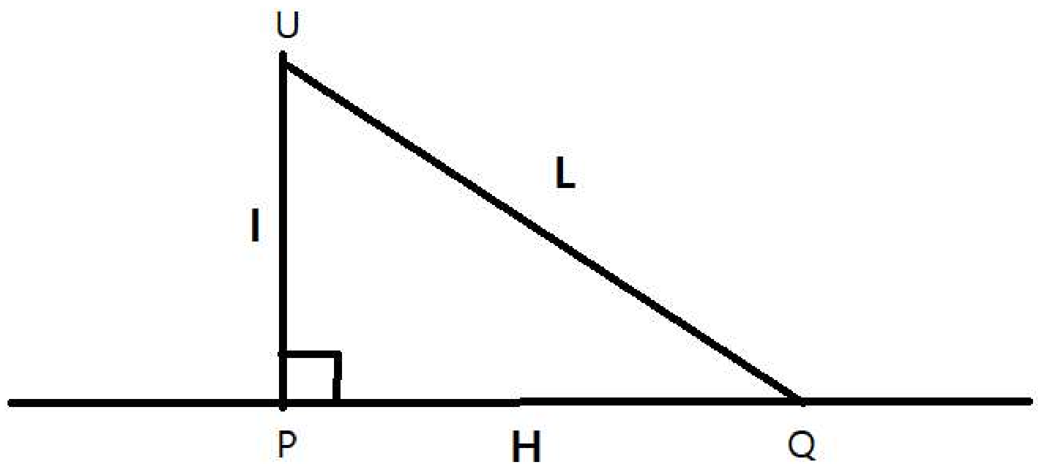

Figure 1

3.5 The speed of light and the size of a black hole using Newton's formula

As we don't yet know the equation for the relationship between R and v, let's apply Newton's formula as an approximation.

According to Newton's formula, $R = \frac{GM}{v^2}$ (where M be mass, and G be gravitational constant) holds, so if we apply on $C_G = \sqrt{c_0^2 - 2v^2}$, then

$$C_G = \sqrt{c_0^2 - 2v^2} = c_0\sqrt{1 - 2\frac{v^2}{c_0^2}} = c_0\sqrt{1 - \frac{2GM}{Rc_0^2}}$$

This is exactly the same as the time delay calculated by the general theory of relativity. (Slowing down the speed of light means slowing down the time)

In addition, if $v^2 = \frac{c_0^2}{2}$, then the speed of light should be 0, so it becomes the boundary of the black hole from which light cannot escape. According to Newton's formula, $R = \frac{GM}{v^2}$ holds, so if we apply on $v^2 = \frac{c_0^2}{2}$, then the radius of the black hole will be $R = \frac{2GM}{c_0^2}$, which exactly matches the Schwarzschild radius calculated by the general theory of relativity.

We will now establish the equation of gravity-time to calculate a more accurate value.

4. The equation of gravity-time

4.1 Newton's law of universal gravitation

According to Newton's law of universal gravitation, $F = G\frac{Mm}{R^2}$ holds.

(where, F be the gravitational force, M, m be the masses of the objects, R be the distance between two objects, and G be gravitational constant)

This means that gravity causes a change in momentum, that space is three-dimensional, so its magnitude is inversely proportional to the square of the distance and proportional to each mass. (the principle of gravity independency may imply ‘proportional to each mass’)

If two point masses are in an pan-inertial frame and are stationary with respect to each other, the above equation holds. However, when an object starts to move with respect to the other, the above equation must be modified because the gravity-times of the two objects passe differently and the action of gravity changes.

4.2 The gravity-time equation of motion

4.21 Momentarily Co-moving Reference Frame (MCRF)

Momentarily Co-moving Reference Frame denotes an pan-inertial frame of reference which, for an infinitesimal time interval, moves in the same direction, at the same speed in respect to an object (or a reference frame).

In the general(or special) theory of relativity, it is an inertial frame, but in the theory of gravity-time, it is a pan-inertial frame.

We will observe the change of momentum acting on an object using MCRF.

4.22 The concept of graviton for explanation

Graviton is not real, but it makes it easy to understand the action of gravity, so the concept of graviton is introduced for convenience of explanation.

A graviton moves at the speed of gravity and functions to transmit momentum. Suppose that the amount of momentum transmitted is proportional to the number of graviton, and let p be the amount of momentum transmitted by one.

