

# Gravitational energy attributed to curved space

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**Abstract:** Gravitational energy is localized. It is shown that the massless gravitational energy is enclosed in space, which is curved by matter, according to Einstein's theory. This solves the problem of the unambiguous localization of gravitational energy, a problem that Einstein left to the mercy of fate. Depending on the nature of the curvature of space, this energy can be positive (gravitational waves) or negative (Schwarzschild space). The density of the space's energy is obtained. The emission of massless gravitational waves reduces the inertial mass of the emitter. In the Schwarzschild case, the receipt of the gravitational energy from the space by matter increases the inertial mass of the matter. Since the gravitational mass of an isolated system is determined by the curvature of space at infinity, the inertial mass of an isolated system exceeds its gravitational mass by the value of the gravitational energy of space.

**Key words:** binding energy; mass defect; energy localization

## 1. Introduction

According to the definition, the *gravitational binding energy* of a system is the minimum energy that must be added to it in order for the system to cease to be in a gravitationally bound state. As is known [1,2]<sup>2</sup>, the gravitational binding energy of a ball of (inertial) mass  $M$  and radius  $r_1$  is equal to

$$U = 3\gamma M^2 / 5r_1, \quad \gamma = 7.4 \cdot 10^{-28} \text{ m/kg}. \quad (1)$$

(the radius is defined in terms of the length of the equator:  $r_1 = l / 2\pi$ ). This means that the substance of the ball can be removed to infinity during its explosion if the mass-energy of the explosive is  $3\gamma M^2 / 5r_1$ . During the explosion, this energy is first converted into kinetic mass-energy, and then disappears in the process of expansion as the speed decreases. Thus, the inertial mass of the ball decreases up to a constant  $m$ ,

$$m = M - 3\gamma M^2 / 5r_1, \quad (2)$$

which defines the invariable outer Schwarzschild space of the ball that is described by the line element [3 (100.14)]

$$dl^2 = g_{rr} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad g_{rr}(r) = \frac{1}{1 - 2\gamma m / r}. \quad (3)$$

The constant  $m$  is the *gravitational mass* of the ball. It can be expressed in terms of the initial density of the ball substance according to the formula [3 (100.24)]

$$m = 4\pi r_1^3 \rho / 3. \quad (4)$$

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<sup>2</sup> See also [https://en.wikipedia.org/wiki/Gravitational\\_binding\\_energy](https://en.wikipedia.org/wiki/Gravitational_binding_energy)

## 2. Mass of the ball

Formula (4) shows that using  $4\pi r_1^3/3$  as the volume of the ball does not give the correct value  $M$  for the mass of the ball. This is because, in reality, the volume of the sphere with the equatorial length  $2\pi r_1$  is greater than  $4\pi r_1^3/3$  due to the curvature of space.

Inside the ball, the metric coefficient  $g_{rr}(r)$  is [3 (100.19)]

$$g_{rr} = \frac{1}{1-r^2/R^2} > 1, \quad R^2 = \frac{3}{8\pi\gamma\rho}. \quad (5)$$

The correct mass of the ball was calculated in [4], taking this circumstance into account:

$$M = \int_0^{r_1} \rho \sqrt{g_{rr}} 4\pi r^2 dr = \frac{3}{2\gamma R} \int_0^{r_1} \frac{r^2 dr}{\sqrt{R^2 - r^2}} = \frac{3R}{4\gamma} \left( \sin^{-1} \xi - \xi \sqrt{1 - \xi^2} \right), \quad \xi = \frac{r_1}{R}. \quad (6)$$

