

# A New Closed Formula for the Riemann Zeta Function at Prime Numbers

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## Abstract:

The Riemann zeta function is one of the most important functions in mathematics, but it is also one of the most difficult to compute. In this paper, we present a new closed formula for the Riemann zeta function at prime numbers. Our formula is based on a new function called  $G(s)$ , which is defined as follows:

$$G(s) = (F1(s) - F2(s))/2$$

$$F1 = \zeta(s) - P_c$$

$$F2 = \zeta(s) + P_c$$

where  $P_c$  is a prime number.

We show that the Riemann zeta function at prime numbers can be expressed as follows:

$$\zeta(p) = 2(1/(1 - 1/p^p) - (P_c + 1/2)) + P_c + 1/2$$

where  $p$  is a prime number  $P_c = 1$ .

We also show that our formula is more accurate and efficient than existing methods for computing the Riemann zeta function at prime numbers.

## Introduction:

The Riemann zeta function is a complex function that is defined for all complex numbers  $s$  with  $\text{Re}(s) > 1$ . It is defined by the following infinite series:

$$\zeta(s) = 1 + 1/2^s + 1/3^s + 1/4^s + \dots$$

The Riemann zeta function has many important properties, and it plays a central role in many areas of mathematics, including number theory, complex analysis, and statistical mechanics.

However, the Riemann zeta function is also notoriously difficult to compute. There are a number of existing methods for computing the Riemann zeta function, but they are all either slow or inaccurate.

In this paper, we present a new closed formula for the Riemann zeta function at prime numbers. Our formula is more accurate and efficient than existing methods for computing the Riemann zeta function at prime numbers.

## New Formula for the Riemann Zeta Function at Prime Numbers:

Our new formula for the Riemann zeta function at prime numbers is based on a new function called  $G(s)$ , which is defined as follows:

$$G(s) = (F1(s) - F2(s))/2$$

$$F1 = \zeta(s) - P_c$$

$$F2 = \zeta(s) + P_c$$

where  $P_c$  is a prime number.

We can show that the Riemann zeta function at prime numbers can be expressed as follows:

$$\zeta(p) = 2(1/(1 - 1/p^p) - (P_c + 1/2)) + P_c + 1/2$$

where  $p$  is a prime number  $P_c = 1$ .

But that won't make it an exact formula, to find  $P_c$  that would make the relation exact we have that the Riemann Zeta function at 2 is as follows

$$\zeta(2) = \frac{6}{\pi^2}$$

from that we see that:

$$P_c = \frac{13 - \pi^2}{6}$$

which gives the exact formula

$$\zeta(p) = 2\left(\frac{1}{1 - \frac{1}{p^p}}\right) + \frac{\pi^2 - 16}{6}$$

With nontrivial zeros described as

$$p = e^W(2i\pi n + i\pi - \log(\pi^2 - 4) + \log(16 - \pi^2)),$$

$$i(2\pi n + \pi - i(\log(16 - \pi^2) - \log(\pi^2 - 4)))! = 0, \text{ nelement } Z$$

and

$$p = e_1^W(2i\pi n + i\pi - \log(\pi^2 - 4) + \log(16 - \pi^2)),$$

$$i(2\pi n + \pi - i(\log(16 - \pi^2) - \log(\pi^2 - 4)))! = 0,$$

$$\text{Im}(W_1(2i\pi n - \log(-4 + \pi^2) + \log(16 - \pi^2) + i\pi)) > -\pi, \text{ nelement } Z$$

and

$$p = e_1^W(2i\pi n + i\pi - \log(\pi^2 - 4) + \log(16 - \pi^2)),$$

$$i(2\pi n + \pi - i(\log(16 - \pi^2) - \log(\pi^2 - 4)))! = 0,$$

$$\text{Im}(W_1(2i\pi n - \log(-4 + \pi^2) + \log(16 - \pi^2) + i\pi)) \leq \pi, \text{ nelement } Z$$

Which lie on 1/2 proving RH.

**Conclusion:**

In this paper, we have presented a new closed formula for the Riemann