Assume a point mass is in a pan-inertial frame, and consider a particle at rest with respect to the point mass. The point mass and the particle have the same flow of gravity-time because the relative velocity is zero. If N gravitons are emitted towards the particle during $\triangle t$(gravity-time interval) from the point mass, then N gravitons are also reached in the particle during $\triangle t$. At this time, the change in the momentum of the particle observed by the MCRF of the particle is :

$\triangle P = pN$

Now, suppose that the particle has the same position as the case above, but moves with a relative velocity $\vec{v}$ ($|\vec{v}| = v$) with respect to the point mass. According to the principle of constancy of the speed of gravity, a graviton arrives at the speed of gravity even for a moving particle, and the change in momentum observed in the MCRF of the particle is proportional to the number of arriving graviton.

However, the moving particle have a different time flow with respect to the point mass. The time interval $\triangle t$ passed at the point mass appears to have passed by $\gamma\triangle t$ when observed from the MCRF of the particle. (where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}}$, c_0 : the speed of gravity).

An observer in the MCRF of the particle sees N gravitons arriving during $\gamma\triangle t$. Therefore, during the same time interval $\triangle t$, N/γ gravitons arrive, and the change in momentum observed in the MCRF is : $\triangle P^{'}$ = pN/γ =$\triangle P/\gamma$

The change in momentum per unit time (that is, acceleration) due to gravity acting on a moving object is smaller (=$1/\gamma$) than that of a stationary object.

This explanation is the reason for deriving the gravity-time equation of motion below as a kind of thought experiment.

4.23 The gravity-time equation of motion

Assuming that a particle is small and does not affect the motion of a point mass, and that a point mass is in an pan-inertial frame, then the gravity-time equation of motion is:

$$-\frac{\nabla\phi(\vec{x})}{\gamma} = \vec{\alpha} \quad \text{(where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}}, \ c_0 \text{ : the speed of gravity).}$$

ϕ : the gravitational potential created by a point mass of mass M in an pan-inertial frame ($\phi = -\frac{GM}{R}$, where G be gravitational constant, R be the distance)

$\vec{x}$: position of a particle when viewed from the pan-inertial frame of a point mass

$\vec{v}$ ($|\vec{v}| = v$) : velocity of a particle when viewed from the pan-inertial frame (with its gravity-time) of a point mass

$\vec{\alpha}$: acceleration of a particle, observed from the MCRF (with MCRF's gravity-time) of the particle

This equation is intuitive, but since the left side is expressed based on the pan-inertial frame of a point mass and the right side is expressed based on the MCRF of the particle, it is necessary to unify the reference frame to make it into a differential equation.

So, it will be made into a differential equation based on the pan-inertial frame of a point mass. In addition, the representations in other pan-inertial frame can be obtained by Lorentz transformation.

4.24 The equation based on the pan-inertial frame of a point mass

4.241 Lorentz transformation *([4]Møller, C.(1952), p.41)*

Let S, S' be pan-inertial frames, suppose the relative velocity of S' with respect to S is $\vec{v} = (v_x, v_y, v_z)$ (viewed from S).

Let the location in each pan-inertial system be $\vec{r} = (x, y, z)$ and $\vec{r}' = (x', y', z')$ respectively, and the gravity-time be t, t' respectively, the Lorentz transformation is :

$$\vec{r}' = \vec{r} + ((\vec{r} \cdot \vec{v})\frac{\gamma_v - 1}{v^2} - t\gamma_v)\vec{v}, \quad t' = \gamma_v(t - \frac{\vec{r} \cdot \vec{v}}{c_0^2})$$

(where $v = |\vec{v}|$, $\gamma_v = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c_0^2}}}$, c_0 : the speed of gravity).

4.242 Lorentz transformation of acceleration

([5] Kopeikin,S.; Efroimsky, M.; Kaplan, G. (2011), p.141)

Suppose there is an acceleration $\vec{a}$ of an object with momentary velocity $\vec{u}$, as viewed from pan-inertial frame S (with its gravity-time t). The acceleration $\vec{a}'$ when this object is observed from the pan-inertial frame S'(with its gravity-time t') can be obtained directly from the Lorentz transformation of Sec

4.241, which is :

$$\vec{a}' = \frac{\vec{a}}{\gamma_v^2(1-\frac{\vec{v}\cdot\vec{u}}{c_0^2})^2} - \frac{(\vec{a}\cdot\vec{v})(\gamma_v-1)\vec{v}}{v^2\gamma_v^3(1-\frac{\vec{v}\cdot\vec{u}}{c_0^2})^3} + \frac{(\vec{a}\cdot\vec{v})\vec{u}}{c_0^2\gamma_v^2(1-\frac{\vec{v}\cdot\vec{u}}{c_0^2})^3}$$

