Inhomogeneous distribution of the universe's matter density as a physical basis for MOND's acceleration a_0, cosmological redshift and expansion

Antonio Francino[1], Gabriel Simón[2]

Abstract

One of the most effective theories for dark matter is Milgrom's Modified Newtonian Dynamics, where a modified law of gravity based on a fixed acceleration scale a_0 is postulated that provides a correct description of the gravitational fields in galaxies. However, the significance of a_0 is unknown, and the whole theory is generally viewed as a phenomenological description of the observations. Based on Newton's gravitational law as applied to a uniform continuous mass we posit a non-homogeneous distribution of mass at cosmological scales that would give rise to a constant acceleration that agrees with MOND's a_0. The implications for MOND as a viable theory of dark matter and for the problem of dark energy are discussed. In particular, relativistic high rotational velocities would be achieved at the border regions of the universe that would generate a transverse Doppler redshift that scales linearly with distance and migh provide an alternative explanation for the observed redshifts and expansion.

[1] Corresponding author francino@clinic.cat

[1] Department of Cardiology, Hospital Clinic Barcelona.

[2] Instituto de Oftalmología Gabriel Simón, Barcelona & Madrid, Spain.

[2] Weldon Engineering School, Purdue University, USA.

Both authors contributed equally to this work

Modified Newtonian Dymanics (MOND) is a Newtonian-derived hypothetical model of gravity proposed 40 years ago by Mordehai Milgrom to explain the multiple gravitational anomalies observed in galaxies and galaxy clusters [1-3]. They are summarized and conventionally explained through the existence Dark Matter, an elusive new form of matter that interacts only gravitationally and is not included in the Standard Model of Particle Physics. While no such particles have yet been found, the search goes on and MOND usually plays a secondary role in the list of candidate explanations for dark matter. One of the reasons is that a_0, the distinctive feature of MOND, does not correspond to any physical entity, and –it is argued- was postulated solely as a means to obtain a gravitational law that fits the observations. It is sometimes called a phenomenological explanation.

While a_0 agrees to within one order of magnitude with the acceleration calculated at the border regions of the observable universe from the simple Newtonian gravitational formula and is also found to relate to Hubble's constant and to the square root of the cosmological constant L, in both cases scaled by the speed of light *c*, no physical representation of such an acceleration has been devised, and most physicists would agree that it represents another constant of nature, whose role would be to relate fundamental gravitational phenomena in the low-acceleration regime, implying probably some modification of the laws of gravity.

The Newtonian ball model of gravity

A generally accepted assumption of all current astrophysical models is the Cosmological Principle, the idea that the universe at large scales is both homogeneous and isotropic. While it may still be isotropic and strong constraints have been set on the range of variation in matter density, the homogeneity condition has little theoretical supporting evidence. Based on original ideas of Isaac Newton, we shall argue that the universe can be modelled as a nearly homogeneous continuous distribution of mass that obeys simple dynamics embodied in the Universal Law of Gravitation. As Newton found in the late 1600s [4], when a continuous distribution of mass with constant density is allowed to evolve according to such law, an acceleration appears that is null at the center and increases outwards in linear proportion to radial distance until it reaches, for a distance equal to the radius of the ball, the exact same value as predicted by conventional Newtonian gravity.

$$F_B = G\ M\ m\ r\ /\ R^3$$

as opposed to a point-mass gravitational field:

$$F_N = G\ M\ m\ /\ R^2$$

where F_B (the force in the Newtonian ball model) and F_N (Newton's conventional point-mass gravitational force) are the force on a test particle with mass m placed at a distance r from the center of the R-ball, or at a distance R from the central point-mass M, respectively. The acceleration for the ball with mass M is then

$$Acc_B = G\, M\, r / R^3 \qquad (1)$$

and solving for G

$$G = Acc_B\, R^3 / M\, r \qquad (2)$$

We now define G' as $4\pi G$ and substitute it for G above. The resulting expression is mathematically equivalent, though it may facilitate the visualization of upcoming considerations.

