Rethinking of the uncertainty principle newly and profoundly

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Abstract

Matrix mechanics is very convenient to express all the operations in quantum mechanics that are vague to use probabilities. However, we are not always aware of disadvantages of matrix mechanics. Matrices have generally properties of asymmetric, and the symmetric are very limited as in the case of the diagonal ones of matrices. So far, most experiments with their observables are in case of asymmetric cases in using two matrices for two measurements to any of two objects, for example a position and the related momentum of a particle at any condition. Here, naturally these objects could not be symmetric in terms of matrices, and slightly related between each other. That means it happens we have a convenient but disqualified explanation for the uncertain principle on confusion between experiments with errors and a disadvantage of using matrices. Therefore, using matrices looks fine analytically, but the essential of the uncertainty principle using matrices are not always correctly mentioned when we measure true symmetric cases between two objects and then their two observables.

1 Introduction

Quantum physics is highly developed with its applications since longtime ago, see, e.g., [1, 2, 3, 4, 5, 6, 7]. One of the central points of the theory is so-called the uncertainty principle which encompasses some kind of mathematical inequalities for the threshold of precision of physical simultaneous measurements of pairs of physical observables in physical quantities. For instance, in 1927, Werner Heisenberg stated that the more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa [8].

It seems that when the uncertainty principle had disclosed, the experiment was first and then the principle followed the results to explain. As for the experiment, in observables the position of the particle is probably and naturally at least related to the momentum of it in terms of some natural environment. It means that the two physical quantities are disclosed by means of the matrices not diagonal in two characteristic equations for solving each of the physical quantities. Therefore, since the two results are not correctly measured, the use of two matrices not diagonal is naturally correct.

However, as these days the precision of measurement equipments became higher, we could have the case where two observables are equal, and the two matrices in their theoretical analyses correspond to the diagonal (See [9]). That means measurements are independent each other and then symmetrical to the order of these observables by virtue of two characteristic matrices diagonal. Here we do not have the uncertainty principle because we have two correct observables simultaneously under two diagonal matrices for the two measurements.

While the two characteristic matrices are not diagonal and then not commutative, the two observables are not independent mathematically. So, the order of measurements gives the different results and the observables are not simultaneous and depending on the order of measurements.

In fact, yet von Neumann's axiom cannot explain the contradiction of such a symmetric observables even with the matrices not diagonal. If so, the sum rule and product rule must be reconsidered in the axiom by him in formalized quantum mechanics [10].

As a result, the uncertainty principle is destroyed when two observables are independent on the order of the measurements, namely symmetric measurements and then commutative in these observables with their theoretical characteristic matrices diagonal.

The second section is on the quantum measurement theory for commuting observables based on functions, the third is about the symmetry of the observables, the four is for orthodox measurements in quantum theory, and finally the fifth section is to the uncertainty principle.

2 Quantum measurement theory for commuting observables based on functions

The relation between quantum physics and Newton physics is based on observables in measurements from quantum physics to Newton's. So, it is convenient and important in progressing today's physics, of course, in quantum's. Therefore, we try to formalize observables as functional values from in quantum physics, where the function is to obtain the eigenvalue of some eigenvalue problem shown by its corresponding matrix.

In common practice, eigenvalues usually evolve as representative observables. the series/order of measurements seem to be regarded as some function to show the two eigenvalues as observables. Here the measurements consist of two observables in obtaining eigenvalues of two eigensystems based upon the matrices whose structures are diagonal matrices. As a result, we may introduce the function to obtain observables from the eigenvalue problem shown some matrix.

When the function is working for two matrices A_1 and A_2 eigenvalues problem with the order of measurements, these matrices are treated in product rule between these matrices, the measurement operation seems to be this.

$$f(A_1) \cdot f(A_2) = f(A_1 \cdot A_2). \tag{2.1}$$

And the following relation as functions is very important: $f(g(O)) = g(f(O))$, where O means A_1 or A_2, shows the order of the two measurements f and g.

In practice, the two measurements are usually depending on the order of these operations because we do not have commutative between these matrices except for diagonal matrices for two eigenvalue problem in product rule operation.