4.243 The gravity-time equation of motion, represented as a differential equation

Let $\vec{r}(t)=(x,y,z)$ be the location of a particle and t be gravity-time, viewed from S the pan-inertial frame of a point mass. We want to know the differential equation of $\vec{r}(t)$. By definition, $\vec{u}=\frac{d\vec{r}}{dt}$, $\vec{a}=\frac{d\vec{u}}{dt}$.

Then, the acceleration $\vec{a}'$ of the particle observed in the pan-inertial frame S' moving at the relative velocity $\vec{v}=(v_x,v_y,v_z)$ with respect to S is given by the equation of Sec 4.242 as follows :

$$\vec{a}' = \frac{\vec{a}}{\gamma_v^2(1-\frac{\vec{v}\cdot\vec{u}}{c_0^2})^2} - \frac{(\vec{a}\cdot\vec{v})(\gamma_v-1)\vec{v}}{v^2\gamma_v^3(1-\frac{\vec{v}\cdot\vec{u}}{c_0^2})^3} + \frac{(\vec{a}\cdot\vec{v})\vec{u}}{c_0^2\gamma_v^2(1-\frac{\vec{v}\cdot\vec{u}}{c_0^2})^3}$$

On the other hand, since the MCRF of the particle is moving with the relative velocity $\vec{u}=\frac{d\vec{r}}{dt}$ with respect to S, the $\vec{a}'$ in the case of $\vec{v}=\vec{u}$ is the acceleration observed at the MCRF of the particle.

Therefore, $\vec{\alpha}$ of Sec 4.23 is :

$$\vec{\alpha} = \vec{a}' \text{ (where } \vec{v}=\vec{u}) = \gamma_u^2\vec{a} - \frac{\gamma_u^3(\vec{a}\cdot\vec{u})(\gamma_u-1)\vec{u}}{u^2} + \frac{\gamma_u^4(\vec{a}\cdot\vec{u})\vec{u}}{c_0^2}$$

and $-\frac{\nabla\phi(\vec{x})}{\gamma}$ of Sec 4.23 is : $-\frac{\nabla\phi(\vec{x})}{\gamma} = -\frac{\nabla\phi(\vec{r})}{\gamma_u}$ (where $|\vec{u}|=u$)

Thus, the equation of Sec 4.23 becomes as follows :

$$-\frac{\nabla\phi(\vec{r})}{\gamma_u} = \gamma_u^2\vec{a} - \frac{\gamma_u^3(\vec{a}\cdot\vec{u})(\gamma_u-1)\vec{u}}{u^2} + \frac{\gamma_u^4(\vec{a}\cdot\vec{u})\vec{u}}{c_0^2}$$

Let $\vec{u} = \dot{r}(t)$, and $\vec{a} = \ddot{r}(t)$, then the equation of Sec 4.23 becomes :

$$-\frac{GM}{\gamma_u|\vec{r}(t)|^3}\vec{r}(t) = \gamma_u^2\ddot{r}(t) - \frac{\gamma_u^3(\ddot{r}(t) \bullet \dot{r}(t))(\gamma_u - 1)\dot{r}(t)}{|\dot{r}(t)|^2} + \frac{\gamma_u^4(\ddot{r}(t) \bullet \dot{r}(t))\dot{r}(t)}{c_0^2}$$

(where $\gamma_u = \frac{1}{\sqrt{1-\frac{|\dot{r}(t)|^2}{c_0^2}}}$, M be the mass of a point mass, G be gravitational constant, and $\dot{r}(t_0) = \vec{u_0}$, $\ddot{r}(t_0) = \vec{a_0}$ as initial conditions)

By solving this differential equation, $\vec{r}(t)$ can be found, and the equations in other pan-inertial frames can be obtained by Lorentz transformation.

4.3 The gravity-time equation of uniform circular motion

4.31 The equation of uniform circular motion

Let $\vec{r}(t) = (x, y, z)$ be the location of a particle and t be gravity-time, viewed from S the pan-inertial frame of a point mass.

Since it is a uniform circular motion, then the speed $|\vec{u}| = |\dot{r}(t)|$ is constant.