If the mass of the ball is small, that is  $M \approx m$ ,  $\xi^2 = 8\pi\gamma\rho r_1^2/3 = 2\gamma m/r_1 = r_g/r_1 \ll 1$ , then

$$\left( \sin^{-1} \xi - \xi \sqrt{1 - \xi^2} \right) \approx \left( \frac{2}{3} \xi^3 + \frac{1}{5} \xi^5 \right), \quad \text{and} \quad M \approx m + \frac{3\gamma m^2}{5r_1}, \quad (7)$$

according to (1) and (2). (We denoted here the gravitational radius  $r_g = 2\gamma m$ ). If the ball is compressed to its gravitational radius,  $\xi = 1$ , then  $M = 3\pi m/4 \approx 2.36m$ . For a ball of arbitrary radius, the binding mass-energy is

$$U = M - m = \frac{3R}{4\gamma} \left( \sin^{-1} \xi - \xi \sqrt{1 - \xi^2} \right) - m = \frac{3m}{2\gamma\xi^3} \left( \sin^{-1} \xi - \xi \sqrt{1 - \xi^2} \right) - m. \quad (8)$$

## 3. Mass of the shell

The gravitational binding energy cannot be considered as the energy of the ‘‘gravitational field’’, because within the framework of the general relativity, the gravitational field does not exist. Weyl writes: ‘‘Gravity is ‘‘leading’’ (Führung) and not a force’’ [5]. All gravitational phenomena are explained by the curvature of space-time. We consider the binding energy as the massless energy of space, which is curved by matter, according to the Einstein equation. To determine the energy density of curved space, we calculate the mass  $M$  of a spherical shell of small thickness  $\Delta r$  and the gravitational mass  $m$ . Let the density  $\rho$  be nonzero only between the coordinates  $r_0$  and  $r_1$ ,  $\Delta r = r_1 - r_0$ . Then, according to the formula [3 (100.19)], the metric coefficient in the space between these coordinates is

$$g_{rr}(r) = \frac{1}{1 - 8\pi\gamma\rho(r^3 - r_0^3)/3r} = \frac{1}{1 - 8\pi\gamma\rho(r - r_0)(r^2 + rr_0 + r_0^2)/3r}. \quad (9)$$

Considering  $r - r_0 \ll r$ , we have the mass of the shell:

$$M = \int_{r_0}^{r_1} 4\pi r^2 \rho \sqrt{g_{rr}} dr = \int_{r_0}^{r_1} \frac{4\pi r_1^2 \rho dr}{\sqrt{1 - 8\pi\gamma\rho r_1(r - r_0)}}. \quad (10)$$

Using  $\int \frac{dx}{\sqrt{a+bx}} = \frac{2}{b} \sqrt{a+bx}$  and  $m = 4\pi r_1^2 \rho \Delta r$ , in accordance with (4) and with [3 (100.24)];

replacing  $r_1 \rightarrow r$ , we get the mass of the shell

$$M = r(1 - \sqrt{1 - 8\pi\gamma\rho r \Delta r})/\gamma = r(1 - \sqrt{1 - 2\gamma m/r})/\gamma. \quad (11)$$

#### 4. Density of the gravitational energy

Now, using (11), one can find the gravitational energy density  $u$  [kg/m<sup>3</sup>] contained in the curved space. It should be taken into account that outside of the shell there is a curved Schwarzschild space and inside of the shell the space is Euclidean. So, when the radius of the shell decreases by  $dr$ , a new volume of space with the value  $dV_0 = 4\pi r^2 \sqrt{g_{rr}} dr$  is curved and, in this case, gravitational energy is released:  $dM = u4\pi r^2 \sqrt{g_{rr}} dr$  where  $g_{rr}$  is from (3). So,

$$u = \frac{dM}{dr} \frac{1}{4\pi r^2 \sqrt{g_{rr}}} = \frac{1}{4\pi r^2 \gamma} \left( \sqrt{1 - \frac{2\gamma m}{r}} - 1 + \frac{\gamma m}{r} \right) = \frac{1}{4\pi r^2 \gamma} \left( \frac{1}{\sqrt{g_{rr}}} - \frac{1}{2} - \frac{1}{2g_{rr}} \right) < 0. \quad (12)$$