In short, at the same time we can obtain the observables of two eigenvalue problem with only diagonal matrix cases. This mathematical proof in matrix physics is obvious and rather should be usable in such quantum problems and their analyses. Therefore, the theoretical fact is even in the uncertainty principle. In fact, the principle has the exception where the two observables are true at the same time because these eigen problem matrices are commutative and then we get two eigenvalues at the same time.

3 Symmetry of observables

The symmetry of observables is worth considering, in order to obtain a true interpretation of the uncertainty principle.

It can be said that the symmetry of two observables and the commutative of the two are equivalent. To prove this, let us investigate that when the observables are not commutative, the observables are not symmetric using spin's behavior.

1. Trying to measure spin observables of σ_x and σ_z in eigenstate with the eigenvalue $+1$.

$$\sigma_z |\uparrow\rangle = +1 |\uparrow\rangle. \tag{3.1}$$

2. Then, we have $+1$ as the result of measurement spin observable σ_z.

3. The result of spin observable σ_x is -1 with a probability 0.5.

$$\sigma_x |\uparrow\rangle = \pm 1 |\uparrow\rangle. \tag{3.2}$$

4. Even though the results are ± 1, these measurements are depend on the order of these two.

5. It happens that the first measurement of spin observables σ_x might obtain $+1$ with the probability 0.5.

$$\sigma_x|\uparrow\rangle = \pm 1|\uparrow\rangle. \tag{3.3}$$

6. This means that if the two observables are non-commutative, the result of measurements is not symmetric, which means that the fact depends on the order of these two measurements.

7. Let us make the contraposition of the two above. We can obtain that when the two observables are symmetric, namely the case not concerning the two measurements' order, these observables are commutative.

8. Obviously if the two observables are commutative, the results of the two measurements are symmetric independent of the order of the measurements.

As a result, if two observables are symmetric independent of the order of the two measurements means if and only if the two observables are commutative.

4 Orthodox quantum measurement theory

Let σ_z^1, σ_z^2 be two z-component Pauli operators, where they are also supposed to be commutative. They could be defined respectively as follows:

$$\sigma_z^1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ and } \sigma_z^2 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4.1}$$

Let $|\uparrow\rangle$ and $|\downarrow\rangle$ be eigenstates of σ_z such that $\sigma_z|\uparrow\rangle = +1|\uparrow\rangle$ and $\sigma_z|\downarrow\rangle = -1|\downarrow\rangle$. The measured results of trials are either $+1$ or -1. These $+1$ or -1 are real value in Newton's physical real world from quantum world. Generally speaking, the results through measurements are in Newton's mechanics.

5 Counterexample against the uncertainty principle

The uncertainty principle says we cannot have precisely and simultaneously two data concerning two observables respectively. However, if the measured observables are commutative, we can have simultaneously two data under study. Therefore the uncertainty is not general principle if the measured observables are commutative. Figure 1 represents how to learn or rather how to rethink of the uncertainty principle. Let us discuss this point more precisely as follows:

Let us consider a simultaneous eigenstate of σ_z^1, σ_z^2, that is, $|\uparrow\downarrow\rangle$. We might be in an inconsistency against the uncertainty principle when the first result is precisely