$$G' = (Acc_B\, 4\pi\, R^3) / (M\, r) \qquad [G' := 4\pi G] \qquad (3)$$

Multiplying both parts of the right-hand quotient by a factor of three,

$$G' = 3\, Acc_B\, 4/3\, \pi\, R^3 / M\, r \qquad (4)$$

and since $4/3\, \pi\, R^3 / M$ is the inverse of the matter density for the spherical volume,

$$G' = 3\, (Acc_B / r) \cdot (1/\rho)$$

$$G' = 3\, Acc_B / (r \cdot \rho) \qquad (5)$$

where ρ is now the average, not necessarily constant matter density at radial distance r. It is well known that the Newtonian model for gravity in solid spheres is valid not only for spheres with uniform density, but for any sphere in which density depends only on radial distance, i.e, for any spherically symmetric distribution of matter.

Looking at equation (5) we see that in such a ball model of the universe, if ρ is constant, then the quotient (Acc_B / r) must be constant, which agrees with the Newtonian view but does not help us understand the existence of a constant acceleration pervading the whole universe that at the same time agrees with the Newtonian acceleration at its border regions, as MOND postulates and available evidence from observed galaxies suggests.

We therefore let ρ vary with radial distance and assume that it is the product in the denominator of equation 5 $(r \cdot \rho)$ that is constant. In other words, we let density decay inversely with radial distance. We immediately see that since both G' and the product $(r{\cdot}\rho)$ are constant, so must be Acc_B, and this acceleration agrees with MOND's universal acceleration a_0 and with the calculated Newtonian acceleration at the border regions of the ball to whithin one order of magnitude, as can be easily checked. Indeed, feeding in the accepted values for the mass of the observable universe (10^{53} Kg), radial distance (10^{26} m) and G, it turns out that the acceleration perceived at the border regions of the observable universe is about $3.4 \cdot 10^{-10}\ m{\cdot}s^{-2}$, quite close to the reported value for a_0 ($1.2\cdot10^{-10}$). According to the Newtonian ball model and assuming $r\cdot\rho$ constant, this same aceleration would be present as a background curvature in the whole universe, explainining its local influence in all galaxies, not just as a constant of nature, but as a real acceleration that would determine the observed accelerations through a geometrical averaging with the local, Newtonian-derived acceleration.

The range of variation in mass density that would be expected depends on how far we are from the central region of the universe, and can be approximately estimated.

From Eq (1), taking $Acc_B = a_0 = 1.2\cdot10^{-10}$ ms^{-2}; $R_U = 4.4\cdot10^{26}$ m; $G' = 8.38\cdot10^{-10}$ $m^3\cdot Kg^{-1}\cdot s^{-2}$, we have

$r\cdot\rho = 0.4295$ $Kg\cdot m^{-2}$

$\rho = 0.4295 / r$

Assuming we are in a mid-radius region, $R_0 = 2.2\cdot10^{26}$ m and making dr = 1 Mpc = $3.1\cdot10^{22}$ m, it turns out that the expected decrease in density per Mpc at a radius half the universe's radius would be:

$d\rho = -0.4295 \cdot R_0^{-2} \cdot dr$

$d\rho = 6.29 \cdot 10^{-30}$ $Kg/m^3/Mpc$

This is aproximately 1% of the accepted baryonic mass density of the universe (4.6% of the critical density 10^{-26} Kg/m^3, or $4.6\cdot10^{-28}$ Kg/m^3). For regions closer to the center, the predicted relative variations are larger. In more external regions they would become much smaller and practically unmeasurable.

Observational evidence for the distribution of mass density in the universe is scant. The large-scale average density of the universe, known as the cosmic density parameter, Ω, depends on its composition and, according to the ΛCDM model, is very close to the critical mass density Ω_c, the one required to make the universe flat. The density of matter, including dark matter would amount to about 28% of the global density ($\Omega_m = 0.28$), while the density of baryonic matter is though to comprise a bare 4.6% of the total density. Distribution of average density as a function of distance is generally assumed to follow the general trend of decreasing as the radius increases, reflecting the overall dilution of matter on larger scales, but observations are dominated by a complex hierarchical structure, the so-called cosmic web, that hinders a precise estimation. As a result, no reliable data are currently available.