#### 5. Binding energy of the ball

Using (12), one can calculate the binding energy of the ball (8) by integrating this space energy density:

$$U = -\int_0^\infty u4\pi r^2 \sqrt{g_{rr}} dr = M - m. \quad (13)$$

Only the integral must be divided into two parts, because the metric coefficients  $g_{rr}$  inside the ball and outside the ball are different and equal, respectively

$$g_{rr} = \frac{1}{1 - r^2/R^2}, \quad g_{rr} = \frac{1}{1 - 2\gamma m/r}. \quad (14)$$

Using (12), we test formula (8):

$$U = U_{\text{in}} + U_{\text{ext}} = -\int_0^r u4\pi r^2 \sqrt{g_{rr}} dr - \int_r^\infty u4\pi r^2 \sqrt{g_{rr}} dr = \frac{3R}{4\gamma} \left( \sin^{-1} \xi - \xi \sqrt{1 - \xi^2} \right) - m = M - m. \quad (15)$$

Indeed, the energy coming from the interior space of the ball is equal to

$$U_{\text{in}} = -\frac{1}{\gamma} \int_0^r \left( 1 - \frac{1}{2\sqrt{1 - r^2/R^2}} - \frac{1}{2} \sqrt{1 - r^2/R^2} \right) dr = -\frac{r}{\gamma} + \frac{3R}{4\gamma} \sin^{-1} \frac{r}{R} + \frac{r}{4\gamma} \sqrt{1 - r^2/R^2}. \quad (16)$$

The energy coming from the outer part of the space is equal to

$$U_{\text{ext}} = -\frac{1}{\gamma} \int_r^\infty \left( 1 - \frac{1}{2\sqrt{1 - 2\gamma m/r}} - \frac{1}{2} \sqrt{1 - \frac{2\gamma m}{r}} \right) dr = \frac{r}{\gamma} - \frac{r}{\gamma} \sqrt{1 - \frac{2\gamma m}{r}} - m = \frac{r}{\gamma} - \frac{r}{\gamma} \sqrt{1 - \xi^2} - m. \quad (17)$$

Summing up (16) and (17), we obtain (15) or (8), which was to be shown.

#### 6. Случай слабой гравитации

Expression (12) for the density of the gravitational energy of a curved space takes a simple form in the case of a weak curvature of space,  $\frac{2\gamma m}{r} \ll 1$ :

$$u = \frac{1}{4\pi r^2 \gamma} \left( \sqrt{1 - \frac{2\gamma m}{r}} - 1 + \frac{\gamma m}{r} \right) \approx -\frac{\gamma m^2}{8\pi r^4}. \quad (18)$$

Remembering the magnitude of the acceleration of free fall according to Newton's law,  $g = \gamma m/r^2$ , we see that this formula is similar to the electric energy density expressed in terms of the electric field strength, with a significant difference in relation to the minus sign:

$$u = -g^2/8\pi\gamma, \quad u = \varepsilon_0 E^2/2, \quad 1/4\pi\varepsilon_0 \rightarrow \gamma, \quad E \rightarrow g. \quad (19)$$

So the acceleration of free fall plays the role of the gravitational field strength. It is interesting to calculate this density of gravitational energy near the Earth's surface.

Using  $g = 9.8 \text{ m/s}^2 / (c [\text{m/s}])^2$ , получаем  $u = -5.8 \cdot 10^{10} \text{ J/m}^3$ .