That is : $\frac{d}{dt}|\dot{r}(t)|^2 = 2(\dot{r}(t) \bullet \ddot{r}(t)) = 0$

Applying this to the gravity-time equation of motion of Sec 4.243,

it is : $-\frac{GM}{\gamma_u|\vec{r}(t)|^3}\vec{r}(t) = \gamma_u^2\ddot{r}(t)$

Now, since it is a uniform circular motion, it does not lose generality if it is assumed $\vec{r}(t) = r(\cos(wt), \sin(wt))$.

Applying this to the equation above, it is :

$$-\frac{GM}{\gamma_u r^3}\vec{r}(t) = -\gamma_u^2 w^2\vec{r}(t) \rightarrow r = \frac{GM}{u^2}(1-\frac{u^2}{c_0^2})^{\frac{3}{2}} \quad \text{(where } u = |\vec{u}| = |\dot{r}(t)|)$$

4.32 The size of a black hole and the speed of light

Now, let's calculate the size of the black hole and the speed of light in a gravitational field, approximated in Sec 3.4.

The speed of light in a gravitational field is : $C_G = \sqrt{c_0^2 - 2u^2}$

The boundary of a black hole is when $u^2 = \frac{c_0^2}{2}$ holds, so the size of a black hole is : $r = \frac{GM}{u^2}(1-\frac{u^2}{c_0^2})^{\frac{3}{2}} = \frac{GM}{\sqrt{2}\,c_0^2}$

This is $\frac{1}{2\sqrt{2}}$ times the Schwarzschild radius.

If $u = f(r)$ is obtained by solving the equation of uniform circular motion of $r = \frac{GM}{u^2}(1-\frac{u^2}{c^2})^{\frac{3}{2}}$, the speed of light in a gravitational field is :

$$C_G = \sqrt{c_0^2 - 2u^2} = c_0\sqrt{1-\frac{2u^2}{c_0^2}} = c_0\sqrt{1-\frac{2f(r)^2}{c_0^2}}$$

4.4 The free-fall motion : the inside of a Black hole

A star (point mass) of mass M becomes a black hole of radius $\frac{GM}{\sqrt{2}\,c_0^2}$.

Consider the case in which a particle is in free fall along an extension of the radius outside the star. When viewed from S the pan-inertial frame of the star with gravity-time t of S, the location of this particle can be expressed as a radius. That is, when the particle's location $\vec{x} = \vec{r}(t)$, velocity $\vec{u} = \dot{r}(t)$, and acceleration $\vec{a} = \ddot{r}(t)$ are assumed, the location, velocity, and acceleration can all be expressed as a radius vector.

So let $\vec{r}(t) = r\hat{r}$, $\vec{u} = -u\hat{r}$, $\vec{a}(t) = -a\hat{r}$, then $\frac{dr}{dt} = -u$, $\frac{du}{dt} = a$

(where $u = |\vec{u}|$, $\hat{r} = \dfrac{\vec{r}(t)}{|\vec{r}(t)|}$)

Applying this to the gravity-time equation of motion of Sec 4.243,

it is : $$-\frac{GM}{\gamma_u |\vec{r}(t)|^3} r(t) = -\gamma_u^3 a \quad \Rightarrow \quad -\frac{GM}{\gamma_u r^2} = -\gamma_u^3 a$$

$$\therefore\ -\frac{GM}{\gamma_u r^2} = -\gamma_u^3 a = \gamma_u^3 u \frac{du}{dr} \quad (\because\ a = \frac{du}{dt} = \frac{dr}{dt}\frac{du}{dr} = -u\frac{du}{dr})$$

$$\Leftrightarrow\ -\frac{GM}{r^2} dr = \frac{u}{(1-\frac{u^2}{c_0^2})^2} du \quad \Leftrightarrow \int -\frac{GM}{r^2} dr = \int \frac{u}{(1-\frac{u^2}{c_0^2})^2} du$$

$$\Leftrightarrow\ \frac{GM}{r} + C = \frac{c_0^2}{2(1-\frac{u^2}{c_0^2})}$$

$$\therefore\ u = c_0 \sqrt{1 - \frac{r c_0^2}{2(GM + rC)}}$$

(If the fall starts from r_0, then $u=0$ at r_0, so $C = \dfrac{c_0^2}{2} - \dfrac{GM}{r_0}$)

On the other hand, since $-\dfrac{GM}{\gamma_u r^2} = -\gamma_u^3 a$, then $a = \dfrac{c_0^4 GM}{4(GM + rC)^2}$

Since $GM + rC$ is a linear function of r and $0 \le r \le r_0$, then

$$GM + rC \ge Min\,(GM, \frac{r_0 c_0^2}{2}) = \rho$$

Therefore, $$a = \frac{c_0^4 GM}{4(GM + rC)^2} \le \frac{c_0^4 GM}{4\rho^2}$$

In conclusion, the acceleration (or force) of a free-falling particle is finite rather than infinite even inside a black hole.