Several authors [5 - 9] notably Peebles, Karachentsev, Nuza and others have probed into the mass distribution in the vicinity of our Milky Way and found that, on average, its density is significantly lower than the average for the whole universe. We would thus be in a local region of low density, the Local Void, which makes the observations not representative of the whole. The interpretation of the results is also compounded by the influence of dark matter and structure formation, two processes of which we know little.
In two important studies [5, 6] the authors examined the distribution of the mean density of matter in spheres of various radii in our Local Universe and found that matter density up to about 50

Mpc decays with distance. The authors conclude that density is on average lower than the global density for the universe ($\Omega_{m,local}$ = 0.08 vs Ω_m = 0.28) and tends to an asymptotic minimum value. However, looking at the data in the figures, we speculate that they might also be consistent with a 1/r decay in that range. However, as the authors point out, larger scale distances are needed to avoid local variations, probably 100 Mpc at least. In the papers, uncertainties in the range up to 90 Mpc seem too large to draw a conclusion.
Also, it was shown that a reliable measurement of the variations in baryonic-mass density around the Milky Way could be used to gauge our proximity to the center of the universe.

Another interesting observation is the striking ressemblance of equation (5) with the Friedmann equation. The Friedman equation can be expressed as [10]

$$a''/a = - 4/3 \cdot \pi \cdot G\ (\rho + 3p/c^2) + \Lambda c^2/3 \qquad (6)$$

And making a customary simplification that consists of replacing

$\rho \longrightarrow \rho - \Lambda c^2/8\pi G$

$p \longrightarrow p + \Lambda c^4/8\pi G$

we have

$$H^2 = (a'/a)^2 = 8/3 \cdot \pi G \rho - kc^2/a^2 \qquad (7)$$

Assuming flat space (k=0) and substituting G' for G (G' = 4πG) results in

$$G' = 4\pi G = (3/2) \cdot H^2 / \rho \qquad (8)$$

which reminds us of Eq 5:

$$G' = 3 \cdot Acc_B / (r \cdot \rho)$$

In the last expression, since dimensions of Accel / r are $1/T^2$, we have

$$G' = 3 \cdot (1 / t)^2 \cdot (1 / \rho) \qquad (9)$$

If we now interpret 1/t as the constant rate of expansion H, it turns out that Eq (1) can be viewed as equivalent to

$$G' = 3 \cdot H^2 / \rho \qquad (10)$$

which differs from the Friedman equation by a factor of 2. The reason for the discrepancy we ignore, but it has happened before in astrophysics that a classical, non-relativistic approach has been later superseded by the appropriate relativistic version that differs from it by a factor of two, e.g., in the old pre-Einstein estimation of the lensing of light from Newtonian gravity by Johann Soldner in 1804.

Thus, the hypothesis of a decreasing matter density that scales inversely with distance seems a reasonable one and, from Newtonian mechanics, this would lead to a constant background cosmic acceleration that agrees with MOND's a_0 and would account for the rotation curves in galaxies. The observed accelerations around galaxies below a_0 would then be the average of the Newtonian and the background a_0. This would now be understood as a real physical phenomenon related to the interaction of two competing accelerations, not only a mathematical construct.

We cannot discuss here the other predictions of MOND related to dark matter. We would rather refer the reader to the works of the original author [1-3]. We should point out however that the discrepancies of MOND with the observations in galaxy clusters might be addressed by an averaging of the gravitational fields due to the cluster itself and to nearby galaxies [11]. Despite our previous reports, we cannot presently trust any of the proposed explanations for the accelerations observed in colliding clusters like the Bullet. As for the CMB, we have argued elsewhere [11] that our understanding of the CMB has some problems that limit its ability to be used as the gold standard to adjudicate prospective fundamental theories. We'd like to draw attention to one of these problems, namely the strong and regular anisotropy observed in the CMB, the so-called CMB dipole, that is generally disregared as originated from the movement of our galaxy with respect to the CMB, out of the Hubble flow. Such anisotropy has been measured as equivalent to 380 Km/s and is generally erased by subtracting this value from the original images. Recent data indicate though that the velocity might be about 600 Km/s [12], much higher than expected, which makes it difficult to explain and might call for a re-evaluation of the anisotropy problem.