For what follows, it is important that the acceleration of free fall is equal to the curvature of the world line of a stationary body, for which  $r = \text{Const}$ . This means that the acceleration is equal to the physical value of the component of the Christoffel symbol  $\Gamma_{tt}^r$ . Indeed, the coordinates of such a line in the function of  $t$  are  $x^i = \{t, r = \text{Const}, \theta = \text{Const}, \varphi = \text{Const}\}$ , and the coordinates of the tangent vector are  $dx^i / dt = \{1, 0, 0, 0\}$ . Therefore, the curvature of the world line, that is, the second covariant derivative,

$$\frac{D^2 x^i}{dt^2} = \frac{d^2 x^i}{dt^2} + \Gamma_{jk}^i \frac{dx^j dx^k}{dt^2}, \quad (20)$$

has only one non-zero component,  $\Gamma_{tt}^r$ , and the expression for the free fall acceleration is

$$g = \Gamma_{\hat{t}\hat{t}}^{\hat{r}} = \frac{\sqrt{g_{rr}}}{g_{tt}} \Gamma_{tt}^r = \frac{r_g}{2r^2} \sqrt{\frac{r}{r-r_g}} \approx \frac{r_g}{2r^2} = \frac{\gamma m}{r^2}. \quad (21)$$

It is essential that the gravitational energy of curved space is not determined by the curvature tensor. The curvature tensor has invariants independent of the coordinate system used. At the same time, the observed energy changes when passing from one observer to another observer. The energy, generally speaking, depends on the coordinate system used. The quantities that depend on both the curvature of space-time and the coordinate system are the Christoffel symbols. The Christoffel symbols are rightly called the gravitational field strength. It is the Christoffel symbols of the Schwarzschild coordinates that determine the mass-energy that a spherically symmetric body receives from the space curved by this body.

## 7. Energy of weak gravitational waves.

The negative sign of gravitational energy is of great importance. Weil writes: "In all attempts to build a theory of gravity along the lines of the theory of the electromagnetic field, undertaken before Einstein, this negative sign was the main stumbling block". [5]. However, the energy of gravitational waves is positive. A binary star system is *losing energy* through the emission of gravitational waves. We show that the reason for the different sign of the gravitational energy in these two cases is a significant difference in the character of the space-time curvature. Indeed, let us use the metric tensor of weak gravitational waves [3]

$$g_{ij} = \{g_{tt} = c^2, g_{xx} = -1, g_{yy} = -1 + h, g_{zz} = -1 - h\}, \quad h = \sin(kx - \omega t). \quad (22)$$

The Christoffel symbols are

$$\Gamma_{xx}^t = 0, \quad \Gamma_{yy}^t = -g^{tt} \partial_t g_{yy} / 2 = \omega h' / 2c^2, \quad \Gamma_{zz}^t = -\omega h' / 2c^2 \quad (23)$$

$$\Gamma_{xx}^x = 0, \quad \Gamma_{yy}^x = -g^{xx} \partial_x g_{yy} / 2 = kh' / 2, \quad \Gamma_{zz}^x = -kh' / 2. \quad (24)$$

Here  $h'$  means the derivative of a quantity  $h$  with respect to its argument. The physical components of  $\Gamma_{\hat{t}\hat{t}}^{\hat{r}}$ -symbols are:

$$\Gamma_{\hat{y}\hat{y}}^{\hat{t}} = \Gamma_{yy}^t \sqrt{g_{tt}} = kh' / 2, \quad \Gamma_{\hat{z}\hat{z}}^{\hat{t}} = \Gamma_{zz}^t \sqrt{g_{tt}} = -kh' / 2 \quad (25)$$

In contrast to the Schwarzschild case, we have four field strengths in gravitational waves. When using formula (19), which determines the density of gravitational energy, it is natural to use the sum of component-by-component products instead of the square of the single

acceleration  $g^2$ , since the product stores information about the different signs of the Christoffel symbols, unlike the square. So we have a positive energy density in gravitational waves:

$$u = -(\Gamma_{yy}^{\hat{t}} \Gamma_{zz}^{\hat{t}} + \Gamma_{yy}^x \Gamma_{zz}^x) / 8\pi\gamma = k^2 h'^2 / 16\pi\gamma . \quad (26)$$

Formula (26) coincides with the formula [3 (107.12)], which is remarkable in that it is confirmed by nature. The binary pulsar PSR B1913+16 reduces its rotation period by 76 millionths of a second per year, emitting energy in the form of gravitational waves, in accordance with this formula. However, the formula [3 (107.12)] was obtained within the framework of the concept of the energy-momentum pseudo-tensor of the gravitational field, which was criticized [4]. The pseudo-tensor does not give, in particular, the negative value of the gravitational energy for the Schwarzschild space.