5. A gravitational field and an accelerated reference frame

5.1 A free-falling elevator

For convenience of explanation, consider a flat gravitational field where the magnitude of the gravitational force (per unit mass, or acceleration) is constant. According to the general theory of relativity, this kind of gravitational field is equivalent to an accelerated reference frame, and the inside of an elevator freely falling is equivalent to an inertial frame.

In an inertial frame, an object on which no force acts is either at rest or to move with a constant velocity. Viewed from inside the elevator, the object must be at rest or to move with a constant velocity. Now, let's compare object A which is stationary, and object B which is moving at a constant velocity in the direction perpendicular to the direction of gravity, when viewed from inside the elevator. Since the inside of the elevator is an inertial system, B must continue to move in the direction perpendicular to the direction of gravity, and the heights of the two objects measured from the elevator floor must remain constant. This is the conclusion of the general theory of relativity.

Let us now examine this case with the theory of gravity-time . According to the theory of gravity-time, stationary objects appear to remain stationary when viewed from inside the elevator. However, the case will be different when one object moves as described above.

If the object B moves as described above, its relative speed (viewed outside) to the star (the source of gravity) is greater than that of the object A. So according to the gravity-time equation of motion, the magnitude of the gravitational force acting on B becomes smaller, and the velocity in the direction of gravity also changes.

Therefore, the object B also decreases its speed (viewed outside) in the direction of gravity, and moves in the direction of the elevator's ceiling when compared to the object A. This is a different conclusion from the general theory of relativity.

In conclusion, according to the theory of gravity-time, the gravitational field has different properties from an accelerated reference frame, and the gravitational field is not reduced to an accelerated reference frame.

5.2 Is gravity-time well defined?

We examined earlier in Sec 2.4 that if a gravitational field is equivalent to an accelerated reference frame, ‘constancy of the speed of gravity’ and ‘gravity independency’ are incompatible. However, the theory of gravity-time derived from these two principles shows that a gravitational field is not the same as an accelerated reference frame.

A gravitational field has a limit on the speed of transmission of gravity by the principle of constancy of the speed of gravity. In contrast, in an accelerated reference frame, the same acceleration is applied at all points at the same time regardless of the state of motion, so it is similar to the transmission speed of the acceleration being infinite.

From these differences, a gravitational field and an accelerated reference frame are clearly distinguished, and consequently gravity-time is well defined.

6. Prediction

6.1 Gravitational wave is faster than light in a gravitational field

We examined earlier in Sec 3.4 that light slows down in a gravitational field. Hence, gravitational wave, an instance of gravity's propagating, is faster than light in a gravitational field.

6.2 Gravity works weakly on a moving object compared to a stationary one

The gravity-time equation of motion of Sec 4.23 shows that gravity works weakly on a moving object compared to a stationary one. Theoretically, if you move in a free-falling elevator, you will go up ‘a little’(=little) to the ceiling of the elevator.

7. Conclusion : About Time

Since time is a tool for measuring change, the measurement of time is closely related to the reasons that cause change.

Something, of which propagating the same speed as light in an inertial frame, is divided into two cases in a gravitational field. One is not affected by the gravitational field at all, and the other is affected by the gravitational field.

What is not affected by the gravitational field is gravity, and what is affected by the gravitational field is light. (In Sec 3.4, the fact that light travels diagonally assumes that light slows down in the gravitational field) If something other than light is affected by a gravitational field, it is also measured at the same speed as light in the gravitational field. (In Sec 3.4, we just assume 'light travels diagonally', so we can assume 'something travels diagonally') This shows that there are two possible units of measuring time in the universe.

The forces that can cause acceleration in space are gravity and electromagnetic force. Since force is the essence of change, it is probably closely related to time. The following is just an interpretation or speculation, but it can be thought that the forces that can cause acceleration in the universe are gravity and electromagnetic force, and the time corresponding to the change in gravity and in electromagnetic force is gravity-time and light-time respectively.

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