We therefore conclude that

1. In a modified Newtonian ball model of the universe, a continuously decreasing matter density that scales as 1/r, as opposed to the uniform distribution from the Cosmological Principle, would give rise to a constant universal physical acceleration that agrees with MOND's a_0.

2. This would provide a physical basis for MOND and support it as a viable interpretation of the dark matter problem.

3. The resulting matter-density distribution may be hard to verify experimentally, for the densities involved, as well as the variations incurred are very low. A variation in mass density around 1% per Mpc is expected.

Cosmological acceleration and redshift in relation to the universe's expansion

We now turn our attention to the mysterious empirical relation observed between a_0 and the parameters that reflect the universe's expansion, H_0 and Λ.

Indeed, the numerical value of MOND's a_0 has been found to be approximately

$$a_0 \sim (c / 2\pi) \cdot H_0 \sim (c^2 / 2\pi) \cdot \mathrm{SQRT}(\Lambda/3)$$

Why is that? What is the intimate relation of a_0 to the accelerated expansion of the universe?

Motivated by hte previous considerations and the various inconsistencies in the current cosmological models, namely the discrepancies in the measurements of the rate of expansion -the so-called Hubble tension-, the existence of galaxies much older than allowed by our working ideas on galaxy formation, and the failure to identify the origin of the accelerated expansion, an alternative explanation is sought for the original observations that led to the idea of an expanding universe. According to the extensively confirmed Hubble Law ($v = H_0D$), a redshift is observed for stars and galaxies that is linearly related to distance and suggests recessional velocities that would cause light to redshift through a Doppler mechanism.

Despite its evident internal logic and agreement with multiple observations, we shall make here no assumptions on homogeneity, isotropy, nor expansion. Our arguments will be checked against the basic observational facts. Namely, the Hubble Law, the approximate isotropy observed in the universe, and the existence of a pervasive background low-energy radiation in the form of the CMB. Ideally, the model should also provide an explanation for the accelerated expansion in recent epochs, as described by Riess, Perlmutter and Schmidt in 1998, as well as for the anisotropy observed in the CMB, its dipole.

In the ball model of the universe, a 1/R matter density distribution leads to a constant background acceleration a_0, sometimes called cosmological acceleration a_L, that has been measured at $1.2 \cdot 10^{-10}$ ms^{-2} [1-3]. Centripetal acceleration as a function or radial distance is then

$$v^2/r = a_0 = \text{constant} \qquad (11)$$

$$v = \mathrm{SQRT}(a_0 \cdot r) \qquad (12)$$

so that velocities increase as the square root of distance.

For the estimated radius of the observable universe ($R = 4.4 \cdot 10^{26}$ m) we have that in a non-relativistic approximation, rotational velocities in the external shells would be approximately

$$v = SQRT(1.2\cdot 10^{-10} * 4.4\cdot 10^{26})\ m/s \sim 2.29\cdot 10^{8}\ m/s \sim c \qquad (13)$$

If we now take the transverse Doppler redshift that would be observed from light emitted by those galaxies at the edge of the observable universe [Wikipedia article 'Redshift']

$$1 + z = 1 / SQRT(1 - (v_T^2/c^2)) \qquad (14)$$

where v_T is the velocity in the direction perpendicular to light trajectory. Redshift goes to infinity as v_T approaches c.

For shorter distances and lower velocities ($v << c$), the approximate formula is

$$z \sim 1/2\ (v_T/c)^2 \qquad (15)$$

Thus, for sub-relativistic velocities and shorter distances, since velocity increases as the square root of distance and redshift scales as the square of velocity, redshift would increase linearly with distance. Distance here is measured from the emitting galaxy to the center of the universe.