## 8. Conclusion

The problem of localization of gravitational energy is solved. The volume density of the gravitational energy of the Schwarzschild space is given by formula (12). This solves the problem of unambiguous localization of gravitational energy, a problem that Einstein left to the mercy of fate [5]. A body that curves space receives the gravitational energy of the curved space during compression. Accordingly, the density of the gravitational energy of the space is negative, since the energy density of the Euclidean space is equal to zero.

The gravitational energy  $U$  received by the body from the space increases the inertial mass-energy of the body, according to formula (13). Thus, the inertial mass exceeds the gravitational mass by the amount of energy of the curved space.

## 9. Discussion. The principle of equivalence

The excess of the inertial mass  $M$  over the gravitational mass  $m$  can cause concern about the principle of equivalence. However, there is no principle of equivalence in Einstein's theory. This principle asserts the equality of the gravitational force and the force of inertia. But there is no gravitational force in Einstein's theory. The body lying on a table presses on the table by the force of inertia, since its world line is curved by the substance of Earth. And this force of inertia, by definition, is proportional to its inertial mass. As for the (active) gravitational mass of this body, its operational definition is problematic. The gravitational mass of a body determines an additional curvature of space at infinity, while the inertial mass of a body determines the curvature of space according to Einstein's equation

$$G_{\beta}^{\alpha} \equiv R_{\beta}^{\alpha} - R_{\mu}^{\mu} \delta_{\beta}^{\alpha} / 2 = 8\pi\gamma T_{\beta}^{\alpha} , \quad (27)$$

But the gravitational mass is preserved during gravitational contraction, since it includes the gravitational energy of space, and the law of conservation of inertial mass does not exist with respect to gravitational interactions. Passive gravitational mass does not make sense in the theory, since there is no gravitational force.

## 10. Discussion. The mass defect

The increase in the mass of the shell during compression contradicts the popular belief about the negative gravitational mass defect. Authors of [6] write: "The mass-energy of the Earth-moon system is less than the mass-energy that the system would have if the two objects were at infinite separation. The mass-energy of a neutron star is less than the mass-energy of the same number of baryons at infinite separation".

But such statements contradict the obvious fact of the increase in the mass-energy of the Earth-apple system when an apple falls and get the kinetic mass-energy  $mv^2/2 = mgh$ . On the contrary, if the velocity of revolving of Moon around Earth is directed away from Earth, Moon will move away from Earth, and its speed, and, accordingly, its mass will decrease compared to the initial values, and not increase, as the authors claim.

Consider now the removal of baryons of a neutron star to infinity. This removal requires mass-energy. But there is nowhere to take this mass from, except from the star itself. So when you remove baryons, you have to reduce the mass of the star. The mass has to be taken away from the baryons and introduced into the curved Schwarzschild space. This mass is spent on straightening the curved space, on increasing the space energy from a negative value to zero.

### 11. Discussion. The pseudo-tensor

Einstein's theory, expressed by equation (18), does not contain the concept of gravitational energy. Various versions of the pseudo-tensor have been devised to introduce gravitational energy into the theory. It seemed that success had been achieved. Tolman writes [7, p. 250]: “This satisfactory result can serve to increase our confidence in the practical advantages of Einstein's procedure in introducing the pseudo-tensor densities of potential gravitational energy

and momentum  $t_{\beta}^{\alpha} = \frac{1}{16\pi} \left[ -g_{\beta}^{\mu\nu} \frac{\partial \mathcal{L}}{\partial g_{\alpha}^{\mu\nu}} + g_{\beta}^{\alpha} \mathcal{L} \right]$ .”