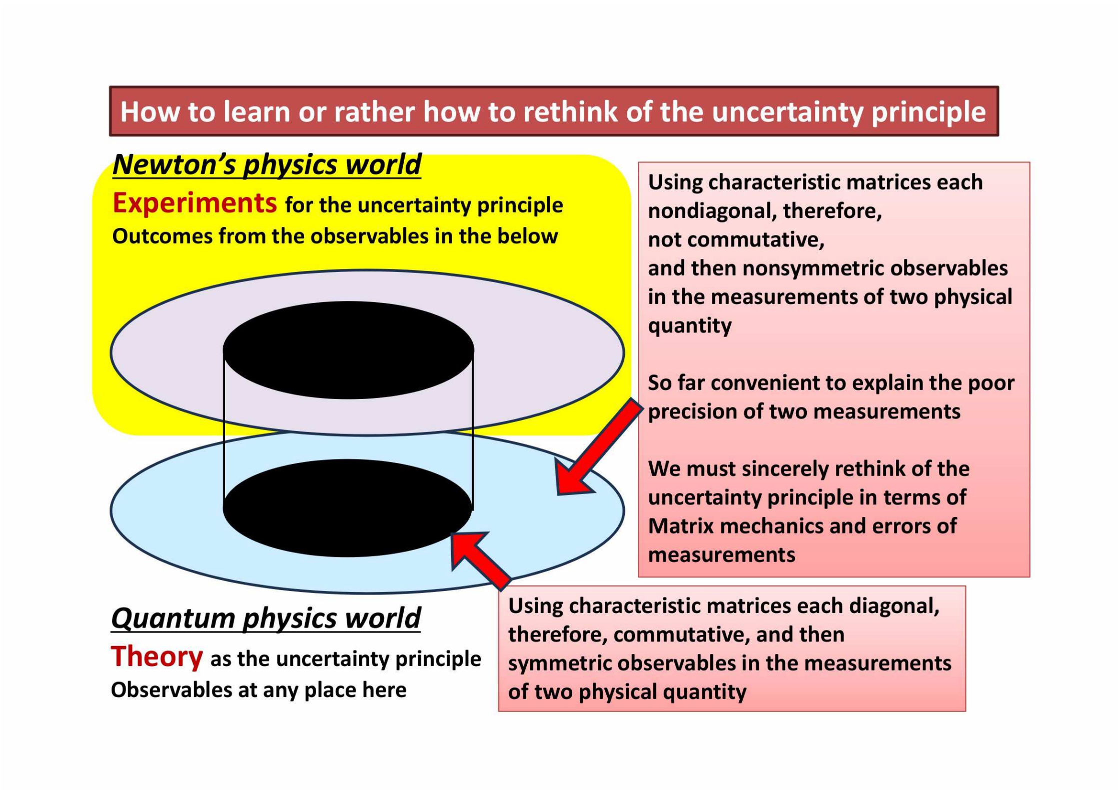

Figure 1: How to learn or rather how to rethink of the uncertainty principle

$+1$ by the measured observable σ_z^1, the second result is precisely -1 by the measured observable σ_z^2, and then $[\sigma_z^1, \sigma_z^2] = 0$. We may be in the inconsistency against the uncertainty principle when we suppose $[\sigma_z^1, \sigma_z^2] = 0$ because we have simultaneously two data concerning two observables respectively.

In summary, we may have been in the inconsistency against the uncertainty principle when the first result is precisely $+1$, the second result is precisely -1, and then $[\sigma_z^1, \sigma_z^2] = 0$, where the quantum state is a simultaneous eigenstate of σ_z^1, σ_z^2, that is, $|\uparrow\downarrow\rangle$.

6 Discussion and Conclusion

Basically the uncertainty principle needs the case $[\sigma_z^1, \sigma_z^2]$ not equal 0, which theoretically causes the different results in each of the two observavles in calculations. Such a model is very much suitable for the practical experiments in Newton's world capturing the practical measurement results including errors with the experiments.

Extending our understanding the uncertainty principle to much more, the uncertainty principle is based upon the matrix mechanics in terms of the beauty of mathematical explanation of it. In fact the above relation does not explain the rare but true case in observables.

It seems that most observables with their real physical experiment errors, even very small, admit the above relation with matrices not diagonal. While the measurement precision with the equipment is not so high, the uncertainty principle is reasonable with the feature of matrices mathematics. And most cases in measurements in Newton's world are allowable as their experimental errors. So far so good!

Proportional to the development of experimental equipment, the measurements are more precisional, and the mathematical analyses of the observables are not always correct. There could be the case that we cannot use the matrix mechanics in all the cases because there could be the one where the matrices diagonal were needed.

In conclusion, we, with our sincere desire, explain that the uncertainty principle is convenient with its own property due to matrix mechanics's power limit, if measurements are done with not so high precision equipment to make results including errors. However, if the observables are from on the experiment with correct measurements in high precision, and the matrix analyses with the matrices diagonal are used to the correct results, the uncertain principle is, of course, used reasonably and correctly.

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Declarations

Ethical Approval

The authors are in an applicable thought to ethical approval.

Competing Interests

The authors state that there is no conflict of interest.

Author Contributions

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