Light coming from distant galaxies, whether at the edge of the universer or closer to us, must then overcome the gravitational potential between its source point and us and, by so doing, it is subject to gravitational blueshift, which amounts approximately

$$z = \Delta U / c^2 \qquad (16)$$

For a constant acceleration a_0, the difference in gravitational potential at distance r is:

$$\Delta U = a_0 \cdot r \qquad (17)$$

Where ΔU is the difference in gravitational potential berween the galaxy and us and r is the distance between both. Redshift is then given by

$$z = a_0 \cdot r / c^2 \qquad (18)$$

This formula does not require correction from general relativity, since it is derived from the equivalence principle [Wikipedia, article 'Gravitational redshift']. We notice that the expressions for transverse-Doppler and gravitational shifts both have c^2 in the denominator and the product ($a_0 \cdot r$) in their numerator (Eqs 12, 15, 18). In the former, distance is computed from the emitting galaxy to the center of the universe; in the second, it is distance from us. Both are of the same order but, unless we are at the very center of the universe, the former is much larger than the latter.

Thus, galaxies in this ball model of the universe emit light that reaches us with a net redshift that is the difference of (14 or 15) and (18).

For sub-relativistic speeds and distances, both (15) and (18) scale linearly with radial distance. Their difference would also scale with distance, with a constant that is the diference of the respective constants. At very large distances, the transverse Doppler redshift as described by the proper relativistic formula (14) overshoots and increases exponentially. The exact calculation of shift is complex and cannot be provided here, but a net redshift that increases linearly with distance and accelerates exponentially for very large distances would mimick an expanding universe and, for the farthest galaxies, an accelerated expansion.

The previous discussion focused on the observation of redshift from galaxies lying in the peripheral regions of the universe. Consistency with the isotropy condition requires that we examine what would be observed when looking in the other direction, towards the central regions. A series of complex phenomena combine to offer a picture that looks globally similar but includes new intervening elements.

First, in the intermediate zone between us and the center, matter density and the number of stars should be larger, and more light should be recorded. In this region, a gravitational redshift that scales with distance from us is expected to predominate. Rotational velocities and transverse redshift would quickly decrease and become negligible. Moreover, as the orbits of galaxies in this region are no longer perpendicular to us, transverse and longitudinal redshift would be weak and alternating between each other. As a result, only gravitational redshift would be consistently recorded.

Second, in the region between the center and the external shell away from us (on the other side from the center), transverse and gravitational redshift would approximately balance each other, causing a relative decrease in the measured redshift which would likely cause a distorted estimation of distances. Furthermore, light emitted from these regions would be eventually redshifted from gravity in the last part of its journey to us, approaching from the center.

Lastly, galaxies in extreme distant regions on the other side of the universe would emit strongly redshifted light from high relativistic rotational velocites and the consequent transverse Doppler redshift would be orders of magnitude higher than gravitational blueshift. It might thus generate -as was the case on this side of the universe- a distinctive low-energy diffuse radiation that might mimick the observations from the CMB.

The global picture would be one of near-isotropy, with both sides of the universe away from us emitting predominantly redshifted light. However, since the phenomena implicated are different, the resulting redshift should be of different magnitude. This might explain the anisotropy of the CMB dipole, that is currenly attributed to the proper motion of our galaxy with respect to the Hubble flow.

In summary we may conclude that

1. A constant cosmological acceleration in line with MOND's a_0 is consistent with a ball model of gravity in the universe and would generate rotational velocities that increa-

se as the square root of radial distance from the center, reaching relativistic speeds at the border regions.

2. Such velocities would generate a transverse Doppler redshift that -when subtracted from gravitational blueshift- scales with radial distance and might mimick recessional velocities and expansion.

3. The apparent isotropy of the universe can be explained from the this redshift, which would be observed from all directions.

4. Whether looking into the outward, peripheral regions of the universe or towards its center and beyond, strongly redshifted light coming from the external shell is expected to predominate and outperform the redshift from the intermediate regions, possibly giving rise to images similar to those observed in the CMB.

Discussion

Several authors, most notably Lombriser, Buchert, Roukema *et a*l [13-15] have proposed that the expansion of the universe might be an apparent phenomenon caused by distortions in the gravitational fileds at cosmic scales, but the models are not complete, are difficult to test and, in some cases, they include radical unobserved features like a variation in the mass of particles.