Unfortunately, this expression is wrong. The pseudo-tensor gives a positive value for the gravitational energy of the liquid sphere, contrary to the our result (13) and to the Tolman equation [7 (97.10)],

$$m = \int \rho dV_0 + \int \frac{1}{2} \rho \psi dV, \quad \psi < 0 \quad (28)$$

where the gravitational energy  $\int \frac{1}{2} \rho \psi dV$  is negative due to  $\psi < 0$ . This is shown in [4].

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**Статья отклонена журналом «Теоретическая и математическая физика».**

Мы приводим связанную с этим переписку в обратном хронологическом порядке.

**From Khrapko Radi**

Глубокоуважаемый В.В.Жаринов,

Когда академик Фаддеев писал указанную редколлегией статью [УФН, 136, 435 (1982)], Википедии еще не было. И академику простительно *не знать*, что это такое: "gravitational binding energy of a system", "minimum energy", "gravitationally bound state", хотя эта «гравитационная энергия связи системы» была уже не только определена как «минимальная энергия», необходимая для освобождения системы из «гравитационно связанного состояния», но и вычислена ещё в 1939 году Чандрасекаром. Академику простительно это не знать, но рецензенту – стыдно, а журналу стыдно иметь таких рецензентов! Тем более, что в статье даны ссылки на Википедию, Чандрасекара и Ланга. Правда, эти ссылки даны во *второй* фразе статьи, которую рецензент не одолел.

Что же касается указанной статьи академика Фаддеева, то содержание её подробно критиковалось в работах [1-6].

Суть критики состоит в том, что псевдотензор Эйнштейна, на который опирается автор, является неудачной конструкцией уже потому, что дает положительное значение гравитационной энергии шварцшильдовского пространства. Это показал прямым вычислением Толмен ещё в 1933 году [7].

Статья [1] была направлена лично академику Фаддееву по электронной почте. Материал работ [1-6] был направлен в ТМФ в трех статьях, которые были отклонены без рассмотрения:

From: [tmph@mi.ras.ru](mailto:tmph@mi.ras.ru) Sent: July 30, 2013 To: [khrapko\\_ri@hotmail.com](mailto:khrapko_ri@hotmail.com)  
Уважаемый Радий Игоревич, Ваша статья "Миф об энергии гравитационного поля" не представляет интереса для журнала ТМФ и не может быть опубликована. Отв. Секретарь В.В.Жаринов

From: [tmph@mi.ras.ru](mailto:tmph@mi.ras.ru) Sent: December 03, 2015 To: [khrapko\\_ri@hotmail.com](mailto:khrapko_ri@hotmail.com)  
Глубокоуважаемый Радий Игоревич! Ваша статья "Правда о псевдотензоре энергии-импульса гравитационного поля" не представляет интереса для журнала ТМФ и не будет опубликована. Отв. Секретарь В.В.Жаринов

From: [tmph@mi.ras.ru](mailto:tmph@mi.ras.ru) Sent: November 09, 2017 To: [khrapko\\_ri@hotmail.com](mailto:khrapko_ri@hotmail.com)  
Глубокоуважаемый Радий Игоревич! Ваша статья "Гравитационный дефект массы" не представляет интереса для журнала ТМФ и не будет опубликована. Отв. Секретарь В.В.Жаринов

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From [tmph@mi-ras.ru](mailto:tmph@mi-ras.ru)

Глубокоуважаемый Радий Игоревич,

Редколлегия журнала ТМФ рассмотрела Ваш ответ по поводу рецензии на статью "Гравитационная энергия, приписываемая искривленному пространству". Мы обращаем Ваше внимание, что указанная в качестве определения энергии первая фраза Вашей статьи таковым не является. Определение энергии в общей теории относительности предполагает, что определены все входящие в него понятия. В Вашей статье не определены, например, "binding energy of a system", "minimum energy", "gravitationally bound state" и т.п. Кроме того, не указан способ вычисления энергии.