On the other hand, while other possibilities have been considered for an interpretation of redshift other than recessional velocities, such as the 'tired light' hypothesis, none has gained traction, mainly due to the fact that they contradict our most fundamental theories, like Special Relativity.

The present semi-quantitative model is based on the assumption of a matter density that decreases inversely with distance, a reasonable hypothesis that is supported by MOND and the observed rotational velocities in galaxies. The assumption could soon be tested by the JWST and other observatories. It offers a picture of a static, rotating universe that would generate the phenomena of redshift and background low-energy radiation that we observe today and constitute the backbone of modern cosmology. Unfortunately, the universe could essentially no longer be expanding nor the Big Bang could take place 13 billion years ago. On the plus side, the matter-density composition of the universe might be understood without gaps, and the law of conservation of energy would no longer be violated at cosmic scales. The universe might turn out to be much older and stable than previously though and, though static, it would offer an ample range of exciting features to work into and speculate.
And yet, extreme caution is advised when contemplating these hypotheses. Our current models are self-consistent and offer a complete picture of the events up to the first nanoseconds of the origin. Even if countered by a few important but minor discrepancies, our current cosmological models work. The present ideas are offered just as an alternative motivated by reasonable arguments, many of them due to other authors and predecessors. They might be worth being looked into and examined and, even if they end up being ruled out from disagreement with observations, the task ahead remains unchanged, which consists of uncompromisingly seeking truth and bettering our understanding.

Acknowledgements

We acknowledge the constructive criticism from various physicists who have read the manuscript and preferred to remain anonymous. Their observations helped us reshape and restate many of our ideas.

Our brother (AF) and friend (GS) Josep M Francino has offered unwavering help in the typesetting, graphics and simulations.

To Mordehai (Moti) Milgrom, for inventing MOND and letting us envision some of its weirdest consequences.

References:

1. Milgrom, M. 1983, ApJ, 270, 365

2. Milgrom, M. 2009, ApJ, 698(2), 1630

3. Milgrom, M. 2014, Scholarpedia, 9(6):31410. http://www.scholarpedia.org/article/The_MOND_paradigm_of_modified_dynamics

4. Isaac Newton, 1687. Principia Mathematica Philosophiae Naturalis. Book 1, Propsition LXXII, Theorem XXXII, and Proposition LXXIII, Theorem XXXIII.

5. Nuza SE, Kitaura F-S, Hess S et al (2014). The cosmic web of the Local Universe: cosmic variance, matter content and its relation to galaxy morphology. ArXiv 1406.1004v2 [astro-ph.CO]. See especially Figure 3.

6. Karachentsev I D (2012). Missing Dark Matter in the Local Universe. ArXiv 1204.3377v1 [astro-ph.CO]. See especially Figs 3 and 4.

7. Hess S, Kitaura F-S, Gottlöber S (2013) MNRAS.

8. Peebles P J E, Principles of Physical Cosmology. Princeton University Press, 1993.

9. Peebles P J E, Nusser A (2010). Nearby galaxies as pointers to a better theory of cosmic evolution. Nature 465, 565-569.

10. Wikipedia, article *Friedmann equations*.

11. Francino A, Simón G (2022) Fluid Spacetime a MOND-Based Alternative Perspective on The Problems of Dark Matter Dark Energy and Black Holes. OSP Journal of Physics and Astronomy 3: JPA-3-13.

12. *CMB Dipole: Speeding through the Universe*, NASA Science Mission Directorate, Apr 3, 2022. https://science.nasa.gov/cmb-dipole-speeding-through-universe.

13. Lombriser, Lucas 2023. Cosmology in Minkowski Space. *Class Quantum Grav*, 40 155055.

14. Buchert, Thomas 2008. Dark energy from structure. *General relativity and Gravitation* 40; 467-527

15. Roukema, Boudewijn F; Ostrowski, Jan J; Buchert, Thomas 2013: Virialisation-induced curvature as a physical explanation for dark energy. ArXiv:1303.4444v3 [astro-ph.CO].