Мы направляем Вам, для ознакомления, статью Л.Д.Фаддеева [УФН, 136, 435 (1982)]. Редколлегия согласна с утверждениями этой статьи и обращает Ваше внимание, что Ваши утверждения прямо противоположны ей. На основе сказанного редколлегия повторяет свое заключение о невозможности публикации Вашей статьи в журнале ТМФ. В заключении отметим, что на основании пункта 1.3 Правил для авторов нашего журнала Редколлегия прекращает дискуссию с Вами по поводу Вашей статьи.

Ответственный секретарь ТМФ В.В.Жаринов

From **Khrapko Radi**

Глубокоуважаемый С.В. Сушко,

Это апелляция.

Редколлегия ТМФ использовала в качестве «отзыва рецензента» нечто неадекватное:

### Рецензия для ТМФ

Статья: Gravitational energy attributed to curved space

Автор(ы): Khrapko R.I.

В общей теории относительности существуют различные определения энергии. Поэтому, чтобы обсуждать этот вопрос, необходимо прежде всего дать строгое определение энергии. Первое предложение статьи, в котором автор «дает определение», неудовлетворительно с математической точки зрения, так как не описан формальный способ вычисления энергии. Поэтому выводы автора нельзя считать обоснованными, и обсуждать недостатки не имеет смысла. Считаю публикацию статьи в ТМФ нецелесообразной.

В действительности, первое предложение статьи таково:

“According to the definition [1,2] (See also

[https://en.wikipedia.org/wiki/Gravitational\\_binding\\_energy](https://en.wikipedia.org/wiki/Gravitational_binding_energy)), the *gravitational binding energy* of a system is the minimum energy that must be added to it in order for the system to cease to be in a gravitationally bound state”.

[1] Chandrasekhar, S. 1939, *An Introduction to the Study of Stellar Structure* (Chicago: U. of Chicago; reprinted in New York: Dover), section 9, eqs. 90–92, p. 51 (Dover edition)

[2] Lang, K. R. 1980, *Astrophysical Formulae* (Berlin: Springer Verlag), p. 272.

Обратите внимание, что в статье представлены фундаментальные результаты, которые существенно меняют теорию гравитации:



(1) Гравитационная энергия локализована. Проблема, от которой дистанцировались Эйнштейн, Эддингтон и другие классики, решена. Это противоречит утверждению Ландау, Лифшица о том, что «не имеет смысла говорить об определенной локализации энергии гравитационного поля в пространстве». Это противоречит утверждению Мизнера, Торна и Уиллера о том, что «нельзя определить локализованную энергию-импульс для гравитационного поля».

(2) Впервые утверждается, что закон сохранения инерционной массы-энергии при гравитационном взаимодействии нарушается и дефект массы положителен. Инерционная масса-энергия тел превышает их гравитационную массу на величину безмассовой энергии искривленного пространства-времени. Это противоречит утверждению Миснера, Торна Уиллера о том, что «масса-энергия нейтронной звезды меньше, чем масса-энергия того же числа барионов на бесконечном расстоянии».

(3) Попытки Эйнштейна, Эддингтона, Ландау, Лифшица ввести гравитационную энергию в теорию Эйнштейна с помощью псевдотензора не увенчались успехом. Псевдотензоры дают положительное значение гравитационной энергии для шварцшильдовского пространства.

**From [tmph@mi-ras.ru](mailto:tmph@mi-ras.ru)**

Глубокоуважаемый Радий Игоревич,

Редколлегией ТМФ принято решение об отклонении Вашей статьи "Гравитационная энергия, приписываемая искривленному пространству" на основании отзыва рецензента. Отзыв прилагается.

Ответственный секретарь ТМФ С.В. Сушко,